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Energy Sector Pricing: On the Role of Neglected Nonlinearity*

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Abstract

Modern economies have been subjected to a number of shocks during the past several years such as the burst of the Internet bubble, terrorist attacks, corporate scandals, the war in Iraq, the uncertainty about energy prices, and the recent subprime mortgage crisis. In particular, during the last few years, the energy shock has caused concerns for potential stagflation for both the United States and numerous other countries. We perform numerous univariate tests for non-linearity and chaotic structure using price data from the energy sector to resolve whether the sector's fundamentals or exogenous shocks drive these prices.

JEL classification: C22, C45, D40, G10, Q40.

Keywords: Energy, Nonlinearity, Chaotic Dynamics.

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1 Introduction

Modern economies have been subjected to a number of shocks during the past several years such as the burst of the Internet bubble, terrorist attacks, corporate scandals, the war in Iraq and the substantial increase in the volatility of energy prices and the most recent subprime mortgage crisis. In particular, during the last few years, the energy shock has caused concerns for potential stagflation for both the U.S. and numerous other countries. Initial explanations for the energy crisis highlight the increased energy demand by fast growing emerging nations such as China and India, but also underscore existing supply constraints because of lagging investments to productive capacity.

Even with its relative diminished significance, the energy sector continues to play a critical role in the global economy. With the price of crude oil hovering around \$130 per barrel during late May 2008, from around \$20 per barrel several years ago, several consequences could be anticipated. Even with an inelastic demand, consumers and businesses will attempt to reduce the quantity demanded through conservation or divert other consumption expenditures towards energy or both. Of course, the recessionary consequences of this recent significant increase in the price of oil will also depend on how monetary policy responds.

In the past, when oil prices rose prior to recessions so did interest rates, and as has been argued by Bernanke *et al.* (1997) it was the increase in the interest rate that led to the downturn. Although this view has been challenged by Hamilton and Herrera (2004), who argue that contractionary monetary policy plays only a secondary role in generating the contractions in real output and that it is the increase in the oil price that directly leads to contractions, it is interesting to note that short-term interest rates have been declining (rather than increasing) during the recent increases in the price of oil. This decoupling of short-term interest rates from the price of oil will likely have significant implications for monetary policy.

In this paper, to study the behavior of energy pricing we collected daily data for five energy products — crude oil, gasoline, heating oil, propane, and natural gas — over the period from 1994 to mid-January 2008. We discuss the interrelationships among these five products and perform numerous univariate tests for nonlinearity and chaotic structure to resolve the issue of whether the energy sector's fundamentals are endogenous or exogenous shocks drive these prices.

The paper is organized as follows. In Section 2 we offer a rapid historical development of the energy industry to illustrate its economic dynamics. This is followed by an assessment of the impact of the energy sector on the U.S. economy to simply reemphasize the often-cited fact of the currently diminished role of energy in contrast to its elevated significance three decades ago. In Section 6 we present the data and investigate the univariate time series properties of the crude oil, gasoline, heating oil, propane, and natural gas time series. In section 7 we discuss a number of tests for nonlinear structure, apply each of these tests to each of the five energy price series, and present and discuss the empirical results. The final

section provides a brief summary and conclusion.

2 Historical Overview

At the beginning of the 20th century, Americans had the main pricing power, predominately John D. Rockefeller and Standard Oil. After the breakup of Standard Oil in 1911, pricing power remained primarily with the U.S. oil companies. The U.S. oil companies regulated output production to match seasonal demand in order to reduce price fluctuations. Around the early 1970s, however, the global leadership of U.S. oil firms ended when American excess crude oil capacity was finally absorbed by rising demand.

At the same time, the marginal pricing of oil moved to the Persian Gulf, which for so long had taken place on the Gulf Coast of Texas. To benefit from their newly acquired pricing power, many producing nations in the Middle East nationalized their oil companies. The full magnitude of their pricing power became evident only during the oil embargo of 1973. During that period, crude oil prices at Ras Tanura, Saudi Arabia, rose to more than \$11 per barrel significantly above the \$1.80 per barrel that had been reasonably constant from 1961 to 1970. A further surge in oil prices occurred during the 1979 Iranian Revolution. Greenspan (2005) elaborates these issues more fully.

The higher prices of the 1970s brought to an end the rapid period of growth in U.S. oil consumption. The use of oil had obviously grown fast in the decades immediately after World War II. In particular, between 1945 and 1973, consumption of petroleum products rose at an average annual rate of 4-4.5%, in excess of growth of real GDP. However, between 1973 and 2004, oil consumption grew, on average, only 0.5 percent per year, far short of the rise in real GDP.

Oil price shocks have influenced U.S. business cycles since the end of World War II, although the relationship seems to have weakened during the 1990s. The economy appears to respond asymmetrically to oil price shocks; rising oil prices hurt economic activity more than falling oil prices help it. Hamilton (2003) reviews an extensive economics literature that relates oil price shocks to aggregate economic activity. More recently, Elder and Serletis (2008) and Rahman and Serletis (2008) examine the direct effects of oil price uncertainty on real economic activity, using recent advances in the financial econometrics literature. Their main result is that uncertainty about the real price of oil has had a negative and significant effect on real economic activity in the United States over the post 1975 period.

The United States economy today is not any more so energy dependent as three decades earlier. Much of the progress in reducing the energy intensity was achieved by 1985 and continued to today. This more-modest rate of decline in energy intensity should not be surprising, given the generally lower level of real oil prices that prevailed between 1985 and 2000. With real energy prices again on the rise, more-rapid decreases in the intensity of use in the years ahead seem virtually inevitable. World markets for oil and natural gas have

been subject to a degree of strain over the past few years. Increased demand and lagging additions to productive capacity have combined to eliminate a significant amount of the slack in energy markets that was essential in containing energy prices between 1985 and 2000.

Monetary policy was prominent among early explanations of how oil price shocks affected aggregate economic activity, but it was gradually supplanted by real business cycle theory which attributed the effects to a classic supply shocks rather than monetary policy. Basic supply shock effects can account for only a portion of the intense effect that oil price shocks have on aggregate economic activity. Possible explanations for the intensity are restrictive monetary policy, adjustment costs, coordination externalities and financial stress. The weak response of economic activity to oil price decreases was seen as a breakdown in the relationship between oil price movements and the economy. So far economic research has not been able to distinguish between the contributions of adjustment costs, coordination problems and financial uncertainty. Analysts are less able to prescribe the best course of action for energy policy. Given the asymmetric response of aggregate economic activity to oil price shocks and the effects of uncertainty about the price of oil on the level of economic activity, an energy policy that leans against movements in international oil prices would seem justified.

3 The Impact of Energy

World oil demand rose sharply during the global recovery period of 2003-2007, with the United States and China responsible for much of the increase. OPEC has been reluctant to increase its production sufficiently to lower prices, citing concerns about seasonal decreases in consumption and the possibility of increased supply from Iraq and non-OPEC sources. China's oil consumption per dollar of GDP in 2004 was twice as much as in the United States and if the share increases then the improvements in world oil-intensity will be less pronounced than the improvements in individual countries viewed separately (see Federal Reserve Bank of Dallas, 2004). China alone was consuming over 1 million more barrels of oil each day in 2004 than it did in 2000, which is double the increased demand coming from the United States over that same period (Hamilton, 2004).

The OPEC price increases do appear to have had significant impacts in many U.S. macro-economic indicator variables in data after 1973. The effects of the price declines of the 1980s are smaller and harder to characterize. These results have potentially important implications for the large body of research which utilizes oil prices as an instrumental or explanatory variable (Hooker, 1996). Also the weaker U.S. dollar against other major currencies since early 2002 affects oil prices. Because the dollar has generally declined, prices in other currencies have not risen by nearly as much. A lower-valued dollar increases the ability of foreign buyers to pay dollars for oil. At the same time, OPEC attempts to maintain its international purchasing power by raising the dollar price of oil as the dollar declines in value. Research shows that a 10% reduction in the value of the U.S. dollar against the currencies of other

oil-consuming countries leads to a 7.5% increase in the dollar price of oil.

Economists expect the effects to be milder regarding the past increases in oil and natural gas prices, and several factors account for the difference. Firms have more experience with energy price shocks and can better predict how other segments of the economy will respond, reducing coordination problems. The sectoral and regional economic effects of higher oil and natural gas prices will be uneven. Energy-intensive industries will incur higher costs and experience reduced profit margins, while energy producers will be helped. Regions with the highest concentrations of energy-intensive industries will be hurt, and regions with energy-producing industries will be helped. Substantial worldwide investment in oil production, LNG facilities, pipelines and the electricity grid will be needed to keep energy prices from rising above their current course (Hooker, 2002)

Evidence does not support the hypotheses that declining energy intensity or deregulation of energy producing and consuming industries played an important role. Monetary policy did not itself become less accommodative of oil shocks, but may have helped create a regime where inflation is less sensitive to price shocks (Hooker, 2002). Monetary policy contributes about 40% to the drop in output following a rise in oil prices according to a benchmark calibration, which approximates the Federal Reserve's behavior since 1979 (Leduc and Sill, 2004). Oil price shocks create the potential for a monetary policy response that exacerbates the basic effects of an oil supply shock. Interest rates are not a good way to assess the stance of monetary policy when there is a supply shock. If the monetary authority wants to lessen the inflationary consequences of rising oil prices, it can tighten policy, which will temporarily aggravate the losses in real GDP. If the authority is willing to accept higher inflation it can temporarily boost GDP through expansionary policy.

According to Hamilton (2003), the recent behavior of oil prices is very different from what has been observed in the past. First, oil prices went up because of an increase in demand. This is quite a different situation from other historical oil shocks that were caused by military conflicts that physically disrupted the production or delivery of petroleum, forcing consumers and firms to make less use of this vital input. The United States had to share the increased supply with other consuming nations. The second way that the recent oil price spike differs from those that preceded earlier U.S. recessions is that a good part of the recent increase is merely a correction to an earlier dramatic drop in oil prices. There were similar corrections (an oil price spike following an earlier downturn) in 1987 and 1994 with no apparently adverse economic effects. By contrast, those episodes that were followed by recessions were invariably associated with dramatic new highs, not simply a correction to an earlier decrease.

4 Energy Interrelationships

Prices of spot crude oil and natural gas have risen sharply recently, in response to constrained supply and the firming of overall demand. As Brown and Yücel (2007) argue “for many

years, fuel switching between natural gas and residual fuel oil kept natural gas prices closely aligned with those for crude oil. More recently, however, the number of U.S. facilities able to switch between natural gas and residual fuel oil has declined, and over the past five years, U.S. natural gas prices have been on an upward trend with crude oil prices but with considerable independent movement. Natural gas market analysts generally emphasize weather and inventories as drivers of natural gas prices.”

In fact, natural gas and oil prices had a stable relationship until 2000, with natural gas adjusting to movements in crude oil. In the past few years, however, natural gas prices have decoupled from oil prices, and the relationship between the two has become unstable. Moreover, the U.S. natural gas industry has been unable to expand production or to increase imports from Canada. International trade in natural gas has also been insufficient to equalize prices across markets, with U.S. natural gas prices since 2002 being significantly higher than prices abroad. As a result, significant segments of the North American gas-using industry are in a weakened competitive position.

It is to be noted, however, that in addition to expanded supplies from abroad, North America still has many significant and unexploited sources of gas production such as in Alaska and the northern territories of Canada. Negotiations over the construction of pipelines connecting these northern supplies to existing delivery infrastructure are currently under way (see Federal Reserve Bank of Dallas, 2004). Moreover, new technologies are facilitating U.S. production of unconventional gas reserves. In fact, according to projections from the Energy Information Administration, most of the growth in the domestic supply of natural gas over the next twenty years will come from unconventional sources.

Finally, from the point of view of energy policy, the substitutability/complementarity relationship among different sources of energy is of prime importance. In this regard, Serletis and Shahmoradi (2008) investigate interfuel substitution possibilities in energy demand in the United States in the context of two semi-nonparametric flexible functional forms — the Fourier and the AIM. They conclude that “the interfuel elasticities of substitution in the United States are (in general) consistently and believably below unity, revealing the limited ability of the U.S. economy to substitute one source of energy for another and suggesting that crude oil will continue to maintain its major role as a source of energy in the near future. Moreover, the low Morishima elasticities of substitution between coal and natural gas and coal and crude oil suggest that there are at least some old industries in the United States that are unable (or unwilling) to adopt new and diverse sources of energy.”

5 Hypothesis

Even if the impact of energy prices has decreased during the past thirty years it remains significant and asymmetric, as recently argued by Elder and Serletis (2008) and Rahman and Serletis (2008). In view of the overall price inelasticity of energy prices, price increases

cause higher economic costs than price decreases cause benefits. Even if we assume perfect randomness in price changes, the net economic impact of this asymmetry is negative. If price changes are not random but follow nonlinear deterministic patterns such information may allow economists to better evaluate the overall impact of energy prices on the U.S. and even the global economy.

This paper performs state-of-the-art univariate tests to uncover the structure of energy prices for five products of the energy complex — crude oil, gasoline, heating oil, propane, and natural gas. Mandelbrot and Hudson (2004) give a detailed description regarding the presence of nonlinear determinism in financial markets. Empirical evidence of chaotic dynamics in financial data such as stock market indexes, foreign exchange rates, macroeconomic time series and several others have been performed by various researchers, including recently Kyrtsov and Vorlow (2005) and in much more detail earlier by Brock and Malliaris (1989) and Scheinkman and LeBaron (1989). However, there is very little empirical work done to study nonlinear chaotic determinism in energy markets — see, however, Serletis and Gogas (1999) and Serletis and Andreadis (2004).

The analysis presented in the earlier sections of this paper documents that initially U.S. oil firms that dominated the global production market attempted to regulate production to meet seasonal variations in the demand for oil and thus to avoid excess price volatility. As OPEC assumed production leadership during the 1970s, energy pricing was guided by a strategy to preserve real prices growing at a reasonable annual rate of growth. Weakness in the U.S. dollar in terms of the Japanese yen and the then Deutsch mark motivated the price increases during the late 1970s to mid-1980s.

However, as the U.S. economy grew rapidly during the 1982-1990 period, it diversified sufficiently to cause the role of energy to partially diminish in terms of its economic impact. The first war in the Gulf during 1990-1991 and the price increases then did cause the first U.S. recession in almost eight years and the subsequent remarkable recovery from 1991 to the bursting of the internet bubble once again demonstrated the decreasing impact of energy. Even the dramatic increases in energy prices since 2004 reveal that global oil producers wish to capitalize on the presence of a global economic boom by keeping oil prices at historically high levels.

This rapid overview highlights that the industrial organization of the energy markets is such that production is highly oligopolistic while demand has recently accelerated due to the global industrial boom led by China, India and other emerging economies who have joined the United States, Europe and Japan to orchestrate a coordinated and sustainable global growth. In view of these economic realities the hypothesis that energy prices follow nonlinear deterministic behavior instead of pure random walks makes sense.

6 The Data

We use daily spot prices, provided by www.barchart.com, on crude oil (CL), gasoline (HU), heating oil (HO), propane (PN), and natural gas (NG). The sample period is January 3, 1994 to January 25, 2008 — a total of 3,509 observations. Figures 1-5 plot the logged levels and the logarithmic first differences of the series.

The first step in conducting nonlinear analysis is to test for stochastic trends (unit roots) in the autoregressive representation of each individual time series. In doing so, we use four alternative testing procedures to deal with anomalies that arise when the data are not very informative about whether or not there is a unit root.

In the first three columns of panel A of Table 1, we report p -values for the augmented Weighted Symmetric (WS) unit root test [see Pantula *et al.* (1994)], the augmented Dickey-Fuller (ADF) test [see Dickey and Fuller (1981)], and the nonparametric, $Z(t_{\hat{\alpha}})$, test of Phillips (1987) and Phillips and Perron (1988). These p -values (calculated using TSP 4.5) are based on the response surface estimates given by MacKinnon (1994). As discussed in Pantula *et al.* (1994), the WS test dominates the ADF test in terms of power. Also, the $Z(t_{\hat{\alpha}})$ test is robust to a wide variety of serial correlation and time-dependent heteroskedasticity. For the WS and ADF tests, the optimal lag length was taken to be the order selected by the Akaike information criterion (AIC) plus 2 — see Pantula *et al.* (1994) for details regarding the advantages of this rule for choosing the number of augmenting lags. The $Z(t_{\hat{\alpha}})$ test is done with the same Dickey-Fuller regression variables, using no augmenting lags. Based on the p -values for the WS, ADF, and $Z(t_{\hat{\alpha}})$ test statistics reported in panel A of Table 1, the null hypothesis of a unit root in levels cannot in general be rejected for each of the variables.

Given that unit root tests have low power against relevant (trend stationary) alternatives, we also follow Kwiatkowski *et al.* (1992) and test for level and trend stationarity to distinguish between series that appear to be stationary, series that appear to be integrated, and series that are not very informative about whether or not they are stationary or have a unit root. KPSS tests for level and trend stationarity are presented in columns 4 and 5 of panel A of Table 1. As can be seen, the t -statistic $\hat{\eta}_{\mu}$ that tests the null hypothesis of level stationarity is large relative to the 5% critical value of .463 given in Kwiatkowski *et al.* (1992). Also, the t -statistic $\hat{\eta}_{\tau}$ that tests the null hypothesis of trend stationarity exceeds the 5% critical value of .146 [also given in Kwiatkowski *et al.* (1992)]. Hence, combining the results of our tests of the stationarity hypothesis with the results of our tests of the unit root hypothesis, we conclude that all the series have at least one unit root.

To test the null hypothesis of a second unit root, in panel B of Table 1 we test the null hypothesis of a unit root (using the WS, ADF, and $Z(t_{\hat{\alpha}})$ tests) as well as the null hypotheses of level and trend stationarity in the first (logged) differences of the series. Clearly, all the series appear to be stationary in first differences, since the null hypothesis of a unit root is rejected and the null hypotheses of level and trend stationarity cannot be rejected. The decision of the order of integration of the series is documented in the last column of Table

1. Table 2 reports summary statistics of the logarithmic first differences of the series.

7 Methodology

7.1 The McLeod-Li, Engle, and Tsay Tests

The McLeod and Li (1983) portmanteau test for non-linear dependence is conducted by examining the Box-Ljung Q statistic of the squared residuals after filtering with an ARMA process. Instead of using the residuals from a linear representation, the raw data can be examined through the use of the k autocorrelation coefficients for $\{x_t\}$, $\{|x_t|\}$, and $\{x_t^2\}$. The Q statistic for each of these three transformed data series can be used to examine the presence of serial correlation. For example, it has been suggested that if $\rho(k) = \rho^2(k)$ for all k , then the time series x_t is linear.

Under the null hypothesis that the prewhitened series x_t is an i.i.d process, McLeod and Li (1983) show that, for a fixed L ,

$$T^{1/2}\rho^2(k) = [\rho^2(1), \dots, \rho^2(L)]$$

is asymptotically a multivariate unit normal. Consequently, for L sufficiently large, the usual Box-Ljung statistic

$$Q = T(T+2) \sum_{j=1}^L \frac{[\rho^2(k)]^2}{T-j}$$

is asymptotically $\chi^2(L)$ under the null hypothesis of a linear generating mechanism for the data.

Engle (1982) proposed a Lagrange multiplier test that explicitly examines for non-linearity in the second moments (in particular, for ARCH-type disturbances). This involves regressing the squared residuals from an autoregression on x_t , against a constant and p lagged values of the squared residuals, as follows

$$\hat{\varepsilon}_t^2 = a_0 + \sum_{j=1}^p a_j \hat{\varepsilon}_{t-j}^2 + u_t$$

If there are no ARCH-type effects, the estimated coefficients a_1 through a_p would be equal to zero, meaning that this regression will have little explanatory power and the coefficient of determination, R^2 , will be very low. If the sample size is T , under the null hypothesis of no ARCH-type errors, the test statistic $T \times R^2$ converges to a χ_p^2 distribution. If $T \times R^2$ is sufficiently large, rejection of the null hypothesis that the coefficients of the lagged squared residuals are all equal to zero is equivalent to rejecting the null hypothesis of no ARCH-type errors.

Finally, the Tsay (1986) test is a generalization of the Keenan (1985) test. It explicitly looks for quadratic serial dependence in the data. While the Engle (1982) test examines evidence for non-linearity in the variance, the Tsay test checks for non-linearity in the mean, i.e., neglected nonlinearity. Let us briefly describe the Tsay (1986) test. Let the $K = k(k - 1)/2$ column vectors V_1, \dots, V_K contain all the possible crossproducts of the form $x_{t-i}x_{t-j}$, where $i \in [1, k]$ and $j \in [1, k]$. Also let v_{ti}^* denote the projection of v_{ti} on the subspace orthogonal to x_{t-1}, \dots, x_{t-k} , i.e., the residuals from a regression of v_{ti} on x_{t-1}, \dots, x_{t-k} . The parameters $\gamma_1, \dots, \gamma_K$ are then estimated by applying OLS to the regression equation

$$x_t = \gamma_0 + \sum_{i=1}^K \gamma_i v_{ti}^* + \eta_t$$

and the Tsay test statistic is the usual F statistic for testing the null hypothesis that $\gamma_1, \dots, \gamma_K$ are all zero.

The results from the application of the McLeod and Li, Engle, and Tsay tests are shown in Table 3, based on bootstrapped as well as asymptotic distributions, for all five return series — crude oil, gasoline, heating oil, propane, and natural gas. Clearly, the null hypothesis of linearity in the variance is rejected by the McLeod and Li (1983) test in all five energy markets. The null hypothesis of no ARCH-type errors is also rejected by the Engle (1982) test, except for the propane market where the null hypothesis is rejected only for $p = 1$. Finally, the Tsay (1986) tests rejects the null hypothesis of linearity in the mean in all five energy markets.

7.2 Mackey-Glass-GARCH Modeling

Based on the major feature of non-linear dynamical models to describe complex intrinsic structures of real economic systems, Kyrtsou (2005, 2006) has built a mixed non-linear model regrouping rich non-linearities in mean and variance, namely the Generalized Mackey-Glass-GARCH model (hereafter GMG-GARCH). In addition to the initial version of this model presented by Kyrtsou and Terraza (2003), the deterministic part of the latter model gives highly non-linear dynamics. Besides the existence of complex deterministic structures, the structural breaks appear as a potential source of non-linearity in economic and financial series.

In the aim to consider possible variations in the estimated coefficients of the GMG-GARCH due to the presence of such breaks, we use ~~a new~~ a new implementation. The structural breaks are detected employing the Zivot and Andrews (1992) method. More specifically, break points are found on October 6, 1997 for crude oil, on August 21, 1997 for gasoline, on November 17, 1997 for heating oil, on December 17, 1996 for propane, and on

January 20, 1997 for natural gas. We use the GMG-GARCH, as follows

$$R_t = \sum_{i=1}^n \alpha_i \frac{R_{i,t-\tau}}{1 + R_{i,t-\tau}^c} - \sum_{i=1}^n \delta_i R_{i,t-1} + \sum_{i=1}^n b_i R_{i,t-j} (1 - R_{i,t-j}) + \varepsilon_t;$$

$$\varepsilon_t \sim N(0, h_t);$$

$$h_t = a_0 + a_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1},$$

where $i = 1, \dots, n$ is the number of samples when a structural break occurs (for example, for one break, $n = 2$), τ and j are delays, and c is a constant. R_t denotes the return series and h_t the GARCH variance. For the estimation of the GMG-GARCH model we use the quasi-maximum likelihood (QML) procedure. The parameters τ , j , and c giving the best fitting are chosen on the basis of likelihood ratio tests and the Schwarz information criterion. Theoretical explanations and more detailed description of the GMG-GARCH model can be found in Kyrtsou (2005, 2006) and Kyrtsou and Serletis (2006).¹

The results of the estimation of the GMG-GARCH model are shown in Table 4. Looking closely at the estimates, we can see that recognizable nonlinear dynamics are archived only for the crude oil series. The entire GMG-GARCH model is significant during the second period, i.e. after the structural break point occurring on October 6, 1997. For the rest of the series, some coefficients appear to be significant here and there; nonetheless such findings are not conclusive taking into account the suggested framework. Besides, they cannot exclude the presence of some kind of linearities in the pre- and post-break period. In this spirit, we can say that the gasoline and heating oil series present similar dynamics since the detected neglected nonlinearity disappears after the break. The propane series does the opposite while the natural gas seems to be unaffected by the presence of the break.

7.3 The Hinich Bispectral and Bicorrelation Tests

7.3.1 Bispectral Tests

Hinich (1982) developed a statistical test for determining whether a sampled stationary time series $\{x(t)\}$ is linear. This is a direct test for linearity and also a test for Gaussianity; it is possible that $\{x(t)\}$ is linear without being Gaussian, but all of the stationary Gaussian time series are linear. The Hinich (1982) test involves estimating the bispectrum of a stationary time series. If the process generating the data is linear then the skewness of the bispectrum will be constant. If the test rejects constant skewness then a non-linear process is implied.

¹Simulation experiments on this modelling can be found in Kyrtsou and Malliaris (2009). For a multivariate setting see Kyrtsou and Labys (2006, 2007) and Kyrtsou and Vorlow (2009).

Let's present a brief description of the Hinich (1982) bispectrum-based linearity and Gaussianity tests. Consider a third order stationary time series $\{x(t)\}$, where the time unit t is an integer. The third-order cumulant function of $\{x(t)\}$ is defined to be

$$C_{xxx}(r, s) = E \left[x(t+s)x(t+r)x(t) \right],$$

for each (r, s) when $E[x(t)] = 0$, in which $s \leq r$ and $r = 0, 1, 2, \dots$. Because third-order cumulants are hard to interpret, the bispectrum, which is the double Fourier transform of the third-order cumulant function, $C_{xxx}(r, s)$, is calculated. The bispectrum at frequency pairs (f_1, f_2) is defined as

$$B_x(f_1, f_2) = \sum_{r=-\infty}^{\infty} \sum_{s=-\infty}^{\infty} C_{xxx}(r, s) \exp \left[-i2\pi(f_1 r + f_2 s) \right],$$

assuming that $|C_{xxx}(r, s)|$ is summable.

The symmetries of $C_{xxx}(r, s)$ translate into symmetries of $B_x(f_1, f_2)$ that yield a principal domain for $B_x(f_1, f_2)$ given by $\Omega = \{0 < f_1 < 0.5, f_2 < f_1, 2f_1 + f_2 < 1\}$. Since the (ordinary power) spectrum of $x(t)$ at frequency f , $S_x(f)$, is given by

$$S_x(f) = \sigma^2 |A(f)|^2,$$

the skewness function of $\{x(t)\}$, $\psi(f_1, f_2)$, is defined by

$$\psi^2(f_1, f_2) = \frac{|B_x(f_1, f_2)|^2}{S_x(f_1)S_x(f_2)S_x(f_1 + f_2)},$$

for all f_1 and f_2 in Ω and $A(f) = \sum_{s=0}^{\infty} \alpha(s) \exp(-i2\pi fs)$.

Linearity and Gaussianity of $\{x(t)\}$ can be tested using a sample estimator of the skewness function. In particular, linearity of $\{x(t)\}$ is tested through the null hypothesis that the skewness function, $\psi(f_1, f_2)$, is constant over all frequencies. Gaussianity of $\{x(t)\}$ is tested through the null hypothesis that $\psi(f_1, f_2)$ is zero over all frequencies.

Columns 1 and 2 of Table 5 present p -values for Hinich's (1982) bispectrum-based Gaussianity and linearity tests. The results reject the null hypothesis of Gaussianity in all five energy markets (see column 1). Although Gaussianity and linearity tests are linked, a rejection of Gaussianity does not necessarily rule out linearity. However, the p -values in column 2 reject the null hypothesis of a linear generating mechanism in all five energy markets, suggesting the existence of nonlinear dependencies within the daily returns.

7.3.2 Bicorrelation Tests

Hinich (1996) proposed a modified version of the Box Pierce (1970) portmanteau Q -statistic for autocorrelation and a third order portmanteau statistic, which can in a sense be viewed

as a time domain analogue of the bispectrum test. A full theoretical derivation of the test statistics and a number of Monte Carlo simulations to assess their size and power are given in Hinich (1996) and Hinich and Patterson (1995).

Let $\{x(t)\}$ denote the sampled data process, where the time unit t is an integer. In this paper the time series will be daily energy returns. The method is to break the observed series into equal length frames and apply a number of statistics to each frame, generating a multivariate time series of frame statistics which are then used to test for linear and nonlinear serial dependencies. In particular, if n is the window length, then the k th window is $\{x(t_k), x(t_k + 1), \dots, x(t_k + n - 1)\}$. The next window is $\{x(t_{k+1}), x(t_{k+1} + 1), \dots, x(t_{k+1} + n - 1)\}$, where $t_{k+1} = t_k + n$. We define $z(t_k)$ as the standardized observations (created by subtracting the sample mean of the window, and dividing by its standard deviation) at time $t = k$, that is,

$$z(t_k) = \frac{x(t_k) - \mu_x}{\sigma_x},$$

where μ_x and σ_x^2 are the sample mean and sample standard deviation of the window. The null hypothesis for each window is that $x(t)$ are realizations of a stationary pure noise process that has zero bicoherence. The alternative hypothesis is that the process in the window is random with some non-zero correlations or non-zero bicoherences.

The C (or correlation) statistic, which has been developed for the detection of linear serial dependencies, is defined as

$$C = \sum_{r=1}^L \left[C^2(r)/(T - r - 1) \right] \sim \chi^2(L), \quad (1)$$

where

$$C(r) = \sum_{k=1}^{T-s} z(t_k)z(t_{k+r})$$

is the sample correlation.

The H statistic, which has been developed for the detection of nonlinear serial dependencies, tests for certain forms of nonlinearity using third-order correlations. It is defined as

$$H = \sum_{s=2}^L \sum_{r=1}^{s-1} \left[G^2(r, s)/(T - s) \right] \sim \chi^2(L(L - 1)/2), \quad (2)$$

where

$$G(r, s) = \sum_{k=1}^{T-s} z(t_k)z(t_{k+r})z(t_{k+s})$$

is the (r, s) sample bicoherence.

In (1) and (2), the number of lags L is specified as $L = T^c$ with $0 < c < 0.5$, where c is a parameter under the choice of the analyst. Based on results from Monte Carlo simulations [see Hinich and Patterson (1995)], the use of $c = 0.4$ is recommended in order to maximize the power of the tests whilst ensuring a valid approximation to the asymptotic theory even when T is small.

The C statistic in (1) is asymptotically distributed, under the null of pure white noise, as a chi-square with L degrees of freedom for large T if $L = T^c$ with $0 < c < 0.5$. It is closely related to the Box-Pierce portmanteau test statistic which detects correlated (non white) noise — see Box and Pierce (1970). Usually, the Box and Pierce Q -statistic for autocorrelation is applied to the residuals of a fitted ARMA model, but the C statistic is applied to the standardized observations, $z(t_k)$. Moreover, the Box and Pierce test does not specify the number of lags L to be used; that decision is left to the analyst. The C statistic specifies $L = T^c$ with $0 < c < 0.5$.

The H statistic in (1) is asymptotically distributed, under the null that the observed process is pure white noise (i.i.d.), as a chi-square with $L(L - 1)/2$ degrees of freedom for large T if $L = T^c$ with $0 < c < 0.5$. It tests for certain forms of nonlinearity using third-order correlations and is considered as a generalization of the Box and Pierce portmanteau test. In particular, the test is of a null of pure white noise against an alternative that the process has m non-zero correlations or bicorrelations in the set $0 < r < s \leq L$, i.e. that there exists second or third order dependence in the data generating process, and relies on the property of pure noise that it has zero bicovariance. The test is particularly useful in detecting nonlinear dependencies, since it has much better small-sample properties, and does not have such stiff data requirements as many of its competitors, such as the BDS test [Brock *et al.* (1996)] for a useful survey.

Columns 3 and 4 of Table 5 present p -values for the correlations (C) and bicorrelations (H) test statistics. The results show that the null of pure noise is strongly rejected by both the C and H statistics in all five energy markets.

7.4 Chaos Tests

Finally, we test for chaos by applying the recently developed methods by Whang and Linton (1999), Linton and Shintani (2003), and Shintani and Linton (2004) and construct the standard error for the Nychka *et al.* (1992) dominant Lyapunov exponent — see Serletis and Shintani (2003) for a detailed discussion of the methodology and an application to the U.S. stock market or Serletis and Shintani (2006) for an application to U.S. monetary aggregates.

Lyapunov exponent point estimates, along with p -values for the null hypothesis $H_0 : \lambda \geq 0$, are reported in Table 6, for the logarithmic first differences of the series. The results are presented for dimensions 1 through 6, with the optimal value of the number of hidden units in the neural net being chosen by minimizing the BIC criterion — see, for example, Serletis and Shintani (2006) for more details.

As can be seen, the reported Lyapunov exponent point estimates are negative and in every case we reject the null hypothesis of chaotic behavior. Of course, the failure to detect low-dimensional chaos does not preclude the possibility of high-dimensional chaos in these series — see, for example, Barnett and Serletis (2000). The presence, however, of dynamic noise makes it difficult and perhaps impossible to distinguish between (noisy) high-dimensional chaos and pure randomness. Thus, as Granger (1991, p. 268) put it, “it will be a sound, pragmatic strategy to continue to use stochastic models and statistical inference.”

7.5 Recurrence Quantification Analysis

Recurrence Quantification Analysis (RQA) is a relatively new analytical tool for the study of non-linear dynamical systems developed by Webber and Zbilut (1994) and then applied to theoretical time series by Trulla *et al.* (1996) and Zbilut *et al.* (2000). The RQA methodology can be summarised as follows: the embedding matrix corresponding to the studied series is constructed by the method of time delays — see Takens (1981). Thus, $x_t^m = (x_t, x_{t+\tau}, \dots, x_{t+(m-1)\rho})$ are the artificial vectors, where $t = 1, \dots, T - (m - 1)\rho$, T is the number of observations, m is the embedding dimension, and ρ is the time delay. Next, using a Euclidean norm, distances D in n -space between individual $i - j$ pairs are calculated. A RP is a graphical representation of the distances matrix $D_{i,j}$, by darkening the point at coordinates (i, j) that corresponds to a distance value between i and j vectors lower than a predetermined critical radius ε . The plot is symmetric ($D_{i,j} = D_{j,i}$) and the main diagonal is always darkened ($D_{i,j} = 0, i = j$). The main feature of RPs is that if the series is fully deterministic, the system’s attractor will be revisited by the trajectory sometime in the future. To be more specific, in the case of periodic signals we have very long diagonal lines while for chaotic signals we obtain short and no diagonal lines.

Unfortunately, the structures that are found in the RPs are not substantiated mathematically. Answering to these limitations the RQA considers that it is possible to quantify the information supplied by the RPs and using certain simple pattern recognition algorithms to mirror this information into a set of measures namely the RQA variables. In order to perform a crosschecking procedure of the previous test results we will employ one of the RQA indicators, the so-called percent of determinism (%DET). %DET measures the proportion of recurrent points forming diagonal line structures. The presence of these lines reveals the existence of a deterministic structure.

The results from the RQA computation are presented in Figures 6-10. As can be seen, distinct signs of determinism cannot be identified. However, as it is mentioned in the previous section interactions between dynamical noise and non-linear deterministic dynamics make indistinguishable the true generating mechanism of data — for some examples, see Kyrtsou *et al.* (2004).

Some applications of the RQA in macroeconomic and financial time series and discussion can be found in Kyrtsou and Vorlow (2005), Kyrtsou and Terraza (2009), Strozzi *et al.*

(2002), Belaire-Franch (2004), and Belaire-Franch *et al.* (2002).

8 Conclusion

We have discussed a number of (widely used) univariate tests from dynamical systems theory to distinguish between deterministic and stochastic origin for time series. We have applied these tests to daily observations for crude oil, gasoline, heating oil, propane, and natural gas, over the period from 1994 to mid-January 2008. We have found evidence consistent with nonlinear dependencies in each of these energy markets, suggesting that successful nonlinear modeling of energy prices would produce a richer notion of energy market fluctuations than linear time series models allow. Furthermore, taking into account the informative power of neglected nonlinear dynamics, contributes to a better understanding of energy price behavior, especially when structural breaks and other shocks occur.

Of course, testing for the presence of a deterministic nonlinear structure is a first step in determining if a stochastic model is valid. As Kyrtsov and Serletis (2006, p. 167) put it, “from an economic perspective, however, the interest in macroeconomic and financial time series is in their relationship with other series. Because the properties of univariate series are different from those of their multivariate relationships, the development of multivariate tests for deterministic nonlinear structure appears to be an area for potentially productive future research.”

Finally, regarding the relationship between energy prices and real economic activity (not investigated in this paper), recent empirical work has focused on the role of uncertainty about oil prices and its effect on real economic activity. For example, Elder and Serletis (2008), Elder and Serletis (this issue), Rahman and Serletis (2008), and Serletis and Rahman (this issue) use recent advances in the financial econometrics literature and present evidence that increased uncertainty about the change in the price of oil is associated with a lower average growth rate of real economic activity, with this result being robust to a number a different specifications, alternative measures of the price of oil, alternative measures of the level of economic activity, as well as alternative sample periods.

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Figure 1. Crude Oil Prices and Returns (in Basis Points)

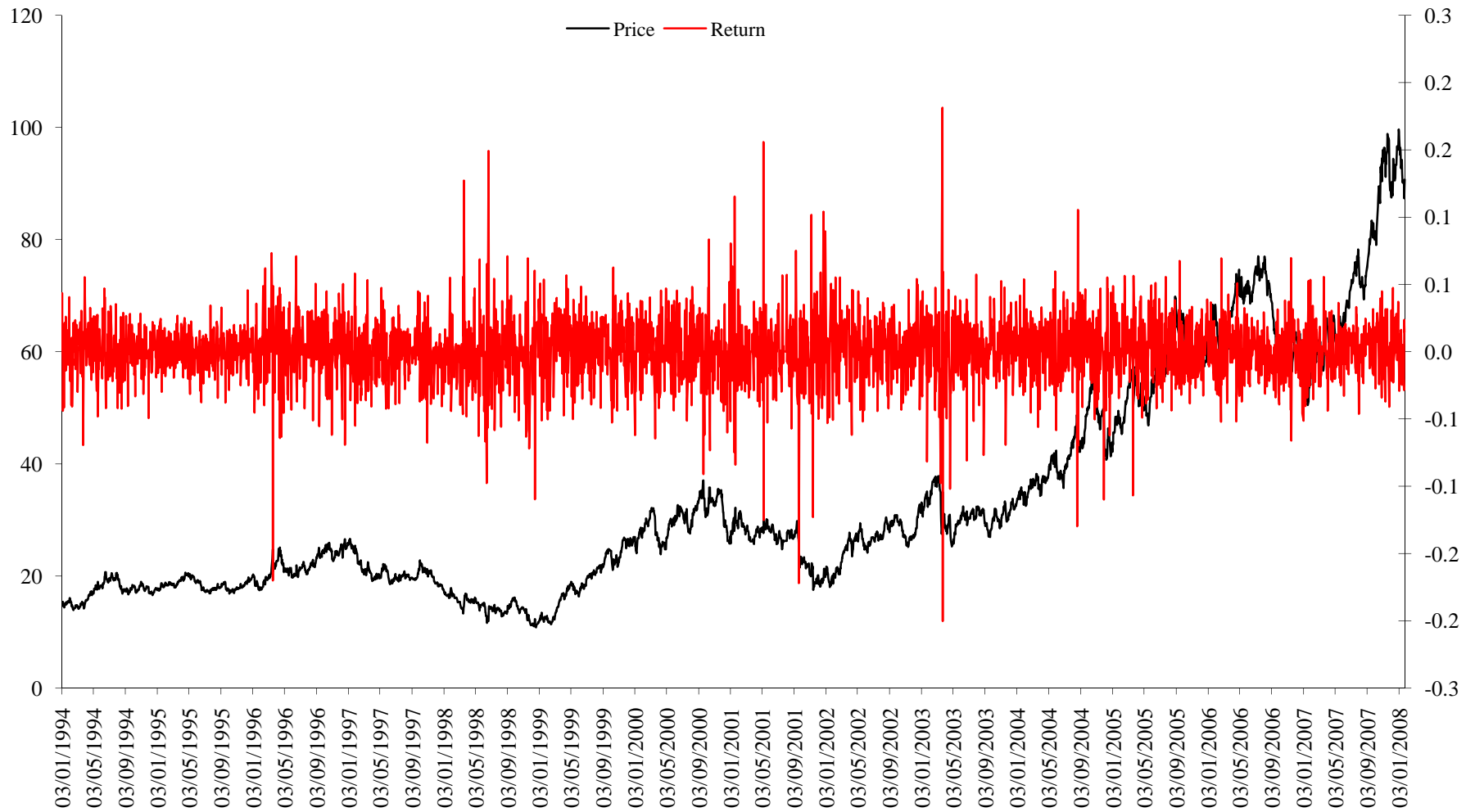


Figure 2. Gasoline Prices and Returns (in Basis Points)

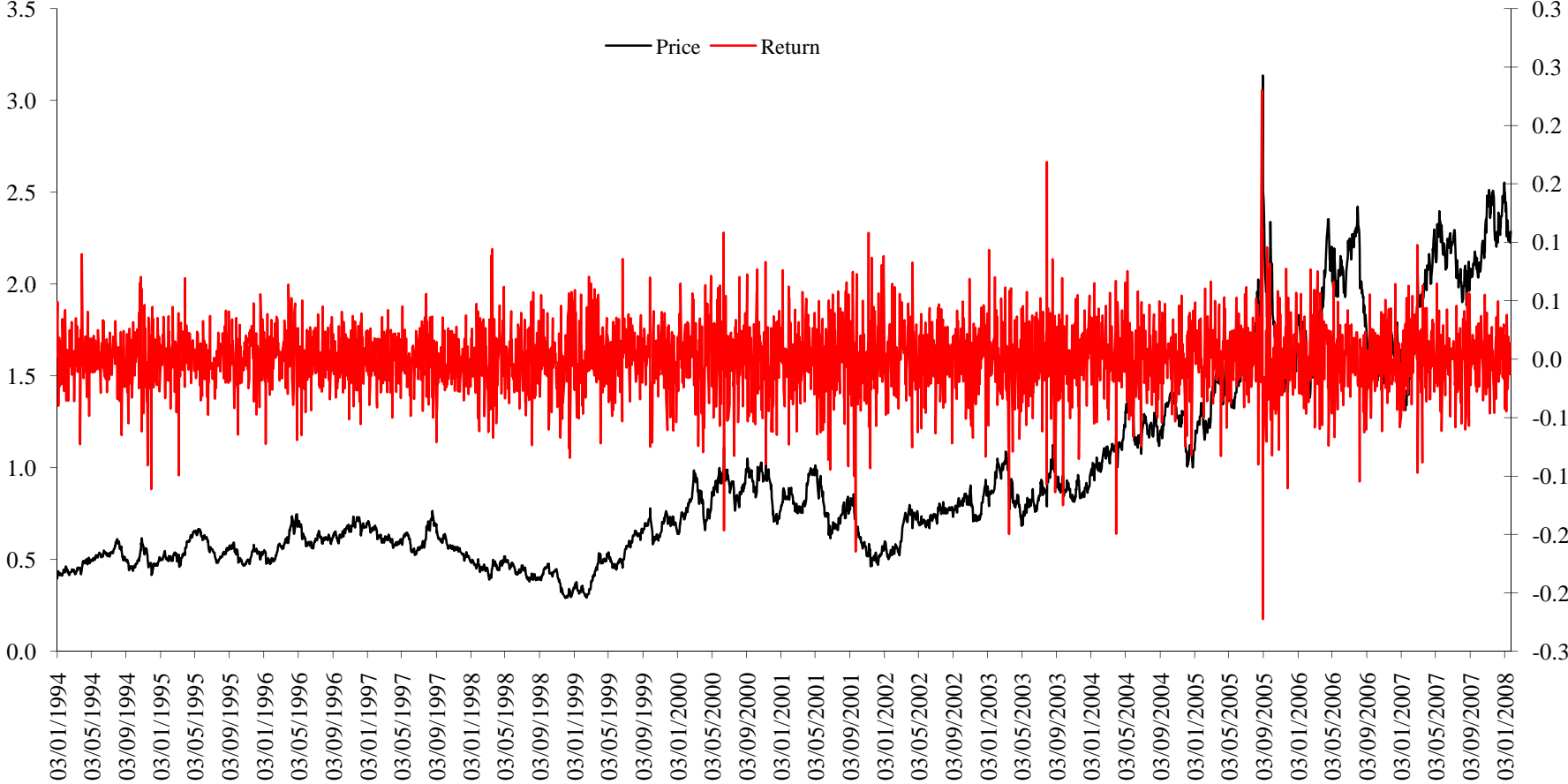


Figure 3. Heating Oil Prices and Returns (in Basis Points)

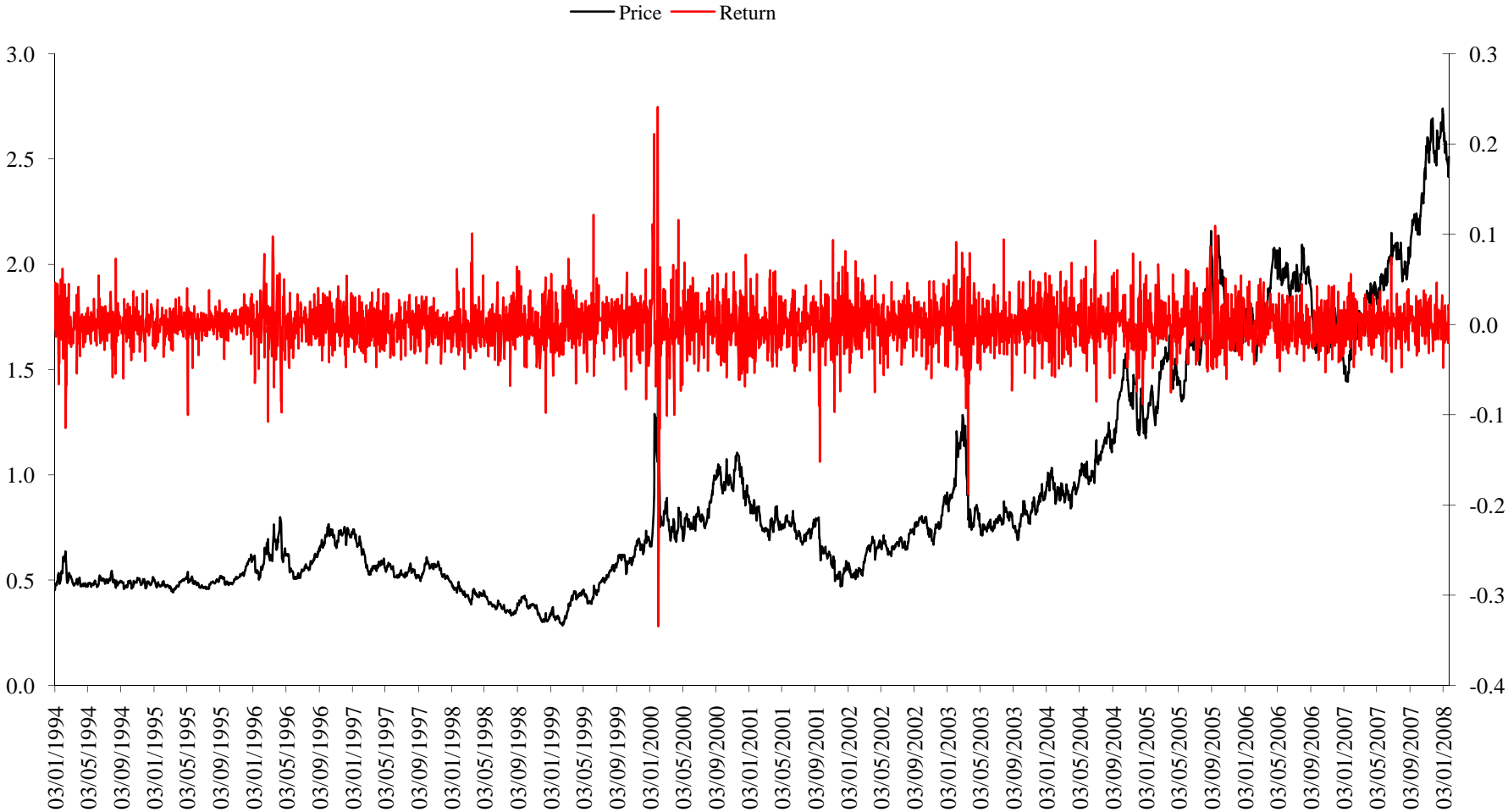


Figure 4. Natural Gas Prices and Returns (in Basis Points)

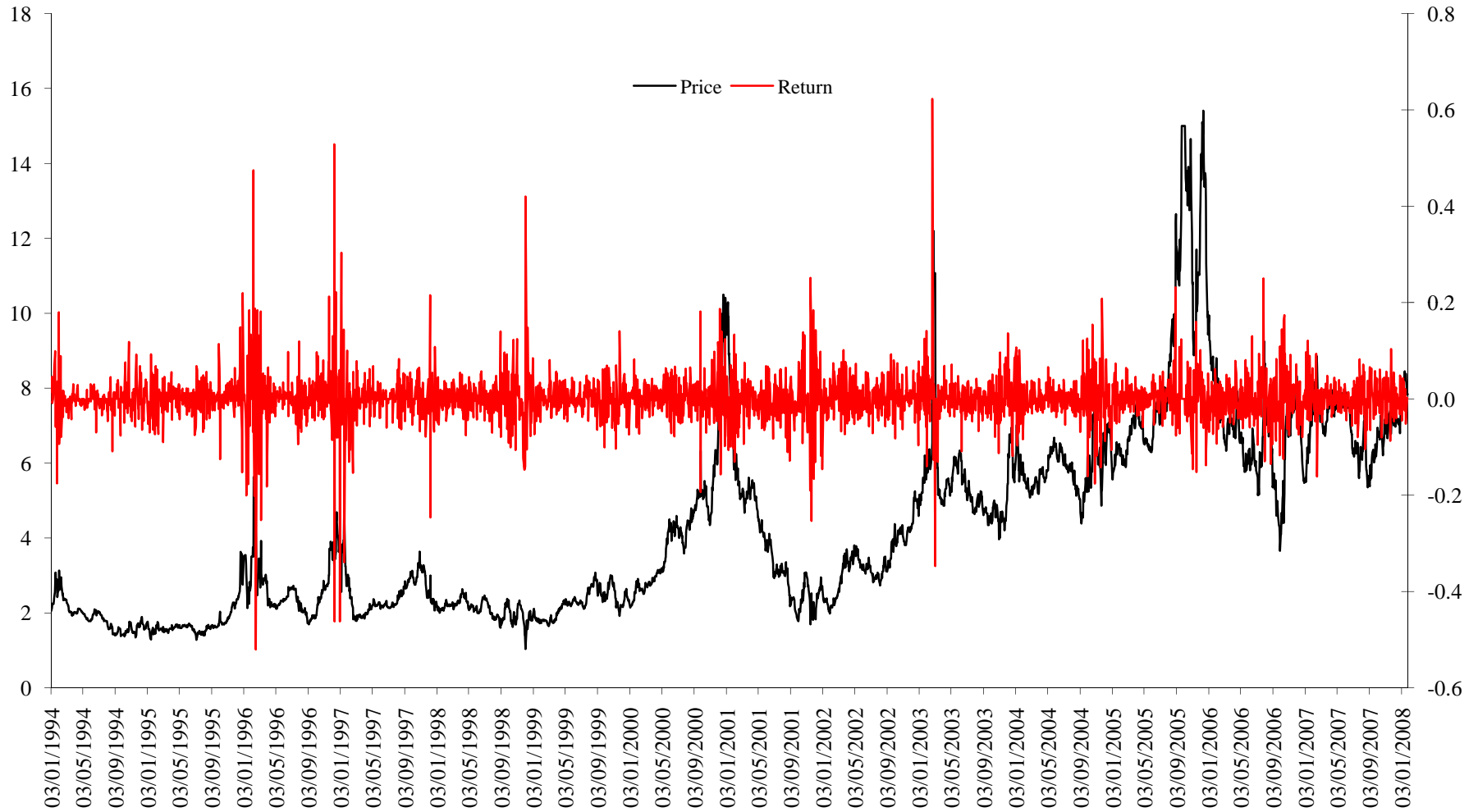


Figure 5. Propane Prices and Returns (in Basis Points)

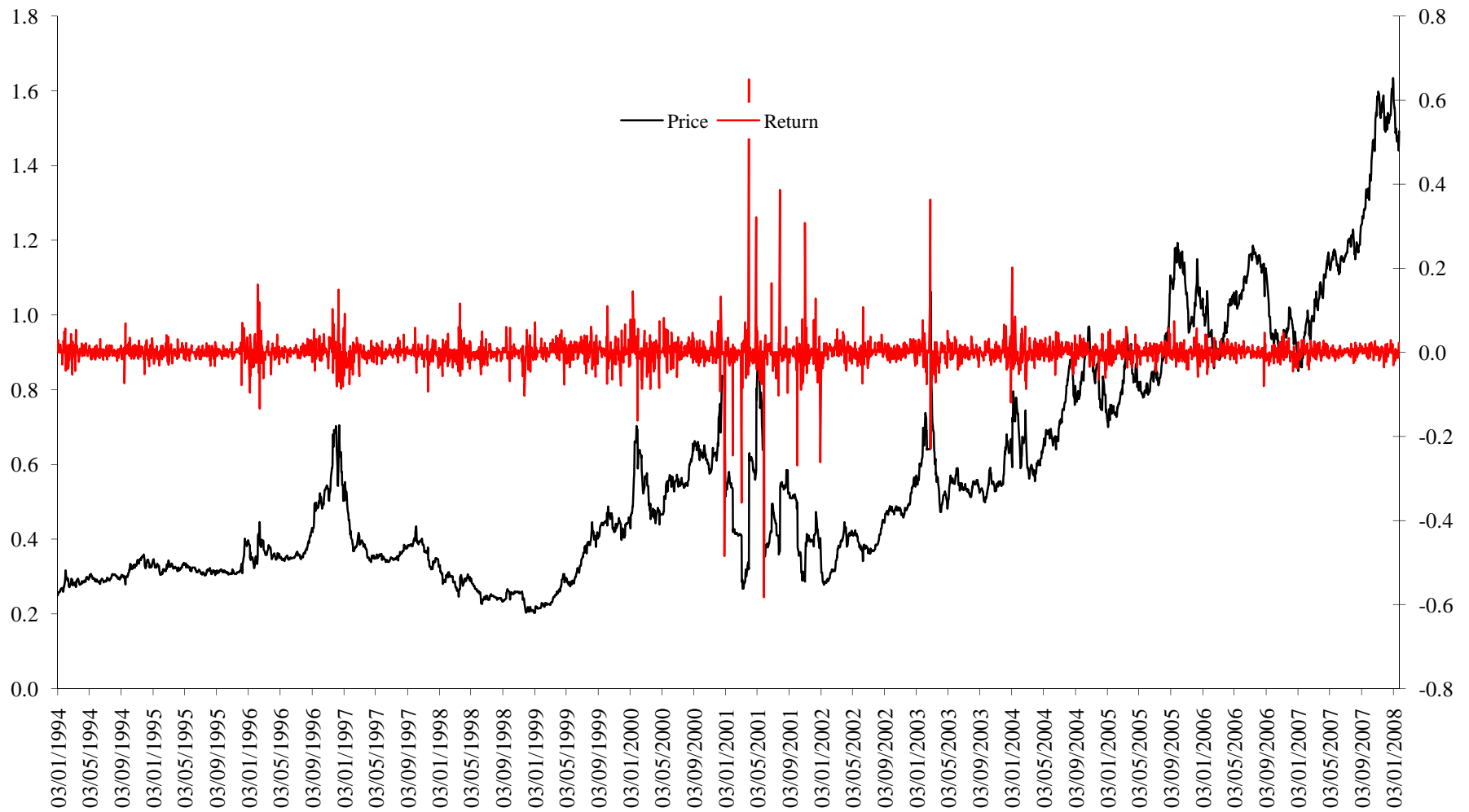


TABLE 1. UNIT ROOT TEST RESULTS

Variable	A. Log levels					B. First differences of log levels					Decision
	p -values			KPSS		p -values			KPSS		
	WS	ADF	$Z(t_{\hat{\alpha}})$	\hat{n}_{μ}	\hat{n}_{τ}	WS	ADF	$Z(t_{\hat{\alpha}})$	\hat{n}_{μ}	\hat{n}_{τ}	
Crude oil	.298	.405	.264	40.320*	6.286*	.000	.000	.000	.075	.027	I(1)
Gasoline	.053	.127	.053	39.228*	5.912*	.000	.000	.000	.033	.019	I(1)
Heating oil	.235	.274	.161	38.495*	6.138*	.000	.000	.000	.068	.019	I(1)
Propane	.019	.052	.017	37.447*	4.472*	.000	.000	.000	.032	.017	I(1)
Natural gas	.007	.008	.000	38.913*	1.135*	.000	.000	.000	.015	.013	I(1)

Notes: Numbers in the WS, ADF, and $Z(t_{\hat{\alpha}})$ columns are tail areas of unit root tests. An asterisk (next to a t -statistic) indicates significance at the 5 percent level. The 5 percent critical values for the KPSS \hat{n}_{μ} and \hat{n}_{τ} test statistics [given in Kwiatkowski *et al.* (1992)] are .463 and .146, respectively.

TABLE 2. SUMMARY STATISTICS OF ENERGY RETURNS

Series	Sample mean	Standard error	Skewness	Kurtosis (excess)	Jarque-Bera (<i>p</i> -value)
Crude oil	.0005	.0236	-.3324	6.7519	.000
Gasoline	.0004	.0286	-.2648	4.1264	.000
Heating oil	.0004	.0263	-.3648	13.8970	.000
Propane	.0005	.0318	.3112	122.7150	.000
Natural gas	.0003	.0493	.4216	26.7148	.000

TABLE 3. MCLEOD-LI, ENGLE, AND TSAY TESTS

Series		McLeod-Li ($L = 24$)	Engle ($p = 5$)	Tsay ($k = 5$)
<i>Crude oil</i>	Bootstrap	$\leq .001$	$\leq .001$	$\leq .001$
	Asymptotic	$\leq .001$	$\leq .001$	$\leq .001$
<i>Gasoline</i>	Bootstrap	$\leq .001$	$\leq .001$	$\leq .001$
	Asymptotic	$\leq .001$	$\leq .001$	$\leq .001$
<i>Heating oil</i>	Bootstrap	$\leq .001$	$\leq .001$	$\leq .001$
	Asymptotic	$\leq .001$	$\leq .001$	$\leq .001$
<i>Propane</i>	Bootstrap	.017	.121	.071
	Asymptotic	$\leq .001$.797	.006
<i>Natural gas</i>	Bootstrap	$\leq .001$	$\leq .001$	$\leq .001$
	Asymptotic	$\leq .001$	$\leq .001$	$\leq .001$

Notes: Sample size $T = 3508$. Numbers are p -values.

TABLE 4. GMG-GARCH ESTIMATION RESULTS

Coefficient	Crude oil	Gasoline	Heating oil	Propane	Natural gas
α_1	13.79 (0.98)	30.14 (2.02)	1.80 (2.11)	-0.03 (0.46)	1.07 (0.80)
δ_1	-12.43 (0.99)	-29.17 (2.00)	0.05 (1.57)	0.11 (1.07)	-0.95 (0.72)
b_1	-1.35 (0.79)	-0.82 (0.94)	-1.81 (2.10)	-0.06 (1.34)	-0.14 (3.80)
α_2	15.79 (3.53)	5.66 (1.02)	-0.06 (0.31)	0.31 (2.29)	0.23 (0.22)
δ_2	-14.86 (3.40)	-5.31 (0.99)	-0.007 (0.35)	0.18 (2.81)	-0.19 (0.19)
b_2	-0.93 (2.09)	-0.32 (0.65)	0.03 (0.17)	0.11 (1.47)	-0.07 (3.02)
α_0	$3.81e - 05$ (4.87)	$3.48e - 05$ (4.21)	$2.36e - 05$ (4.34)	$5.55e - 04$ (3.38)	$3.31e - 05$ (2.00)
α_1	0.09 (3.38)	0.07 (3.78)	0.09 (5.59)	0.99 (2.44)	0.13 (7.68)
β_1	0.84 (28.64)	0.88 (33.95)	0.87 (45.36)	-	0.86 (37.20)
Model parameters	$c = 2, \tau = j = 1$	$c = 2, \tau = j = 1$	$c = 2, \tau = j = 2$	$c = \tau = 2, j = 5$	$c = j = 2, \tau = 1$

Notes: Sample size $T = 3508$. Numbers in parentheses are absolute t -statistics.

TABLE 5. BISPECTRAL AND BICORRELATIONS TESTS

Series	Bispectral tests		Bicorrelation tests	
	Gaussianity	Linearity	C-statistic	H-statistic
Crude oil	$\leq .0001$	$\leq .0001$	$\leq .0001$	$\leq .0001$
Gasoline	$\leq .0001$	$\leq .0001$	$\leq .0001$	$\leq .0001$
Heating oil	$\leq .0001$	$\leq .0001$	$\leq .0001$	$\leq .0001$
Propane	$\leq .0001$	$\leq .0001$	$\leq .0001$	$\leq .0001$
Natural gas	$\leq .0001$	$\leq .0001$	$\leq .0001$	$\leq .0001$

Notes: Sample size $T = 3508$. Numbers are p -values.

TABLE 6. LYAPUNOV EXPONENT ESTIMATES

NLAR lag (m)	Number of hidden units								
	$k = 1$			$k = 2$			$k = 3$		
	BIC	$\hat{\lambda}$	p -value	BIC	$\hat{\lambda}$	p -value	BIC	$\hat{\lambda}$	p -value
<i>Crude oil</i>									
1	-7.503	-2.635	$\leq .001$	-7.500	-2.897	$\leq .001$	-7.501	-2.977	$\leq .001$
2	-7.500	-2.068	$\leq .001$	-7.500	-2.131	$\leq .001$	-7.499	-2.157	$\leq .001$
3	-7.495	-1.263	$\leq .001$	-7.494	-1.575	$\leq .001$	-7.489	-1.365	$\leq .001$
4	-7.487	-0.943	$\leq .001$	-7.487	-1.408	$\leq .001$	-7.489	-0.864	$\leq .001$
5	-7.490	-0.648	$\leq .001$	-7.487	-0.630	$\leq .001$	-7.481	-0.625	$\leq .001$
6	-7.479	-0.923	$\leq .001$	-7.477	-0.733	$\leq .001$	-7.484	-0.911	$\leq .001$
<i>Gasoline</i>									
1	-7.094	-3.698	$\leq .001$	-7.092	-2.882	$\leq .001$	-7.091	-2.634	$\leq .001$
2	-7.107	-2.297	$\leq .001$	-7.106	-1.843	$\leq .001$	-7.104	-1.889	$\leq .001$
3	-7.093	-0.945	$\leq .001$	-7.089	-1.041	$\leq .001$	-7.078	-1.038	$\leq .001$
4	-7.097	-0.668	$\leq .001$	-7.094	-0.693	$\leq .001$	-7.092	-0.912	$\leq .001$
5	-7.085	-0.651	$\leq .001$	-7.083	-0.617	$\leq .001$	-7.092	-0.672	$\leq .001$
6	-7.092	-0.483	$\leq .001$	-7.086	-0.500	$\leq .001$	-7.076	-0.393	$\leq .001$
<i>Heating oil</i>									
1	-7.275	-2.809	$\leq .001$	-7.290	-2.838	$\leq .001$	-7.289	-3.533	$\leq .001$
2	-7.278	-1.851	$\leq .001$	-7.276	-2.192	$\leq .001$	-7.307	-1.870	$\leq .001$
3	-7.274	-1.045	$\leq .001$	-7.303	-1.046	$\leq .001$	-7.292	-0.943	$\leq .001$
4	-7.326	-0.655	$\leq .001$	-7.329	-0.963	$\leq .001$	-7.311	-0.823	$\leq .001$
5	-7.281	-0.618	$\leq .001$	-7.331	-0.551	$\leq .001$	-7.294	-0.787	$\leq .001$
6	-7.285	-0.419	$\leq .001$	-7.332	-0.578	$\leq .001$	-7.330	-0.578	$\leq .001$
<i>Propane</i>									
1	-6.885	-3.611	$\leq .001$	-6.885	-2.931	$\leq .001$	-6.884	-1.873	$\leq .001$
2	-6.880	-1.221	$\leq .001$	-6.885	-0.904	$\leq .001$	-6.877	-0.898	$\leq .001$
3	-6.886	-0.848	$\leq .001$	-6.882	-0.958	$\leq .001$	-6.878	-0.785	$\leq .001$
4	-6.874	-0.944	$\leq .001$	-6.869	-0.744	$\leq .001$	-7.007	-1.395	$\leq .001$
5	-6.883	-0.602	$\leq .001$	-6.899	-0.419	$\leq .001$	-6.892	-0.445	$\leq .001$
6	-6.885	-0.390	$\leq .001$	-6.945	-0.312	$\leq .001$	-7.004	-0.356	$\leq .001$
<i>Natural gas</i>									
1	-6.011	-3.583	$\leq .001$	-6.024	-3.736	$\leq .001$	-6.019	-2.667	$\leq .001$
2	-6.023	-1.067	$\leq .001$	-6.021	-0.910	$\leq .001$	-6.021	-0.965	$\leq .001$
3	-6.008	-1.158	$\leq .001$	-6.032	-1.348	$\leq .001$	-6.025	-1.293	$\leq .001$
4	-6.033	-0.278	$\leq .001$	-6.052	-0.298	$\leq .001$	-6.043	-0.265	$\leq .001$
5	-6.000	-0.671	$\leq .001$	-6.039	-0.645	$\leq .001$	-6.045	-0.225	$\leq .001$
6	-6.037	-0.326	$\leq .001$	-6.040	-0.333	$\leq .001$	-6.082	-0.258	$\leq .001$

Notes: Sample size $T = 3508$. The largest Lyapunov exponent estimates, $\hat{\lambda}$, are presented with p -values for $H_0 : \lambda \geq 0$.

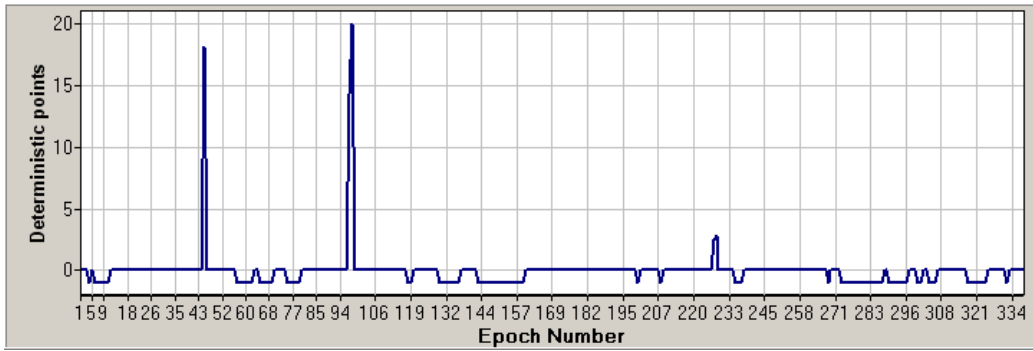


Figure 6. %determinism for the crude oil returns series. RQA parameters: embedding dimension = 12, delay = 3, line definition = 2 points, length of each epoch = 100 points, data shift = 10, radius $\epsilon = 10$.

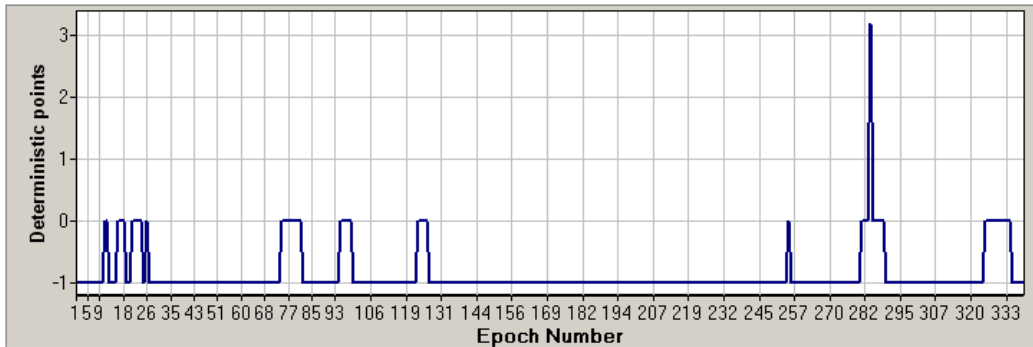


Figure 7. %determinism for the gasoline returns series. RQA parameters: embedding dimension = 15, delay = 2, line definition = 2 points, length of each epoch = 100 points, data shift = 10, radius $\epsilon = 10$.

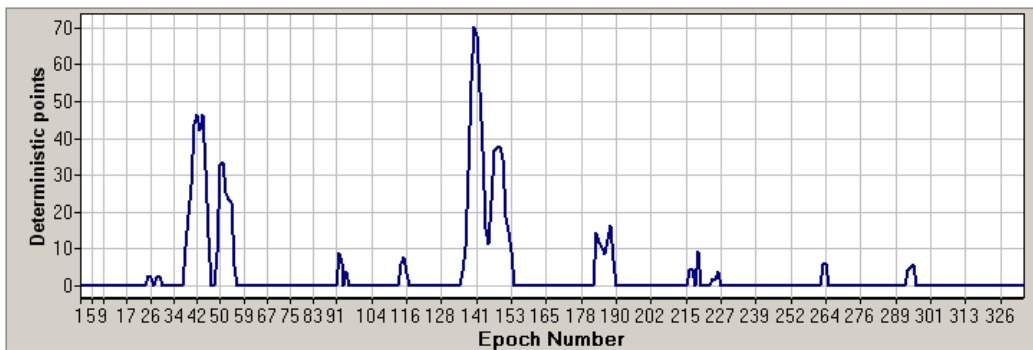


Figure 8. %determinism for the heating oil returns series. RQA parameters: embedding dimension = 14, delay = 6, line definition = 2 points, length of each epoch = 100 points, data shift = 10, radius $\epsilon = 10$.

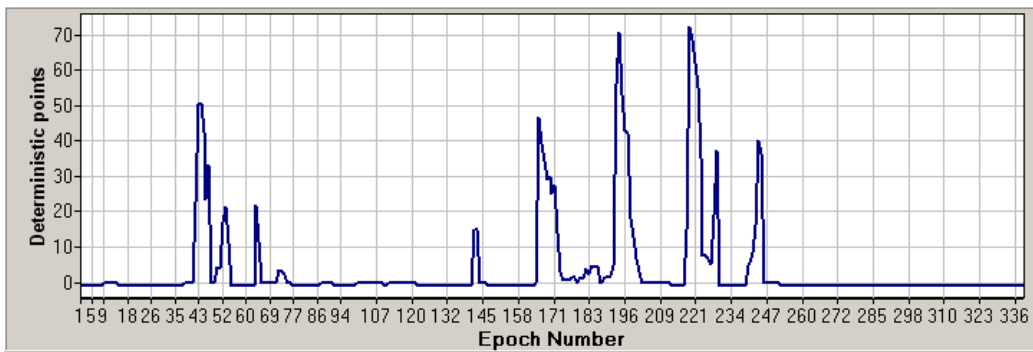


Figure 9. %determinism for the propane returns series. RQA parameters: embedding dimension = 10, delay = 2, line definition = 2 points, length of each epoch = 100 points, data shift = 10, radius $\epsilon = 10$.

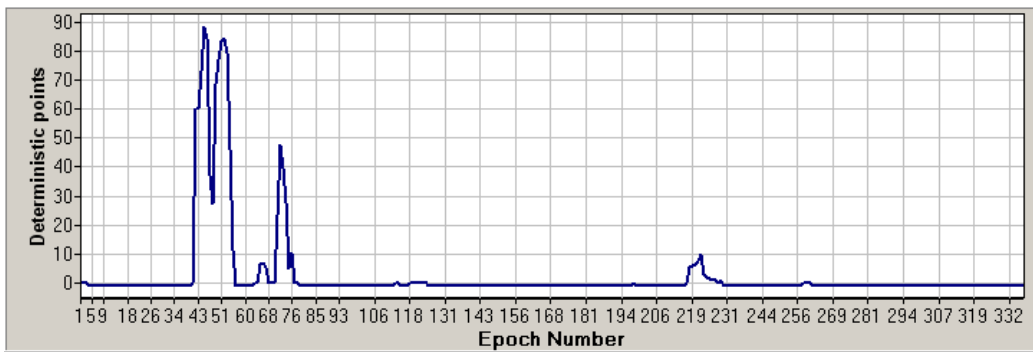


Figure 10. %determinism for the natural gas returns series. RQA parameters: embedding dimension = 14, delay = 2, line definition = 2 points, length of each epoch = 100 points, data shift = 10, radius $\epsilon = 10$.