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A Monte Carlo Study of Pearson and Log-Linear Chi-Square One Sample Tests with Small N

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A MONTE CARLO STUDY OF PEARSON AND LOG-LINEAR
CHI-SQUARE ONE SAMPLE TESTS WITH SMALL N

by

Adam J. Miller II

A Dissertation Submitted to the Faculty of the Graduate School
of Loyola University of Chicago in Partial Fulfillment
of the Requirements for the Degree of
Doctor of Education

May

1979

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Especial thanks are due Dr. Melvin Cohen and McGill University for supplying the source deck and instructions on "How to Use the McGill Random Number Package 'Super-Duper'" that made this study possible within the time and money constraints that prevailed.

VITA

The author, Adam James Miller II, is the son of Adam James Miller, Sr. and Mabel (Hansen) Miller. He was born June 23, 1920, in Chicago, Illinois.

His elementary education was obtained in the public schools of Chicago, Illinois. His secondary education was obtained at St. John's Military Academy, Delafield, Wisconsin, where he graduated in 1937.

In September, 1937, he entered M. I. T., and in June, 1941, he received the degree of Bachelor of Science with a major in Business and Engineering Administration. While attending M. I. T., he also was a special student at the Harvard Graduate School of Education during 1939.

From April, 1941, to February, 1944, he was Assistant Plant Engineer and Assistant to the Superintendent of Hull and Machinery Outfitting at North Carolina Shipbuilding Company, a subsidiary of Newport News Shipbuilding and Drydock Company. From February, 1944, to May, 1946, he served as an Engineering Officer in the United States Navy Reserve. During this period, as a civilian, he supervised the construction of the least expensive Liberty Ship ever built. He also received a commendation for delivering the least expensive pair of LSM ships at any navy yard and converted this same type of ship to

missile launchers in minimum time. The last year of his naval service was spent as Personnel and Engineering Officer for a division of 990 people, having a budget of \$900,000,000.per annum.

In 1941, he purchased a partnership in a Texaco distributorship, devoting part-time to that enterprise as well as devoting himself to other interests, including employment as a Production Engineer at Chicago Screw Company (1946 to 1948). After that, he devoted full-time effort to the gasoline distributorship, becoming senior partner and, therefore, president of the succeeding corporation. From a nadir in 1948, sales were increased to over \$2,000,000 in 1954.

Desiring to return to academia and a teaching career, the author enrolled as a student in Loyola University of Chicago's Graduate School of Business in April, 1970. He received the M.B.A. degree with a major in Quantitative Methods in June, 1971. From June, 1971, to date, he has been a student in Loyola University's Graduate School of Education. In addition, he served as a Lecturer, Educational Foundations, Loyola University, in the fall and spring semesters of 1977-1978.

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CHAPTER I

INTRODUCTION

Until recently most of the literature of the social, behavioral, educational, and philosophical sciences was expository, descriptive, or historical in nature, as described in part by Stephen Issac and William B. Michael.¹ As these authors imply, such approaches lack sophistication and complexity of experimental design, statistical manipulation, and analysis, which was not due to a paucity of excellent books or courses of instruction available in the early 1900's, but rather to a defection from experimentation to essay writing. Campbell and Stanley believed that disillusioned rejection of the scientific method was based upon over-optimistic expectations regarding the experimental approach, difficulty of securing adequate data, and the rejection of favored hypotheses.² Most techniques could handle only a few variables at a time. This lack of ability

¹Stephen Issac and William B. Michael, Handbook in Research and Evaluation (San Diego: Robert R. Knapp, 1971), pp. 17-23.

²Donald T. Campbell and Julian C. Stanley, Experimental and Quasi-Experimental Designs for Research (Chicago: Rand McNally and Company, 1963), pp. 1-6.

to account for extraneous and concomitant variables was the greatest fault of the then available statistical procedures. Obviously much of this disenchantment was also due to the lack of the general reader's competency in the allied disciplines of tests and measurements, which was later verified by S. T. Mayo.¹ However, of equal importance were the negative attitudes of educators toward quantitative thinking.

As more and more behavioral scientists became familiar with the scientific method and the differences between descriptive and inferential statistics, the level of research writing improved in quality. This rejuvenation occurred in the 1930's with the influx of governmental funding due to renewed Army and Navy interest in psychological and educational testing for decision making and personnel selection and classification.^{2, 3}

Needless to say, so far as the physical sciences were concerned, experimental designs and analyses were more advanced in stature than those of the socio-behavioral sciences. This disparity was evidenced by the test statistics that were used to verify or reject the null hypotheses that

¹Samuel T. Mayo, Pre-Service Preparation of Teachers in Educational Measurement (United States Department of Health, Education, and Welfare, 1967), pp. 61-62.

²Robert L. Ebel, Essentials of Educational Measurement (Englewood Cliffs, N. J.: Prentice-Hall, 1972), pp. 3-27.

³W. Allen Wallis and Harry V. Roberts, Statistics: A New Approach (Glencoe, Ill.: The Free Press, 1965), pp. 19-20.

were proposed and investigated. An examination of the publications of this period would demonstrate that the work in the physical sciences involved analysis of variance and covariance, factorial designs of various types, factor or discriminant analysis, and various multivariate analyses. Unfortunately, educational and psychological audiences were not yet ready to understand and interpret this kind of advanced research. Most research reports in such a vein were concerned with differences between the means of the groups or the measures of relationships of the groups studied. Of course, present readers recognize these approaches as reports utilizing the t-test statistic¹ and the chi-square test statistic² for contingency or cross-break tabulation, almost entirely with two categories.

As the review of related literature will demonstrate, reader and researcher competency has advanced to the point where the physical and the behavioral scientists are no longer so divergent in knowledge of the components of research design and analysis as they formerly were. In 1938-1939, when Philip J. Rulon was first involved in promulgat-

¹The "t" variable and test statistic are discussed in many basic statistic texts, such as T. H. Wonnacott and R. J. Wonnacott, Introductory Statistics (New York: John Wiley and Sons, 1969). These authors give an historical perspective to the statistic introduced by Gossett, writing under the pseudonym, "Student", later validated by R. A. Fisher.

²Karl Pearson, "Experimental Discussion of the Chi-square Test for Goodness of Fit," Biometrika, 1932, 24, pp. 351-381.

ing his formula for calculating split-half test reliability¹, a grant was received from the World Book Company and the Committee on Scientific Aids to Learning to research the effectiveness of a series of phonographic recordings in terms of knowledge, comprehension, motivation, and attitude changes. Despite the fact that Rulon and his assistants were all well versed in behavioral research techniques and statistics, it was decided to report the results utilizing multiple t-tests. The rationale was that the number of consumers of the monographs would be greater than if analysis of variance or factorial designs had been used.²

The revival of interest in the scientific and statistical approach and the concurrent increased recognition of socio-behavioral science as a science was based upon the evolution of the digital computer - the parent of "the knowledge and information explosion". The first hint that the logic and apparatus of the physical sciences could be applied to the third force - behavioral sciences - was the realization that electro-mechanical devices could be applied to problems other than those of science and engi-

¹Philip J. Rulon, "A Simplified Procedure for Determining the Reliability of a Test by Split-halves," Harvard Educational Review, 1939, 9, pp. 99-103.

²Philip J. Rulon and others, "A Comparison of Phonographic Recordings with Printed Materials," Harvard Educational Review, 1943, 13, a series of 4.

neering. In 1937 Vannevar Bush designed a differential analyzer at M. I. T. capable of negating the criticism that educational and psychological research was based only upon a small number of variables. The differential analyzer could handle 27 variables. This fact opened a broad vista to the speedy solution of technological and engineering problems. It was only a matter of time that digital computers would become refined and generally available to all disciplines. This revitalized the socio-behavioral studies whose potency had been previously restricted by the number of variables that could be considered in that ultimate mechanism - man.

Eventually, software packages for statistical inference and hypothesis testing were developed to the degree that the average student could conduct meaningful research analyses of both simple and complex designs. Most of these packages are concerned with parametric statistics that are well understood and conceptualized. However, the assumptions used in these techniques are often forgotten or ignored. Fortunately, much research has been conducted that demonstrates the degree to which these assumptions, such as independence of the variables and the parametric form of the distribution, can be violated and still result in a robust procedure, especially when the sample size is large and the central limit theorem applies.

Lindgren, for example, states:

Statistical problems involving normal distributions arise in many applications in which a population is adequately (if sometimes only approximately) represented by a normal distribution. The mathematics involved in treating normal populations is especially tractable and therefore highly developed and procedures derived on the assumption of normality frequently turn out to be 'robust' - Their applicability is somewhat insensitive to moderate departures from normality.¹

Many studies of various experimental designs have been made by Raymond O. Collier and Frank B. Baker to compare the power of the F-test under permutation (randomization) versus the normal theory power evaluation.² Although the designs considered were mainly randomized block and repeated measures designs, the findings are applicable to the simple one sample tests used in this study since the F-test statistic is a ratio of two chi-squares. The essential findings were that the normal theory power evaluations only slightly overestimated those arrived at by permutation.

Conversely, nonparametric statistics do not usually make any assumptions except that the random variables be independent, and with the recent revisions such as those made

¹Bernard W. Lindgren, Statistical Theory, 1st ed. (New York: The MacMillan Co., 1960), p. 315.

²Raymond O. Collier, Jr. and Frank B. Baker, "Some Monte Carlo Results on the Power of the F-test Under Permutation in the Simple Randomized Block Design," Bio-metrika, 1966, 53, pp. 199-203; "Analysis of Experimental Designs by Means of Randomization, a Univac 1103 Program," Behavioral Science, 1961, 6, p. 369; and others referenced later in this study.

to the SPSS, SPS, BIOMED, and other packages, behavioral scientists can now compute a variety of nonparametric statistics from One-sample Chi-square tests to Kruskal-Wallis One-way Analysis of Variance.¹

Although nonparametric statistics are often conceived as being "quick and dirty" second cousins to the parametric analogues, they should be considered as very useful tools of the practicing educator and the behaviorist, particularly when the investigator cannot make his measurements on an interval or ratio scale. It was previously noted that parametric statistics also require certain basic assumptions. The conditions which must be satisfied to make a parametric test most powerful are at least these:

1. The observations must be independent.
2. The observations must be drawn from normally distributed populations.
3. The populations have have the same variance (or a known ratio of variances).
4. The variables must have been measured in at least an interval scale.
5. For the F-test of analysis of variance, the means of these normal and homoscedastic populations must have effects that are additive.

¹Norman H. Nie and C. Hadlai Hull, et al, Statistical Package for the Social Sciences (New York: McGraw-Hill, Revision 7, 1977).

When the assumptions are fewer and weaker for a particular model, the conclusions that result can be generalized more, but the test of the null hypothesis is weaker. Siegel resolves this question of test selection by introducing the concept of power efficiency when the sample size available is such that a test with the larger sample is as powerful as another having a smaller sample size.¹ For example, if $N = 30$ in both cases, test A may be more powerful than test B. However, test B may be more powerful with $N = 30$ than is test A with $N = 20$. In this case, the experimenter does not have to choose between broad generality and power if the sample size can be enlarged for test A. This relationship is stated as follows: The power efficiency of test B = $(100)N_a/N_b$ percent. Thus the assumptions and scaling problems of parametric statistics can be avoided if there is a sufficiently large sample. This argument leads to a vital point in this study: What is the minimal sample size that can be used for selected tests involving the chi-square goodness of fit statistic?

Other vital points covered in this study are the chi-square statistic itself and the chi-square probability

¹Sidney Siegel, Nonparametric Statistics for the Behavioral Sciences (New York: McGraw-Hill, 1956), pp. 1-34.

distribution. The following chapter on a review of the literature will give some indication of the abundant use of the chi-square test for contingency tables. However, the primary concern is to compare the Pearsonian chi-square test and the log-linear maximum likelihood chi-square test. The use of the chi-square test is the least complex of the nonparametric tests. The one sample case will be the basis of the general discussion.

In order to obtain sufficient precision, a large number of one sample cases must be used to establish any of the premises made in this study. The Monte Carlo Method of simulation of an empirical experiment will be used to procure random sampling, as explained by J. M. Hammersley and D. C. Handscomb,¹ Jack P. C. Kleijnen,² Y. A. Schreider,³ and I. M. Sobol.⁴ In simulations of this type, researchers most often generate their random variables from a uniform or normal distribution. Donald E. Knuth states that random numbers should not be generated with a method chosen at

¹J. M. Hammersley and D. C. Handscomb, Monte Carlo Methods (Methuen and Company, 1964), pp. 1-42.

²Jack P. C. Kleijnen, Statistical Techniques in Simulation, Part I (New York: Marcel Dekker, Inc., 1974), pp. 1-48.

³Yu. A. Schreider, The Monte Carlo Method, trans. by G. J. Tee (Oxford: Pergamon Press, 1966), pp. 1-91.

⁴I. M. Sobol, The Monte Carlo Method (Chicago: University of Chicago Press, 1974), pp. 7-30.

random; some theory should be used as a basis for the generator.¹ This research purports the use of the gamma distribution of order $\nu/2$, which is Pearson's chi-square distribution with ν degrees of freedom. The rationale for this selection is the concern with one case samples of small size and where there is a great likelihood that such samples would be skewed rather than normal or uniform in distribution. Furthermore, use of random variables generated according to chi-square distributions of varying degrees of freedom and expected frequencies should substantiate Siegel's claim that nonparametric techniques are distribution free.²

Since the one sample case assumes nothing except that the random variables are independent, the concept of robustness - that is, the insensitivity of the violation of assumptions for a statistical procedure - does not enter into this study. As Siegel states:

The literature does not contain much information about the power function of the χ^2 test. Inasmuch as this test is most commonly used when we do not have a clear alternative available, we are usually not in a position to compute the exact power of the test.

When nominal measurement is used or when the data con-

¹Donald E. Knuth, The Art of Computer Programming, vol. 1: Fundamental Algorithms; vol. 2: Seminumerical Algorithms; 7 vols. (Reading: Addison-Wesley Company, 1968 - 1973), 2:5.

²Siegel, p. 3.

sist of frequencies in inherently discrete categories, then the notion of power-efficiency of the χ^2 test is meaningless, for in such cases there is no parametric test that is suitable. If the data are such that a parametric test is available, then the χ^2 test may be wasteful of information.

It should be noted that when $df > 1$, χ^2 tests are insensitive to the effects of order, and thus when a hypothesis takes order into account, χ^2 may not be the best test.¹

The alternative to investigating the power function or the robustness of this test statistic is to analyze the "goodness of fit" for the samples that are generated. This rationale establishes the problem that will be researched.

¹Siegel, p. 47.

STATEMENT OF THE PROBLEM

As the review of related literature will demonstrate, prior research has made several comparisons of nonparametric tests and parametric tests based on the uniform, normal, exponential, and Poisson distributions. Therefore, the first problem is to devise an algorithm for a distribution whose use has been neglected, such as the chi-square distribution. The program should be capable of being easily understood, efficient in terms of micro-seconds necessary to generate the random numbers, and capable of producing these numbers with a high quantitative measure of randomness. The output should be of such form that the two types of chi-square statistics can be easily identified and sorted. The probabilities of each type of statistic should also be printed out concurrently with the cell frequencies that are generated for each iteration. Furthermore, the program should be comprehensive in nature, so that random numbers of good quality can be generated for distributions other than the chi-square if it is later found to be desirable to make comparisons between distributions.

An article by Fienberg about model fitting and goodness of fit tests was the impetus for this research which compares Pearson's chi-square test statistic, henceforth indicated as $X^2(P)$, and the log-linear likelihood

test, henceforth indicated as $X^2(L)$.¹ Fienberg's article points out that for small samples it is not clear whether $X^2(P)$ or $X^2(L)$ is superior. The observations resulting from research by other writers will be covered in the section entitled REVIEW OF RELATED LITERATURE.

For the purpose of this study, two variables will be manipulated: (1) the number of categories from 4 to 8; (2) the expected cell frequencies 3, 5, and 10. Such action will result in one sample cases of sizes 12 to 80. Upon the basis of these one sample cases that are generated, a comparison will be made first to decide the superiority of $X^2(P)$ or $X^2(L)$ for small sample sizes and, in addition, secondarily will permit the following equal area model hypotheses to be evaluated:

$$\begin{array}{ll}
 H_0 = P_1 = P_{01} & H_1 = \text{not } H_0 \\
 P_2 = P_{02} & \\
 \cdot & \cdot \\
 \cdot & \cdot \\
 \cdot & \cdot \\
 P_k = P_{0k} &
 \end{array}$$

Furthermore, an ancillary third investigation will be to express $X^2(P)$ and $X^2(L)$ as a function of the expected cell frequencies, $E(x)$, the degrees of freedom and α regions.

¹Stephen E. Fienberg, "The Analysis of Multidimensional Contingency Tables," Ecology, 1970, 51, pp. 419-433.

Since tests for goodness of fit are concerned with the probabilities in the upper tail of the distribution, this is the main criterion under which $X^2(P)$ and $X^2(L)$ will be compared. Where cumulative multinomial probabilities have been published for some of the small samples that will be generated, this information will also be given in order to make a more comprehensive decision about the errors involved in the approximations that are most commonly used.

Although the basic premise of this dissertation is that the parent populations are skewed, comparisons resulting from Gaussian random number generations will be made since great disparity is apparent for the same sample sizes, degrees of freedom, and expected values used in the chi-square distributions.

CHAPTER II

REVIEW OF RELATED LITERATURE

INTRODUCTION

In an important paper, Tukey posed many unsolved problems in experimental statistics, particularly in the area of client and consumer relationships with respect to complexity, inference, and assumptions.¹ While advising that the complexity of experimental statistics will clearly increase, he stated that the methodology should be tailored to the needs of the user. He writes:

'What should be done' is almost always more important than 'what can be done exactly'. Hence new developments in experimental statistics are more likely to come in the form of approximate methods than in the form of exact ones.

This is of interest, since in this study various one sample problems will be manipulated using Pearson's approximation to the chi-square distribution, the maximum likelihood ratio, and the exact multinomial probabilities.

Tukey goes on to state:

In every statistical area, we almost certainly need methods admitting one more nuisance parameter, methods of one higher level of robustness and de-parametrization, methods with both of these desiderata. Here we may turn

¹John W. Tukey, "Unsolved Problems of Experimental Statistics," Journal of the American Statistical Association, 1954, 49, pp. 707-731.

the carpet back to see the dirt - it is a large carpet trying to cover much dirt. We have a reasonably wide variety of procedures for analyzing counted data which assume pure binomial variation - contingency tables, chi-square, and ω^2 goodness of fit tests, Kolmogorov-Smirnov bounds on the population distribution and so on.

The crux of this study emphasizes some of Tukey's problems and questions, such as: "Statistics must continually study the behavior of its techniques when their conventional assumptions are not true." For example, many techniques assume homogeneity of variance, utilize a normality assumption almost exclusively as a means of predicting the stability of estimated variance, and discuss the efficiency of estimation assuming an underlying normal distribution. What about those experiments that do not meet these assumptions?

Tukey also presents some provocative questions that are related to this current study:

What are we trying to do with goodness of fit tests? (Surely not to test whether the model fits exactly, since we know that no model fits exactly!)

Why isn't someone writing a book on one and two sample techniques?¹

Tukey's questions are now easier to answer. At the same time that Tukey was presenting his position, Cochran was espousing on the X^2 test of goodness of fit.² As a search of the literature would demonstrate, this problem

¹Tukey, p. 721.

²William G. Cochran, "The X^2 Test of Goodness of Fit," Annals of Mathematical Statistics, 1952, 23, pp. 315-345.

has been investigated in almost all aspects from 1900 when Pearson invented the test until the present. Formerly, the users tended to be more restrictive in their selection of α levels, subject to selecting rigid cut-off points for hypothesis testing, overly conservative, and selective in the choice of application or model fitting.

Since the 1950's, many standard texts have included chapters on nonparametric statistics and one and two sample techniques. Siegel's text is often utilized in this area, as referenced in the first chapter of this paper. For the student and user of statistical theory, there is Hays¹, as well as Walsh.² For the more advanced, Lindgren³, Mood, Graybill and Boes⁴, and also Johnson and Kotz⁵ are suggested.

¹William L. Hays, Statistics for the Social Sciences, 2d ed. (New York: Holt, Rinehart and Winston, Inc., 1973).

²John E. Walsh, Handbook of Nonparametric Statistics (Princeton, N. J.: D. Van Nostrand Co., Inc., 1962).

³Bernard W. Lindgren, Statistical Theory, 1st ed. (New York: The MacMillan Co., 1960).

⁴Alexander M. Mood, Franklin A. Graybill, and Duane C. Boes, Introduction to the Theory of Statistics, 3rd ed. (New York: McGraw-Hill, 1974).

⁵Norman L. Johnson and Samuel Kotz, Distributions in Statistics, 4 vols. (New York: John Wiley and Sons, 1970).

Many journal articles and dissertations have concerned themselves with the X^2 test, particularly with respect to contingency tables, categorization, expected cell and sample size, substitutes for the Pearsonian X^2 statistic, model fitting, and the like. Therefore, because there is an abundance of publications in this area and because they pertain to and influence the use of the X^2 statistic in the one sample case, these articles will be reviewed in succeeding sections. Also, sections will be presented on the chi-square and the multinomial distributions, recent work on one sample cases, and the Monte Carlo experimental methodology.

LITERATURE ON DISTRIBUTION THEORY

The four-volume series by Johnson and Kotz on Distributions in Statistics, referred to in the previous section, seems destined to be an authoritative and definitive work in the statistical field and can be expected to become a standard reference, just as the articles of Cochran¹ and those of Lewis and Burke² on the chi-square test have become. Needless to say, the replies to the Lewis and Burke criticisms by Edwards³, Pastore⁴, and Peters⁵, and the recapitulation of these replies by Lewis and Burke⁶ form a part of this body of knowledge on the chi-square test methodology.

¹William G. Cochran, "The X^2 Test of Goodness of Fit," pp. 315-345.

²Don Lewis and C. J. Burke, "The Use and Misuse of the Chi-square Test," Psychological Bulletin, 1949, 46, pp. 433-489.

³A. L. Edwards, "On the Use and Misuse of the Chi-square Test - The Case of the 2 x 2 Contingency Table," Psychological Bulletin, 1950, 47, pp. 341-346.

⁴N. Pastore, "Some Comments on 'The Use and Misuse of the Chi-square Test'," Psychological Bulletin, 1950, 47, pp. 338-340.

⁵Charles C. Peters, "The Misuse of Chi-square - A Reply to Lewis and Burke," Psychological Bulletin, 1950, 47, pp. 331-337.

⁶Don Lewis and C. J. Burke, "Further Discussion of the Use and Misuse of the Chi-square Test," Psychological Bulletin, 1950, 47, pp. 347-355.

Before proceeding to discuss current literature about the one sample case, it would seem advantageous to review statistical distributions and the chi-square applications that are discussed historically and to consider the trends of current investigations. After publication, Kotz and Johnson made a subjective historical appraisal of over 2500 papers in the literature when they prepared their series on "Distributions in Statistics".¹

They pointed out that originally distributions arose in connection with real-life situations and that in the latter part of the 19th century and early part of the 20th century, the studies were divided into two categories. One subdivision was the determination of sampling distributions based on variables having established distributions. The other was the study of systems of distributions with reference to use in model fitting. While the first of these has displayed prolonged interest that still continues in more and more complexity, model fitting is presently attracting revived interest. The works of Fienberg, Goodman, and Haberman, which are reviewed later, evoked this present investigation, the algorithm, and the Monte Carlo experiment.

¹Samuel Kotz and Norman L. Johnson, "Statistical Distributions: A Survey of the Literature, Trends, and Prospects," American Statistician, 1973, 27, pp. 15-17.

Kotz and Johnson state that during the period from 1925 to 1939 a number of new distributions were derived as variants of classical distributions and that this period was followed by a decade of interest in establishment of tables, approximations, and frequency moment estimators. From 1950 to 1959 there was a considerable interest in "robustness". This area is still under investigation as statisticians are displaying increased concern with multivariate analysis and maximum likelihood estimation. The value of this study of the chi-square statistic is supported by the number of articles that Kotz and Johnson have tabulated in the 1960-1969 period. In that period, references to the gamma, exponential, and non-central X^2 distributions even exceed those of the normal distribution. A multidimensional study by McNamee that is of particular pertinency to this study is reviewed later with respect to sample size and to expected and observed cell size.¹

Quoting from an early journal article by Lancaster,² Johnson and Kotz place Pearson's X^2 approximation in a historical perspective that is often overlooked by all but

¹Raymond Joseph McNamee, "Robustness of Homogeneity Tests in Parallelepiped Contingency Tables" (Ph.D. Dissertation, Loyola University of Chicago, 1973), pp. 1-134.

²H. O. Lancaster, "Forerunners of the Pearson X^2 ," Australian Journal of Statistics, 1966, 8, pp. 117-126.

applied mathematical statisticians. Briefly, Lancaster states:

Manipulations leading to a chi-square distribution or something much like it, have a history going back well before Karl Pearson's classic 1900 paper, in which the chi-square distribution was used to approximate the null distribution of the chi-square statistic for goodness of fit.

Descriptions are given of a Bayesian derivation by Laplace of a gamma distribution for a precision parameter in a very special case; of a somewhat similar manipulation by Bienayme (1838) in a trinomial context; of Bienayme's asymptotic development (1852) of the gamma distribution for the sum of squared errors (not residuals) in the linear hypothesis context; of related work by Ellis (1844); and of Helmert's well known derivations (1875-1876) of the chi-square distributions for the (normed) sums of squared errors and residuals in the normal linear hypothesis case.

The gamma distribution derived by Laplace was the posterior distribution of the precision constant ($h = \frac{1}{2} \sigma^{-2}$) that causes the area of the Gaussian probability function to equal one, given the values of n independent normal variables with zero mean and standard deviation σ (assuming a uniform prior distribution for h). The origin of the Bayesian approach by Laplace was undoubtedly encouraged by Thomas Bayes' essay, published posthumously in 1763.¹ Where Bayes excelled in logical penetration, using the

¹Sir Ronald A. Fisher, Statistical Methods for Research Workers (New York: Hafner Publishing Co., 1958), 13th ed., pp. 20-21 citing Thomas Bayes, "An Essay Toward Solving a Problem in the Doctrine of Chances," Philosophical Transactions, 1763, liii, pp. 370-418.

theory of probability as an instrument of inductive reasoning, Laplace was a master of the analytical technique. He introduced the principle of inverse probability where the deduction of inferences respecting populations resulted from observations respecting samples. Fisher was adverse to this technique.

Similar work by Bienayme obtained the continuous χ^2 distribution as the limiting distribution of the discrete random variable $\sum_{i=1}^k (N_i - np_i)^2 (np_i)^{-1}$ when $(N_1 \dots N_k)$ have a joint multinomial distribution with parameters $n, p_1, p_2 \dots, p_k$. This will be discussed later in a following section as applied to this paper.

Laplace's work on the normal distribution was extended by Poisson, Bienayme, and Todhunter. Later, Sheppard studied the theme advanced by Bienayme of the distribution of a linear form in the class frequencies of a multinomial distribution and considered possible tests of goodness of fit for the multinomial distribution. As a test of goodness of fit, Sheppard proposed to work out the value of the difference of the observed frequency from the expected frequency for each cell of a contingency table and to see how often it exceeded its probable error. The similarity of this approach to that of Pearson is obvious, and he obtained his solution based upon the variance-covariance matrix rather than the matrix of a generalized contingency table proposed by Sheppard. Many others, such as Bravais, Schols,

and Edgeworth, developed the study along the lines of the joint multivariate normal distribution.¹ However, this study is restricted to the approximations to the multinomial distribution, and succeeding sections will be essentially concerned with these relationships and problems.

¹H. O. Lancaster, The Chi-squared Distribution (New York: John Wiley & Sons, 1969), pp. 2-3.

CHI-SQUARE DISTRIBUTIONS AND STATISTICS

A simplified explanation of the chi-square distribution may make later discussions of the distribution easier for the uninitiated reader to understand. Such explanations are presented in many basic textbooks, and a comprehensive presentation has been made by Glass and Stanley.¹ In order to construct the distribution whose mathematical curve was derived by Pearson in 1900, it is necessary to assume a huge population of scores that are essentially normally distributed with mean 0 and standard deviation 1. One then selects n scores X_n at random and calculates the standard score for each of them. The next step is to square each z score and sum them as follows:

$$z_1^2 + z_2^2 + \dots + z_n^2 = \chi^2.$$

Having selected many thousands of sets of X_n , one can then calculate the corresponding χ_n^2 and construct a frequency polygon of the values so obtained. If this frequency polygon is smoothed after many thousand values of χ_n^2 have been recorded and if the scale of the ordinate is adjusted so that the area under the curve is 1, the graph of the chi-square distribution with

¹Gene V. Glass and Julian C. Stanley, Statistical Methods in Education and Philosophy (Englewood Cliffs, N. J.: Prentice-Hall, 1970), pp. 228-232.

n degrees of freedom will be obtained. The area under the curve is set equal to 1 so that the distribution is a probability distribution, approximately the exact continuous multinomial distribution.

The χ^2 distribution is the basis of a test statistic which is used for many purposes but is essentially used for the chi-square test of goodness of fit. As Cochran states:¹

In the standard applications of the test, the n observations in a random sample from a population are classified into k mutually exclusive classes. There is some theory or null hypothesis which gives the probability p_i that an observation falls into the ith class ($i = 1, 2, \dots, k$). Sometimes the p_i are completely specified by the theory as known numbers, and sometimes they are less completely specified as known functions of one or more parameters $\alpha_1, \alpha_2, \dots$ whose actual values are unknown. The quantities $m_i = np_i$ are called the expected numbers, where

$$\sum_{i=1}^k p_i = 1 \quad \sum_{i=1}^k m_i = n$$

The starting point in the theory is the joint frequency distribution of the observed numbers x falling in the respective classes. If the theory is correct, these observed numbers follow a multinomial distribution with p as probabilities.

The test criterion for the null hypothesis that the theory is correct, proposed by Pearson, is:

$$\chi^2 = \sum_{i=1}^k \frac{(x_i - m_i)^2}{m_i} = \sum_{i=1}^k \frac{x_i^2}{m_i} - n$$

A more common notation is:

$$\chi^2 = \sum_{i=1}^k \frac{(n_i - NP_i)^2}{NP_i} \quad \text{or} \quad \sum_x \frac{(O_x - E_x)^2}{E_x} = \chi^2(P) = \sum_{\text{All cells}} \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

where $p_i = \phi_i$; $\sum n = N$ (the total sample size); and $E(x) = m$.

¹William G. Cochran, "The Chi-square Test of Goodness of Fit," p. 315.

Similarly, the multinomial probability is expressed as:

$$P(n_1, n_2, \dots, n_k) = \frac{N!}{n_1! n_2! \dots n_k!} p_1^{n_1} p_2^{n_2} \dots p_k^{n_k} \quad 1$$

There is a different chi-square distribution for each integer value of n (1, 2, 3, . . .). The properties of the curve depend upon the value of n , usually indicated as ν , the degrees of freedom. Glass and Stanley provide a partial description of the family of chi-square distributions:

1. The mean of a chi-square distribution with ν degrees of freedom is equal to ν .
2. The mode of χ^2 is at the point $\nu - 2$ for $\nu = 2$ or greater.
3. The standard deviation of χ^2 is $\sqrt{2\nu}$.
4. The skewness of χ^2 is $\sqrt{8/\nu}$. Hence every chi-square distribution is positively skewed, but the asymmetry becomes very slight for large degrees of freedom.
5. As the degrees of freedom become large, χ^2 approaches more nearly a normal distribution with mean and standard deviation of $\sqrt{2\nu}$.

An important theorem that will be emphasized in the review of several journal articles that follows is:

If $\chi_{\nu_1}^2$ has a chi-square distribution with ν df. and if $\chi_{\nu_2}^2$ has a chi-square distribution with ν_2 df. and is independent of $\chi_{\nu_1}^2$, then $\chi_{\nu_1}^2 + \chi_{\nu_2}^2$ has a chi-square

¹In later sections $X^2(P)$ will represent the Pearsonian chi-square, $X^2(L)$ the likelihood ratio chi-square and (M) the multinomial probability.

²Glass and Stanley, pp. 231-232.

distribution with $U_1 + U_2$ df. This theorem is used in model fitting, partitioning, analysis of association, and other methodologies.

The importance of the chi-square variate is particularly evident when one considers that the t , χ^2 , and F distributions are all based on the normal distribution and are interrelated as:

$$t_v^2 = \frac{z^2}{\chi_v^2/v} = \frac{\chi_1^2/1}{\chi_v^2/v} = F_{1,v} \quad \text{and} \quad F_{v,\infty} = \frac{\chi_v^2}{v} .$$

APPLICATIONS AND CRITICISMS OF
THE X^2 STATISTIC

Most of the early relevant literature has to do with the chi-square test and degrees of freedom, sample size, the misuse of the test, and possible substitutes for the statistic. As Cochran points out, the most common of all uses of the X^2 test is for the 2×2 contingency table, and a review of this $r \times c$ table is indicative of the errors and conflicts that have prevailed for many years. For example, in the 2×2 tables, Pearson attributed 3 degrees of freedom to X^2 , whereas it should receive only 1, $(r-1)(c-1)$.¹ Pearson made this correction at about the same time that Fisher was trying to verify Pearson's work using the multinomial as an exact test.² Dissonance of this type pervades the literature on chi-square and depends upon the kind of tables being considered, that is, whether one is considering a random sample from only one population, or if two populations are being compared, or if the two populations have fixed marginal totals in repeated sampling. This complexity increases as the dimensions of the contingency tables in-

¹Cochran, "X² Test," p. 319, and Lancaster, "The X² Distribution," pp. 170-178.

²Fisher, p. 96.

crease, as demonstrated in the dissertation of R. J. McNamee that has been previously mentioned.

The X^2 test and distribution is used in many experimental situations; however, the major applications are in testing the goodness of fit, independence, and homogeneity. Although this paper is concerned with a basic example of goodness of fit, the one sample case, many of the problems and concepts of the other applications are pertinent to this research. The theoretical frequencies and the corresponding sample size is a major consideration of most of the writers already cited. Other concepts are the normal approximation to the binomial, hypergeometric, and Poisson distribution, maximum likelihood, minimum X^2 , moments, and cumulants.

Lewis and Burke discuss at great length the rule of thumb of having 5 or 10 as the expected cell frequencies. They state:¹

Many users and would-be users of the chi-square test gain erroneous impressions from what they read about limitations on the size of theoretical frequencies. A textbook says that frequencies of less than 10 are to be avoided. This statement is often interpreted to mean not that 10 is a limiting value to be exceeded whenever possible, but that 10 is a value around which the various theoretical frequencies may fall; and if an occasional frequency happens to be as low as 4 or 5, that is all right because other frequencies will be larger than 10 and everything will average out in the end. A textbook that gives 5 as the suggested minimum tends to encourage the retention of impossibly small theoretical frequencies. And so does a text

¹Lewis and Burke, "Use and Misuse of the X^2 Test," pp. 486-487.

which states, in effect, that Yates' correction for continuity should be applied if the cell frequencies are 5 or less and precision is desired. This implies not only that frequencies of less than 5 are quite acceptable, but also that Yates' correction is an antidote for small frequencies. Both implications are fallacious.

Yule and Kendall state:¹

In the first place, N must be reasonably large. . . It is difficult to say exactly what constitutes largeness, but as an arbitrary figure we may say that N should be at least 50, however few the number of cells.

No theoretical cell frequency should be small. Here again it is hard to say what constitutes smallness, but 5 should be regarded as the very minimum, and 10 is better.

Hoel gives 5 as the recommended minimal value of the theoretical or expected frequency, but he emphasizes the importance of having a fairly large value of the total N by stating that, if the number of categories or cells is less than 5, the individual expected values should be larger than 5.² On the other hand, Cramer recommends a minimal value of 10 and states that, if the expected values, even after grouping, are less than 10, the chi-square should not be applied.³

Cochran recognizes these differences in opinion,

¹G. U. Yule and M. G. Kendall, An Introduction to the Theory of Statistics, 12th ed. (London: Griffin, 1940), p. 422.

²P. G. Hoel, Introduction to Mathematical Statistics (New York: Wiley & Sons, 1947), p.191.

³H. Cramer, Mathematical Methods of Statistics (Princeton: Princeton University Press, 1946), p. 420.

but he states that the value of the minimal expectation also depends upon the application of the test and the level of significance that has been selected as the criterion. For example, in the goodness of fit tests of bell-shaped curves such as the normal distribution, the expectations in the tails are small, and there is little disturbance to the 5% level when a single expectation is as low as $1/2$. Cochran suggests using Fisher's exact multinomial test for 2×2 contingency tables in samples up to size 30. In tests in which all expectations are small, Cochran refers to the results of Neyman and Pearson, which support the contention that the tabular X^2 is tolerably accurate, provided that all expectations are at least 2. He also imposes the constraint that the degrees of freedom are less than 15. If the degrees of freedom exceed 60, Cochran suggests using the normal approximation to the exact distribution using Haldane's expressions for the mean and variance.

Most educational research does not have an excessive number of degrees of freedom or a large sample size and, since this paper is concerned with the nonparametric one sample case, Siegel's position on small expected frequencies should be considered. When there are only 2 categories, k , each expected frequency should be at least 5. When k categories are greater than 2, the chi-square test for the one sample case should not be used when more than 20 percent of

the expected frequencies are smaller than 5 or when any expected frequency is smaller than 1. Expected frequencies sometimes can be increased by combining adjacent categories, but only if these resulting categories are meaningful. If one starts with but two categories or has but two categories after combining and has an expected frequency of less than 5, then the binomial test should be used rather than the chi-square test.

The modification of the rule of 5 is made in this study since McNamee found that the chi-square test for first order interaction is quite robust as far as sample size is concerned, when the expected frequency for each cell is as small as 3. He also found that if the cells have a minimum value of 1, the chi-square for second order interaction is within the limits of error for the 400 iterations used in his study.¹ This lower value is not used in this study since it is designed around the one sample case.

It should be obvious that the goodness of fit test is the primary emphasis of this monograph and a simple definition is in order. Goodness of fit tests are used to test the hypothesis that nature is in a certain specified state when the alternative hypothesis is the general one that nature is not in that state. As previously mentioned,

¹McNamee, pp. 104-105.

the χ^2 test is most generally used. As cited by Lancaster:

In the series, Mathematical Contributions to the Theory of Evolution, Karl Pearson introduced a number of theoretical statistical distributions, which were new to statistics, and among which the Type III is, after an appropriate choice of scale and origin, the distribution of χ^2 or alternatively the gamma-distribution. Given any particular set of empirical data, it became necessary to distinguish those distributions which fitted it closely from those which did not. Pearson realised that the normal curve had too often been accepted uncritically as fitting empirical data.

Pearson had been much concerned with generalizing the univariate normal distribution to the general normal correlation; so that, it appeared natural for him to provide a normal approximation to the multinomial distribution. . . The symbol, χ^2 , was first introduced by Pearson (1896), where it was written in place of $x^T R^{-1} x$ for brevity.

Pearson's contributions to statistical theory were numerous but, perhaps, the greatest of them was the χ^2 test of goodness of fit, which has remained one of the most useful of all statistical tests. Pearson (1900a) states 'the object of this paper is to investigate a criterion of the probability on any theory of an observed system of errors, and to apply it to the determination of goodness of fit in the case of frequency curves'.¹

It is self evident that the statistic can be applied to studies of parent populations other than that of the normal distribution. Various texts, such as those of Fisher, Lancaster, Lindgren and others, demonstrate the use of the chi-square test for the Poisson, exponential, hypergeometric and other distributions particularly in contrast to the estimates derived from maximum likelihood, likelihood ratio,

¹Lancaster, "The Chi-Square Distribution," p. 3.

moment and cumulant generation, and other tests.

The statistics that are derived from the sum of the powers are based upon the concepts of moments and cumulants. The first moment, m' , is the arithmetic mean and is usually written \bar{x} , and it follows that the moments of the higher powers of a random variable or of a distribution are the expectations of the powers of the random variable which has the given distribution. If X is a random variable, the r^{th} moment of X , usually denoted by μ'_r , is defined as $\mu'_r = E[X^r]$.¹ It follows that the second moment about the mean is the variance, the third moment is a measure of the skewness, and the fourth moment is the kurtosis.

The moments are properties resulting from a moment generating function which is defined by letting X be a random variable with density $f_x(\cdot)$. The expected value of e^{tx} is defined as the generating function if the expected value exists for every value of t in some interval $-h < t < h$; $h > 0$. The logarithm of the moment generating function is defined as the cumulant function of X . The r^{th} cumulant, denoted by k_r , is the coefficient of $t^r/r!$ in the Taylor

¹Mood, p. 73.

series expansion of the cumulant generating function.¹

This discussion of moments and cumulants is not absolutely necessary to this current study except insofar as it may be required in the explanation of the results and because of its reference importance in the literature about the chi-square distribution.

A formal presentation of the maximum likelihood principle is beyond the scope of this paper, and is mentioned here briefly since it is involved in one of the statistics that is used in the calculations resulting from the various sets of data that are generated according to selected Type III gamma distributions. Furthermore, the maximum likelihood estimator method is the basis of rigorous proofs used by mathematical statisticians since these estimators meet the requirement that they are unbiased, consistent, efficient, and sufficient. Fisher makes a point of distinguishing between probability and the mathematical quantity that is appropriate for making statistical inferences among different populations. Lindgren explains maximum likelihood as follows:

Suppose first that the population of interest is discrete, so that it is meaningful to speak of the probability that $\mathbf{X}=\mathbf{x}$, where \mathbf{X} denotes a sample (X_1, \dots, X_n)

¹Mood, p. 80.

and x a possible realization (x_1, \dots, x_n) . This probability that $\mathbf{X}=x$ depends on x , of course, but it also depends on the state of nature θ which governs. As a function of θ for given x , it is called the likelihood function:

$$L(\theta) = P_0(\mathbf{X} = x).$$

The principle of maximum likelihood requires first that a value $\theta = \hat{\theta}$ be found which furnishes the 'best explanation' of a given result that is observed. That is, holding x fixed, we allow θ to wander over the various possible states of nature and select one, $\hat{\theta}$, which maximizes the probability $L(\theta)$ of obtaining the result actually observed. Then, having found a state $\hat{\theta}$ that best explains the observed result x , we take the action that would be best if $\hat{\theta}$ really were the true state. This best action for a given state of nature is naturally determined by the loss function (or, equivalently, by the regret function) as that action which minimizes the loss (or regret).

Because the best explanation of a given x depends on that x , held fixed during the maximization of $L(\theta)$, the minimizing θ depends on x . It defines a function of the observations - a statistic. The rule of taking the action that minimizes $l(\theta, a)$ is then a decision function, an assignment of an action to each possible outcome of the sampling experiment.¹

A goodness of fit test that evolves from the above principle is that of the likelihood ratio test. For a one sample case when the hypothesis is that nature is in a certain specified state and the alternative hypothesis is that nature is not in that state, the null hypothesis is:

$$H_0: p_1 = \pi_1, \dots \quad \text{and} \quad p_k = \pi_k$$

where π_1, \dots, π_k are specified numbers on the interval $[0, 1]$ whose sum is 1 and k parameters p_1, \dots, p_k are restricted by the condition that their sum equals 1.

¹Lindgren, pp. 188-189.

The basis for testing H_0 is a random sample of n observations on X with the joint probability function

$$f(x;p) = p_1^{f_1} p_2^{f_2} \dots p_k^{f_k}$$

which depends on the observations (X_1, \dots, X_n) only through the corresponding frequencies. The likelihood ratio test for $p = \pi$ against $p \neq \pi$ is the ratio of $L(\pi)$, the maximum on the simple hypothesis that $p = \pi$, and $L(\hat{p})$, the maximum on $H_0 + H_1$, where $p = \hat{p}$ and $\hat{p}_i = f_i/n$. It is expressed as

$$\lambda = \frac{L(n)}{L(\hat{p})} = \frac{\pi_1^{f_1} \dots \pi_k^{f_k}}{(f_1/n)^{f_1} \dots (f_k/n)^{f_k}} = n^n \prod_{i=1}^k \left(\frac{\pi_i}{f_i} \right)^{f_i} \quad 1$$

The null hypothesis is rejected for $\lambda < \text{constant}$, the value of which is determined by the α selected. Since the calculation of the distribution is prolonged and is based upon a multiplication product, the logarithm is used for the large sample distribution $-2 \log \lambda$. This distribution is asymptotically chi-square with $k - 1$ degrees of freedom and the rejection limit is the $100(1 - \alpha)$ th percentile of that distribution. The similarity of the above statistic and the log-linear likelihood test, $X^2(L) = 2 \sum (\text{observed}) \log (\text{observed/expected})$, which is investigated in this Monte Carlo study should be easily and readily recognized.²

¹Lindgren, p. 295.

²Journal articles by Feinberg, Goodman, and Haberman that utilize $X^2(L)$ as a test statistic are referenced in the Bibliography.

A brief presentation of tests that are competitive alternatives to X^2 is made because of their recurrence in the literature. The method of maximum likelihood consists in multiplying the log of the number expected in each category by the number observed, summing for all categories and finding the value of θ for which the sum is the positive maximum solution of the differentiation of the resulting quadratic equation. The method of minimum X^2 is arrived at by differentiating for the smallest positive solution resulting from the comparison of observed with expected frequencies and calculating the discrepancy, X^2 , between them.¹

The ω^2 test has been constructed and developed by Cramer', von Mies, and Smirnov in order to avoid the grouping of continuous data that is necessary with X^2 and still resembles the X^2 test in that the tests are not directed against any specific alternative hypothesis. Neyman's smooth test also postulates that the cumulative frequency (assumed continuous) is known from the null hypothesis. If the frequency functions which are continuous and depart in a gradual and regular manner from the null hypothesis, the variates will not follow a rectangular distribution in the interval (0,1) whereas these variates would follow a rectangular distribution when the deviations from the null

¹Fisher, pp. 304-305, and Lancaster, pp. 136-139.

hypothesis are erratic or discontinuous. The X^2 test is not directed specifically at either class.¹ As it can be noted, tests other than X^2 often have certain restrictions as to their application or information necessary to their use. This factor coupled with the overwhelming familiarity of users of statistical methods and the consuming audience with the X^2 results in application of this test statistic for all but very specific problems.

¹Cochran, pp. 335-339.

LITERATURE BASIC TO THE PROBLEM

ERIC and Psychological Abstract searches reveal that there has been a paucity of research regarding the chi-square test for the one sample case, particularly for samples randomly drawn from distributions that are not normal in form. However, it is obvious that many other chi-square investigations are applicable to the problem that is herein proposed.

Guenther has been actively involved in chi-square tests for hypotheses concerning multinomial probabilities, the power and sample size for such tests.¹ Three cases are presented: (1) the hypothesis which specifies all the multinomial probabilities, (2) the hypothesis of independence, and (3) the hypothesis of homogeneity. He points out that if these hypotheses are false, the statistic has approximately a noncentral chi-square distribution with the same degrees of freedom but also a noncentrality parameter λ . Haynam, Govindarajulu, and Leone have prepared tables of the non-central chi-square distribution designed for easy solution

¹William C. Guenther, "Power and Sample Size for Approximate Chi-Square Tests," American Statistician, 1977, 31, pp. 83-85.

to these power problems.¹

The results of these works emphasize the large sample size necessary for the tests to have appreciable power. An article by Meng and Chapman further reports on the noncentrality parameter for $r \times c$ contingency tables.² Again, the power of these tests was approximated on a large sample basis. The concept of noncentrality is introduced here only insofar as it may be necessary to explain some of the results of this study should the null hypothesis be rejected for the small sample sizes that are used.

Categorization for this experiment has been explained in the chapter on the statement of the problem and the means by which the results from the random numbers generated by the Monte Carlo study which is used and is explained in the next chapter concerning the design. However, since questions arise as to the effectiveness of using equal area or linear score models, Kerlinger's rules of categorization are of interest at this point. Categorization is another word for partitioning, which is referred to in many articles that use

¹G. E. Haynam, Z. Govindarajulu, and F. C. Leone, "Tables of the Cumulative Non-Central Chi-Square Distribution," Selected Tables in Mathematical Statistics, Vol. 1, eds. H. L. Harter and D. B. Owens, (Chicago: Markham Publishing Co., 1970).

²Rosa C. Meng and Douglas G. Chapman, "The Power of Chi Square Tests for Contingency Tables," Journal of the American Statistical Association, 1966, 61, pp. 965-975.

analysis of variance or multiple contingency tables as a means of methodology. Emphasizing that the first step in any analysis is categorization, Kerlinger lists five rules of categorization:

1. Categories are set up according to the research problem and purpose.
2. The categories are exhaustive.
3. The categories are mutually exclusive and independent.
4. Each category (variable) is derived from one classification principle.
5. Any categorization scheme must be on one level of discourse.¹

Kittelton and Roscoe studied the power and robustness of the chi-square and Kolmogorov statistics with both the linear score scale and equal area models.² They found that the traditional procedure for testing goodness of fit to normal used a linear score scale model in which the chi-square approximation of the multinomial cell limits were defined by dividing a standard score scale into equal parts. The criticism of this method is that the expected frequencies in the tails of the distribution tend to be very small with samples of reasonable size, such as $n = 100$ or less.

¹Fred N. Kerlinger, Foundations of Behavioral Research, 2d ed. (New York: Holt, Rinehart & Winston, 1973), pp. 137-143.

²Howard M. Kittelson and John T. Roscoe, "An Empirical Comparison of Four Chi-Square and Kolmogorov Models for Testing Goodness of Fit to Normal" (paper presented to AERA, Chicago, 1972), pp. 1-8.

When the sample sizes are small, as in this experiment, an alternative chi-square model has been suggested by many authors. In these cases, the cell limits are defined by dividing the area under the curve into equal parts - an equal area model. Not only does this model overcome the problem of small expected frequencies in the tails, it also increases the power of the chi-square approximation by having uniform expected frequencies in each division. Mann and Wald investigated the power of the chi-square test with regard to the distance of the observed and expected distribution and found that the optimum power for the goodness of fit test for continuous distribution is achieved when the expected frequencies are equal.¹ Williams elaborated on their results together with useful numerical tabulations.²

Watson suggested the equal area model for the chi-square test of goodness of fit but also suggested that the number of cells should be at least ten.³ Kempthorne also

¹H. B. Mann and A. Wald, "On the Choice of the Number of Class Intervals in the Application of the Chi Square Test," Annals of Mathematical Statistics, 1942, 13, pp. 306-317.

²C. A. Williams, Jr., "On the Choice of the Number and Width of Classes for the Chi Square Test of Goodness of Fit," Journal of the American Statistical Association, 1950, 45, pp. 77-86.

³G. S. Watson, "The Chi-Square Goodness of Fit Test for Normal Distributions," Biometrika, 1957, 44, pp. 336-348.

avored the equal area model, but his findings were based in part upon a Monte Carlo study when the number of cells (k) was set equal to the sample size (n).¹ An extensive empirical study by Roscoe and Byars demonstrated an acceptable approximation with expectancies as small as one when testing goodness of fit to uniform.² They found that the approximation is not quite so good with uniform hypotheses, but did not examine goodness of fit to normal. The main contribution was that the average expected frequencies had to be increased for lower α levels for uniform distributions and also for those distributions that varied from the uniform; otherwise, the approximations tended to be liberal.

Kittelson and Roscoe randomly generated ten thousand uniformly distributed sets of samples for each combination of sample size and number of cells under study. Sample sizes were 10, 20, 30, and 50. The cell sizes were set equal to 6, 10, and 20 with the number of cells also being set equal to 50 for samples of size 50. The null

¹Kempthorne, "The Classical Problem of Inference - Goodness of Fit," Fifth Berkeley Symposium on Mathematical Statistics and Probability, 1967, 1, pp. 235-249.

²J. T. Roscoe and J. A. Byars, "An Investigation of the Restraints with Respect to Sample Size Commonly Imposed on the Use of the Chi-Square Statistic," Journal of the American Statistical Association, 1971, 66, pp. 755-759.

hypothesis was sampling from normal distribution and testing against the normal distribution. The false hypothesis was sampling from uniform distribution and testing from normality. The chi-square equal area models proved to be superior to the chi-square linear score model and to both of the Kolmogorov tests. The chi-square equal area model was erratic with samples of size 10; however, an acceptable approximation was achieved with all other sample sizes ($n = 20, 30, \text{ and } 50$).

Whitney made several comparisons of various non-parametric tests and parametric tests based on the normal distribution and non-normal alternatives, rectangular, double rectangular, triple rectangular, and Cauchy distributions.¹ With sample sizes of 5, 10, and 50, and an underlying normal distribution, the normal approximation to the binomial showed greater power than the "t" test, and the "t" test was more powerful than the sign test. Under the assumption of a rectangular distribution, the normal test was considerably better than the sign test. With a double rectangular distribution, the normal test has high power while the sign test is of little value when there are

¹D. R. Whitney, "A Comparison of the Power of Non-Parametric Tests and Tests Based on the Normal Distribution Under Non-Normal Alternatives" (Ph.D. dissertation, Ohio State University, 1948).

only small increases in the mean but has greater power when the increases are large.

When Whitney selected a triple rectangular distribution with a density function that was highly peaked and had a fair amount in the tails, the sign test had more power than the normal or "t" tests. However, if the distribution was flattened, the normal or "t" tests were more powerful. With a Cauchy distribution, Whitney found the sign test consistent, and the normal or "t" tests were inconsistent. In his summary, Whitney states:

Alternatives in which the probability is heavily concentrated about the mean or median favor the sign test over the normal test and the "t" test.¹

This research is of a similar nature in that the chi-square distribution is a violation of the normal assumption that is often made. The chi-square test is a popular nonparametric test statistic, and the methodology of considering the hypothesis for each quantile of the distribution is analogous to the Kolmogorov-Smirnov test statistic with its step function and the consideration of violating the upper and lower bounds of the selected function. Details of the design of the experiment and additional review of related literature are contained in the following chapters.

¹Whitney, p. 4.

CHAPTER III

DESIGN OF THE STUDY

THE ALGORITHM

The choice of an algorithm with which to generate random variables from chi-square distributions using methods to generate these variables that utilize proven techniques and that are already known is pivotal to the study. After review of two of the renowned volumes of Pearson and Hartley¹ and tables by Harter,² it was found and confirmed by the IBM Research Division³ that the algorithm established by Knuth, cited below, had all the necessary attributes.

Except for differences in notation, Knuth's formula for the chi-square distribution is the same as that found in the preceding works by other authors. His algorithm is as follows:

The chi-square distribution with ν degrees of freedom, also called the gamma distribution of order $\nu/2$. We have

$$F(x) = \frac{1}{2^{\nu/2} \Gamma(\nu/2)} \int_0^x t^{\nu/2-1} e^{-t/2} dt, \quad x \geq 0$$

¹E. S. Pearson and H. O. Hartley, Biometrika Tables for Statisticians, 2 vols. (Cambridge: Cambridge University Press, 1956-1972).

²H. Leon Harter, "A New Table of Percentage Points of the Chi-Square Distribution," Biometrika, 1964, 51, pp. 231-234.

³IBM Research Division, Yorktown Heights, New York, 10598.

If $U = 2k$ where k is an integer, set $X = 2(Y_1 + Y_2 + \dots + Y_k)$, where the Y 's are independent random variables with the exponential distribution, each with mean 1. If $U = 2k + 1$, set $X = 2(Y_1 + \dots + Y_k) + Z^2$, where the Y 's are as before, and Z is an independent random variable with the normal distribution (mean zero, variance one).¹

As can be noted, the chi-square distributed random variables are dependent upon exponential distributed random variables when the degrees of freedom are even and upon both normal and exponential distributed random variables when the degrees of freedom are odd. This permits the selection of various subroutines to generate the variables.

As Quenouille has noted, the increased popularity of Monte Carlo methods has increased the supply of random observations.² Until recently, these observations were available only in the form of random numbers,³ random normal deviates,⁴ correlated random normal deviates,⁵ and serially

¹Knuth, vol. 2, p. 115.

²M. H. Quenouille, "Tables of Random Observations from Standard Distributions," Biometrika, 1959, 46, pp. 178-181.

³M. G. Kendall and B. Babington Smith, Tables of Random Sampling Numbers (Cambridge: Cambridge University Press, 1939; L. H. C. Tippett, Random Sampling Numbers (Cambridge: Cambridge University Press, 1927); and Rand Corporation, A Million Random Digits with 100,000 Normal Deviates (Glencoe, Illinois: Free Press, 1955).

⁴H. Wold, Random Normal Deviates (Cambridge: Cambridge University Press, 1954).

⁵E. C. Fieller, T. Lewis, and E. S. Pearson, Random Correlated Normal Deviates (Cambridge: Cambridge University Press, 1955).

correlated random number and normal deviates.¹ In order to draw random observations from any distribution, it was necessary to calculate the distribution function and the transformation of rectangularly distributed observations using this function. Since these two steps could require considerable calculations, Quenouille constructed tables that relate estimated values of non-normal observations to the corresponding values obtained with the same observations transformed to normality. One thousand random observations are provided from each distribution. Obviously, one thousand random variables is an insufficient quantity for anything but a pilot study, but Quenouille's work has been referenced here for the researcher that might wish to write his own program and to provide a mathematical background source.

The contents of the tables are:

x_1 - random normal deviates.

x_2 - random rectangular deviates.

x_3 - random deviates from a distribution whose logarithm was normally distributed.

x_4 - random deviates from the exponential distribution.

x_5, x_6, x_7 - random deviates from an Edgeworth

¹M. G. Kendall, "Tables of Autoregressive Series," Biometrika, 1949, 36, pp. 267-289.

Type A expansion with various k values.

x_g - random observations from the two-sided exponential distribution.

A set of subroutines that are readily available and undoubtedly come to mind first for use in the algorithm and study are those of IBM - the "Scientific Subroutine Packages".¹ The subroutine RANDU could be used for uniformly distributed random numbers and transformed to exponentially distributed numbers. The subroutine GAUSS computes normally distributed random numbers with a given mean and standard deviation. Together these operations would allow the use of the Knuth algorithm for chi-square distributed random variables. However, the necessity of performing the transformations would result in longer computer time and more lines of printout or storage. Other subroutines from SSP that are of interest but that can be circumvented are:

NDTR which computes $y = P(x) = \text{PROB.}(X \leq x)$ where X is a random variable distributed normally with mean zero and variance one.

NDTRI computes $x = p^{-1}(y)$ such that $y = P(x) = \text{PROB.}(X \leq x)$ where X is again a random variable distributed

¹IBM, SSP ("Scientific Subroutine Packages"), Form H20-0205-3, rev. 2/14/69, pp. 68, 77, 78, 81, and 83.

normally with mean zero and variance one.

CDTR computes $P = P(x) = \text{PROB.}(X \leq x)$ where X is a random variable following the chi-square distribution with continuous parameter m .

CHISQ calculates degrees of freedom for a given contingency table A of observed frequencies with n rows (conditions) and m columns (groups).

Knuth examined several techniques for generating normal deviates and favors Marsaglia's rectangle-wedge-tail method as being an extremely efficient program with small average running time.¹ Since this study will necessitate the generation of 540,000 random numbers from chi-square distributions and entail the generation of at least 1,602,000 exponentially distributed and 216,000 to 756,000 normally distributed random variables, speed and accuracy are paramount. Knuth describes three methods of generating normal deviates and states:

The polar method is rather slow, but it has essentially perfect accuracy, and it is very easy to write a program for the polar method if we assume square root and logarithm subroutines are available. Teichroew's method is also easy to program, and it requires no other subroutines; therefore it takes considerably less total memory space. Teichroew's method is only approximate, although in most applications its accuracy (an error bounded by 2×10^{-4} when $|R| \leq 1$) is quite satisfactory. Marsaglia's method is considerably faster than either

¹Knuth, vol. 2, pp. 105-108.

of the others, and like the polar method it gives essentially perfect accuracy. It requires square root, logarithm, and exponential subroutines, and an auxiliary table of 100-400 constants, so its memory space requirement is rather high; yet its speed more than compensates for this on a large computer. A program for Marsaglia's method is considerably more difficult to prepare, but a general-purpose subroutine based on Algorithm M will be a valuable part of any subroutine library.¹

Just as in the case of the normal distribution, there is an extremely fast rectangle-wedge-tail method available for the exponential distribution based on a decomposition of the frequency function.

Inasmuch as The McGill Random Number Package "Super-Duper" facilitated the design of this experiment to such great extent, directions about how to use the package, as well as an off-line print-out of the source package, are included in Appendix A.²

The uniform number generator (which is either called directly or else is built into the normal and exponential generators) combines a multiplicative congruential generator and a shift register generator. The congruential generator uses the multiplier 69069, found after a search of millions of multipliers to have nearly optimal lattice structure in 2, 3, 4, and 5 dimensions - much better than any of the

¹Knuth, vol. 2, pp. 113-115.

²G. Marsaglia, K. Ananthanarayanan, and N. Paul, School of Computer Science, McGill University, Montreal, Quebec, Canada, National Research Council of Canada (NRC-A7901).

highly touted but poorly justified multipliers used for the past 20 years. But, even though the congruential generator is as good as a congruential generator can be, it is still not good enough, and it has been combined with a shift register generator on 32 bits (right shift 15, left shift 17). The bit patterns produced by the two separate generators are added as binary vectors - that is, exclusive or addition. Combining the two generators produces a sequence with period about 5×10^{18} .

Having established the accuracy and speed of the methods of Marsaglia, et al, and having confirmed Knuth's evaluation of the randomness of the algorithm that will be used in the generation of chi-square distributed random variables, it is now fitting to discuss the Monte Carlo methodology, the number of iterations, the calculations of the category cut-off values, the computer program, and the test statistics and their evaluation.

MONTE CARLO METHODOLOGY

The Monte Carlo method, often called the method of statistical trials, is a method of solving problems of computational mathematics by simulation of random quantities. The methodology comprises that branch of mathematics which is considered essentially experimental rather than analytical. The problems are of two types - probabilistic or deterministic, depending upon whether or not they are concerned with the behavior and outcome of random processes or variables.¹ Kleijnen makes an interesting observation about the use of analytical and numerical solutions:

An analytical solution uses properties known from that part of mathematics called 'analysis' which comprises differential and integral calculus. It gives a solution in the form of a formula that holds for various possible values of the independent variables and parameters. . .

A numerical solution substitutes numbers for the independent variables and parameters of the model and manipulates these numbers. Many numerical techniques are iterative, i.e., each step in the solution gives a better solution using the results from previous steps. . . Two special numerical techniques are the Monte Carlo method and simulation.²

In the same vein, Hammersley states:

It should almost go without saying, if it were not so important to stress, that whenever in the Monte Carlo estimation of a multiple integral we are able to per-

¹Sobol, p. 1.

²Kleijnen, p. 5.

form part of the integration by analytical means, that part should be so performed. As in some kinds of gambling, it pays to make use of one's knowledge of form.¹

Electronic computers are to be credited with modern day Monte Carlo methods, and, as Sobol points out, the accepted birth date of the methodology is 1949, and the American mathematicians, Neyman and Ulam, are considered its originators.² However, Schreider points out that historically the first example of a computation by a Monte Carlo method is Buffon's celebrated problem of needle tossing, which he described in 1777 in his treatise Essai d'Arithmetique Morale.³ This resulted in a method for computing the quantity $1/\pi$. Where K is the number of times that the dropped needles cross parallel lines on a ruled plane and N is the number of times the needles are tossed, then according to the Law of Large Numbers, $K/N \approx 1/\pi$.

The generation of random variables of various distribution can be obtained by transformation of independent uniformly distributed variables as described in the preceding section about the algorithm. Where L is the number of pairs of coordinates out of a possible N pairs, an estimate of the computation of the probability p is based upon the integral of the area which can be represented by $L/N \approx p = \int_0^1 f(x) dx$. The estimate of the error obviously depends on

¹Hammersley and Handscomb, p. 74.

²Sobol, p. 1.

³Schreider, p. 4.

the number N of tests. Note that no conditions need be imposed on the smoothness of the function $f(x)$, in order for this method of computing the integral to be applicable. It is sufficient that $f(x)$ be measurable and bounded. Errors will be "smoothed out" by the use of large N . This indicates that the use of an electronic computer is of utmost importance in order to calculate the desired statistics with forecastable precision.

Since this study is based upon small sample sizes N ranging from 12 to 80, precision must be obtained by utilizing a large number of iterations where the error δ of the Monte Carlo method for the computation of the probability of an event A is of the order $\delta \sim 1/\sqrt{N}$. It is evident that a reduction of the error is associated with a significant increase in the number of tests. The discussion of the selection of the number of iterations follows in a succeeding section based upon Chebyshev's Theorem.

The random variables that are generated are discrete and can assume the values defined by the table

$$X = \begin{pmatrix} x_1 & x_2 & \dots & x_n \\ p_1 & p_2 & & p_n \end{pmatrix} \quad \text{where } x_1, x_2, \dots, x_n \text{ are the}$$

possible values of the variable X , and p_1, p_2, \dots, p_n are the probabilities corresponding to them. The Monte Carlo method assumes that the variables are continuous and

the probability that X lies in the arbitrary interval (a', b') contained in $[a, b]$ is equal to the integral $P(a' < x < b') = \int_a^{b'} p(x) dx$.

The conditions that prevail for both discrete and continuous random variables are that the density $p(x)$ is non-negative and that the integral, or sum, of the density over the whole interval is equal to 1. It should be noted that, as the size of the intermediate intervals is reduced, the discrete distribution approaches the continuous distribution as the limit. On the basis of a single trial one cannot precisely predict the values that X and the corresponding probability will assume. The more the trials there are, that is, the larger the sample, the more precise the prediction will be.

Kleijnen and Knuth discuss the generation of random variables for Monte Carlo studies at some great length. Kleijnen concludes that there are no foolproof generators, and at the present the best one seems to be the multiplicative generator. Since the major part of this experiment is based upon numerical solutions and the corresponding Monte Carlo method, Knuth's recommendation is followed, and Marsaglia's multiplicative congruential generator with a shift register generator has been adopted.

THE NUMBER OF ITERATIONS

The Monte Carlo methodology in this experiment is used to approximate the probability distribution obtained from calculating the Pearsonian and log-linear chi-square statistics at the various quantiles represented by the cut-off points of the various categories used in each sample set, i.e., $k = 4$ to 8 , and the theoretical distribution above the ten percent level of significance. As previously stated, the number of iterations t determines the precision of the estimates.

The determination of the number of iterations N is based upon a procedure used by Kavanagh¹ and more recently by McNamee.² Let $X^2(P)$ be the Pearsonian statistic and $X^2(L)$ be the log-linear statistic calculated by the following formulae:

$$X^2(P) = \sum_{\text{all cells}} (\text{observed} - \text{expected})^2 / (\text{expected})$$
$$X^2(L) = 2 \sum_{\text{all cells}} (\text{observed}) \log(\text{observed}/\text{expected})$$

McNamee's approach was that he was interested in the 90th percentile or less of the theoretical distribution and

¹J. A. Kavanagh, "A Monte Carlo Study of the Polynomial Discriminant Method for Pattern Recognition" (Ph.D. Dissertation, University of Minnesota, 1972), p. 26.

²McNamee, pp. 40-42.

the comparison with the statistic which was calculated as $\chi^2(P)$ above. Considering the following notation used in this discussion where p is an estimate of y :

$$y = p = \begin{cases} 1 & \text{for } \{ \chi^2(P) \mid \chi^2(P) \geq \chi^2_{.90} \} \\ 0 & \text{for } \{ \chi^2(P) \mid \chi^2(P) < \chi^2_{.90} \} \end{cases}$$

Where t (or n) independent iterations or observations of y (or p) and where p is the probability that $\chi^2(P) \geq \chi^2_{.90}$, then $\sum_{i=1}^t y_i$ is binomially distributed with the parameter p . Mathematical proofs since Pearson¹ up to Tate and Hyer's² report on the comparison of the multinomial and chi-square tests, apply The Central Limit Theorem to approximate the distribution of \bar{y} (P) as the number of observations (n) gets larger. All the proofs assume at some point that the observed frequencies are distributed normally about the expected frequencies with a mean of P and variance of $P(1 - P)/n$. From this the following probability statement can be made where Z_{α} is found in standard normal tables.

¹Karl Pearson, "On the Criterion That a Given System of Deviations from the Probable in the Case of a Correlated System of Variables Is Such That It Can Be Reasonably Supposed to Have Arisen from Random Coupling," Philosophical Magazine Series, 1900, 50, pp. 157-175.

²Merle W. Tate and Leon A. Hyer, "Significance Values for an Exact Multinomial Test and Accuracy of the Chi-Square Approximation, Final Report" (Bureau of Research, Office of Education, Washington D. C., 1969), BR-8-B-023, pp. 1-75.

$$\Pr \left(\frac{|P - P_0|}{\sqrt{P_0 (1 - P_0/n)}} \leq Z_{1 - \alpha/2} \right) = \alpha$$

$Z \stackrel{d}{=} N(0,1)$

The worst situation considered in McNamee's study was when $P_0 = .9$ and the variance was $(.9)(.1)/n$. Obviously, the worst condition that could exist would be when $P_0 = .5$ and the variance $(.5)(.5)/n$. In this above-mentioned study, the experimenter was satisfied when $|P - P_0| = d = .03$ ninety-five percent of the time and $P - P_0 \pm .03$, the estimate of the true P_0 ninety-five percent of the time for the true value $P_0 = .90$. The number of iterations was calculated to be 385 and subsequently 400 iterations were used in the study. This resulted in precision values (d) of .02 for $P_0 = .95$ and .009 for $P = .99$.

Pilot computer runs were made for this study of the one sample test using 400 iterations. However, after the program was rewritten for efficiency and speed, it was decided to use 1000 iterations, even though this would only increase the precision minimally, namely to .02 for $P_0 = .90$, .01 for $P_0 = .95$, and .006 for $P_0 = .99$ using the same 95% criteria of the previously described experiment.

Based upon the work of Slakter,¹ 10,000 random samples should be generated for each empirical distribution.

¹M. J. Slakter, "Comparative Validity of the Chi-Square and Two Modified Chi-Square Goodness-of-Fit Tests for Small but Equal Expected Frequencies," Biometrika, 1966, 53, pp. 619-622.

Similar calculations for this experiment would have meant ten times as many computations as were used and the generation of over 23,580,000 random variables. Tate and Hyer¹ used 65,536 sets of outcomes for the relatively simple multinomial distribution, $N = 8$, $k = 4$ and the ϕ equal. Their total grant required in excess of 8,000,000 sets of outcomes. It is self evident that there is insufficient time or money to extend this study to a like scope.

¹Tate and Hyer, p. 5.

CATEGORIZATION AND PROGRAMMING

The arguments for the use of the equal area model for the chi-square test of goodness of fit were discussed extensively in the previous chapter, pages 42-46. The samples of random variables that are generated in this study are categorized accordingly. The only difficulty that arises in such categorization is the establishment of the chi-square category cut-off points for many of the various percentiles that are not generally tabulated. Although interpolation of values of χ^2 is explained in BTS 1,¹ the procedures suggested in BTS 2² are used since this volume includes many tabulations calculated by Harter,³ which are more comprehensive than previous tabulations in that the tables are entered with ν and P, rather than Q; include additional entries for P; and have eight significant figures for low values of ν and P rather than the usual six significant figures.

Where 3-decimal accuracy is adequate, linear interpolation is usually sufficient and particularly where $\nu < 30$. However, for greater accuracy, and in line with established

¹Pearson and Hartley, BTS 1, pp. 13-16.

²Pearson and Hartley, BTS 2, pp. 140-142 and pp. 382-385.

³Harter, p. 234.

practice, the study uses percentage points accurate to four decimal points, particularly since the study is for samples of small size and no more than seven degrees of freedom. In order to interpolate the untabulated percentage points, Pearson's five-point Lagrangian interpolation formulae are used. BTS 2 table 69 contains the coefficients which are used to break down the gaps between the standard quantiles $x(P)$ and are presented in eight different grids, which are based upon the standard P-values for which x is tabled and the P-value for which x is required.¹

The interpolated value: $x_p = \sum_{i=1}^5 L_i x_{p_i}$ or $x(P) = L_1 x(P_1) + L_2 x(P_2) + L_3 x(P_3) + L_4 x(P_4) + L_5 x(P_5)$.

For example, in order to interpolate the value for the first category cut-off point, when the sample has six categories and five degrees of freedom, one should use grid 4 since $x(P) = 1/6$ of the area = .1667, and this value is near the center of the tabled values in grid 4. The calculations are as follows:

P_i	0.05	0.10	0.20	0.30	0.40
L_i	-.024019	.242443	1.054338	-.350307	.077545
$\chi^2(P_i)$	1.14548	1.61031	2.34253	2.99991	3.65550

¹Pearson and Hartley, BTS 2, pp. 382-385.

Summing the products $L_i \chi^2(P_i)$, it is found that $\chi^2(.16|5) = 2.0651897$. Subtracting this from $\chi^2(.20|5) = 2.34253$, dividing the difference by 400, multiplying this quotient by 67, and adding the product to $\chi^2(.16|5)$ result in the linear interpolation for $\chi^2(.1667|5) = 2.112$. This was checked by calculated $\chi^2(.17|5)$ and interpolating downward to .1667 with comparable results.

With the algorithm selected and with the categorization process established, the succeeding steps in the design are to write an assembler-fortran program to generate the random variables, calculate $X^2(P)$ and $X^2(L)$ for the various sample sets, and then refine the program for speed and ease of evaluation. First, the fortran subroutine for calculating the Pearsonian chi-square probabilities, $X^2(P)$, was adapted from the SPSS package, and fortran statements for the calculation of the log-linear chi-square probabilities, $X^2(L)$, were appended to the random number generator that has been previously discussed. Part of a sample run of 40 iterations is given in Appendix B and demonstrates how lengthy and time consuming the basic design could be, while also showing how the algorithm is used for a chi-square distribution with four degrees of freedom and expected frequencies of three. Column one consists of the Y_1 random numbers that are generated from an exponential distribution, column two consists of the Y_2 variables, and column three is the desired chi-square random variable which is twice the sum of

Y_1 and Y_2 . The cell frequencies are displayed along with the Pearsonian and the log-linear chi-square values and the apropos probabilities. It should be noted that this sample of 40 iterations required 986 lines, evidence that the program had to be condensed for 1000 iterations of the fifteen sample sets that are studied.

Appendix C shows the output of an intermediate stage in the evolution of the final program. This sample run is for a chi-square distribution having seven degrees of freedom and expected cell frequencies of three. For the sake of brevity, only the fortran statements and some of the output is reproduced in this appendix, which displays how the program branches to $X = 2(Y_1 + \dots + Y_k)$ for distributions having an even number of degrees of freedom or branches to $X = 2(Y_1 + Y_k) + Z^2$ for odd numbered degrees of freedom. As before, the Pearsonian and log-linear chi-square values and probabilities are printed. When the log-linear statistic is indeterminate because of a zero cell frequency, this fact is flagged by the printing of a series of asterisks.

Appendix D is an example of the final condensed version of the program that was evolved and part of the output when 1000 iterations were used for each sample set. It should be noted that the intermediate calculations are not printed and that the output is sorted according to the probabilities of the Pearson test statistic. This procedure

facilitates the tabulation and comparison of $X^2(P)$ and $X^2(L)$.

Calculations by the use of an electronic hand calculator were made of randomly selected sample sets at each stage of the programming evolution to verify the accuracy of the computer work. In addition, the nonparametric subroutine of SPSS VII for one case samples was used for further proof of the program. A single example of this is shown in Appendix E and demonstrates the value of the program that was designed when many cases must be studied and speed and efficiency are of utmost concern.

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EVALUATION

The primary hypothesis:

$$H_0 : P_1 = P_{01}$$

$$H_1 = \text{not } H_0$$

$$P_2 = P_{02}$$

. .

. .

. .

$$P_k = P_{0k}$$

will be evaluated for $X^2(P)$ and $X^2(L)$ by tabulating the frequencies that appear within the equal area proportions for both test statistics when the number of categories were selected a priori from 4 to 8 and the expected cell frequencies were 3, 5, and 10.

For example, when $K = 4$, the hypothesized proportions falling within each category are 25 percent. Using 1000 iterations, 250 sample sets should fall within each category. However, when zero cell frequencies occur, $X^2(L)$ is indeterminate, as can be recognized from the test statistic: $X^2(L) = 2 \sum (\text{observed}) \log(\text{observed}/\text{expected})$. When this occurs, the number of indeterminate iterations will be noted on the tabulation and the percentages adjusted accordingly. The percentage of error that occurs will also be displayed for both test statistics. Since 10 percent is

usually the criteria of goodness of fit when a researcher is "model fitting", as in ANOVA, ANOCOVA, etc., the null hypothesis will be accepted if the experimental frequencies are within this 10 percent limit. In order to identify the effect of small sample size, the tabulations will be presented separately for each K category and the three expected frequencies, $E(x) = 3, 5, \text{ and } 10$.

For each N and K, where $\phi = 1/K$, the chi-square probabilities for $X^2(P)$ and $X^2(L)$ will be tabled in the .005-.009, .010-.050, and .051-.100 regions since these regions are the ones most often used in making statistical inferences in the one sample case. These tabulations will also show the exact multinomial probabilities that are available from the Tate and Hyer study previously noted. Except for the special case of the binomial distribution, there had previously been no tables of the multinomial. The reason for this is that there are too many parameters in the general case to permit construction of manageable tables, and the expansion of the multinomial is long and laborious for all but small N. However, in the case of the equal area model being studied, the ϕ are equal, just as in the Tate and Hyer ¹ study, and the size of the tables and the labor is greatly reduced. This is particularly true if the calculations are made using logarithms and log factorials when

¹Tate and Hyer, pp. 1-8 and pp. 25-75.

computing the multinomial probability of an outcome. Tate and Hyer used a digital computer program that was written by David L. March¹ for a CDC 6400 computer to further reduce the complexity of the procedures.

Since the Tate study of the multinomial distribution was limited to N of 30 or less and ϕ of $1/7$ or more, a complete comparison of the multinomial probability of $P(M)$ and $X^2(P)$ and $X^2(L)$ is possible for only part of this study.

¹David L. March, Multin Program, 1968, Lehigh University Computing Center, Bethlehem, Pa.

CHAPTER IV

RESULTS OF THE STUDY

INTRODUCTION

This chapter presents the results of the study which were derived using Monte Carlo methodology to generate random numbers from gamma distributions of order $\nu/2$, which are Pearson's chi-square distributions with ν degrees of freedom. This decision was based on the desire to verify Siegel's claim that nonparametric techniques are distribution free, whereas researchers most often generate their random variables from uniform or normal distributions. By varying the degrees of freedom from 3 to 7, distributions of varying measures of skewness were generated. Sample size was manipulated by using expected cell frequencies of 10, 5, and 3.

As originally programmed, see Appendix B, this research required 1,080,000 random numbers constructed from 1,602,000 exponentially and 756,000 normally distributed random variables and necessitated 745,500 lines of computer output and 3 hours 55 minutes 52.5 seconds of CPU time, even though the efficient McGill Random Number Package "Super-Duper" was used. The intermediate program, shown as Appendix C, cut the CPU time but increased the lines of out-

put to over 4,000,000. The program that was finally evolved required only 42,345 lines of output and 41 minutes 38.9 seconds of CPU time, a drastic reduction. A sample of this program is displayed as Appendix D.

The efficiency of the Monte Carlo program made it possible to examine various one sample cases with expected values of 10, 5, and 3 in each cell. These numbers were proposed in many articles reviewed in CHAPTER II and will be discussed further in the following sections, particularly those on the use of Fienberg's $X^2(L)$ and the equal area models with categories of 4, 5, 6, 7 and 8, wherein the total sample size is restricted, as noted in the STATEMENT OF THE PROBLEM.

The rationale for the use of α regions rather than point estimates is explained in the section on the goodness of fit. The results obtained from the use of Pearson's chi-square, $X^2(P)$, the log-linear likelihood ratio, $X^2(L)$, and the multinomial, (M), for these regions are reported as a function of the expected cell frequencies, $E(x)$, and the number of categories, k . Generally, $X^2(P)$ is as desirable a test statistic as the multinomial (M). However, the deviations that do exist have interesting implications which will be discussed in the pertinent sections.

$\chi^2(P)$ OR $\chi^2(L)$ FOR SMALL SAMPLES

Since so little research has been reported regarding the chi-square test for the one sample case, examples of the one sample case and the constraints imposed in this study are reiterated to differentiate this experiment from many other chi-square investigations. Siegel explains the function of the chi-square one sample test as follows:

Frequently research is undertaken in which the researcher is interested in the number of subjects, objects, or responses which fall in various categories. For example, a group of patients may be classified according to their preponderant type of Rorschach response, and the investigator may predict that certain types will be more frequent than others. Or children may be categorized according to their most frequent modes of play, to test the hypothesis that these modes will differ in frequency. Or persons may be categorized according to whether they are 'in favor of', 'indifferent to', or 'opposed to' some statement of opinion, to enable the researcher to test the hypothesis that these responses will differ in frequency.

The χ^2 test is suitable for analyzing data like these. The number of categories may be two or more. The technique is of the goodness-of-fit type in that it may be used to test whether a significant difference exists between an observed number of objects or responses falling in each category and an expected number based on the null hypothesis.

In order to be able to compare an observed with an expected group of frequencies, we must of course be able to state what frequencies would be expected. The null hypothesis states the proportion of objects falling in each of the categories in the presumed population. That is, from the null hypothesis we may deduce what are the expected frequencies.¹

¹Siegel, pp. 42-43

A key phrase is 'the presumed population'. Much of the literature that has been reviewed is based upon sampling from uniform or normal populations although the seed for this research was Fienberg's statement that, for small samples, it is not clear whether $X^2(P)$ or $X^2(L)$ is the superior statistic, and that study was based upon sampling from Poisson distributions.¹

As Fienberg explained, the same maximum likelihood estimates for the expected cell counts for log-linear models can be obtained under a variety of different sampling procedures. The most simple such sampling procedure which can be assumed is that one where the observed cell counts have independent Poisson distributions with the expected counts as their means. However, since this experiment is concerned with small sample sizes and small expected cell frequencies, the basic assumption was made to sample from skewed chi-square distributions of various degrees of freedom.

In support of Cochran,² Siegel states that "The chi-square test for the one sample case should not be used when more than 20 percent of the expected frequencies are smaller than 5 or when any expected frequency is smaller than 1."³ McNamee showed that the chi-square test for

¹Fienberg, pp. 421-425.

²Cochran, "X² Test," p. 319.

³Siegel, p. 46.

first order interaction is quite robust with expected values as small as 3.¹ This conclusion and the well-known rules of 5 or 10 decided what expected values would be covered in this study.

The results of 1000 iterations for the 15 different combinations of 5 categories and 3 expected frequencies are displayed in Table 1 on the following pages, listed in ascending order of k categories with $\phi = 1/k$ and $E(x) = 10, 5,$ and 3. The theoretically expected frequency for each category is $1000/k$. The observed frequency for each equal proportion is listed along with the percentage deviation from the theoretical frequency, % e, for $X^2(P)$ and $X^2(L)$. The frequencies of $X^2(L)$ that are indeterminate, Indet, are listed for the corresponding proportions. The frequencies of chi-square probabilities generated from a normal distribution and the deviation from the theoretical frequencies are listed as f Gauss and % e. This latter sampling procedure will be discussed in the following section, EVALUATION OF THE HYPOTHESIS OF EQUAL PROPORTIONS.

Since the .10 level is the usual α level for goodness of fit tests, it has been selected as the criterion for the comparison of the two test statistics, $X^2(P)$ and $X^2(L)$. For example, when $K = 4$, $\phi = 1/4$, $E(x) = 10$, and $N = 40$, the deviations for the 4 categories are less than

¹McNamee, p. 104.

10 percent, as shown in Table 1, and $X^2(P)$ or $X^2(L)$ result in similar decisions of inference or hypothesis testing for the one sample case. Both statistics support the null hypothesis that the samples came from chi-square populations. Likewise, when $K = 5$, $\phi = 1/5$, $E(x) = 10$, and $N = 50$, an experimenter would be likely to use $X^2(P)$ or $X^2(L)$. Only when $K = 4$ and $E(x) = 5$ and when $K = 6$ and $E(x) = 10$, does $X^2(L)$ display a better fit to the sampled chi-square distributions than $X^2(P)$, and this could be due to experiment-wise error since 45 sample sets were generated.

The most obvious disadvantage to the use of $X^2(L)$ for small samples is the increasing number of the statistic that are more and more indeterminate as the number of categories are increased, and the expected frequencies are decreased. The pattern of the number of these indeterminate test statistics also reflects the skewness and kurtosis of the populations since the categories with the higher proportions, i.e., the right-hand tail, have increasing frequencies of indeterminate results.

The number of zero observations that are encountered in the higher probabilities of the chi-square and the arithmetic of the test statistic, $X^2(L) = 2 \sum (\text{observed}) \log(\text{observed}/\text{expected})$, portends that many calculations would be indeterminate since the logarithm of zero divided by a number is indeterminate. In the case of contingency tables, this effect can be negated to a large degree by transposing

rows or columns, as in the study by McNamee.¹ A point not emphasized is that such transposition changes the identification of the corresponding interactions. Fienberg, Goodman, Haberman and others utilizing transposition modify the models to reflect the alteration or deletion of some interactions.² The rejection of $X^2(L)$ for use in one sample cases does not detract from Fienberg's use in the analysis of multidimensional contingency tables, since $X^2(L)$ can be used in the selection of suitable models, via an iterative technique of partitioning.

Further reference to the use of $X^2(P)$ and $X^2(L)$ is made in a succeeding section in which the statistics are compared to the exact multinomial in three α regions from .005 to .100 with the implications for goodness of fit.

¹McNamee, p. 66

²Fienberg, pp. 426-431.

TABLE 1

EVALUATION OF HYPOTHESES OF EQUAL AREA MODELS

$K = 4, \phi = 1/4, N = 40, 20, \text{ and } 12$

Expected Frequency 250 per Category

$N = 40$

P_0	$f X^2(P)$	% e	$f X^2(L)$	% e	Indet	f Gauss	% e
P.25	257	+2.8	248	-0.8		242	3.2
P.50	234	-6.4	246	-1.6		210	16.0
P.75	265	+6.0	248	-0.8		276	10.4
P1.00	244	-2.4	258	+3.2		272	8.8

TABLE 1 - Continued

N = 20

P_o	$f X^2(P)$	% e	$f X^2(L)$	% e	Indet	f Gauss	% e
P. _{.25}	332	+32.8	265	+6.0		305	+22.0
P. _{.50}	141	-43.6	224	-10.4		159	-36.4
P. _{.75}	300	+20.0	271	+8.4		294	+17.6
P. _{1.00}	227	-9.2	225	-10.0	15	242	-3.2

TABLE 1 - Continued

N = 12

P_0	$f X^2(P)$	% e	$f X^2(L)$	% e	Indet	f Gauss	% e
P.25	223	-10.8	223	-10.8		197	-21.2
P.50	302	+20.8	195	-22.0		322	+28.8
P.75	207	-17.2	314	+25.6		228	-8.8
P _{1.00}	268	+7.2	150	-40.0	118	253	+1.2

TABLE 1 - Continued

K = 5, $\phi = 1/5$, N = 50, 25, and 15

Expected Frequency 200 per Category

N = 50

P_0	f $X^2(P)$	% e	f $X^2(L)$	% e	Indet	f Gauss	% e
P. _{.20}	215	+7.5	199	-0.5		208	+4.0
P. _{.40}	178	-11.0	181	-9.5		180	-10.0
P. _{.60}	207	+3.5	193	-3.5		199	-.5
P. _{.80}	197	-1.5	216	+8.0		216	+8.0
P. _{1.00}	203	+1.5	211	+5.5		197	-1.5

TABLE 1 - ContinuedN = 25

P_o	f $X^2(P)$	% e	f $X^2(L)$	% e	Indet	f Gauss	% e
P _{.20}	207	+3.5	184	-8.0		232	+16.0
P _{.40}	116	-42.0	160	-20.0		127	-36.5
P _{.60}	262	+31.0	215	+7.5		244	+22.0
P _{.80}	196	-2.0	204	+2.0		190	-5.0
P _{1.00}	219	+9.5	218	+9.0	19	207	+3.5

TABLE 1 - Continued

N = 15

P_o	f $X^2(P)$	% e	f $X^2(L)$	% e	Indet	f Gauss	% e
P _{.20}	189	-5.5	189	-5.5		186	-7.0
P _{.40}	232	+16.0	170	-17.5		206	+3.0
P _{.60}	167	-16.5	223	+11.5	6	179	-10.5
P _{.80}	176	-12.0	139	-30.5	39	204	+2.0
P _{1.00}	236	+18.0	112	-44.0	167	225	+12.5

TABLE 1 - Continued

$K = 6, \phi = 1/6, N = 60, 30, \text{ and } 18$

Expected Frequency: 166-2/3 per Category

N = 60

P_0	f $X^2(P)$	% e	f $X^2(L)$	% e	Indet	f Gauss	% e
P. _{.167}	144	-13.6	150	-10.0		149	-10.6
P. _{.333}	205	+23.0	174	+4.4		185	+11.0
P. _{.500}	151	-9.4	171	+2.6		138	-17.2
P. _{.667}	186	+11.6	178	+6.8		184	+10.4
P. _{.833}	157	-5.8	165	-1.0		168	+0.8
P. _{1.00}	157	-5.8	162	-2.8		176	+5.6

TABLE 1 - ContinuedN = 30

P_0	$f X^2(P)$	% e	$f X^2(L)$	% e	Indet	f Gauss	% e
P.167	182	+9.2	182	+9.2		183	+9.8
P.333	187	+12.2	129	-22.6		162	-2.8
P.500	109	-34.6	176	+5.6		117	-29.8
P.667	196	+17.6	155	-7.0		210	+26.0
P.833	171	+2.6	185	+11.0	2	159	-4.6
P1.00	155	-7.0	156	-6.4	15	169	+1.4

TABLE 1 - Continued

N = 18

P_0	f $X^2(P)$	% e	f $X^2(L)$	% e	Indet	f Gauss	% e
P.167	157	-5.8	145	-13.0		177	+6.2
P.333	112	-32.8	124	-25.6		102	-38.8
P.500	171	+2.6	168	+0.8	9	181	+8.6
P.667	227	+36.2	170	+2.0	47	206	+23.6
P.833	174	+4.4	119	-28.6	70	161	-3.4
P _{1.00}	159	-4.6	43	-74.2	105	173	+3.8

TABLE 1 - Continued

$K = 7, \phi = 1/7, N = 70, 35, \text{ and } 21$

Expected Frequency 142-6/7 per Category

N = 70

P_0	f $X^2(P)$	% e	f $X^2(L)$	% e	Indet	f Gauss	% e
P _{.143}	119	-16.7	112	-21.6		154	+7.8
P _{.286}	118	-17.4	122	-14.6		122	-14.6
P _{.429}	131	-8.3	141	-1.3		118	-17.4
P _{.571}	150	+5.0	144	+0.8		134	-6.2
P _{.714}	143	-0.1	136	-4.8		158	+10.6
P _{.857}	177	+23.9	171	+19.7		178	+24.6
P _{1.00}	162	+13.4	174	+21.8		136	-4.8

TABLE 1 - ContinuedN = 35

P_0	f $X^2(P)$	% e	f $X^2(L)$	% e	Indet	f Gauss	% e
P. _{.143}	112	-21.6	114	-20.2		133	-6.9
P. _{.286}	126	-11.8	117	-18.1		149	+4.3
P. _{.429}	111	-22.3	126	-11.8		116	-18.8
P. _{.571}	184	+28.8	166	+16.2		176	+23.2
P. _{.714}	170	+19.0	139	-2.7	2	161	+12.7
P. _{.857}	137	-4.1	158	+10.6	11	113	-20.9
P. _{1.00}	160	+12.0	138	-3.4	29	152	+6.4

TABLE 1 - ContinuedN = 21

P_o	f $X^2(P)$	% e	f $X^2(L)$	% e	Indet	f Gauss	% e
P. _{.143}	93	-34.9	130	-9.0		100	-30.0
P. _{.286}	159	+11.3	127	-11.1		172	+20.4
P. _{.429}	181	+26.7	134	-6.2	18	167	+16.9
P. _{.571}	102	-28.6	109	-23.7	24	95	-33.5
P. _{.714}	186	+30.2	103	-27.9	62	198	+38.6
P. _{.857}	138	-3.4	90	-37.0	77	145	+1.5
P. _{1.00}	141	-1.3	42	-70.6	84	123	-13.9

TABLE 1 - Continued

K = 8, $\phi = 1/8$, N = 80, 40, and 24

Expected Frequency 125 per Category

N = 80

P_0	f $X^2(P)$	% e	f $X^2(L)$	% e	Indet	f Gauss	% e
P .125	32	-74.4	29	-76.8		120	-4.0
P .250	49	-60.8	46	-63.2		114	-8.8
P .375	68	-45.6	73	-41.6		130	+4.0
P .500	71	-43.2	77	-38.4		118	-5.6
P .625	103	-17.6	90	-28.0		137	+9.6
P .750	150	+20.0	133	+6.4		142	+13.6
P .875	159	+27.2	176	+40.8		106	-15.2
P 1.00	368	+194.4	374	+199.2	2	133	+6.4

TABLE 1 - Continued

N = 40

P_0	$f X^2(P)$	% e	$f X^2(L)$	% e	Indet	f Gauss	% e
P _{.125}	53	-57.6	54	-56.8		93	-25.6
P _{.250}	79	-36.8	76	-39.2		150	+20.0
P _{.375}	93	-25.6	84	-32.8		130	+4.0
P _{.500}	89	-28.8	92	-26.4		86	-31.2
P _{.625}	109	-12.8	106	-15.2		135	+8.0
P _{.750}	150	+20.0	137	+9.6	3	143	+14.4
P _{.875}	202	+61.6	182	+45.6	20	131	+4.8
P _{1.00}	225	+80.0	201	+60.8	68	132	+5.6

TABLE 1 - ContinuedN = 24

P_o	$f X^2(P)$	% e	$f X^2(L)$	% e	Indet	f Gauss	% e
P.125	77	-38.4	79	-36.8		103	-17.6
P.250	113	-9.6	114	-8.8		157	+25.6
P.375	54	-56.8	31	-75.2	8	60	-52.0
P.500	142	+13.6	132	+5.6	28	178	+42.4
P.625	148	+18.4	89	-28.8	58	150	+20.0
P.750	136	+8.8	83	-33.6	58	118	-5.6
P.875	128	+2.4	66	-47.2	89	121	-3.2
P1.00	202	+61.6	34	-72.8	131	113	-9.6

EVALUATION OF THE HYPOTHESES OF EQUAL PROPORTIONS

Table 1, which is included in the previous section, not only provides the tabulation necessary to answer the question posed by Fienberg as to whether $X^2(P)$ or $X^2(L)$ is superior for one sample tests with small N , but also evaluates the hypotheses:

$$\begin{array}{ll}
 H_0 : P_1 = P_{01} & H_1 = \text{not } H_0 \\
 P_2 = P_{02} & \\
 \cdot & \cdot \\
 \cdot & \cdot \\
 \cdot & \cdot \\
 P_k = P_{0k} &
 \end{array}$$

The proportions under the null hypotheses are of equal area and therefore are associated with equal probabilities and uniform expected frequencies for each of the 5 different categories enumerated with k varying from 4 to 8. The criterion for each sample set is that the percentage of error should not exceed 10% if H_0 is to be accepted.

The table displays that H_0 would be rejected for the great majority of one sample cases generated from chi-square distributions whether $X^2(P)$ or $X^2(L)$ was used as the test statistic. This preponderant rejection was not anticipated although the section, LITERATURE BASIC TO THE

PROBLEM, was concerned with the use of equal area models resulting from sampling from uniform distributions. Roscoe and Byars stated that the chi-square tests of goodness of fit are not so good with non-uniform hypotheses.¹ Watson tested for goodness of fit to normal but suggested that the number of cells should be at least 10.² Kempthorne tested for goodness of fit to normal but set the number of cells equal to the sample size.³ Dahiya found that the chi-square approximation tends to be liberal if the value of K is set too high and is larger than n.⁴

Because of the results obtained when sampling from chi-square distributions, Gaussian random numbers were also generated and are tabulated in Table 1 for the same sample sizes, degrees of freedom, and expected values used for $X^2(P)$. The frequencies tabled as f Gauss and corresponding deviations from the expected frequency in each category, % e, were calculated utilizing $X^2(P)$. The test statistic $X^2(L)$ was omitted from the study of the Gaussian since examination of the computer print-out disclosed that many of the samples

¹Roscoe and Byars, pp. 755-759.

²Watson, pp. 336-348.

³Kempthorne, "The Classical Problem of Inference," pp. 235-249.

⁴R. C. Dahiya, "On the Pearson Chi-squared Goodness of Fit Test Statistic," Biometrika, 1971, 58, pp. 685-686.

were indeterminate, although fewer in number than those from the chi-square sampling.

The null hypotheses of equal proportions were rejected for the goodness of fit except for $K = 5$, $\phi = 1/5$, $E(x) = 10$, and $N = 50$. It should be noted that this same sample set had similar results for $X^2(P)$ and $X^2(L)$. In general, the sampling from normal resulted in lower percentage error than did sampling from chi-square distributions. A major point to be considered is that of the small sample size studied. The most important implication found in this part of the design is the increase of error found in the larger number categories. This will be discussed in a later chapter in conjunction with the results of the tests for goodness of fit when $X^2(P)$ and $X^2(L)$ are compared with the exact multinomial probability (M) for the α regions in the upper tail of the distributions.

COMPARISONS OF $\chi^2(P)$, $\chi^2(L)$ AND (M) AT
VARIOUS LEVELS OF SIGNIFICANCE

Tests in which a comparison of an observed probability distribution is made with a theoretical distribution like the Poisson, binomial, normal or others, are called goodness of fit tests. As previously stated, one of the most commonly used test statistics is the chi-square test. As Snedecor explains, "The chi-square test is a large sample approximation, based on the assumption that the distribution of the observed members in the classes are not far from normal. This assumption fails where some or all of the observed numbers are very small."¹

Small sample sizes and small expected frequencies have already been reviewed, and the problem is raised here only because, in the more extreme cases, it is possible to work out the exact distribution of chi-square. The probability that f_i observations fall in the i^{th} class is given by the multinomial distribution. Tate and Hyer have tabulated the exact cumulative probabilities for a multinomial such that expected frequencies vary from 1 to not less than 5 in the case where the expected frequencies are equal, which is equivalent to the equal area model; they have also

¹George W. Snedecor and William G. Cochran, Statistical Methods, 6th ed. (Ames, Iowa: Iowa State University Press, 1967), pp. 235-242.

studied the accuracy of the conventional chi-square goodness of fit tests in the often used levels of significance.¹

In designs of greater complexity than the one sample case, such as contingency tables, analysis of variance and the like, the chi-square statistic is usually involved. However, the question of sample size and expected frequencies still continues to be a matter of discussion in these cases. Cochran suggests using Fisher's exact multinomial for 2 x 2 contingency tables in samples up to size 30. In tests in which all expectations are small, the contention is that the tabular χ^2 is tolerably accurate, provided that all expectations are at least 2. A constraint is also imposed that the degrees of freedom are less than 15. If the degrees of freedom exceed 60, it is suggested that the normal approximation to the exact distribution be used.²

The above usage of the chi-square statistic is referred to at this point in order to introduce the concept of "Lack of Fit". "Lack of Fit" occurs when the sum of squares contains at least two sources of variation. According to Cochran and Cox, the first contribution is due to experimental errors, which make the values deviate from the true response surface. The second is that there be inflation of

¹Tate and Hyer, pp. 25-72.

²Cochran, " χ^2 Test," pp. 329-334.

the values due to the failure of the linear equation to represent the correct shape of the response surface. Likewise, a chi-square proportion evaluation can show a lack of fit if the parent population is not uniform or normal.¹

Since the usual test for goodness of fit is concerned with the probabilities of the various test statistics in the upper tail of the studied distributions rather than the evaluation of equal proportions, the frequencies that were observed for $X^2(P)$ and $X^2(L)$ for 3 α regions and 1000 iterations of the 15 different sample sets generated are presented for comparison in Table 2. The frequencies that are probable for 1000 multinomial outcomes are also listed for comparison in the same α regions.

The rationale for using probability regions rather than point estimates is multiple. As Kempthorne points out, "A point estimate alone is of little value because we are in the position of having a sample of one from a population of which we do not know the spread. We do not know, therefore, how close we are likely to be to the true value."² A reader is more frequently concerned with making an interval estimate in order to know the probability of this confidence in-

¹William G. Cochran and Gertrude M. Cox, Experimental Designs, 2d ed. (New York: John Wiley & Sons, 1957), p. 340.

²Oscar Kempthorne, The Design and Analysis of Experiments (Huntington, N. Y.: Robert E. Krieger, 1973), p. 28.

terval containing the true value. Cochran's criterion is:

. . . compare the exact P and the P from the X^2 table, when the null hypothesis is true, in the region in which the tabular P lies between 0.05 and 0.01. This criterion is not ideal, but it does appraise the performance of the tabular approximation in the borderline region between statistical significance and nonsignificance. A disturbance is regarded as unimportant if when the P is 0.05 in the X^2 table, the exact P lies between 0.04 and 0.06, and if when the tabular P is 0.01, the exact P lies between 0.007 and 0.015. These limits are, of course, arbitrary; some would be content with less conservative limits. ¹

As Skipper, Guenther, and Nass contend, .05 is not sacred. They say:

. . . there is a need for social scientists to choose levels of significance with full awareness of the implications of Type I and Type II error for the problem under investigation . . . the tendency to dichotomy resulting from judging some results significant and others 'non-significant' can be misleading both to professionals and lay audiences . . . a more rational approach might be to report the actual level of significance, placing the burden of interpretative skill upon the reader. Such a policy would also encourage scientists to give higher priority to selecting appropriate levels of significance for a given problem. ²

This approach to hypothesis testing is similar to that used in commerce and industry where the use of the prob-value, short for probability value, is prevalent. The prob-value is defined as the probability that the sample value

¹Cochran, "X² Test," pp. 328-329.

²James K. Skipper, Jr., Anthony L. Guenther, and Gilbert Nass, "The Sacredness of .05: A Note Concerning the Uses of Statistical Levels of Significance in the Social Sciences," American Sociologist, 1967, 2, pp. 16-18.

would be as extreme as the value actually observed/ H_0 , and the reader would reject H_0 iff prob-value $< \alpha$.¹

Furthermore, Tate and Hyer ignore the more extreme outcomes that have probabilities less than .005 since they felt that such extreme values seldom occur and, if they do, are most often a result of experimental or sampling errors. The regions that were selected not only encompass α levels that are often used for tests of significance but are also of mathematical necessity. The calculation of the cumulative multinomial probability of an outcome results in an exact probability. Only by grouping these probabilities can there be any meaningful comparison with the corresponding Pearson chi-square statistic for a set of outcomes. For example, the cumulative multinomial probability is .043 for an outcome of a random sample of 15 in 5 categories of 1, 0, 6, 5, 3 (order is immaterial) and the null hypothesis of all $\phi = 1/5$ would be rejected at the 4.3 percent level. $X^2(P)$ for the same outcome is 8.66667 with the tabular probability approximately .0745 so that the null hypothesis would still be rejected for the .100 goodness of fit criterion. However, it should be noted that the probability is in a different region. As Tate and Hyer found, the median percentage agreement between the exact (M) and the approxi-

¹T. H. Wonnacott and R. J. Wonnacott, Introductory Statistics (New York: John Wiley & Sons, 1969), pp. 179-181.

mation $X^2(P)$ probabilities in the regions $< .010$, $.010-.050$, $.051-.100$ and $> .100$ was 68. On average, the probabilities fell in the same region about 2/3 of the time. However, as in the point example above, the most apparent source of error was the number of outcomes yielding the same X^2 , but having varying multinomial probabilities and, on the other hand, the number of outcomes having the same multinomial probabilities, but yielding varying X^2 .¹

Table 2, which follows, is easily read. The α region $.005-.009$ has an expected frequency, $f_e = 5$; the $.010-.050$ region has $f_e = 41$, while the $.051-.100$ region has $f_e = 50$. These are the same no matter how many categories, K , are involved. The number of observations of $X^2(P)$, $X^2(L)$, and (M) for 1000 iterations are enumerated according to the sample sizes. The source of $X^2(P)$ and $X^2(L)$ is the random number generator print-out. Since (M) is an exact probability, f_e for the 7 sample sets, each containing the 3 α regions of interest, is easily calculated from Tate and Hyer's tables.² For each sample set, the lowest and highest probability within the 3 regions is ascertained. By subtracting the lowest from the highest, the probability range within each region is determined. When this result is multi-

¹Tate and Hyer, pp. 2, 13.

²Tate and Hyer, pp. 28-72.

plied by 1000, the frequency within each region is obtained for this study.

There is very little difference in the observed frequencies of $X^2(P)$ and $X^2(L)$ except when there are large numbers of indeterminate $X^2(L)$. This occurs in all five categories when $E(x) = 3$ and when $E(x) = 5$ and $K = 8$. This bias favors the selection of the Pearson chi-square statistic over the log-linear likelihood ratio statistic unless it is known a priori that zero or small cell frequencies are unlikely to be observed. Specifically, $X^2(P)$ has a tendency to have fewer values in the .005-.050 region than $X^2(L)$ but more in the .051-.100 region. If the statistic is being used for goodness of fit tests, either one could be used with the preceding constraints. $X^2(P)$ is generally less than the theoretical frequencies expected in the three regions, when K is less than 7. This would cause a researcher sometimes to make a Type II error and fail to reject the null hypothesis when the null hypothesis was false.

The results tabulated for $K = 8$, $\phi = 1/8$, $N = 80$, 40, and 24, are worthy of special consideration since the observed results are so divergent from the theoretical and since they also support many of Tate and Hyer's conclusions. They also found that:

1. When f_e were five or fewer, the mean errors increased as the number of categories increased.

2. The percentage error of $X^2(P)$ decreased as the (M) probabilities increased over the .005-.100 region.

3. If close approximations to the exact probabilities are needed, the $X^2(P)$ test is not satisfactory when $E(x)$ are fewer than about 10, and, even when they are more than 10, the approximation may at times be poor. On the other hand, if one is interested only in whether the cumulative probability associated with an outcome in a multinomial distribution is less or greater than .05, the chi-square test performs reasonably well with expectations as small as 1.

4. The use of the chi-square in place of the multinomial involves at least 2 types of error, one arising from the approximations that are made in deriving the chi-square function from the multinomial, the other from the fact that the former is a continuous function, while the latter is discrete.

5. All of the proofs of the chi-square distribution assume at some point that the observed frequencies in a category, O_i , are distributed normally about E_i in the i^{th} category. This means that E_i must be greater than zero to preclude positive skewness and large enough to temper discreteness. The question in the application of chi-square to frequency data is that of how large E_i must be to make the assumption of normality in categories tenable.

CHAPTER V contains more specifics as to why some of the preceding divergent results were observed, differences with established authorities, and implications for future research.

TABLE 2

COMPARISONS OF FREQUENCIES OF $X^2(P)$, $X^2(L)$, AND (M)
 FOR 1000 ITERATIONS
 IN VARIOUS PROBABILITY REGIONS

$K = 4, \phi = 1/4, N = 40, 20, \text{ and } 12$

<u>Expected Freq.</u>	<u>.005-.009</u>	<u>.010-.050</u>	<u>.051-.100</u>
	<u>5</u>	<u>41</u>	<u>50</u>
<u>N = 40</u>			
$X^2(P)$	4	45	49
$X^2(L)$	5	51	41
(M)	Not Available		
<u>N = 20</u>			
$X^2(P)$	4	35	48
$X^2(L)$	3	37	57
(M)	5	40	43
<u>N = 12</u>			
$X^2(P)$	0	35	37
$X^2(L)$	1	4	48
(M)	3	25	37

TABLE 2 - ContinuedK = 5, $\phi = 1/5$, N = 50, 25, and 15

<u>Expected Freq.</u>	<u>.005-.009</u>	<u>.010-.050</u>	<u>.051-.100</u>
	<u>5</u>	<u>41</u>	<u>50</u>
<u>N = 50</u>			
X ² (P)	3	54	56
X ² (L)	3	57	46
(M)	Not Available		
<u>N = 25</u>			
X ² (P)	3	36	57
X ² (L)	0	43	50
(M)	5	41	45
<u>N = 15</u>			
X ² (P)	0	34	64
X ² (L)	0	12	17
(M)	3	33	42

K = 6, $\phi = 1/6$, N = 60, 30, and 18

<u>Expected Freq.</u>	<u>.005-.009</u>	<u>.010-.050</u>	<u>.051-.100</u>
	<u>5</u>	<u>41</u>	<u>50</u>
<u>N = 60</u>			
X ² (P)	2	40	47
X ² (L)	6	46	44
(M)	Not Available		
<u>N = 30</u>			
X ² (P)	3	43	29
X ² (L)	7	33	38
(M)	5	40	49
<u>N = 18</u>			
X ² (P)	3	25	50
X ² (L)	0	3	13
(M)	5	41	46

TABLE 2 - Continued

$$K = 7, \phi = 1/7, N = 70, 35, \text{ and } 21$$

<u>Expected Freq.</u>	<u>.005-.009</u>	<u>.010-.050</u>	<u>.051-.100</u>
<u>N = 70</u>			
X ² (P)	5	41	50
X ² (L)	6	44	51
(M)	7	41	55
(M)	Not Available		
<u>N = 35</u>			
X ² (P)	3	43	53
X ² (L)	2	42	49
(M)	Not Available		
<u>N = 21</u>			
X ² (P)	5	46	72
X ² (L)	0	2	23
(M)	5	40	45

$$K = 8, \phi = 1/8, N = 80, 40, \text{ and } 24$$

<u>Expected Freq.</u>	<u>.005-.009</u>	<u>.010-.050</u>	<u>.051-.100</u>
<u>N = 80</u>			
X ² (P)	5	41	50
X ² (L)	22	137	111
(M)	34	123	136
(M)	Not Available		
<u>N = 40</u>			
X ² (P)	13	73	79
X ² (L)	7	67	79
(M)	Not Available		
<u>N = 24</u>			
X ² (P)	6	56	64
X ² (L)	1	5	16
(M)	Not Available		

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS FOR

FUTURE RESEARCH

CONCLUSIONS

This study supports the continuing controversy, highlighted in 1949, that began with the publications by Lewis and Burke relative to the use and misuse of the chi-square test. The bibliography reflects the many well-known statisticians who have concerned themselves with the resolution of the required sample size, expected frequencies, sampling techniques, categorization and application of the chi-square statistic to tests of goodness of fit; contingency tables, both simple and multi-dimensional; analysis of variance, univariate and multivariate; analysis of covariance; and many other experimental designs of the block and lattice types.

From the results shown in CHAPTER IV, it is evident that $X^2(P)$ is to be preferred to $X^2(L)$ for small sample sizes and small expected cell frequencies because $X^2(L)$ becomes indeterminate when observed zero cells occur. This occurs more and more frequently as the number of categories are increased and the expected frequencies are decreased for

the one sample case. This does not detract from the use of $\chi^2(L)$ when used in model fitting for multi-dimensional contingency tables.

It is interesting to note that Tate and Hyer had originally intended to include only the probabilities for the multinomials, $k = 3, 4, 5$ and total sample sizes N which would yield expected frequencies not more than 5. As they proceeded, it became evident that the often read statement that the chi-square test is satisfactory when the expected frequencies are not less than 5 and the degrees of freedom 2 or more was not supported by their research. The study was extended to larger samples with $k = 3, 4, 5, 6, 7$ and N 's. Unfortunately, only for $k = 3$ did they tabulate results when the expected frequencies were as large as 10 and $N = 30$.

As shown in Table 2, this study supports the Tate and Hyer findings that the chi-square approximation improved as the multinomial probabilities increased over the .005-.100 region, but the errors were greatest in the .005-.009 region. Another important point of agreement is that when the expected frequencies were 5 or fewer, the errors increased as the number of categories increased, and likewise if the expected frequencies decreased. However, the errors did decrease when the expected frequencies were 10 or more. As the expected frequencies increase, the range of the multinomial probabilities of outcomes having identical chi-square probabilities decreases.

At first glance, the above seems to disagree with McNamee's findings that the chi-square test for first order interaction is quite robust as far as sample size is concerned, when the expected frequency for each cell is as small as 3. He also found that if the cells have a minimum value of 1, the chi-square for second order interaction was within the .03 limit of error allowed. However, this disagreement is only valid if close approximation to exact probabilities is needed, such as reported in a preceding section, EVALUATION OF THE HYPOTHESES OF EQUAL PROPORTIONS. Table 1 discloses that the chi-square test is not satisfactory when expected frequencies are fewer than about 10, and, even when they are more than 10, the approximation may be poor. However, the chi-square test performs reasonably well with small expectations, even as small as 1, if the researcher is interested only in the probability values of the multinomial or chi-square distribution in the upper right hand tail and wishes to know if the probability of an outcome is less than or greater than .05. The rule of 5 is no better than the rule of 1 when chi-square is used to test the hypothesis that the parameters of a multinomial distribution have specified values against the alternative that at least one parameter is not as specified.

In another study comparing multinomial and chi-square probabilities for samples with unequal ϕ , El Shanawany found that most chi-square probabilities would lead to the

same conclusion as the multinomial probabilities if one accepted the null hypothesis when P was greater than .05, remained in doubt when P was between .05 and .01, and rejected the hypothesis when P was less than .01.¹

Snedecor² and Cochran³ explain the reason for some of the divergent results tabulated in Tables 1 and 2, particularly in chi-square distributions with the larger number of categories and smaller expected frequencies. Obviously, the chi-square distributions that were generated for the Monte Carlo methodology had different degrees of skewness and kurtosis. A measure of the amount of skewness in a population is given by the average value of $(X - \mu)^3$, taken over the population. This quantity is called the third moment about the mean and, when divided by σ^3 , to render the measure independent of scale, the result is the coefficient of skewness. Since the mean of the population is seldom known, the sample estimate $\sqrt{b_1}$ or g_1 is usually calculated as follows:

$$\sqrt{b_1} = g_1 = m_3 / (m_2 \sqrt{m_2})$$

where the second moment $m_2 = \sum (X - \bar{X})^2/n$ and the third moment $m_3 = \sum (X - \bar{X})^3/n$. If the sample comes from a nor-

¹M. R. Shanawany, "An Illustration of the Accuracy of the Chi-square Approximation," Biometrika, 1936, 28, pp. 315-345.

²Snedecor and Cochran, pp. 86-89.

³William G. Cochran, Sampling Techniques, 2d ed. (New York: John Wiley & Sons, 1963), p. 43.

mal distribution, g_1 is approximately normally distributed with mean zero and S.D. $\sqrt{(6/n)}$.

Kurtosis is a further type of departure from normality. In a population, a measure of kurtosis is the value of the fourth moment $(X - \mu)^4$ divided by σ^4 . For the normal distribution, this ratio has the value of 3. If the ratio exceeds 3, there is usually an excess of values near the mean with a corresponding depletion of the tails of the distribution curve. Ratios less than 3 result from curves that have a flatter top than the normal. A sample estimate of the fourth moment is given by:

$$g_2 = b_2 - 3 = (m_4/m_2^2) - 3$$

$$\text{where } m_4 = \sum (X - \bar{X})^4/n$$

In large samples, over 1000, from the normal distribution, g_2 is normally distributed with mean zero and S.D. $\sqrt{24/n}$.

In samples from non-normal populations, the quantities g_1 and g_2 are used as estimates of the population values. The measures of skewness and kurtosis both go to zero when the sample size increases as expected from The Central Limit Theorem. It should be noted that kurtosis is damped much faster than the skewness. The purpose of this rather lengthy discussion is to emphasize the effects that this study's sampling from non-normal distributions had on the variance in those samples.

As Cochran points out, "One effect of non-normality is that the estimated variance may be more highly variable from sample to sample than we expect . . ." The variance of s^2 in random samples can be expressed as:

$$V(s^2) = \frac{2\sigma^4}{n-1} \left\{ 1 + \frac{n-1}{n} \cdot \frac{g_2}{2} \right\}$$

"The factor outside the brackets is the variance of s^2 in samples from a normal population. The term inside the brackets is the factor by which normal variance is multiplied when the population is non-normal." Readers may well recognize that the above formula states that the variance of s^2 is the sum of variance when the parent population is normal and $\frac{K_4}{n}$, the fourth cumulant divided by the sample size. It should be noted that the skewness does not affect the stability of s^2 : the important factor is the fourth moment in the parent population. Further reference to the effects of skewness and kurtosis is made in the next section recommending future extension of some Collier and Baker studies.

RECOMMENDATIONS FOR FUTURE RESEARCH

Since chi-square tests for hypotheses concerning multinomial probabilities are among the most frequently used statistical procedures, the current study suggests many lines that should be investigated further. This research should be duplicated, using sampling from other distributions, particularly the uniform and the Poisson, and the results compared to ascertain if the error patterns would be similar. The Tate and Hyer study should be extended to include more categories and larger expected frequencies, at least 10 for these additional categories and those already tabled for $k = 4, 5, 6$ and 7 . These studies should shed some light upon that area where Cochran suggests using the chi-square for samples with small expected frequencies but fewer than 15 degrees of freedom.

Mayo states that there is difficulty in finding a comprehensive treatment of contingency analysis in the literature and that "Especially conspicuous by its absence is an explanation of how to interpret interaction when the null hypothesis of independence is rejected."¹ McNamee's research was undoubtedly inspired by this. It is suggested

¹Samuel T. Mayo, "Interactions Among Categorical Variables," Educational and Psychological Measurement, 1961, 21, p. 840.

that McNamee's research on the "Robustness of Homogeneity Tests in Parallelepiped Contingency Tables" be extended to include sampling from other than the uniform distribution, preferably sampling from normal, chi-square, and Poisson distributions, in order to get more generalized results.

More complex designs, such as the completely randomized or randomized block, should be broken down into smaller contingency tables and tested by use of the chi-square statistic as suggested by both Snedecor and Cochran.¹ When the initial chi-square test shows a significant value, subsequent tests should be made that may help to explain the high values of chi-square. These subsequent tests should take into account the various studies by Baker and Collier listed in the bibliography which cover the effect of skewness and kurtosis in randomized block designs. Baker and Collier compared results under normal theory and under permutation theory. It is suggested that these studies be extended to include sampling from the chi-square distributions and the Poisson.

This study and its methodology could be adapted to the study of the fit to response surfaces as described by Cochran and Cox.² However, much of the research suggested

¹Snedecor and Cochran, pp. 127 and 242.

²Cochran and Cox, pp. 335-368.

above could be facilitated if the multinomial tables were enlarged.

Research methodologists must not be unmindful of the fact that social, behavioral, and educational scientists currently constitute a growing majority of intermediate consumers of statistical literature.

While it is known that analytical and computer studies are oriented toward the mathematical statisticians, it must also be realized that this group comprises a minority. Scientific recognition of pre-eminent authorities is necessary for mathematical approaches to analytical investigation. Therefore, the present study was intended to encourage renewed analytical concentration upon the questions raised by this empirical research.

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APPENDIX A

HOW TO USE THE MCGILL RANDOM NUMBER PACKAGE "SUPER-DUPER"

To get uniform, normal or exponential random variables, call as ordinary FORTRAN functions. For example,

$$U=UNI(0)$$

will produce a uniform random variable U in the half-open interval $(0,1)$, while

$$V=VNI(0)$$

will produce a uniform variate V in the open interval $(-1,1)$.

Similarly

$$X=RNOR(0)$$

will produce a standard normal random variable X and

$$Y=REXP(0)$$

will produce a standard exponential variate Y .

In each case the arguments of UNI , VNI , $RNOR$ and $REXP$ are dummy integers that are ignored by the subroutine. Thus either $X=RNOR(3)$ or $X=RNOR(362436)$ will cause a normal random variable to be stored in the memory location of X .

The package also includes a provision for random integers:

$$K=IUNI(0)$$

will produce a random integer in the range $0 \leq K < 2^{31}$, while

$$L=IVNI(0)$$

will produce a random signed integer in the full possible range of the 360 machine: $-2^{31} \leq L < 2^{31}$.

The uniform number generator (which is either called directly or else is built into the normal and exponential genera-

tors) combines a multiplicative congruential generator and a shift register generator. The congruential generator uses the multiplier 69069, found after a search of millions of multipliers to have nearly optimal lattice structure in 2, 3, 4 and 5 dimensions - much better than any of the highly touted but poorly justified multipliers used for the past 20 years. But even though the congruential generator is as good as a congruential generator can be, it is still not good enough, and we have combined it with a shift register generator on 32 bits (right shift 15, left shift 17). The bit patterns produced by the two separate generators are added as binary vectors - that is, exclusive or addition. Combining the two generators produces a sequence with period about 5×10^{18} .

The program has built-in starting values for those who forget, or don't care, to assign their own starting values. To assign starting values IS and JS to the congruential and shift register sequences, one uses

```
CALL RSTART(IS,JS),
```

where IS and JS are any two integers within the allowable range of 360 FORTRAN.

Those wanting to use a pure congruential generator with multiplier 69069 may do so by

```
CALL RSTART(IS,0)
```

while those wanting a pure shift register generator would

```
CALL RSTART(0,JS).
```

Do not:

CALL RSTART(0,0)

unless you want a sequence of 5×10^{18} zeros.

The package comes as a source deck, containing one program written in IBM/360 Assembler Language (which may be assembled using the BPS Basic Assembler or any higher level assembler, e.g., E, F or G) and two small FORTRAN Function Subprograms (which require a compiler at the FORTRAN G or higher level).

If you plan to use the package very often, you should have an object deck produced when the source deck is compiled, to simplify and speed up subsequent use. Better yet, if you find the program as useful as we have designed it to be, you may take steps to have it included in your subroutine library so that RSTART, UNI, VNI, RNOR, REXP, IUNI and IVNI may be called as standard functions in the same way as ALOG, COS, SIN, etc.

Timing for the Super-Duper Random Number Package, with some comparisons:

	<u>UNIFORM RANDOM VARIABLES</u>	360/75 0/S	370/155 RAX
		All times in micro-seconds	
X=UNI(0) (Super-duper)		28	38
X=VNI(0) (Super-duper)		30	40
	<u>NORMAL RANDOM VARIABLES</u>		
1. X=SQRT(-*ALOG(UNI(0)))*COS(3.141593*UNI(0))		234	420
2. Polar method		153	230
3. X=RNOR(0) (Super-duper)		45	65
	<u>EXPONENTIAL RANDOM VARIABLES</u>		
1. X=-ALOG(UNI(0))		99	163
2. X=REXP(0) (Super-duper)		49	71

MCGILL RANDOM NUMBER PACKAGE "SUPER-DUPER"

SUMMARY OF CALLING PROCEDURES

FORTRAN STATEMENT	RESULT
U=UNI(0)	U is assigned a normalized floating value in $0 \leq u < 1$, uniform distribution.
U=UNI(1) U=UNI(2) U=UNI(71)	Same result as above. The integer argument of UNI is ignored by the subroutine, as are the integer arguments of VNI, RNOR, REXP, IUNI, IVNI below.
V=VNI(0)	V is assigned a normalized floating point value in the interval $-1 < v < 1$, uniform distribution
X=RNOR(0)	X is assigned a normalized floating point value with the normal (Gaussian) density, mean zero, variance 1.
Y=REXP(0)	Y is assigned a normalized floating point value with the exponential density e^{-y} , $y > 0$.
K=IUNI(0)	K is assigned a random integer value in the range $0 \leq K < 2^{31}$, uniform distribution.
L=IVNI(0)	L is assigned a random integer value, uniform in the range $-2^{31} \leq L < 2^{31}$.
CALL RSTART(I,J)	This call statement should be used before the above functions are called; it starts the congruential generator (multiplier 69069) with I or I + 1, depending on whether I is odd or even, and the shift register generator with J mod 2048. If CALL RSTART(I,J) is not used, the subroutine will use the built-in starting values for I and J. One can make the uniform generator a pure congruential generator by CALL RSTART(I,0) where I is any integer $\neq 0$, and a pure shift register generator with CALL RSTART(0,J) and J $\neq 0$. Avoid CALL RSTART(0,0) - it will produce a sequence of zeros. If CALL RSTART(I,J) is used with both I and J not zero, the generator combines a congruential generator and a shift register generator and has period 5×10^{18} .
Here I and J are any two integers you care to choose, e.g., your social insurance number and your birth date written backwards.	

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1 *
2 * MCGILL UNIVERSITY SCHOOL OF COMPUTER SCIENCE 0C10
3 * 0020
4 * RANDOM NUMBER GENERATOR PACKAGE - 'SUPER-DUPER' 0030
5 * ----- 0040
6 * ----- 0050
7 * UNIFORM,NORMAL AND EXPONENTIAL RANDOM NUMBER GENERATOR 0060
8 * 0070
9 * G. MARSAGLIA, K.ANANTHANARAYANAN, N.PAUL. 0080
10 * 0090
11 * REGISTER USAGE 0100
12 * ----- 0110
13 * GPR 0 - STORES RESULT OF IUNI,IVNI 0120
14 * GPR 1 - (REGB) CALCULATION OF RESULTS 0130
15 * GPR 2 - (REGC) CALCULATION OF RESULTS 0140
16 * GPR 3 - (REGD) CALCULATION OF RESULTS 0150
17 * GPR13 - ADDRESS OF SAVE AREA OF CALLING PROGRAM,OR OF THIS 0160
18 * PROGRAMS'S SAVE AREA ON CALL TO RNORTH OR REXPTH 0170
19 * GPR14 - CONTAINS RETURN ADDRESS. 0180
20 * GPR15 - USED AS BASE REGISTER. 0190
21 * GPR 0 - RESULT OF UNI,VNI,REXP,RNOR. 0200
22 * 0210
23 * 0220
23 RANDOM START 0 DEFINE ENTRY POINTS 0230
24 ENTRY RSTART CALL RSTART(I1,I2) 0240
25 ENTRY UNI U=UNI(0) 0250
26 ENTRY VNI V=VNI(0) 0260
27 ENTRY RNOR X=RNOR(0) 0270
28 ENTRY REXP Y=REXP(0) 0280
29 ENTRY IUNI K=IUNI(0) 0290
30 ENTRY IVNI J=IVNI(0) 0300
31 EXTRN RNORTH FORTRAN FUNCTIONS REQUIRED-RNORTH(I) 0310
32 EXTRN REXPTH REXPTH(I) 0320
33 REGB EQU 1 0330
34 REGC EQU 2 0340
35 REGD EQU 3 0350
36 * 0360
37 * CALL RSTART(I1,I2) I1,I2 ARE USED FOR STARTING THE TWO 0370
38 * SEQUENCES 'MCGN' AND 'SRGN'. 0380
39 USING RSTART,15 0390
40 RSTART STM REGB,REGD,24(13) SAVE REGISTERS 1,2,3 0400
41 LM REGC,REGD,0(1) LOAD ADDRESSES OF I1,I2 INTO REGC,REGD 0410
42 L REGC,0(REGC) LOAD VALUE OF I1 INTO REGC 0420
43 LTR RECC,REGC 0430
44 BC 8,ST1 IF ZERO,STORE AT 'MCGN',ELSE 0440
45 0 RECC,X1 ENSURE ODD,TO KEEP PERIOD OF 'MCGN' LARGE 0450
46 ST1 ST REGC,MCGN STORE AT 'MCGN' 0460
47 L REGD,0(REGD) LOAD I2 INTO REGD 0470
48 LTR RECD,REGD 0480
49 BC 8,ST2 IF ZERO, STORE AT 'SRGN',ELSE 0490
50 N REGD,X7FF TAKE RESIDUE MODJLC 2048 0500
51 0 RECD,X1 AND ENSURE NON-ZERO 0510
52 ST2 ST REGD,SRGN AND STORE AT 'SRGN'. 0520
53 RETRNO LM REGB,REGD,24(13) RESTORE REGISTERS 1,2,3 0530
54 BCF 15,14 AND RETURN 0540
55 * 0550
56 * U=UNI(0) RESULT IS NORMALIZED FLOATING POINT VALUE 0560
57 * UNIFORMLY DISTRIBUTED ON (0.0,1.0). 0570
58 USING UNI,15 0580

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59 UNI STM RECB,REGD,24(13) SAVE REGISTERS 1,2,3 0590
60 RDIGT1 L REGB,SRGN LOAD SRGN INTO REGB 0600
61 LR REGC,REGB AND INTO REGC 0610
62 SRL RECC,15 SHIFT REGC RIGHT 15 BITS 0620
63 XR REGB,REGC AND XOR INTO REGB 0630
64 LR REGC,RFCR COPY REGB INTO REGC 0640
65 SLL REGC,17 SHIFT IT LEFT 17 BITS, 0650
66 XR RECB,REGC AND XOR INTO REGB 0660
67 ST REGB,SRGN SAVE THE NEW 'SRGN' 0670
68 L RECC,MCGN LOAD MCGN INTO REGD 0680
69 M RECC,MULT AND MULTIPLY BY 69069 0690
70 ST REGD,MCGN STORE RESULT,MODULO 2**32, AS NEW 'MCGN' 0700
71 XR REGD,REGB XOR NEW 'MCGN' AND 'SRGN' IN REGD 0710
72 SRL REGD,8 SHIFT REGD RIGHT 8 BITS FOR F.P. FRACTION 0720
73 AL REGD,CHAR ADD CHARACTERISTIC X'40' INTO FIRST BYTE 0730
74 ST RFCR,FWD STORE AT FWD, LOAD INTO FPR 0, 0740
75 LE C,FWD AND ADD NORMALIZED TO ZERC 0750
76 AE 0,Z LEAVING RESULT 'UNI' IN FPR 0. 0760
77 RETRNI LM REGB,REGD,24(13) 0770
78 BCR 15,14 RETURN 0780
79 * 0790
80 * V=VNI(0) RESULT IS NORMALIZED FLOATING POINT VALUE 0800
81 * UNIFORM ON (-1.0,1.0) 0810
82 USING VNI,15 0820
83 VNI STM RECB,REGD,24(13) SAVE REGISTERS 1,2,3 0830
84 RDIGT2 L REGB,SRGN LOAD SRGN INTO REGB 0840
85 LR REGC,REGB AND INTO REGC 0850
86 SRL RECC,15 SHIFT REGC RIGHT 15 BITS 0860
87 XR RECB,REGC AND XOR INTO REGB 0870
88 LR REGC,REGB COPY REGB INTO REGC 0880
89 SLL REGC,17 SHIFT IT LEFT 17 BITS, 0890
90 XR REGB,REGC AND XOR INTO REGB 0900
91 ST REGB,SRGN SAVE THE NEW 'SRGN' 0910
92 L REGD,MCGN LOAD MCGN INTO REGD 0920
93 M RECC,MULT AND MULTIPLY BY 69069 0930
94 ST REGD,MCGN STORE RESULT,MODULO 2**32, AS NEW 'MCGN' 0940
95 XR REGD,REGB XOR NEW 'MCGN' AND 'SRGN' IN REGD 0950
96 SRA REGD,7 SHIFT RIGHT 7 BITS PRESERVING SIGN BIT 0960
97 N REGD,SIGN ZERO OUT LAST 7 BITS OF FIRST BYTE 0970
98 AL REGD,CHAR ADD CHARACTERISTIC X'40' TO FIRST BYTE 0980
99 ST RFCR,FWD STORE AT FWD, LOAD INTO FPR 0 0990
100 LF 0,FWD AND ADD NORMALIZED TO ZERO 1000
101 AE 0,Z LEAVING RESULT 'VNI' IN FPR 0. 1010
102 RETRNI LM REGB,REGD,24(13) 1020
103 BCR 15,14 RETURN 1030
104 * 1040
105 * X=RNDR(0) RESULT IS STANDARD NORMAL VARIATE. 1050
106 * 1060
107 * METHOD 1070
108 * ----- 1080
109 * 1. GENERATE H1H2H3H4H5H6H7H8,8 RANDOM HEXADECIMAL DIGITS. 1090
110 * 1100
111 * 2. IF H1H2 .LT. 68, SET 'RNDR' TO 1110
112 * (NTBL(H1H2)+.H3H4H5H6H7H8)/16, AND QUIT. 1120
113 * 3. IF H1H2 .LT. 00, SET 'RNDR' TO 1130
114 * (-NTBL(H1H2-68)-.H3H4H5H6H7H8)/16, AND QUIT. 1140
115 * 4. IF H1H2H3 .LT. L2F, SET 'RNDR' TO 1150
116 * (NTBL(H1H2H3-CEB)+.H4H5H6H7H8)/16, AND QUIT. 1160

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117 * 5. IF H1H2H3 .LT. F5E, SET 'RNDR' TO 1170
118 * (-NTBL(H1H2H3-E17)-.H4H5H6H7H8)/16, AND QUIT. 1180
119 * 6. ELSE, GENERATE 'RNDR' FROM THE NORMAL TOOTH-TAIL SUBPROGRAM. 1190
120 * 1200
121 * 1210
122 USING RNDR,15 1220
123 RNDR STM REGB,REGD,24(13) SAVE REGISTERS 1,2,3 1230
124 RDIGT3 L REGB,SRGN LOAD SRGN INTO REGB 1240
125 LR REGC,RFCB AND INTO REGC 1250
126 SRL REGC,15 SHIFT REGC RIGHT 15 BITS 1260
127 XR REGB,REGC AND XOR INTO REGB 1270
128 LR REGC,REGB COPY REGB INTO REGC 1280
129 SLL REGC,17 SHIFT IT LEFT 17 BITS, 1290
130 XR REGB,REGC AND XOR INTO REGB 1300
131 ST REGB,SRGN SAVE THE NEW 'SRGN' 1310
132 L REGD,MCGN LOAD MCGN INTO REGD 1320
133 M REGC,MULT AND MULTIPLY BY 69069 1330
134 ST REGD,MCGN STORE RESULT,MODULO 2**32, AS NEW 'MCGN' 1340
135 XR REGC,REGB XOR NEW 'MCGN' AND 'SRGN' IN REGD 1350
136 NRCT SLR REGC,REGC ZERO OUT REGC 1360
137 CL REGD,X68 IF REGD GE 68000000,BRANCH TO 'ND2' 1370
138 RC 11,ND2 1380
139 ND1 SLDL REGC,8 SHIFT FIRST 2 HEX DIGITS INTO REGC 1390
140 IC REGC,NTBL(REGC) FETCH CORRESPONDING BYTE FROM NTBL 1400
141 STC REGC,PSTWRD+1 STORE AS 2ND BYTE OF PSTWRD 1410
142 SRL REGC,8 TAKE REMAINING 24 BITS OF REGD 1420
143 AL REGD,PCHAR FORM FLOATING POINT FRACTION,CHAR X'3F' 1430
144 ST REGD,FRAC AND STORE AT 'FRAC' 1440
145 LE C,PSTWRD ADD 'PSTWRD' AND 'FRAC' 1450
146 AE 0,FRAC LEAVING RESULT IN FPR 0 1460
147 LM REGB,REGD,24(13) 1470
148 BCR 15,14 RETURN 1480
149 ND2 CL REGD,XDC IF REGD GE D0000000,BRANCH TO 'ND3' 1490
150 HC 11,ND3 1500
151 SLDL REGC,8 SHIFT FIRST 2 HEX DIGITS INTO REGC 1510
152 SL REGC,X68R AND SUBTRACT 00000068 1520
153 IC REGC,NTBL(REGC) FETCH CORRESPONDING BYTE FROM NTBL 1530
154 STC REGC,NSTWRD+1 STORE AS 2ND BYTE OF NSTWRD 1540
155 SRL REGD,8 TAKE REMAINING 24 BITS OF REGD 1550
156 AL REGD,PCHAR FORM FLOATING POINT FRACTION,CHAR X'3F' 1560
157 ST REGD,FRAC AND STORE AT 'FRAC' 1570
158 LF 0,NSTWRD SUBTRACT 'FRAC' FROM 'NSTWRD' 1580
159 SE 0,FRAC LEAVING RESULT IN FPR 0 1590
160 LM REGB,REGD,24(13) 1600
161 BCR 15,14 RETURN 1610
162 ND3 CL REGD,XE2F IF REGD GE F2F00000,BRANCH TO 'ND4' 1620
163 HC 11,ND4 1630
164 SLDL REGC,12 SHIFT FIRST 3 HEX DIGITS INTO REGC 1640
165 SL REGC,XCFB AND SUBTRACT 000000CB 1650
166 IC REGC,NTBL(REGC) FETCH CORRESPONDING BYTE FROM NTBL 1660
167 STC REGC,NSTWRD+1 STORE AS 2ND BYTE OF NSTWRD 1670
168 SRL REGD,8 TAKE REMAINING 20 BITS OF REGD 1680
169 AL REGD,PCHAR FORM FLOATING POINT FRACTION,CHAR X'3F' 1690
170 ST REGD,FRAC AND STORE AT 'FRAC' 1700
171 LF C,PSTWRD ADD 'PSTWRD' AND 'FRAC' 1710
172 AE C,FRAC LEAVING RESULT IN FPR 0 1720
173 LM REGB,REGD,24(13) 1730
174 BCR 15,14 RETURN 1740

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175 ND4 CL REGD,XF5E IF REGD GE XF5E0000,BRANCH TO 'NTHTL' 1750
176 HC 11,NTHTL 1760
177 SLCL REGC,12 SHIFT FIRST 3 HEX DIGITS INTO REGC 1770
178 SL RECC,XF17 AND SUBTRACT 00000E17 1780
179 IC REGC,NTBL(RFGC) FETCH CORRESPONDING BYTE FROM NTBL 1790
180 STC REGC,NSTWRD+1 STORE AS 2ND BYTE OF NSTWRD 1800
181 SRL REGD,8 TAKE REMAINING 20 BITS OF REGD 1810
182 AL REGD,PCHAR FORM FLOATING POINT FRACTION,CHAR X'3F' 1820
183 ST REGD,FRAC AND STORE AT 'FRAC' 1830
184 LF 0,NSTWRD SUBTRACT 'FRAC' FROM 'NSTWRD' 1840
185 SE 0,FRAC LEAVING RESULT IN FPR 0 1850
186 LM REGB,REGC,24(13) 1860
187 HCR 15,14 RETURN 1870
188 NTHTL ST RECC,ARG STORE REGD AS ARGUMENT FOR RNRTH ROUTINE 1880
189 STM 14,0,12(13) SAVE ALL REGISTERS FROM 14 TO 3. 1890
190 LR 3,13 COPY PREVIOUS SAVE AREA ADDRESS TO GPR3 1900
191 LA 13,SVAREA LOAD ADDRESS OF SVAREA INTO GPR13 1910
192 ST 13,8(0,3) STORE ADDRESS OF SVAREA IN SAVE AREA 1920
193 ST 3,4(0,13) STORE ADDRESS OF PREVIOUS SAVE AREA 1930
194 LA 1,ARGLST PLACE ADDRESS OF ARGUMENT LIST IN GPR 1 1940
195 L 15,ADNTH 1950
196 BALR 14,15 BRANCH TO SUBPROGRAM 1960
197 LR 13,3 RESTORE ADDRESS OF SAVE AREA IN GPR13 1970
198 MVI 12(13),X'FF' SET RETURN INDICATOR 1980
199 RETRN3 LM 14,REGD,12(13) RESTORE ALL REGISTERS 1990
200 BCR 15,14 RETURN 2000
201 * 2010
202 * Y=REXP(0) RESULT IS STANDARD EXPONENTIAL VARIATE. 2020
203 * 2030
204 * METHOD 2040
205 * 2050
206 * 1. GENERATE H1H2H3H4H5H6H7H8, 8 RANDOM HEXADECIMAL DIGITS 2060
207 * 2070
208 * 2. IF H1H2 .LT. 05, SET 'REXP' TO 2080
209 * (ETBL(H1H2)+.H3H4H5H6H7H8)/16, AND QUIT. 2090
210 * 3. IF H1H2H3 .LT. F17, SET 'REXP' TO 2100
211 * (ETBL(H1H2H3-CFF)+.H4H5H6H7H8)/16, AND QUIT 2110
212 * 4. ELSE,GENERATE 'REXP' FROM THE EXPONENTIAL TOOTH-TAIL SUBPROGRAM 2120
213 * 2130
214 USING REXP,15 2140
215 REXP STM REGB,RLGD,24(13) SAVE REGISTERS 1,2,3 2150
216 RDIGT4 L REGB,SRGN LOAD SRGN INTO REGB 2160
217 LR REGC,REGB AND INTO REGC 2170
218 SRL RLGC,15 SHIFT REGB RIGHT 15 BITS 2180
219 XR REGB,REGC AND XOR INTO REGB 2190
220 LR REGC,REGB COPY REGB INTO REGC 2200
221 SLL REGC,17 SHIFT IT LEFT 17 BITS. 2210
222 XR REGB,REGC AND XOR INTO REGB 2220
223 ST REGB,SRGN SAVE THE NEW 'SRGN' 2230
224 L RECC,MCGN LOAD MCGN INTO REGD 2240
225 M REGC,MULT AND MULTIPLY BY 69069 2250
226 ST RECC,MCGN STORE RESULT,MODULO 2**32, AS NEW 'MCGN' 2260
227 XR REGD,REGB XOR NEW 'MCGN' AND 'SRGN' IN REGD 2270
228 BRCT SLR REGC,REGC ZERO OUT REGC 2280
229 CL REGD,XD5 IF REGD GE D5000000,BRANCH TO 'ED2' 2290
230 HC 11,ED2 2300
231 FDI SLDL REGC,8 SHIFT FIRST 2 HEX DIGITS INTO REGC 2310
232 IC REGC,ETBL(RFGC) FETCH CORRESPONDING BYTE FROM ETBL 2320

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233		STC	REGC,PSTWRD+1	STORE AS 2ND BYTE OF PSTWRD	2330
234		SRL	REGD,8	TAKE REMAINING 24 BITS OF REGD	2340
235		AL	REGD,PCHAR	FORM FLOATING POINT FRACTION,CHAR X'3F'	2350
236		ST	REGD,FRAC	AND STORE AT 'FRAC'	2360
237		LF	C,PSTWRD	ADD 'PSTWRD' AND 'FRAC'	2370
238		AE	O,FRAC	LEAVING RESULT IN FPR 0	2380
239		LM	REGB,REGD,24(13)		2390
240		BCR	15,14	RETURN	2400
241	ED2	CL	REGD,XF17	IF REGD GE F1700000,BRANCH TO 'ETTHTL'	2410
242		BC	11,ETTHTL		2420
243		SLDL	RECC,12	SHIFT FIRST 3 HEX DIGITS INTO REGC	2430
244		SL	REGC,XCFF	AND SUBTRACT 0000CFF	2440
245		IC	RECC,ETRL(REGC)	FETCH CORRESPONDING BYTE FROM ETRL	2450
246		STC	REGC,PSTWRD+1	STORE AS 2ND BYTE OF PSTWRD	2460
247		SRL	REGD,8	TAKE REMAINING 20 BITS OF REGD	2470
248		AL	REGD,PCHAR	FORM FLOATING POINT FRACTION,CHAR X'3F'	2480
249		ST	REGD,FRAC	AND STORE AT 'FRAC'	2490
250		LE	C,PSTWRD	ADD 'PSTWRD' AND 'FRAC'	2500
251		AE	O,FRAC	LEAVING RESULT IN FPR 0	2510
252		LM	REGB,REGD,24(13)		2520
253		BCR	15,14	RETURN	2530
254	ETTHTL	ST	REGD,ARG	STORE REGD AS ARGUMENT FOR REXPTH ROUTINE	2540
255		STM	14,0,12(13)	SAVE ALL REGISTERS FROM 14 TO 3.	2550
256		LR	3,13	COPY PREVIOUS SAVF AREA ADDRESS TO GPR 3	2560
257		LA	13,SVAREA	LOAD ADDRESS OF SVAREA INTO GPR13	2570
258		ST	13,8(0,3)	STORE ADDRESS OF SVAREA IN SAVE AREA	2580
259		ST	3,4(0,13)	STORE ADDRESS OF PREVIOUS SAVE AREA	2590
260		LA	1,ARGLST	PLACE ADDRESS OF ARGUMENT LIST IN GPR 1	2600
261		L	15,ADETH		2610
262		BALR	14,15	BRANCH TO SUBPROGRAM	2620
263		LR	13,3	RESTORE ADDRESS OF SAVE AREA IN GPR13	2630
264		MVI	12(13),X'FF'	SET RETURN INDICATOR	2640
265	RETRN4	LM	14,REGD,12(13)	RESTORE ALL REGISTERS	2650
266		BCR	15,14	RETURN	2660
267	*				2670
268	*		K=IUNI(0)	UNIFORMLY DISTRIBUTED POSITIVE INTEGER.	2680
269	*				2690
270		USING	IUNI,15		2700
271	IUNI	STM	REGB,REGD,24(13)	SAVE REGISTERS 1,2,3	2710
272	RDIGTS	L	REGB,SRGN	LOAD SRGN INTO REGB	2720
273		LR	REGC,REGB	AND INTO REGC	2730
274		SRL	RECC,15	SHIFT REGC RIGHT 15 BITS	2740
275		XR	REGB,REGC	AND XOR INTO REGB	2750
276		LR	RECC,REGB	COPY REGB INTO REGC	2760
277		SLL	REGC,17	SHIFT IT LEFT 17 BITS.	2770
278		XR	REGB,REGC	AND XOR INTO REGB	2780
279		ST	RECH,SRGN	SAVE THE NEW 'SRGN'	2790
280		L	REGD,MCGN	LOAD MCGN INTO REGD	2800
281		M	REGC,MULT	AND MULTIPLY BY 69069	2810
282		ST	REGD,MCGN	STORE RESULT,MCDULO 2**32, AS NEW 'MCGN'	2820
283		XR	RECC,RECH	XOR NEW 'MCGN' AND 'SRGN' IN REGD	2830
284		SRL	REGD,1	SHIFT LEFT 1 BIT,LEAVING SIGN BIT ZERO	2840
285		LR	C,REGC	AND MOVE RESULT 'IUNI' TO GPR 0.	2850
286	PETRNS	LM	REGB,REGD,24(13)		2860
287		BCR	15,14	RETURN	2870
288	*				2880
289	*		J=IVNI(0)	UNIFORMLY DISTRIBUTED INTEGER.	2890
290	*				2900

291	*	METHOD		THE BASIC RANDOM NUMBER IS A COMBINATION	2910
292	*	-----		OF TWO SEPARATELY GENERATED NUMBERS.	2920
293	*			'SRGN' & 'MCGN' AS FOLLOWS.	2930
294	*		1. TEMP=XOR(SRGN,SRGN SHIFTED RIGHT 15 BITS)		2940
295	*		2. SRGN=XOR(TEMP,TEMP SHIFTED LEFT 17 BITS)		2950
296	*		3. MCGN=MCGN*69069,MODULO 2**32		2960
297	*		4. RESULT=XOR(MCGN,SRGN)		2970
298	*				2980
299		USING	IVNI,15		2990
300	IVNI	STM	REGB,REGC,24(13)	SAVE REGISTERS 1,2,3	3000
301	RDIGT6	L	REGB,SRGN	LOAD SRGN INTO REGB	3010
302		LR	REGC,REGB	AND INTO REGC	3020
303		SRL	REGC,15	SHIFT REGC RIGHT 15 BITS	3030
304		XR	REGB,REGC	AND XOR INTO REGB	3040
305		LR	REGC,REGB	COPY REGB INTO REGC	3050
306		SLL	REGC,17	SHIFT IT LEFT 17 BITS,	3060
307		XR	REGB,REGC	AND XOR INTO REGB	3070
308		ST	REGB,SRGN	SAVE THE NEW 'SRGN'	3080
309		L	REGD,MCGN	LOAD MCGN INTO REGD	3090
310		M	REGC,MULT	AND MULTIPLY BY 69069	3100
311		ST	REGD,MCGN	STORE RESULT,MODULO 2**32, AS NEW 'MCGN'	3110
312		XR	REGD,REGB	XOR NEW 'MCGN' AND 'SRGN' IN REGD	3120
313		LR	0,REGD	LEAVE RESULT 'IVNI' IN GPRO	3130
314	RETRN6	LM	'REGB,REGD,24(13)		3140
315		BCR	15,14	RETURN	3150
316	*	CONSTANTS SECTION			3160
317	MULT	DC	F'69069'		3170
318	SRGN	DC	F'01073'		3180
319	X7FF	DC	X'0000C7FF'		3190
320	MCGN	DC	F'12345'		3200
321	X1	DC	X'00000001'		3210
322	FWD	DC	F'0'		3220
323	Z	DC	E'0.0'		3230
324	CHAR	DC	X'40000000'		3240
325	SIGN	DC	X'80FFFFFF'		3250
326	XDS	DC	X'D5000000'		3260
327	XF17	DC	X'F1700000'		3270
328	XGFF	DC	X'00000CFF'		3280
329	XG8	DC	X'68000000'		3290
330	XD0	DC	X'D0000000'		3300
331	XG8R	DC	X'0000C068'		3310
332	XE2F	DC	X'E2F00000'		3320
333	XCE8	DC	X'00000CE8'		3330
334	XF5E	DC	X'F5E00000'		3340
335	XE17	DC	X'00000E17'		3350
336	PSTWRD	DC	X'41AA0C00'		3360
337	NSTWRD	DC	X'C1AA0C00'		3370
338	PCHAR	DC	X'3FC00000'		3380
339	FRAC	DC	F'0'		3390
340	ARG	DS	F		3400
341	ADNTH	DC	A(RNORTH)		3410
342	ADETH	DC	A(REXPTH)		3420
343	ARGLST	DC	X'RC'		3430
344		DC	AL3(ARG)		3440
345	SVAREA	DS	18F		3450
346	NTRL	DC	1X'00'	TABLE USED FOR NORMAL LOCK-UP	3460
347		DC	1X'01'	FIRST PART HAS 104 ELEMENTS	3470
348		DC	2X'02'		3480

349	DC	4X'03'	3490
350	DC	5X'04'	3500
351	DC	1X'09'	3510
352	DC	5X'0A'	3520
353	DC	3X'0E'	3530
354	DC	1X'12'	3540
355	DC	1X'17'	3550
356	DC	5X'0C'	3560
357	DC	5X'01'	3570
358	DC	4X'02'	3580
359	DC	2X'03'	3590
360	DC	1X'04'	3600
361	DC	5X'05'	3610
362	DC	5X'06'	3620
363	DC	5X'07'	3630
364	DC	5X'08'	3640
365	DC	4X'09'	3650
366	DC	4X'0B'	3660
367	DC	4X'0C'	3670
368	DC	4X'0D'	3680
369	DC	1X'0E'	3690
370	DC	3X'0F'	3700
371	DC	3X'10'	3710
372	DC	3X'11'	3720
373	DC	2X'12'	3730
374	DC	2X'13'	3740
375	DC	2X'14'	3750
376	DC	2X'15'	3760
377	DC	2X'16'	3770
378	DC	1X'17'	3780
379	DC	1X'18'	3790
380	DC	1X'19'	3800
381	DC	1X'1A'	3810
382	DC	1X'1B'	3820
383	DC	1X'1C'	3830
384	DC	1X'1D'	3840
385	DC	10X'05'	3850
386	DC	7X'06'	3860
387	DC	5X'07'	3870
388	DC	2X'08'	3880
389	DC	9X'0B'	3890
390	DC	5X'0C'	3900
391	DC	1X'0D'	3910
392	DC	10X'0F'	3920
393	DC	7X'1C'	3930
394	DC	3X'11'	3940
395	DC	12X'13'	3950
396	DC	9X'14'	3960
397	DC	5X'15'	3970
398	DC	2X'16'	3980
399	DC	13X'1E'	3990
400	DC	10X'19'	4000
401	DC	7X'1A'	4010
402	DC	5X'1B'	4020
403	DC	2X'1C'	4030
404	DC	15X'1F'	4040
405	DC	13X'1F'	4050
406	DC	12X'20'	4060

START OF SECOND PART OF NORMAL TABLE
223 ELEMENTS

407	DC	10X'21'
408	DC	9X'22'
409	DC	8X'23'
410	DC	7X'24'
411	DC	6X'25'
412	DC	5X'26'
413	DC	4X'27'
414	DC	3X'28'
415	DC	3X'29'
416	DC	2X'2A'
417	DC	2X'2B'
418	ETBL DC	15X'00'
419	DC	13X'01'
420	DC	9X'02'
421	DC	5X'03'
422	DC	5X'06'
423	DC	8X'08'
424	DC	8X'0A'
425	DC	6X'0C'
426	DC	2X'0F'
427	DC	2X'11'
428	DC	4X'15'
429	DC	1X'19'
430	DC	2X'20'
431	DC	1X'2B'
432	DC	1X'01'
433	DC	4X'02'
434	DC	7X'03'
435	DC	11X'04'
436	DC	10X'05'
437	DC	5X'06'
438	DC	9X'07'
439	DC	1X'08'
440	DC	8X'09'
441	DC	7X'0B'
442	DC	1X'0C'
443	DC	6X'0D'
444	DC	4X'0F'
445	DC	5X'0F'
446	DC	5X'10'
447	DC	3X'11'
448	DC	4X'12'
449	DC	4X'13'
450	DC	4X'14'
451	DC	3X'16'
452	DC	3X'17'
453	DC	3X'18'
454	DC	2X'19'
455	DC	2X'1A'
456	DC	2X'1B'
457	DC	2X'1C'
458	DC	2X'1D'
459	DC	2X'1E'
460	DC	2X'1F'
461	DC	1X'21'
462	DC	1X'22'
463	DC	1X'23'
464	DC	1X'24'

START OF TABLE FOR EXPONENTIALS
FIRST PART HAS 213 ELEMENTS

SECOND PART OF EXPONENTIAL TABLE
455 ELEMENTS

4070
4080
4090
4100
4110
4120
4130
4140
4150
4160
4170
4180
4190
4200
4210
4220
4230
4240
4250
4260
4270
4280
4290
4300
4310
4320
4330
4340
4350
4360
4370
4380
4390
4400
4410
4420
4430
4440
4450
4460
4470
4480
4490
4500
4510
4520
4530
4540
4550
4560
4570
4580
4590
4600
4610
4620
4630
4640

465	DC	1X'25'	4650
466	DC	1X'26'	4660
467	DC	1X'27'	4670
468	DC	1X'28'	4680
469	DC	1X'29'	4690
470	DC	1X'2A'	4700
471	DC	5X'05'	4710
472	DC	2X'07'	4720
473	DC	1X'09'	4730
474	DC	1X'0B'	4740
475	DC	4X'0D'	4750
476	DC	9X'0F'	4760
477	DC	3X'10'	4770
478	DC	10X'12'	4780
479	DC	5X'13'	4790
480	DC	9X'16'	4800
481	DC	6X'17'	4810
482	DC	2X'18'	4820
483	DC	13X'1A'	4830
484	DC	10X'1B'	4840
485	DC	7X'1C'	4850
486	DC	5X'1D'	4860
487	DC	2X'1E'	4870
488	DC	13X'21'	4880
489	DC	11X'22'	4890
490	DC	9X'23'	4900
491	DC	8X'24'	4910
492	DC	6X'25'	4920
493	DC	5X'26'	4930
494	DC	4X'27'	4940
495	DC	2X'28'	4950
496	DC	1X'29'	4960
497	DC	1EX'2C'	4970
498	DC	14X'2D'	4980
499	DC	13X'2E'	4990
500	DC	12X'2F'	5000
501	DC	11X'30'	5010
502	DC	11X'31'	5020
503	DC	10X'32'	5030
504	DC	9X'33'	5040
505	DC	9X'34'	5050
506	DC	8X'35'	5060
507	DC	8X'36'	5070
508	DC	7X'37'	5080
509	DC	7X'38'	5090
510	DC	6X'39'	5100
511	DC	6X'3A'	5110
512	DC	6X'3B'	5120
513	DC	5X'3C'	5130
514	DC	5X'3D'	5140
515	DC	4X'3E'	5150
516	DC	4X'3F'	5160
517	END		5170
518	C ENCR TOOTH FUNCTION		5180
519	FUNCTION RNDRTH(K)		5190
520	DIMENSION C(45)		5200
521	DATA C/Z4CFD2B5F,Z4CFD2B5F,Z4CFAA9AD,Z4CF5A648,Z4CF32496,		5210
522	Z4CEE2131,Z4CF69C1A,Z4CE198B5,Z40DA139E,Z40D28E87,Z40C887BF,		5220

```

523 $ Z40C102A6,Z40B6F0DD,Z40ACF513,Z40A2FE4A,Z4098E78C,Z40916269, 5230
524 $ Z40875BAC,Z407D54D6,Z40734E0D,Z406BC8F6,Z4061C22C,Z405A3D15, 5240
525 $ Z4052B7FE,Z404B32E7,Z4043ADD0,Z403C28B9,Z40372554,Z402FA03D, 5250
526 $ Z402A9CDE,Z40259973,Z4020960E,Z401E145C,Z401910F7,Z40168F45, 5260
527 $ Z40140D93,Z40118BE0,Z3FF0A2E4,Z3FC887BE,Z3FA06C98,Z3F785172, 5270
528 $ Z3F785172,Z3F50364C,Z3F50364C,Z3F50364C/ 5280
529 DATA I1/ZFBC3540C/,I2/ZFE79702E/ 5290
530 IF(K.GT.I1)GO TO 3 5300
531 S=UNI(0) 5310
532 T=UNI(C) 5320
533 B=AINT(7.*(S+T)+37.*ABS(S-T)) 5330
534 X=UNI(0)-UNI(0) 5340
535 RNORTH=.0625*(X+SIGN(B,X)) 5350
536 RETURN 5360
537 3 IF(K.GT.I2)GO TO 5 5370
538 4 RNORTH=2.75*VNI(0) 5380
539 J=16.*ABS(RNORTH)+1. 5390
540 IF(J-14) 6,6,7 5400
541 6 P=(J+J-1)*.1497466E-2 5410
542 GO TO 8 5420
543 7 P=(89-J-J)*.698817E-3 5430
544 8 IF(UNI(0).GT.79.78846*(EXP(-.5*RNORTH*RNORTH) 5440
545 -C(J)-P*(J-16.*ABS(RNORTH)))) GOTO4. 5450
546 RETURN 5460
547 5 V=VNI(0) 5470
548 IF(V.EQ.0) GO TO 5 5480
549 X=SQRT(7.5625-2.*ALOG(ABS(V))) 5490
550 IF(UNI(0)*X.GT.2.75)GO TO 5 5500
551 RNORTH=SIGN(X,V) 5510
552 RETURN 5520
553 END 5530
554 C REXP TOOTH FUNCTION 5540
555 FUNCTION REXPTH(K) 5550
556 DIMENSION C(65) 5560
557 DATA C/Z40F00000,Z40E10000,Z40D40000,Z40C70000,Z40B0000, 5570
558 $ Z40AF0000,Z40AS0000,Z409B0000,Z40910000,Z40890000,Z40800000, 5580
559 $ Z40780000,Z40710000,Z406A0000,Z40640000,Z405E0000,Z40580000, 5590
560 $ Z40530000,Z404E0000,Z40490000,Z40440000,Z40400000,Z403C0000, 5600
561 $ Z40390000,Z40350000,Z40320000,Z402F0000,Z402C0000,Z40290000, 5610
562 $ Z40270000,Z40240000,Z40220000,Z40200000,Z401E0000,Z401C0000, 5620
563 $ Z401A0000,Z40190000,Z40170000,Z40160000,Z40150000,Z40130000, 5630
564 $ Z40120000,Z40110000,Z40100000,Z3FF00000,Z3FE00000,Z3FD00000, 5640
565 $ Z3FC00000,Z3FB00000,Z3FH00000,Z3FA00000,Z3F900000,Z3F900000, 5650
566 $ Z3F800000,Z3F800000,Z3F700000,Z3F700000,Z3F600000,Z3F600000, 5660
567 $ Z3F600000,Z3F500000,Z3F500000,Z3F400000,Z3F400000,Z3F400000/ 5670
568 DATA I1/ZFB4FAA91/ 5680
569 IF(K.GT.I1)GO TO 5 5690
570 1 U1=UNI(C) 5700
571 IF(U1.GT..7917049) GO TO 3 5710
572 T=1.-1.239902*U1 5720
573 REXPTH=-ALOG(T) 5730
574 J=16.*REXPTH+1. 5740
575 IF(UNI(C)*(.0604*T+.0039).GT.T-C(J))GOTO1 5750
576 RETURN 5760
577 3 REXPTH=19.20352*U1-15.20352 5770
578 J=16.*REXPTH+1. 5780
579 EX=EXP(-REXPTH) 5790
580 IF(UNI(0)*(.0604*EX+.0039).GT.EX-C(J))GOTO1 5800

```

```
581 RETURN  
582 5 UI=UNI(C)  
583 IF(U1.EQ.C)GC TC 5  
584 REXPTI=4.-ALOG(U1)  
585 RETURN  
586 END
```

```
5810  
5820  
5830  
5840  
5850  
5860
```

APPENDIX B

H A S P S Y S T E M L O G

```

$ 15.49.19 JOB 771 -- RA18FAC2 -- BEGINNING EXEC - PART 9 - CLASS 1
*15.50.21 JOB 771 IL00011 RA18FAC2 ASM          150K REQ,   150K USED, 00:00:55 ET, 00:00:06 CPU, 11.59%, 0000 (COMP)
*15.50.43 JOB 771 IL00011 RA18FAC2 FOR1        150K REQ,    80K USED, 00:00:21 ET, 00:00:06 CPU, 31.96%, 0000 (COMP)
*15.51.30 JOB 771 IL00011 RA18FAC2 LKED        150K REQ,    96K USED, 00:00:46 ET, 00:00:01 CPU,  3.55%, 0000 (COMP)
*15.51.45 JOB 771 IL00011 RA18FAC2 GO         150K REQ,    38K USED, 00:00:15 ET, 00:00:03 CPU, 26.09%, 0000 (COMP)
$ 15.51.49 JOB 771 -- RA18FAC2 -- END EXECUTION!  973 LINES
    
```

HASP3.1.14 JOB STATISTICS -- 650 CARDS READ -- 986 LINES PRINTED -- 0 CARDS PUNCHED -- 2.50 MINUTES (ELAPSED TIME)


```

//RAIRFAC2 JOB (3101,1,2),MILLER,
// MSGLEVEL=(1,1),CLASS=1,TIME=1
// EXEC ASMGFOR1,PARM,LKED=
XXASMGFOR1 PROC DECK=1
XXASM EXEC PGM=ASMGASM,PARM='B,LO,ES,RL,UM,FX,CO=3,&DECK'
IEF051I SUBSTITUTION JCL - PGM=ASMGASM,PARM='B,LO,ES,RL,UM,FX,CO=3,
XXSYSLIB DD DSN=MATH210,MACLIB,DISP=SHR
XX DD DSN=SYS1,MACLIB,DISP=SHR
XXSYOUT1 DD UNIT=SYSDA,SPACE=(3500,(400,50))
XXSYOUT2 DD UNIT=SYSDA,SPACE=(3500,(400,50))
XXSYOUT3 DD UNIT=SYSDA,SPACE=(3500,(400,50))
//SYSPRINT DD DUMMY
X/SYSPRINT DD SYSOUT=A
XXSYSPUNCH DD SYSOUT=B
XXSYSLIN DD DSN=&LOADSET,UNIT=SYSDA,SPACE=(80,(200,50)),
XX DISP=(MOD,PASS),DCB=(RECFM=F,LRECL=80,BLKSIZE=80)
//ASM.SYSIN DD *

```

```

00000100
00000200
-----
00000300
00000400
00000500
00000600
00000700
-----
00000800
00000900
X00001000
00001100

```

```

IEF236I ALLOC. FOR RAIRFAC2 ASM
IEF237I 251 ALLOCATED TO SYSLIB
IEF237I 150 ALLOCATED TO
IEF237I 157 ALLOCATED TO SYSUT1
IEF237I 155 ALLOCATED TO SYSUT2
IEF237I 157 ALLOCATED TO SYSUT3
IEF237I 301 ALLOCATED TO SYSPUNCH
IEF237I 155 ALLOCATED TO SYSLIN
IEF237I 351 ALLOCATED TO SYSIN

```

```

IEF142I - STEP WAS EXECUTED - COND CODE 0000
IEF205I MATH210.MACLIB KEPT
IEF205I VOL SER NOS= LD3001.
IEF205I SYS1.MACLIB KEPT
IEF205I VOL SER NOS= SYSVOL.
IEF205I SYS77227.T003350.RF000.RAIRFAC2.R0003070 DELETED
IEF205I VOL SER NOS= LD3031.
IEF205I SYS77227.T003350.RF000.RAIRFAC2.R0003071 DELETED
IEF205I VOL SER NOS= LD3029.
IEF205I SYS77227.T003350.RF000.RAIRFAC2.R0003072 DELETED
IEF205I VOL SER NOS= LD3031.
IEF205I SYS77227.T003350.RF000.RAIRFAC2.LOADSET PASSED
IEF205I VOL SER NOS= LD3029.

```

```

IEF373I STEP /ASM / START 1727.1549
IEF374I STEP /ASM / STOP 1727.1550 CPU 0MIN 06.39SEC MAIN 150K LCS UK
IL0002I STEP /ASM / UNIT 251 4 EXCPS
IL0002I STEP /ASM / UNIT 150 26 EXCPS
IL0002I STEP /ASM / UNIT 157 0 EXCPS
IL0002I STEP /ASM / UNIT 155 10 EXCPS
IL0002I STEP /ASM / UNIT 157 2 EXCPS
IL0002I STEP /ASM / UNIT 301 0 EXCPS
IL0002I STEP /ASM / UNIT 155 39 EXCPS
IL0002I STEP /ASM / UNIT 351 519 EXCPS
IL0003I STEP /ASM / EXCPS1 DISK 611 TAPE

```

```

01 UR 519; TP 01 TOTAL 6001
00001200
00001300
00001400
X00001500
00001600

```

```

//FORT.SYSIN DD *
XXF0RT EXEC PGM=IEYF0RT,PARM='LINECNT=60'
XXSYSPRINT DD SYSOUT=A
XXSYSPUNCH DD SYSOUT=B
XXSYSLIN DD DSN=&LOADSET,DISP=(MOD,PASS),
XX DCB=(RECFM=F,LRECL=80,BLKSIZE=80)

```

```

IEF236I ALLOC. FOR RAIRFAC2 FORT
IEF237I 304 ALLOCATED TO SYSPRINT
IEF237I 301 ALLOCATED TO SYSPUNCH
IEF237I 155 ALLOCATED TO SYSLIN
IEF237I 354 ALLOCATED TO SYSIN

```

IEF142I - STEP WAS EXECUTED - COND CODE 0000

```

                IFF2851  SYS77227.1003350.RF000.RA18FAC2.LOADSET      PASSED
                IFF2851  VOL SER NOS= LD3029.
            IFF3731 STEP /FORT / START 77227.1550
            IFF3741 STEP /FORT / STOP 77227.1550 CPU 0MIN 06.92SEC MAIN 80K LCS OK
        IL00021 STEP /FORT / UNIT 384 197 EXCPS
        IL00021 STEP /FORT / UNIT 3C1 0 EXCPS
        IL00021 STEP /FORT / UNIT 155 111 EXCPS
        IL00021 STEP /FORT / UNIT 354 124 EXCPS
        IL00031 STEP /FORT / EXCPS: DISK 1111 TAPE 01 UR 3211 TP 01 TOTAL 4321
                XX LKED EXEC PGM=IEWL,PARM='XREF,LET,LIST',COND=(4,LT,FORT)
                XX SYSLIN DD DSN='SYS1.FORTLIB',DISP=SHR 00001700
                XX DD DSN=SSPLIB,DISP=SHR 00001800
                XXSYSLMOD DD DSN=SGOSET(MAIN),DISP=(NEW,PASS),UNIT=SYSDA, 00001900
                XX SPACE=(TRK,(20,5,1)) X00002000
                XXSYSPRINT DD SYSOUT=A 00002100
                XXSYSDA DD UNIT=SYSDA,DISP=(,DELETE),SPACE=(CYL,(1,1)) 00002200
                XXSYSLIN DD DSN=LOADSET,DISP=(OLD,DELETE), X00002300
                XX DCB=(RECFM=F,LRECL=60,BLKSIZE=60) 00002400
                XX DD DDNAME=SYSIN 00002500
                IFF2361 ALLOC. FOR RA18FAC2 LKED 00002600
                IFF2371 150 ALLOCATED TO SYSLIB
                IFF2371 251 ALLOCATED TO
                IFF2371 157 ALLOCATED TO SYSLMOD
                IFF2371 384 ALLOCATED TO SYSPRINT
                IFF2371 155 ALLOCATED TO SYSDA
                IFF2371 155 ALLOCATED TO SYSLIN
        IFF1421 - STEP WAS EXECUTED - COND CODE 0000
                IFF2851 SYS1.FORTLIB KEPT
                IFF2851 VOL SER NOS= SYSVOL.
                IFF2851 SSPLIB KEPT
                IFF2851 VOL SER NOS= LD3001.
                IFF2851 SYS77227.1003350.RF000.RA18FAC2.GOSET PASSED
                IFF2851 VOL SER NOS= LD3031.
                IFF2851 SYS77227.1003350.RF000.RA18FAC2.R0003079 DELETED
                IFF2851 VOL SER NOS= LD3029.
                IFF2851 SYS77227.1003350.RF000.RA18FAC2.LOADSET DELETED
                IFF2851 VOL SER NOS= LD3029.
            IFF3731 STEP /LKED / START 77227.1550
            IFF3741 STEP /LKED / STOP 77227.1551 CPU 0MIN 01.64SEC MAIN 96K LCS OK
        IL00021 STEP /LKED / UNIT 150 84 EXCPS
        IL00021 STEP /LKED / UNIT 291 14 EXCPS
        IL00021 STEP /LKED / UNIT 157 27 EXCPS
        IL00021 STEP /LKED / UNIT 384 5 EXCPS
        IL00021 STEP /LKED / UNIT 155 18 EXCPS
        IL00021 STEP /LKED / UNIT 155 151 EXCPS
        IL00031 STEP /LKED / EXCPS: DISK 2941 TAPE 01 UR 51 TP 01 TOTAL 2991
                XXGO EXEC PGM=IEWL,PARM='XREF,LET,LIST',COND=(4,LT,FORT),(4,LT,LKED)
                XXSYSDA DD DDNAME=ASMINPT 00002700
                XXSYSPRINT DD SYSOUT=A 00002800
                XXFT05F001 DD DDNAME=FORINPT 00002900
                XXFT06F001 DD SYSOUT=A 00003000
                XXFT07F001 DD SYSOUT=B 00003100
                //GO.FORINPT DD *
                //
                IFF2361 ALLOC. FOR RA18FAC2 GO
                IFF2371 157 ALLOCATED TO PGM=*,DD
                IFF2371 384 ALLOCATED TO SYSPRINT
                IFF2371 355 ALLOCATED TO FT05F001
                IFF2371 385 ALLOCATED TO FT06F001
                IFF2371 3C1 ALLOCATED TO FT07F001
        IFF1421 - STEP WAS EXECUTED - COND CODE 0000
                IFF2851 SYS77227.1003350.RF000.RA18FAC2.GOSET PASSED
    
```

```

IL000021 IFF3741 SIFP /60 VOL SER NU3E LD3031.
IL000021 IFF3741 SIFP /60 START / STOP OMIN 03.92SEC MAIN 36K LCS OK
IL000021 STEP /60 UNIT 197 / 0 EXCPS CPU
IL000021 STEP /60 UNIT 364 / 0 EXCPS
IL000021 STEP /60 UNIT 355 / 2 EXCPS
IL000021 STEP /60 UNIT 385 / 042 EXCPS
IL000021 STEP /60 UNIT 301 / 0 EXCPS OI HR IP
IL000031 EXCPS: / DISK / TABLE 644 DELETED 644
          / SYS 77227.T003350.PF000.HA18FAC2.GUSET 01 TOTAL
          / IFF2851 VOL SER NU3E LD3031.
          / IFF2851 START / STOP OMIN 18.87SEC
          / IFF2851 STEP /60 UNIT 364 / 0 EXCPS CPU
          / IFF3761 JOB /HA18FAC2/ START / STOP OMIN 18.87SEC
          / IFF3761 JOB /HA18FAC2/ STOP
  
```

No. 10

```

0001      DIMENSION INT(5)
0002      READ(5,101,END=50) DF,E
0003      WRITE(6,102) DF,E
0004      101 FORMAT(2F3.0)
0005      102 FORMAT(' DEGREES OF FREEDOM=',F3.0,/, ' EXPECTED FREQ=',F3.0)
0006      ZF=0
0007      DO 20 J=1,40
0008      DO 10 I=1,5
0009      10 INT(I)=0
0010      DO 5 I=1,15
0011      Y1=REXP(0)
0012      Y2=REXP(0)
0013      X=(Y1+Y2)*2
0014      WRITE (6,103) Y1,Y2,X
0015      103 FORMAT(3F10.5)
0016      IF (X.GT.1.049) GO TO 4
0017      INT(1)=INT(1)+1
0018      GO TO 5
0019      4 IF (X.GT.2.753) GO TO 3
0020      INT(2)=INT(2)+1
0021      GO TO 5
0022      3 IF (X.GT.4.045) GO TO 2
0023      INT(3)=INT(3)+1
0024      GO TO 5
0025      2 IF (X.GT.5.989) GO TO 1
0026      INT(4)=INT(4)+1
0027      GO TO 5
0028      1 INT(5)=INT(5)+1
0029      5 CONTINUE
0030      CHISQ=0
0031      CHISQ=0
0032      DO 15 I=1,5
0033      IF (INT(I).NE.0) GO TO 14
0034      ZF=1
0035      GO TO 15
0036      14 CHISQ=CHISQ+INT(I)*ALOG(INT(I)/E)
0037      15 CHISQ=CHISQ+((INT(I)-E)**2)/E
0038      IF (ZF) 18,18,17
0039      17 CHISQ=999
0040      PL=99
0041      DL=999
0042      IERR=999
0043      ZF=0
0044      GO TO 19
0045      18 CHISQ=CHISQ+CHISQ
0046      CALL CDIR(CHISQ,DF,PL,DL,IERR)
0047      19 CALL CDIR(CHISQ,DF,PP,DP,IERR)
0048      20 WRITE(6,100) INT,CHISQ,PP,DP,IERR,CHISQ,PL,DL,IERR
0049      100 FORMAT(' CELL FREQ',5I3, ' CHISQ=',F7.4, ' P=',F7.5, '
A DEG=',F7.4, ' IERR=',I2.3X, 'CHISQ=',F7.4, ' P=',F7.5, ' DE
B=',F7.4, ' IERR=',I2)
0050      50 STOP
0051      END

```

		SUBPROGRAMS CALLED							
SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION
TRCOM#	10C	REXP	110	CDTR	114	ALOG	118		
		SCALAR MAP							
SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION
DF	150	E	154	ZF	158	J	15C	I	160
Y1	164	Y2	168	X	16C	CHISQP	170	CHISQL	174
PL	178	DL	17C	IERRL	180	IEERL	184	PP	188
DP	18C	IEER	190						
		ARRAY MAP							
SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION
INT	194								
		FORMAT STATEMENT MAP							
SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION
101	1A8	102	1AF	103	1DF	100	1E6		

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*OPTIONS IN EFFECT* ID,ERCDTC,SOURCE,NOLIST,NODECK,LOAD,MAP
*OPTIONS IN EFFECT* NAME = MAIN , LINECNT = 80
*STATISTICS* SOURCE STATEMENTS = 51,PROGRAM SIZE = 1586
*STATISTICS* NO DIAGNOSTICS GENERATED
    
```

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C RNROR 100TH FUNCTION
0001 FUNCTION RNRORTH(K)
0002 DIMENSION C(45)
0003 DATA C/240F02B5F,740FD2B5F,740FAA9AD,Z40F5A648,Z40F32496,
$ 240E12131,Z40E69C1A,Z40E19885,Z40DA139E,Z40D28E87,Z40C887BE,
$ 240L162A8,Z40B6FBDD,Z40AC4513,Z40A2EE4A,Z4098E780,Z40916269,
$ 240B75BA0,Z407D54D0,Z40734E0D,Z4068C0F6,Z4061C22C,Z40583015,
$ 24052B7FE,Z404832L7,Z4043A0D0,Z403C28B9,Z40372554,Z402FA03D,
$ 2402A9CDB,Z40259973,Z4020960E,Z401E145C,Z401910F7,Z40168F45,
$ 240140D93,Z401188E0,Z3FF0A2E4,Z3FC887BE,Z3FA06C98,Z3F785172,
$ Z3F785172,Z3F50364C,Z3F50364C,Z3F50364C/
0004 DATA I1/ZFC35400/,I2/ZFE79702F/
0005 IF(K.GI.I1)GO TO 3
0006 S=UNI(0)
0007 T=UNI(0)
0008 H=AINT(7.*(S+T)+37.*ABS(S-T))
0009 X=UNI(0)-UNI(0)
0010 RNRORTH=.0625*(X+SIGN(H,X))
0011 RETURN
0012 3 IF(K.GT.I2)GO TO 5
0013 4 RNRORTH=2.75*VNI(0)
0014 J=16.*ABS(RNRORTH)+1.
0015 IF(J-14) 6,6,7
0016 6 P=(J+J-1)*.1497466E-2
0017 GO TO 8
0018 7 P=(85-J-J)*.698817E-3
0019 8 IF(UNI(0).GT.79.78846*(EXP(-.5*RNRORTH*RNRORTH)
$ -C(J)-P*(J-16.*ABS(RNRORTH)))) GOTO4
0020 RETURN
0021 5 V=VNI(0)
0022 IF(V.EG.0) GO TO 5
0023 X=SQRT(7.5625-2.*ALOG(ABS(V)))
0024 IF(UNI(0)*X.GT.2.75)GO TO 5
0025 RNRORTH=SIGN(X,V)
0026 RETURN
0027 END

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		SUBPROGRAMS CALLED							
SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION
UNI	D4	VNI	D8	EXP	DC	SQRT	E0	ALOG	E4
		EQUIVALENCE DATA MAP							
SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION
RNORTH	I38								
		SCALAR MAP							
SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION
I1	I3C	I2	I40	K	I44	S	I48	T	I4C
D	I50	X	I54	J	I58	P	I5C	V	I60
		ARRAY MAP							
SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION
C	I64								

OPTIONS IN EFFECT IO,EBODIC,SOURCE,NOLIST,NODECK,LOAD,MAP
 OPTIONS IN EFFECT NAME = RNORTH , LINECNT = 60
 STATISTICS SOURCE STATEMENTS = 27,PROGRAM SIZE = 1368
 STATISTICS NO DIAGNOSTICS GENERATED

	C	REXP TOOTH FUNCTION	5540
0001		FUNCTION REXPTH(K)	5550
0002		DIMENSION C(65)	5560
0003		DATA C/240F00000,Z40E10000,Z40D40000,Z40C70000,Z40RH0000,	5570
		\$ Z40AF0000,Z40AS0000,Z409H0000,Z40Y10000,Z40890000,Z40600000,	5580
		1 Z407B0000,Z40710000,Z406A0000,Z40640000,Z405E0000,Z40580000,	5590
		\$ Z405S0000,Z404F0000,Z40490000,Z40440000,Z40400000,Z403C0000,	5600
		\$ Z40390000,Z40350000,Z40320000,Z402F0000,Z402C0000,Z40290000,	5610
		\$ Z40270000,Z40240000,Z40220000,Z40200000,Z401E0000,Z401C0000,	5620
		\$ Z401A0000,Z40190000,Z40170000,Z40160000,Z40150000,Z40130000,	5630
		\$ Z40120000,Z40110000,Z40100000,Z3FF00000,Z3FE00000,Z3FD00000,	5640
		\$ Z3FC00000,Z3FB00000,Z3FB00000,Z3FA00000,Z3F900000,Z3F900000,	5650
		\$ Z3F800000,Z3F800000,Z3F700000,Z3F700000,Z3F600000,Z3F600000,	5660
		\$ Z3F600000,Z3F500000,Z3F500000,Z3F400000,Z3F400000,Z3F400000/	5670
0004		DATA 11/ZFB4FAA91/	5680
0005		IF(K.GT.11)GO TO 5	5690
0006	1	U1=UNI(0)	5700
0007		IF(U1.GT..7917049) GO TO 3	5710
0008		T=1.-1.239962*U1	5720
0009		REXPTRH=-ALOG(T)	5730
0010		J=16.*REXPTRH+1.	5740
0011		IF(UNI(U1)*(0.0604*T+.0039).GT.T-C(J))GO TO 1	5750
0012		RETURN	5760
0013	3	REXPTRH=19.20352*U1-15.20352	5770
0014		J=16.*REXPTRH+1.	5780
0015		EX=EXP(-REXPTRH)	5790
0016		IF(UNI(U1)*(0.0604*EX+.0039).GT.EX-C(J))GO TO 1	5800
0017		RETURN	5810
0018	5	U1=UNI(0)	5820
0019		IF(U1.EQ.0)GO TO 5	5830
0020		REXPTRH=4.-ALOG(U1)	5840
0021		RETURN	5850
0022		END	5860

		SUBPROGRAMS CALLED							
SYMBOL UNI	LOCATION BC	SYMBOL ALOG	LOCATION CO	SYMBOL EXP	LOCATION C4	SYMBOL	LOCATION	SYMBOL	LOCATION
SYMBOL REXPTH	LOCATION ER	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION
SYMBOL II EX	LOCATION EC 100	SYMBOL K	LOCATION FO	SYMBOL UI	LOCATION F4	SYMBOL T	LOCATION FB	SYMBOL J	LOCATION FC
SYMBOL C	LOCATION 104	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION

OPTIONS IN EFFECT IO,EBODIC,SOURCE,NOLIST,NODECK,LOAD,MAP

OPTIONS IN EFFECT NAME = REXPTH , LINECNT = 60

STATISTICS SOURCE STATEMENTS = 22,PROGRAM SIZE = 1076

STATISTICS NO DIAGNOSTICS GENERATED

STATISTICS NO DIAGNOSTICS THIS STEP

F35-LEVEL LINKAGE EUI FOR OPTIONS SPECIFIED NONE
DEFAULT OPTION(S) USED - SIZE=(92160*28672)
***** UOFS NOT EXIST UOI HAS BEEN ADDED TO DATA SLI

$\chi^2_{.20} = 1.107$
 $\chi^2_{.05} = 2.335$
 $\chi^2_{.01} = 6.635$
 $\chi^2_{.001} = 8.163$

$\chi^2 + Y^2 = X^2$

DEGREE OF FREEDOM = *	EXPLOITED FREQ = 3	1	2	3	4	5	6	7	8	9	10
1	1.77000	4.87617									
2	0.56456	3.85577									
3	1.90725	4.90302									
4	0.76441	1.58065									
5	0.25727	1.26655									
6	0.95595	4.57372									
7	0.40684	2.17153									
8	1.16184	3.83046									
9	0.34565	3.41218									
10	0.97768	8.77752									
11	0.32061	6.81012									
12	1.07559	9.65565									
13	0.44498	9.89680									
14	0.56729	1.70609									
15	1.18850	2.31150									

CELL FREQ	P=0.0000	DEN=0.26424	CHISUP=2.0000	IERR=0	CHISQL=1.8645	P=0.23934	DEN=0.1835	IERR=0	CHISUP	IERR
1	1.89833	9.37827								
2	0.00338	9.07651								
3	0.75317	10.77512								
4	0.43594	2.43554								
5	1.42259	4.50462								
6	0.53111	1.40380								
7	4.23764	12.25280								
8	0.44305	1.45473								
9	0.07795	3.38014								
10	1.49012	6.75388								
11	0.31566	1.01120								
12	0.20580	2.36109								
13	1.75156	4.03170								
14	0.19325	3.17671								
15	0.44338	1.83611								

CELL FREQ	P=0.26424	DEN=0.1835	CHISUP=2.0000	IERR=0	CHISQL=2.4057	P=0.33840	DEN=0.1806	IERR=0	CHISUP	IERR
1	1.09293	5.50231								
2	0.37461	1.37427								
3	1.61276	5.68170								
4	0.11090	1.36627								
5	0.25141	4.41690								
6	0.12660	3.26191								
7	2.19721	1.22042								
8	0.60229	5.06229								
9	1.00209	9.27302								
10	0.68091	1.48853								
11	0.14822	1.90335								
12	2.96354	8.95559								
13	0.39094	2.47697								
14	1.05586	3.36330								
15	1.78928	8.47451								

CELL FREQ	P=0.14430	DEN=0.1711	CHISUP=1.3533	IERR=0	CHISQL=1.3592	P=0.14874	DEN=0.1722	IERR=0	CHISUP	IERR
1	1.06122	4.21035								
2	0.69362	2.23034								
3	0.31853	0.65900								
4	0.21274	0.65909								
5	0.07446	2.45254								
6	0.79818	7.97380								
7	0.01286	2.11849								
8	0.38095	2.17449								
9	1.08157	8.54218								
10	0.39029	2.35161								
11	1.57797	4.15825								

0.09864	0.79328	1.01564								
0.11967	1.98170	4.20274								
0.26365	0.97833	3.88395								
0.35642	3.26439	7.24162								
CELL FRF0	3 5 2 2	3	CHISQP= 2.0000	P=0.26424	DEN= 0.1839	IERR= 0	CHISQL= 1.8645	P=0.23934	DEN= 0.1835	IERR= 0
1.50667	0.09820	3.20974								
0.24646	1.58437	3.66164								
0.21741	1.21702	2.87005								
1.00448	0.66516	3.33927								
3.20461	0.77880	7.96681								
3.15670	0.12330	0.55999								
0.30997	0.57407	1.70008								
0.22472	0.13712	0.72366								
0.79714	0.56201	2.71949								
0.92196	1.04636	5.13605								
1.38500	0.95554	4.68107								
0.82643	0.80531	3.38347								
0.87126	0.81140	3.36532								
5.15009	1.87364	14.04746								
0.34210	1.53791	3.86001								
CELL FRF0	1 2 7 2	3	CHISQP= 7.3333	P=0.68071	DEN= 0.0469	IERR= 0	CHISQL= 6.4212	P=0.83018	DEN= 0.0647	IERR= 0
0.46635	0.83164	2.59590								
0.37067	0.91080	2.56306								
1.67992	0.40862	4.15908								
6.62541	1.44771	10.14622								
1.32218	0.39203	3.42342								
0.56119	1.50244	4.12727								
0.05777	1.42782	2.97116								
0.13856	0.47688	1.23048								
3.23039	1.82855	10.11767								
3.24560	2.78537	12.06193								
0.31428	0.22317	1.07489								
0.09086	0.24580	1.87336								
1.69409	0.08808	3.52435								
0.59073	1.93876	5.05898								
0.31196	0.44574	1.51540								
CELL FRF0	3 3 3 3	3	CHISQP= 0.0	P=0.0	DEN= 0.0	IERR= 0	CHISQL= 0.0	P=0.0	DEN= 0.0	IERR= 0
0.89159	0.43732	2.05783								
0.54045	1.01827	3.11744								
0.07806	0.30243	0.74498								
1.54232	0.40480	3.97423								
1.20954	0.07056	2.68019								
1.84620	1.60886	6.91613								
0.89282	2.30958	6.40461								
1.47076	1.59238	6.12630								
2.56462	0.14426	5.41817								
0.04756	1.14766	2.35005								
0.05074	0.90927	1.92001								
0.35527	1.45097	3.61248								
0.75503	0.73006	2.98218								
0.21742	1.24294	2.92671								
0.78323	0.35298	2.35241								
CELL FRF0	1 5 5 1	3	CHISQP= 5.3333	P=0.74523	DEN= 0.0926	IERR= 0	CHISQL= 5.8221	P=0.78716	DEN= 0.0792	IERR= 0
1.51202	0.03774	4.29952								
1.03901	0.47038	3.01810								
1.86029	0.44031	4.60122								
3.13262	2.03271	11.53060								
0.24668	0.45807	1.56550								
1.08073	1.05078	4.20302								
0.20617	0.26738	0.94811								
0.25101	0.67358	1.86670								
3.38882	1.06551	8.90866								

Z test = Uniform Distributions.

1.95970	0.13469	4.10877	5.3333	0.0926	5.1783	0.073051	0.0972	IERR= 0	
2.24333	0.01712	4.52090		CHISQ= 0	CHISQ= 5.1783	P=0.73051	DEN= 0	IERR= 0	
3.24031	3.89228	4.24118	16	0.77097					
2.00819	0.77358	5.56354		0.48445					
1.27493	0.14398	2.75781		0.88326					
0.47204	0.14187	5.27743	6	1.92701					
CELL FRG0	2	4	CHISUP= 5.3333	P=0.74523	DEN= 0	CHISQ= 5.1783	P=0.73051	DEN= 0	
0.25325	0.11623	0.77097		0.56525				IERR= 0	
2.40520	1.00703	0.48445		3.43175					
2.03971	0.70192	0.88326		1.94146					
0.42796	0.53554	3.43175		2.95753					
0.49660	1.21908	1.94146		5.45836					
1.86432	0.16441	2.95753		5.11671					
1.35730	0.12146	5.45836		3.39604					
2.54722	0.09525	5.11671		2.03000					
1.29152	0.48620	3.39604		0.79543					
0.01344	1.00134	2.03000		0.10494					
3.13386	0.26366	0.79543		2.27030					
2.77685	0.31561	0.10494		6.54335					
0.77524	1.03991	2.27030	2	5	CHISUP= 3.3333	P=0.49633	DEN= 0	IERR= 0	
0.42562	2.84605	6.54335	2	2.63847		CHISQ= 3.5906	P=0.53577	DEN= 0.1491	
CELL FRG0	1	4	3	4.83910		CHISQ= 3.5906	P=0.53577	DEN= 0	
0.92401	0.39522	0.32507		0.32824				IERR= 0	
1.92388	0.32507	0.32824		3.44985					
1.79810	1.37602	0.32824		5.29804					
1.35160	0.34785	3.44985		7.89415					
2.04945	0.52857	5.29804		4.32849					
0.98767	0.82792	7.89415		6.19948					
2.19671	1.52543	4.32849		4.33921					
0.64042	1.52343	6.19948		0.50063					
2.41008	0.68876	4.33921		4.58282					
1.58984	0.77977	0.50063		0.35226					
0.08343	0.19654	4.58282		0.48984					
0.69880	1.59261	0.35226		5.33505					
0.15880	6.00733	0.48984		3	CHISUP= 4.6667	P=0.67676	DEN= 0	IERR= 0	
0.07817	0.15675	5.33505	2	3	CHISUP= 4.6667	P=0.67676	DEN= 0	IERR= 0	
0.31751	2.48097	3.10113	3	1	4	CHISUP= 4.6667	P=0.67676	DEN= 0	
CELL FRG0	0	1	4	0.40113		CHISQ= 4.4987	P=0.65730	DEN= 0.1186	
0.72527	0.40113	1.45456		3.39075		CHISQ= 4.4987	P=0.65730	DEN= 0	
0.44226	0.44226	0.35858		6.11983				IERR= 0	
2.21050	0.84942	1.42171		2.83008					
0.46125	1.42171	3.76591		3.47426					
0.073232	0.88272	3.47426		2.68333					
0.01266	1.72748	3.47426		2.52760					
3.32209	1.15195	2.68333		0.75340					
0.19154	1.05013	2.52760		6.43159					
0.02103	1.16298	0.75340		4.43354					
0.12835	0.33339	6.43159		4.88624					
0.14689	0.24148	4.43354		3	CHISUP= 0.8667	P=0.64462	DEN= 0	IERR= 0	
3.43082	0.70527	4.88624	3	4	CHISUP= 0.8667	P=0.64462	DEN= 0	IERR= 0	
1.58315	0.63362	0.21287	2	3	4	CHISUP= 0.8667	P=0.64462	DEN= 0	
2.27304	0.21287	1.29010	2	3	4	CHISUP= 0.8667	P=0.64462	DEN= 0	
CELL FRG0	0	2	3	4	0.40607		CHISQ= 0.6796	P=0.04618	DEN= 0.1210
0.23294	1.29010	2.52674		1.28145				IERR= 0	
0.02284	6.78933	1.28145		3.39075					
0.59227	0.28145	3.39075		6.44303					
0.09290	1.00248	6.44303		0.09434					
3.34660	0.06039	0.20369		2.17322					
3.81869	0.20369	0.15553							
0.93188	0.15553								

0.34474	1.20775	3.10498							
0.22328	0.29231	2.43117							
1.26339	2.59747	7.72172							
0.22931	2.63164	5.72190							
2.15945	1.96307	6.24503							
0.21378	0.73309	1.89374							
0.00901	0.22327	0.46456							
2.18238	0.45617	5.27710							
CELL FREQ	2 3 3 3 4	CHISQP= 0.6667	P=0.04462	DEN= 0.1194	IERR= 0	CHISQL= 0.6796	P=0.04618	DEN= 0.1210	IERR= 0
0.36961	0.53495	1.80912							
0.44083	2.06836	5.01839							
0.09654	0.00143	0.19544							
0.45551	0.80730	2.52563							
0.50793	0.54210	2.10004							
0.29626	0.69554	1.98359							
0.74481	0.33346	2.15655							
0.13163	1.12919	2.52166							
1.06643	0.49966	3.13657							
0.99442	2.34835	6.68554							
0.23075	0.90924	2.27997							
0.05587	1.71629	3.54432							
2.53548	0.60134	6.27365							
1.07852	1.59485	5.34674							
0.01211	0.07445	0.17312							
CELL FREQ	2 7 2 2 2	CHISQP= 6.6667	P=0.84541	DEN= 0.0595	IERR= 0	CHISQL= 5.3747	P=0.74904	DEN= 0.0915	IERR= 0
0.43325	0.65897	2.18444							
0.06009	0.44187	1.00392							
1.35490	0.93617	2.78212							
0.03403	1.30940	2.90665							
0.49553	0.33148	1.65461							
2.39527	1.85459	8.49973							
0.26406	0.25558	1.03929							
0.66985	0.38410	2.10789							
0.05445	0.82120	1.75149							
1.48562	2.50466	7.98056							
0.42791	0.19730	1.25042							
2.12160	1.42262	7.06724							
0.01971	0.11414	0.26769							
0.86021	0.27489	2.27018							
1.52986	1.03837	5.13647							
CELL FREQ	4 5 2 1 3	CHISQP= 3.3333	P=0.49633	DEN= 0.1574	IERR= 0	CHISQL= 3.5906	P=0.53577	DEN= 0.1491	IERR= 0
0.78921	0.13355	1.84552							
0.09629	5.80679	11.40616							
0.05262	1.35575	2.77673							
0.67306	4.28650	9.91914							
1.02449	1.76780	5.58457							
0.61328	1.31729	3.86114							
0.72922	0.22254	1.90432							
1.43352	0.59343	4.05390							
2.81777	1.35556	8.34653							
1.07787	2.87810	7.91195							
0.61570	9.11010	1.45160							
0.33665	0.13596	0.94525							
1.06382	2.28748	0.58260							
1.28032	4.37098	11.30261							
0.19308	0.26439	0.95494							
CELL FREQ	3 2 2 2 6	CHISQP= 4.0000	P=0.59399	DEN= 0.1353	IERR= 0	CHISQL= 3.4522	P=0.51481	DEN= 0.1536	IERR= 0
1.56822	0.04296	0.22236							
0.21760	0.47500	1.38518							
2.53837	0.88000	0.95575							
1.28411	1.28358	5.09539							
1.43953	3.48955	9.85817							

0.22225	0.85126	2.14702	CHISup= 2.0000	P=0.26424	DEN= 0.1839	IERR= 0	CHISQL= 2.4057	P=0.33840	DEN= 0.1806	IERR= 0
0.13148	4.14143	0.58983	1	4	3	4	3	4	3	4
1.67200	0.80872	4.96142	0.50301	0.69121	2.10420	0.89121	2.10420	0.99435	10.24715	0.99435
0.67990	0.71714	2.79330	0.99435	4.12522	1.44361	0.17028	3.02278	0.57892	5.78921	0.57892
1.25516	0.60863	1.88739	0.60863	1.99228	0.19732	0.90232	0.89046	1.28045	0.89046	1.28045
0.44703	3.87667	2.65015	0.44703	2.39048	0.04669	0.17218	0.46895	0.46895	0.46895	0.46895
0.54132	1.08612	2.89915	1.08612	0.27438	0.15921	4.15009	1.31012	0.59117	2.86149	0.59117
1.27740	0.27438	5.09082	0.27438	0.50301	0.59117	4.15009	1.31012	0.59117	2.86149	0.59117
CELL FRFO	1	4	3	4	3	4	3	4	3	4
0.25212	0.16089	0.89121	0.16089	0.99435	10.24715	0.99435	10.24715	0.99435	10.24715	0.99435
0.11310	4.12522	1.44361	4.12522	1.99228	0.19732	0.90232	0.89046	1.28045	0.89046	1.28045
0.17256	1.99228	0.19732	1.99228	0.04669	0.17218	0.46895	0.46895	0.46895	0.46895	0.46895
0.34282	0.04669	0.17218	0.04669	0.15921	4.15009	1.31012	0.59117	2.86149	0.59117	2.86149
2.66154	4.15009	1.31012	4.15009	0.59117	2.86149	0.59117	2.86149	0.59117	2.86149	0.59117
0.10256	0.59117	2.86149	0.59117	0.59117	2.86149	0.59117	2.86149	0.59117	2.86149	0.59117
CELL FRFO	7	1	2	3	4	5	6	7	8	9
0.32121	0.21205	1.09652	0.21205	0.99291	5.70960	1.09652	5.70960	3.00147	2.89981	2.90081
1.89712	0.99291	5.70960	0.99291	1.29801	0.31836	0.89981	2.90081	7.92105	6.19424	19.17939
1.03157	1.29801	0.31836	1.29801	0.62237	7.92105	6.19424	19.17939	1.96878	0.13224	0.93153
3.33816	0.62237	7.92105	0.62237	6.19424	19.17939	1.96878	0.13224	0.93153	1.03153	1.03153
1.42937	6.19424	19.17939	6.19424	3.04445	0.70708	1.03153	1.03153	1.03153	1.03153	1.03153
0.80972	3.04445	0.70708	3.04445	0.56143	0.44111	5.66713	0.66713	5.66713	5.66713	5.66713
0.76167	0.56143	0.44111	0.56143	1.55992	0.77867	3	5	6	7	8
0.27012	1.55992	0.77867	1.55992	0.16771	0.16771	0.16771	0.16771	0.16771	0.16771	0.16771
0.25743	0.16771	0.16771	0.16771	0.16771	0.16771	0.16771	0.16771	0.16771	0.16771	0.16771
2.27542	0.16771	0.16771	0.16771	0.16771	0.16771	0.16771	0.16771	0.16771	0.16771	0.16771
1.67564	0.16771	0.16771	0.16771	0.16771	0.16771	0.16771	0.16771	0.16771	0.16771	0.16771
CELL FRFO	2	3	2	3	2	3	2	3	2	3
0.06065	0.10671	0.06106	0.10671	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106
0.35353	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106
1.61723	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106
0.54065	1.61723	0.06106	1.61723	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106
0.14549	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106
1.47298	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106
3.61916	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106
0.81209	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106
2.28653	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106
0.67417	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106
2.27386	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106
0.63706	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106
0.52090	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106
1.09809	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106
1.09809	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106	0.06106
CELL FRFO	3	1	2	3	4	5	6	7	8	9
1.62276	0.03907	4.92386	0.03907	0.06710	2.08983	1.27722	0.06710	2.08983	1.27722	0.06710
1.27722	0.06710	2.08983	0.06710	1.06822	0.35491	1.06822	0.35491	1.06822	0.35491	1.06822
0.35491	1.06822	0.35491	1.06822	0.35491	1.06822	0.35491	1.06822	0.35491	1.06822	0.35491

CELL	FREQ	CHISQ	P	DEN	IERR	CHISQ	P	DEN	IERR	CHISQ	P	DEN	IERR
0.16718	0.13480	0.13480	1.3333	0.14430	0.1711	0.66493	1.3592	0.14874	0.1722	0.66493	1.3592	0.14874	0.1722
0.13524	1.57671	1.57671	1.3333	0.14430	0.1711	3.40598	1.3592	0.14874	0.1722	3.40598	1.3592	0.14874	0.1722
0.61324	1.23369	1.23369	1.3333	0.14430	0.1711	1.75433	1.3592	0.14874	0.1722	1.75433	1.3592	0.14874	0.1722
0.27633	0.42336	0.42336	1.3333	0.14430	0.1711	1.40344	1.3592	0.14874	0.1722	1.40344	1.3592	0.14874	0.1722
1.92905	0.08316	0.08316	1.3333	0.14430	0.1711	3.22442	1.3592	0.14874	0.1722	3.22442	1.3592	0.14874	0.1722
0.64460	1.30175	1.30175	1.3333	0.14430	0.1711	3.50087	1.3592	0.14874	0.1722	3.50087	1.3592	0.14874	0.1722
0.51640	1.43571	1.43571	1.3333	0.14430	0.1711	4.62401	1.3592	0.14874	0.1722	4.62401	1.3592	0.14874	0.1722
2.33168	0.29937	0.29937	1.3333	0.14430	0.1711	7.53478	1.3592	0.14874	0.1722	7.53478	1.3592	0.14874	0.1722
0.31650	0.83025	0.83025	1.3333	0.14430	0.1711	1.63174	1.3592	0.14874	0.1722	1.63174	1.3592	0.14874	0.1722
0.34746	0.83025	0.83025	1.3333	0.14430	0.1711	2.34240	1.3592	0.14874	0.1722	2.34240	1.3592	0.14874	0.1722
0.21337	0.95783	0.95783	1.3333	0.14430	0.1711	2.04629	1.3592	0.14874	0.1722	2.04629	1.3592	0.14874	0.1722
0.15567	0.86748	0.86748	1.3333	0.14430	0.1711	6.60234	1.3592	0.14874	0.1722	6.60234	1.3592	0.14874	0.1722
0.71075	2.59042	2.59042	1.3333	0.14430	0.1711	5.14831	1.3592	0.14874	0.1722	5.14831	1.3592	0.14874	0.1722
2.53546	0.08830	0.08830	1.3333	0.14430	0.1711	0.52821	1.3592	0.14874	0.1722	0.52821	1.3592	0.14874	0.1722
0.14735	0.06860	0.06860	1.3333	0.14430	0.1711	2.62821	1.3592	0.14874	0.1722	2.62821	1.3592	0.14874	0.1722
0.26766	4.20460	4.20460	1.3333	0.14430	0.1711	4.69887	1.3592	0.14874	0.1722	4.69887	1.3592	0.14874	0.1722
3.56302	1.72209	1.72209	1.3333	0.14430	0.1711	10.57925	1.3592	0.14874	0.1722	10.57925	1.3592	0.14874	0.1722
1.12534	0.91642	0.91642	1.3333	0.14430	0.1711	4.28353	1.3592	0.14874	0.1722	4.28353	1.3592	0.14874	0.1722
0.16506	0.97001	0.97001	1.3333	0.14430	0.1711	2.27135	1.3592	0.14874	0.1722	2.27135	1.3592	0.14874	0.1722
2.62045	0.62077	0.62077	1.3333	0.14430	0.1711	6.62326	1.3592	0.14874	0.1722	6.62326	1.3592	0.14874	0.1722
0.62130	0.24368	0.24368	1.3333	0.14430	0.1711	1.72996	1.3592	0.14874	0.1722	1.72996	1.3592	0.14874	0.1722
2.11106	0.01209	0.01209	1.3333	0.14430	0.1711	4.24628	1.3592	0.14874	0.1722	4.24628	1.3592	0.14874	0.1722
2.11646	0.47615	0.47615	1.3333	0.14430	0.1711	5.26524	1.3592	0.14874	0.1722	5.26524	1.3592	0.14874	0.1722
0.52323	0.14091	0.14091	1.3333	0.14430	0.1711	3.33225	1.3592	0.14874	0.1722	3.33225	1.3592	0.14874	0.1722
3.22372	0.10209	0.10209	1.3333	0.14430	0.1711	6.65182	1.3592	0.14874	0.1722	6.65182	1.3592	0.14874	0.1722
0.00051	0.48999	0.48999	1.3333	0.14430	0.1711	9.96100	1.3592	0.14874	0.1722	9.96100	1.3592	0.14874	0.1722
1.20939	2.52480	2.52480	1.3333	0.14430	0.1711	7.46838	1.3592	0.14874	0.1722	7.46838	1.3592	0.14874	0.1722
1.38280	1.26751	1.26751	1.3333	0.14430	0.1711	5.30066	1.3592	0.14874	0.1722	5.30066	1.3592	0.14874	0.1722
0.13711	0.72092	0.72092	1.3333	0.14430	0.1711	1.71606	1.3592	0.14874	0.1722	1.71606	1.3592	0.14874	0.1722
1.96529	1.14902	1.14902	1.3333	0.14430	0.1711	6.22863	1.3592	0.14874	0.1722	6.22863	1.3592	0.14874	0.1722
0.26437	0.10200	0.10200	1.3333	0.14430	0.1711	9.74433	1.3592	0.14874	0.1722	9.74433	1.3592	0.14874	0.1722
0.64414	3.18423	3.18423	1.3333	0.14430	0.1711	7.57477	1.3592	0.14874	0.1722	7.57477	1.3592	0.14874	0.1722
0.11204	0.21353	0.21353	1.3333	0.14430	0.1711	0.77712	1.3592	0.14874	0.1722	0.77712	1.3592	0.14874	0.1722
0.10424	0.25286	0.25286	1.3333	0.14430	0.1711	0.73421	1.3592	0.14874	0.1722	0.73421	1.3592	0.14874	0.1722
0.87783	2.53711	2.53711	1.3333	0.14430	0.1711	6.62988	1.3592	0.14874	0.1722	6.62988	1.3592	0.14874	0.1722
1.47691	0.32583	0.32583	1.3333	0.14430	0.1711	3.74948	1.3592	0.14874	0.1722	3.74948	1.3592	0.14874	0.1722
0.34320	3.15700	3.15700	1.3333	0.14430	0.1711	7.00041	1.3592	0.14874	0.1722	7.00041	1.3592	0.14874	0.1722
0.83302	1.28876	1.28876	1.3333	0.14430	0.1711	7.24356	1.3592	0.14874	0.1722	7.24356	1.3592	0.14874	0.1722
0.95653	0.91712	0.91712	1.3333	0.14430	0.1711	3.75130	1.3592	0.14874	0.1722	3.75130	1.3592	0.14874	0.1722
0.21584	0.73157	0.73157	1.3333	0.14430	0.1711	6.61481	1.3592	0.14874	0.1722	6.61481	1.3592	0.14874	0.1722
1.15745	1.23159	1.23159	1.3333	0.14430	0.1711	4.60228	1.3592	0.14874	0.1722	4.60228	1.3592	0.14874	0.1722
1.30752	0.17122	0.17122	1.3333	0.14430	0.1711	2.95806	1.3592	0.14874	0.1722	2.95806	1.3592	0.14874	0.1722
0.87491	0.78254	0.78254	1.3333	0.14430	0.1711	3.31369	1.3592	0.14874	0.1722	3.31369	1.3592	0.14874	0.1722
0.01460	0.01184	0.01184	1.3333	0.14430	0.1711	1.25348	1.3592	0.14874	0.1722	1.25348	1.3592	0.14874	0.1722

CHISQ= 1.3333 P=0.14430 DEN= 0.1711 IERR= 0 CHISQ= 1.3592 P=0.14874 DEN= 0.1722 IERR= 0

CHISQ= 1.3333 P=0.14430 DEN= 0.1711 IERR= 0 CHISQ= 1.3592 P=0.14874 DEN= 0.1722 IERR= 0

CHISQ= 8.6667 P=0.53001 DEN= 0.0284 IERR= 0 CHISQ=***** P=***** DEN=***** IERR= 0

CELL FREQ	4	1	4	2	4	CHISQP= 2.6667	P=0.38494	DEN= 0.1757	IERR= 0	CHISQL= 3.0853	P=0.45634	DEN= 0.1649	IERR= 0
1.90494		1.40463		6.73914									
1.39415		0.54466		3.87761									
0.61767		0.03896		1.31326									
0.30679		0.87380		2.3611d									
0.41557		0.89288		2.61688									
0.37123		0.46196		1.66638									
0.25729		1.38813		3.29083									
1.23792		1.36743		5.21071									
0.91823		0.00455		1.84556									
2.19667		1.21204		6.81742									
0.38978		0.37326		1.52008									
0.29150		0.38092		1.34486									
0.32174		1.20484		3.05315									
0.72388		1.08494		3.61765									
2.91444		0.41299		6.65484									
CELL FREQ	3	4	4	1	3	CHISQP= 2.0000	P=0.26424	DEN= 0.1839	IERR= 0	CHISQL= 2.4057	P=0.33840	DEN= 0.1806	IERR= 0
0.93026		0.62314		3.10679									
1.96497		0.46905		4.86804									
1.60509		1.04230		5.29478									
1.68243		1.20599		5.77685									
2.84691		1.08431		7.66243									
1.68829		0.36178		4.16012									
0.56314		2.60385		6.33396									
0.06029		1.02941		2.17940									
0.16532		1.45224		3.23512									
0.99033		0.01440		2.00946									
0.74283		2.17167		5.82899									
0.48956		0.57004		2.11921									
0.20080		0.03169		0.46496									
0.16237		0.45354		1.23160									
1.72502		0.24852		3.94709									
CELL FREQ	2	3	3	5	2	CHISQP= 2.0000	P=0.26424	DEN= 0.1839	IERR= 0	CHISQL= 1.8645	P=0.23934	DEN= 0.1835	IERR= 0
0.80137		0.81360		3.22995									
3.76383		0.43457		8.39679									
1.83149		0.46040		4.56377									
0.19904		0.21415		0.82638									
0.06407		0.66693		1.46201									
2.06042		0.13605		4.35293									
0.79856		0.55219		2.70149									
0.01293		1.52848		3.68281									
0.31511		0.59222		1.81466									
1.43814		0.06378		3.06383									
0.12750		0.50030		1.25560									
0.80745		1.32846		4.27183									
0.01049		2.34517		4.71132									
0.65997		0.02004		1.36062									
0.40851		0.79239		3.20180									
CELL FREQ	4	2	4	4	1	CHISQP= 2.6667	P=0.38494	DEN= 0.1757	IERR= 0	CHISQL= 3.0853	P=0.45634	DEN= 0.1649	IERR= 0
0.00140		0.71988		1.44211									
0.33828		1.93410		4.54477									
0.73049		1.18114		3.82325									
0.47663		0.72180		2.39685									
0.51015		0.66839		2.23708									
1.82216		2.56351		8.65133									
3.36760		1.42134		9.57188									
0.56793		2.42562		5.96706									
0.83837		0.45833		2.59340									
0.89487		1.66133		5.11239									
0.44995		0.17495		1.32980									
0.04276		3.27319		8.71190									
0.48243		0.54282		2.95649									

2.84424	0.48696	0.66140								
CELL FREQ	3	3	CHISUP= 2.6667	P=0.38494	DEN= 0.1757	IERR= 0	CHISQL= 2.9110	P=0.42718	DEN= 0.1698	IERR= 0
0.70074	2.59801	3.35211								
0.75895	0.35620	2.60483								
0.05883	0.54547	3.56935								
1.96533	0.71448	2.42740								
4.26477	3.66166	7.83990								
7.06619	2.30840	7.27851								
0.74084	0.33652	0.84580								
0.71838	0.32076	6.27386								
0.68743	0.53159	3.08569								
0.45672	0.54957	4.86515								
2.06691	0.54957	4.07557								
0.49841	0.09170	0.46480								
0.78129	1.75594	3.52123								
0.50349	0.56227	2.26898								
0.53220	0.57046	2.66473								
CELL FREQ	4	4	CHISUP= 2.6667	P=0.38494	DEN= 0.1757	IERR= 0	CHISQL= 3.0853	P=0.45634	DEN= 0.1649	IERR= 0
0.07804	0.35620	2.60483								
0.94621	0.54547	3.56935								
1.23920	0.71448	2.42740								
1.99925	3.66166	7.83990								
0.25829	2.30840	7.27851								
0.86618	0.33652	0.84580								
1.61618	0.32076	6.27386								
1.03320	0.53159	3.08569								
2.26110	0.54957	4.86515								
1.34176	0.69603	4.07557								
0.23070	0.09170	0.46480								
0.00468	1.75594	3.52123								
CELL FREQ	2	3	CHISUP= 4.6567	P=0.67676	DEN= 0.1131	IERR= 0	CHISQL= 4.4987	P=0.65730	DEN= 0.1186	IERR= 0
0.55197	0.56227	2.26898								
0.80445	0.57046	2.66473								
1.07745	0.57046	3.25576								
0.59333	2.01782	5.23430								
0.45178	0.13697	1.17752								
1.00340	0.50696	3.02071								
2.36059	0.55158	5.82394								
0.01729	1.02559	2.06575								
3.36040	0.44599	1.93138								
0.67107	2.59376	12.02032								
0.20274	0.07158	1.46529								
0.31076	0.08114	1.16175								
1.02144	0.46114	1.70380								
0.625125	0.65626	3.74149								
CELL FREQ	3	2	CHISUP= 3.3333	P=0.49633	DEN= 0.1574	IERR= 0	CHISQL= 3.5906	P=0.53577	DEN= 0.1491	IERR= 0
1.17576	0.32404	3.22557								
0.80497	0.26535	2.41832								
3.44529	0.57929	3.56863								
1.12189	0.09207	8.04916								
0.26543	1.04364	2.61622								
0.07137	1.27654	2.47361								
0.39750	0.22248	2.41997								
1.62620	0.25951	3.77541								
0.82912	0.49618	2.65059								
0.62264	1.37877	4.00292								
1.13836	2.66251	7.60174								

0.05220	0.85448	1.81335								
1.23376	0.04643	2.56039								
0.23162	0.42389	1.31103								
0.36598	1.09016	2.91227								
CELL FREQ	1 7 4 1	2 7	CHISQP= 8.6667	P=0.93001	DEN= 0.0284	IERR= 0	CHISQL= 8.1473	P=0.91367	DEN= 0.0347	IERR= 0
1.73292	0.39203	4.24990								
0.46156	0.30122	1.52555								
0.12046	0.49902	1.23896								
2.00194	0.23232	4.40850								
0.62503	0.03537	1.32080								
0.39214	0.22449	1.23326								
0.22082	0.24754	0.93672								
0.61950	1.49888	4.63916								
2.38067	1.92445	8.61023								
0.64791	1.18707	3.66995								
1.66322	0.57401	4.47447								
2.53974	0.03460	5.14067								
0.57434	1.61197	4.37261								
2.19955	0.14608	4.69126								
1.07549	0.08319	2.31736								
CELL FREQ	5 1 1 7	1 1	CHISQP=10.6667	P=0.96942	DEN= 0.0129	IERR= 0	CHISQL=10.3787	P=0.96549	DEN= 0.0145	IERR= 0
2.51157	1.55017	8.12348								
0.04636	0.17066	0.43403								
2.32194	0.79492	6.23371								
1.20614	0.25946	2.93119								
3.35869	0.31057	7.35851								
0.13165	2.47023	5.20375								
1.24719	0.12385	2.74209								
0.23116	1.43382	3.32997								
0.56840	3.42074	7.97627								
0.10609	4.22075	8.65367								
1.59598	0.60794	4.40784								
2.62630	0.08819	5.42898								
1.20875	0.62918	3.67568								
0.24472	2.86750	6.32385								
0.84350	0.24433	2.17567								
CELL FREQ	1 2 3 3	6	CHISQP= 4.6667	P=0.67676	DEN= 0.1131	IERR= 0	CHISQL= 4.4987	P=0.65730	DEN= 0.1186	IERR= 0
0.36033	1.94145	4.60356								
3.98852	0.66832	9.31367								
0.28360	3.95625	8.47970								
4.08769	1.55942	11.49422								
1.28706	2.21810	7.01051								
0.25374	3.26736	7.04221								
1.98956	0.57333	5.12577								
2.15689	0.26153	4.63083								
3.76064	0.38538	8.29262								
0.22980	0.35626	1.17211								
0.14658	0.91526	2.12508								
0.07385	2.62168	5.39105								
0.16648	0.43383	1.20063								
0.83924	3.26474	8.20836								
2.61766	0.36516	5.84683								
CELL FREQ	2 1 0 5	7	CHISQP=11.3333	P=0.97694	DEN= 0.0096	IERR= 0	CHISQL=*****	P=*****	DEN=*****	IERR= 0
0.87461	1.78532	5.31966								
0.54362	0.29279	1.67262								
0.20411	1.34623	3.10867								
0.39110	0.15612	1.09443								
0.33470	0.91557	2.50044								
0.05636	2.70976	5.53224								
0.78057	0.23060	2.12253								
0.38663	0.34442	1.46211								
0.21257	0.39337	1.21368								

1.46113	0.65013	0.22252	4.6667	0.1131	0	0.67676	P=0.04618	0.6796	0.1210	0
0.28552	1.52331	3.61767	0.42067	4.74333	0	CHISWP=	0.6667	0.6796	0.1210	0
0.42187	0.40944	1.66663	1.71507	3.91225	0	CHISOL=	0.6667	0.6796	0.1210	0
1.52723	0.03975	3.13396	0.26019	1.70911	0	CHISOL=	0.6667	0.6796	0.1210	0
2.73249	0.05915	5.70326	2.41000	13.69928	0	CHISOL=	0.6667	0.6796	0.1210	0
0.74214	0.79025	2.05379	0.92373	2.19969	0	CHISOL=	0.6667	0.6796	0.1210	0
CELL FREQ	3	4	3	3	3	CHISWP=	0.6667	0.6796	0.1210	0
1.95092	0.42067	4.74333	0.42067	4.74333	0	CHISOL=	0.6667	0.6796	0.1210	0
0.89396	1.71507	3.91225	1.71507	3.91225	0	CHISOL=	0.6667	0.6796	0.1210	0
0.63436	0.26019	1.70911	0.26019	1.70911	0	CHISOL=	0.6667	0.6796	0.1210	0
4.43964	2.41000	13.69928	2.41000	13.69928	0	CHISOL=	0.6667	0.6796	0.1210	0
0.17612	0.92373	2.19969	0.92373	2.19969	0	CHISOL=	0.6667	0.6796	0.1210	0
0.84196	1.26597	2.05385	1.26597	2.05385	0	CHISOL=	0.6667	0.6796	0.1210	0
0.73320	1.15063	3.76768	1.15063	3.76768	0	CHISOL=	0.6667	0.6796	0.1210	0
0.60028	0.69574	1.39204	0.69574	1.39204	0	CHISOL=	0.6667	0.6796	0.1210	0
2.00540	0.42730	4.86540	0.42730	4.86540	0	CHISOL=	0.6667	0.6796	0.1210	0
0.03535	0.48635	1.74340	0.48635	1.74340	0	CHISOL=	0.6667	0.6796	0.1210	0
1.18847	2.06938	7.75568	2.06938	7.75568	0	CHISOL=	0.6667	0.6796	0.1210	0
0.86943	1.19445	4.12778	1.19445	4.12778	0	CHISOL=	0.6667	0.6796	0.1210	0
0.01048	0.92018	0.86212	0.92018	0.86212	0	CHISOL=	0.6667	0.6796	0.1210	0
0.00940	0.67174	1.36237	0.67174	1.36237	0	CHISOL=	0.6667	0.6796	0.1210	0
3.50539	0.69162	5.39403	0.69162	5.39403	0	CHISOL=	0.6667	0.6796	0.1210	0
CELL FREQ	4	3	3	3	3	CHISWP=	0.6667	0.6796	0.1210	0

APPENDIX C

```

0001      DIMENSION INT(10),BND(10)
0002      INTEGER BK
0003      60 READ(5,101,END=50) DF,E,BND
0004      IDF=DF+.1
0005      WRITE(6,102) DF,E,(BND(I),I=1,IDF)
0006      I1=IDF+1
0007      KK=DF/2.+.1
0008      ZF=0
0009      ITEMS=(DF+1)*E+.1
0010      ASSIGN 6 TO BK
0011      IF (MOD(IDF,2).EQ.1) ASSIGN 7 TO BK
0012      DO 20 J=1,40
0013      DO 10 I=1,I1
0014      INT(I)=0
0015      DO 9 I=1,ITEMS
0016      Y=0
0017      DO 4 K=1,KK
0018      YTEMP=REXP(0)
0019      WRITE(6,104) YTEMP
0020      8 Y=Y+YTEMP
0021      104 FORMAT(' YTEMP=',F8.4)
0022      GO TO BK,(6,7)
0023      6 Y=Y+Y
0024      GO TO 5
0025      7 Z=RNGR(0)
0026      WRITE(6,105) Z
0027      105 FORMAT(' Z=',F8.4)
0028      Y=Y+Z/2
0029      5 WRITE(6,103) Y
0030      103 FORMAT(' Y=',F8.4)
0031      DO 4 K=1,IDF
0032      IF (Y.GT.BND(K)) GO TO 4
0033      INT(K)=INT(K)+1
0034      GO TO 4
0035      4 CONTINUE
0036      INT(I1)=INT(I1)+1
0037      9 CONTINUE
0038      CHISQP=0
0039      CHISQL=0
0040      DO 15 I=1,I1
0041      IF (INT(I).NE.0) GO TO 14
0042      ZF=1
0043      GO TO 15
0044      14 CHISQL=CHISQL+INT(I)*ALOG(INT(I)/E)
0045      15 CHISQP=CHISQP+((INT(I)-E)**2)/E
0046      IF (ZF) 16,16,17
0047      17 CHISQL=999
0048      PL=99
0049      DL=99
0050      IERRL=999
0051      ZF=0
0052      GO TO 19
0053      16 CHISQL=CHISQL+CHISQP
0054      CALL CDBR(CHISQL,DF,PL,DL,IERRL)
0055      19 CALL CDBR(CHISQP,DF,PL,DF,IERR)
0056      WRITE(6,100) CHISQP,PE,DF,IERR,CHISQL,PL,DL,IERRL,(INT(I),I=1,I1)
0057      WRITE(6,100) CHISQP,PE,DF,IERR,CHISQL,PL,DL,IERRL,(INT(I),I=1,I1)
0058      20 CONTINUE

```

```
0059      GO TO 50
0060      50 END FILE 6
0061      STOP
0062      100 FORMAT(
           A DEN='F6.4,' IERR='I1,' CHISQ='F7.4,' P='F7.5,'
           B.4,' IERR='I1,' CELL FREQ='I013)
0063      101 FORMAT(2F3.0,10F7.4)
0064      102 FORMAT(' DEGREES OF FREEDOM='F3.0,/, ' EXPECTED FREQ='F3.0,/,
           1' BOUNDARY VALUES='10F8.4)
0065      END
```


DEGREES OF FREEDOM= 7.

EXPLCTED FREQ= 3.

BOUNDARY VALUES= 3.1040 4.2550 5.2870 6.3460 8.0920 9.0370 11.3390 .

YTEMP= 0.6681

YTEMP= 1.7700

YTEMP= 1.2433

Z= -0.0221

Y= 7.3633

YTEMP= 1.3873

YTEMP= 1.0643

YTEMP= 0.7644

Z= 1.2259

Y= 7.9347

YTEMP= 0.2573

YTEMP= 0.3760

YTEMP= 0.6559

Z= 0.1309

Y= 2.5956

YTEMP= 0.4060

YTEMP= 0.3789

YTEMP= 1.1618

Z= 0.5659

Y= 4.6154

YTEMP= 0.9450

YTEMP= 0.7519

YTEMP= 0.5777

Z= -1.9371

Y= 0.3228

YTEMP= 0.5845

YTEMP= 0.3206

YTEMP= 3.7522

Z= 0.7631

Y= 10.6969

YTEMP= 0.4450

YTEMP= 0.6031

YTEMP= 0.5073

Z= 1.7242

Y= 5.0036

YTEMP= 0.1855

YTEMP= 0.9703

YTEMP= 1.2860

Z= -0.0235

Y= 4.8853

YTEMP= 0.1779

YTEMP= 0.6604

YTEMP= 0.3854

Z= -1.8003

Y= 5.6883

YTEMP= 1.8860

YTEMP= 1.4501

YTEMP= 0.1905

Z= -0.0651

Y= 7.1342

YTEMP= 0.2789

YTEMP= 0.9056

YTEMP= 0.5873

Z= -1.2514

Y= 5.0677

YTEMP= 1.4981

YTEMP= 0.1732

YTEMP= 0.6347

Z= -0.6709
Y= 4.4981
YTEMP= 1.0580
YTEMP= 1.0929
YTEMP= 0.3125
Z= -0.3121
Y= 6.2248
YTEMP= 1.8281
YTEMP= 1.0128
YTEMP= 0.5723
Z= 1.4858
Y= 9.0340
YTEMP= 4.1808
YTEMP= 0.5212
YTEMP= 1.3531
Z= 1.5418
Y= 14.4994
YTEMP= 0.4856
YTEMP= 0.1246
YTEMP= 0.6339
Z= -1.5472
Y= 5.3688
YTEMP= 0.0302
YTEMP= 4.1003
YTEMP= 0.9634
Z= 0.5159
Y= 8.7088
YTEMP= 0.8038
YTEMP= 0.1482
YTEMP= 1.5147
Z= -0.7133
Y= 5.4421
YTEMP= 0.8915
YTEMP= 0.3409
YTEMP= 0.6288
Z= -0.5455
Y= 4.0390
YTEMP= 1.4476
YTEMP= 1.7897
YTEMP= 0.5437
Z= -0.5615
Y= 7.8771
YTEMP= 0.2248
YTEMP= 0.8930
YTEMP= 0.2100
Z= 1.2127
Y= 4.1412
YTEMP= 0.0094
YTEMP= 0.3165
YTEMP= 0.7388
Z= -1.4875
Y= 4.3459
YTEMP= 3.0051
YTEMP= 0.9816
YTEMP= 1.0129
Z= 0.2966
Y= 10.0874
YTEMP= 0.8571
YTEMP= 0.3801
YTEMP= 2.3025
Z= -1.3815
Y= 9.1621

CHISQP= 6.6667 P=0.53561 DEN=0.1089 IERR=0 CHISQL= 6.9044 P=0.56110 DEN=0.1055 IERR=0 CELL FREQ 1 2 6 4 4 3 3 1
 YTEMP= 0.8532
 YTEMP= 0.3403
 YTEMP= 0.1852
 Z= -1.3800
 Y= 4.6616
 YTEMP= 0.0086
 YTEMP= 0.7993
 YTEMP= 0.1197
 Z= -1.4817
 Y= 4.0506
 YTEMP= 0.9636
 YTEMP= 0.9783
 YTEMP= 0.3564
 Z= -1.3269
 Y= 0.3574
 YTEMP= 1.5067
 YTEMP= 0.0982
 YTEMP= 0.2465
 Z= 0.8969
 Y= 4.5070
 YTEMP= 0.2174
 YTEMP= 1.2176
 YTEMP= 1.0045
 Z= 1.6683
 Y= 7.7294
 YTEMP= 1.1287
 YTEMP= 0.7788
 YTEMP= 3.1567
 Z= 0.6858
 Y= 10.5987
 YTEMP= 0.3166
 YTEMP= 0.5741
 YTEMP= 0.2247
 Z= 1.0121
 Y= 3.2419
 YTEMP= 0.7971
 YTEMP= 0.5626
 YTEMP= 0.9220
 Z= 2.3439
 Y= 10.0103
 YTEMP= 1.3850
 YTEMP= 0.9555
 YTEMP= 0.0264
 Z= -0.3653
 Y= 6.4674
 YTEMP= 0.6713
 YTEMP= 0.6114
 YTEMP= 5.1501
 Z= -1.3735
 Y= 15.5524
 YTEMP= 0.3921
 YTEMP= 1.5379
 YTEMP= 0.4664
 Z= -0.3315
 Y= 4.9027
 YTEMP= 0.3707
 YTEMP= 0.9109
 YTEMP= 1.6709
 Z= -0.6586
 Y= 6.3387
 YTEMP= 6.6258

YTEMP= 1.4477
 YTEMP= 1.3222
 Z= 0.7670
 Y= 19.3789
 YTEMP= 0.5612
 YTEMP= 1.5024
 YTEMP= 0.0578
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 Y= 4.9916
 YTEMP= 0.1386
 YTEMP= 0.4767
 YTEMP= 3.2304
 Z= -1.3286
 Y= 9.4563
 YTEMP= 3.2456
 YTEMP= 2.7054
 YTEMP= 0.3143
 Z= 1.2232
 Y= 14.1866
 YTEMP= 0.6909
 YTEMP= 0.2456
 YTEMP= 1.6941
 Z= 0.4806
 Y= 6.0370
 YTEMP= 0.5907
 YTEMP= 1.2388
 YTEMP= 0.3129
 Z= -2.0692
 Y= 13.0476
 YTEMP= 1.4263
 YTEMP= 0.1164
 YTEMP= 0.8916
 Z= -0.6246
 Y= 3.0630
 YTEMP= 0.5405
 YTEMP= 1.6183
 YTEMP= 0.0701
 Z= 1.4899
 Y= 5.4774
 YTEMP= 1.5823
 YTEMP= 0.4098
 YTEMP= 1.2695
 Z= 0.4831
 Y= 7.2931
 YTEMP= 1.8462
 YTEMP= 1.6089
 YTEMP= 0.8926
 Z= -0.7471
 Y= 9.2535
 YTEMP= 1.4768
 YTEMP= 1.5924
 YTEMP= 2.5648
 Z= 0.1443
 Y= 11.2760
 YTEMP= 0.0476
 YTEMP= 1.1475
 YTEMP= 0.0507
 Z= -0.4093
 Y= 2.6596
 CHISQP= 9.3333
 YTEMP= 0.3553
 YTEMP= 1.4510

P=0.77040 DEN=0.0656 IERR=0 CHISQ=***** P=***** DEN=***** IERR= CELL FREQ 1 2 5 3 4 0 6 3

YTEMP= 0.7550
Z= -0.1736
Y= 5.1527
YTEMP= 0.2174
YTEMP= 1.2429
YTEMP= 0.7832
Z= -0.6430
Y= 4.9006
YTEMP= 1.5120
YTEMP= 0.6377
YTEMP= 1.0390
Z= -0.6576
Y= 6.8100
YTEMP= 1.8003
YTEMP= 0.4403
YTEMP= 3.1326
Z= 1.2484
Y= 12.4250
YTEMP= 0.2507
YTEMP= 0.4881
YTEMP= 1.0807
Z= -0.5506
Y= 4.0343
YTEMP= 0.2062
YTEMP= 0.2672
YTEMP= 0.2510
Z= 0.5544
Y= 1.7575
YTEMP= 3.3888
YTEMP= 1.0655
YTEMP= 1.9597
Z= 0.1347
Y= 12.8462
YTEMP= 2.2433
YTEMP= 0.0171
YTEMP= 3.2403
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Y= 14.7759
YTEMP= 2.0082
YTEMP= 0.7736
YTEMP= 1.2149
Z= 0.0415
Y= 8.1151
YTEMP= 0.4720
YTEMP= 4.1417
YTEMP= 0.2533
Z= 0.0737
Y= 9.7394
YTEMP= 2.4052
YTEMP= 1.0470
YTEMP= 2.0597
Z= -0.1394
Y= 12.2833
YTEMP= 0.4280
YTEMP= 0.5355
YTEMP= 0.4466
Z= -0.7191
Y= 3.4377
YTEMP= 0.8643
YTEMP= 0.1064
YTEMP= 1.2573
Z= 0.0590

Y= 4.6595
 YIFMP= 2.6329
 YIFMP= 0.0453
 YIFMP= 2.5472
 Z= 0.1986
 Y= 10.5903
 YIFMP= 1.2315
 YIFMP= 0.4065
 YIFMP= 0.0130
 Z= -0.5014
 Y= 3.6747
 YIFMP= 3.1339
 YIFMP= 0.2639
 YIFMP= 2.7769
 Z= -0.1906
 Y= 12.3855
 YIFMP= 0.0752
 YIFMP= 1.0599
 YIFMP= 0.4250
 Z= -0.3461
 Y= 3.2413
 YIFMP= 0.9240
 YIFMP= 0.3952
 YIFMP= 1.9239
 Z= -0.2707
 Y= 6.5595
 YIFMP= 1.7881
 YIFMP= 1.3760
 YIFMP= 1.3516
 Z= -0.2726
 Y= 9.1053
 YIFMP= 2.0495
 YIFMP= 0.5986
 YIFMP= 0.9677
 Z= -0.3279
 Y= 7.7389
 YIFMP= 2.1967
 YIFMP= 1.5504
 YIFMP= 0.6404
 Z= -1.0236
 Y= 9.8232
 YIFMP= 2.4110
 YIFMP= 0.0866
 YIFMP= 1.3896
 Z= -1.0468
 Y= 10.1173
 YIFMP= 0.6988
 YIFMP= 1.5426
 YIFMP= 0.1688
 Z= 0.1948
 Y= 4.9584
 YIFMP= 0.0752
 YIFMP= 0.1567
 YIFMP= 0.3175
 Z= -1.0600
 Y= 4.5666
 CHISQ=10.0000
 YIFMP= 0.7253
 YIFMP= 0.4011
 YIFMP= 0.2100
 Z= -0.9840
 Y= 3.6435

P=0.81145 DEN=0.0567 IERR=0 CHISQ=***** P=***** DEN=***** IERR= CELL FREQ 1 4 5 0 3 1 5 5

YIFMP= 0.0443
YIFMP= 0.3586
YIFMP= 2.2105
Z= -0.3494
Y= 5.3488
YIFMP= 0.4512
YIFMP= 1.4217
YIFMP= 0.7323
Z= 0.6202
Y= 5.6152
YIFMP= 0.9127
YIFMP= 1.7245
YIFMP= 3.3221
Z= -0.5795
Y= 10.4542
YIFMP= 0.1915
YIFMP= 1.0501
YIFMP= 0.0910
Z= -0.6103
Y= 3.0379
YIFMP= 0.1484
YIFMP= 0.3353
YIFMP= 0.1469
Z= 1.3040
Y= 2.9275
YIFMP= 3.4305
YIFMP= 0.7853
YIFMP= 1.5831
Z= 0.5711
Y= 11.9241
YIFMP= 2.2305
YIFMP= 0.2127
YIFMP= 0.2329
Z= -0.8526
Y= 6.0790
YIFMP= 0.0298
YIFMP= 2.7485
YIFMP= 0.3623
Z= 1.7785
Y= 9.4442
YIFMP= 0.0930
YIFMP= 1.6025
YIFMP= 3.3466
Z= -0.3684
Y= 10.2202
YIFMP= 3.6105
YIFMP= 0.2037
YIFMP= 0.9311
Z= 0.0305
Y= 9.9074
YIFMP= 0.3447
YIFMP= 1.2078
YIFMP= 0.2233
Z= -0.4923
Y= 3.7939
YIFMP= 1.2634
YIFMP= 2.5975
YIFMP= 0.2293
Z= 1.2313
Y= 9.6964
YIFMP= 2.1595
YIFMP= 1.9631

YTEMP= 0.2150
 Z= -0.1706
 Y= 8.7017
 YTEMP= 0.0090
 YTEMP= 0.2233
 YTEMP= 2.1824
 Z= -0.8937
 Y= 5.6280
 YTEMP= 0.3096
 YTEMP= 0.5349
 YTEMP= 0.4408
 Z= 1.7672
 Y= 5.8139
 YTEMP= 0.0965
 YTEMP= 0.0014
 YTEMP= 0.4555
 Z= 0.7448
 Y= 1.6617
 YTEMP= 0.5079
 YTEMP= 0.5421
 YTEMP= 0.2903
 Z= -0.0705
 Y= 2.6975
 YTEMP= 0.7448
 YTEMP= 0.3335
 YTEMP= 0.1316
 Z= -0.5667
 Y= 2.7410
 YTEMP= 1.0004
 YTEMP= 0.4999
 YTEMP= 0.9944
 Z= -1.8484
 Y= 8.5416
 YTEMP= 0.2307
 YTEMP= 0.9092
 YTEMP= 0.0039
 Z= 1.6538
 Y= 5.1267
 YTEMP= 2.5355
 YTEMP= 0.6013
 YTEMP= 1.0785
 Z= 0.9074
 Y= 9.2540
 YTEMP= 0.0121
 YTEMP= 0.0745
 YTEMP= 0.4433
 Z= 0.5340
 Y= 1.3286
 YTEMP= 0.0001
 YTEMP= 0.4419
 YTEMP= 1.5045
 Z= 0.2237
 Y= 3.7637
 CHISQ= 13.3334
 YTEMP= 0.0040
 YTEMP= 1.3058
 YTEMP= 0.4055
 Z= -0.2165
 Y= 3.9405
 YTEMP= 2.3953
 YTEMP= 1.0040
 YTEMP= 0.2091

P=0.93501 D=0.0220 IERR=0 CHISQ=***** P=***** DEN=***** IERR= CELL FREQ 6 3 1 5 0 2 6 1

Z= 1.7556
Y= 12.1099
YTEMP= 0.6698
YTEMP= 0.3841
YTEMP= 0.0545
Z= -0.3838
Y= 2.3841
YTEMP= 1.4656
YTEMP= 2.5047
YTEMP= 0.4279
Z= 0.2598
Y= 8.9039
YTEMP= 2.1216
YTEMP= 1.4220
YTEMP= 0.0197
Z= 0.0516
Y= 7.1293
YTEMP= 0.8602
YTEMP= 0.2749
YTEMP= 1.5299
Z= -0.5384
Y= 5.6198
YTEMP= 0.7892
YTEMP= 0.1335
YTEMP= 0.0903
Z= 3.0643
Y= 11.4279
YTEMP= 1.3357
YTEMP= 0.6731
YTEMP= 4.2865
Z= -0.5245
Y= 12.8657
YTEMP= 1.7678
YTEMP= 0.6133
YTEMP= 1.3173
Z= -0.1667
Y= 7.8245
YTEMP= 0.2229
YTEMP= 1.4335
YTEMP= 0.5934
Z= -0.3803
Y= 4.8444
YTEMP= 1.3555
YTEMP= 1.6779
YTEMP= 2.8781
Z= 0.9007
Y= 11.6044
YTEMP= 0.1101
YTEMP= 0.3367
YTEMP= 0.1360
Z= 1.4504
Y= 3.2691
YTEMP= 4.3710
YTEMP= 0.1931
YTEMP= 0.2844
Z= -0.7898
Y= 10.3207
YTEMP= 1.0268
YTEMP= 0.0025
YTEMP= 0.0430
Z= 0.2176
Y= 2.2058

YTEMP= 0.4750
YTEMP= 2.5484
YTEMP= 0.8800
Z= -0.7841
Y= 8.5216
YTEMP= 1.2636
YTEMP= 1.4395
YTEMP= 3.4696
Z= 1.1598
Y= 13.7304
YTEMP= 0.8513
YTEMP= 0.1320
YTEMP= 4.1414
Z= 2.2970
Y= 15.5255
YTEMP= 0.8087
YTEMP= 0.6799
YTEMP= 0.7177
Z= -0.0461
Y= 4.4149
YTEMP= 0.3250
YTEMP= 1.2552
YTEMP= 0.0739
Z= -0.8845
Y= 4.0906
YTEMP= 3.8769
YTEMP= 0.5413
YTEMP= 2.3905
Z= 0.8760
Y= 14.4006
YTEMP= 1.0861
YTEMP= 1.2740
YTEMP= 0.2744
Z= 1.6406
Y= 8.1239
YTEMP= 0.5630
YTEMP= 0.1605
YTEMP= 0.6912
Z= -2.8492
Y= 11.3480
YTEMP= 1.4436
YTEMP= 0.1703
YTEMP= 1.9923
Z= -0.4023
Y= 7.3742
YTEMP= 0.2459
YTEMP= 0.1973
YTEMP= 0.5936
Z= 0.1091
Y= 2.0885
CHISQ=11.3333
YTEMP= 0.0583
YTEMP= 0.1722
YTEMP= 1.9008
Z= 1.0967
Y= 5.4854
YTEMP= 0.0679
YTEMP= 0.5912
YTEMP= 0.1131
Z= 0.8176
Y= 2.2129
YTEMP= 0.1726

P=0.87527 DEN=0.0398 IERR=0 CHISQ= 9.6770 P=0.79237 DEN=0.0614 IERR=0 CELL FREQ 3 3 2 1 3 3 1 8

APPENDIX D

```

$ 20.37.52 JOB 711 -- RAIYADA9 -- BEGINNING EXEC - PART 10- CLASS 4
*20.39.12 JOB 711 ILLU011 RAIYADA9 ASM      230K REQ:  230K USED, 00:01:02 ET, 00:00:06 CPU,   9.94%, 0000 (COMP)
*20.39.55 JOB 711 ILLU011 RAIYADA9 FORT    230K REQ:   80K USED, 00:00:42 ET, 00:00:07 CPU,  18.23%, 0000 (COMP)
*20.40.20 JOB 711 ILLU011 RAIYADA9 LKED    230K REQ:   96K USED, 00:00:25 ET, 00:00:01 CPU,   6.29%, 0000 (COMP)
*20.53.16 JOB 711 ILLU011 RAIYADA9 GO     230K REQ:   48K USED, 00:12:55 ET, 00:02:33 CPU,  19.73%, 0000 (COMP)
*20.54.23 JOB 711 ILLU011 RAIYADA9 SORT   230K REQ:  230K USED, 00:01:06 ET, 00:00:03 CPU,   5.19%, 0000 (COMP)
$*20.54.25 JOB 711 -- RAIYADA9 -- END EXECUTION: 1407 LINES
    
```

HASP3.1.14 JOB STATISTICS -- 671 CARDS READ -- 1,416 LINES PRINTED -- 0 CARDS PUNCHED -- 16.54 MINUTES (ELAPSED TIME)

```
//RAI9ADA9 JOB (3203,1,2), 'MILLER',
// MSGLEVEL=(1,1),CLASS=4,TIME=4
// EXEC ASMGFORT,PARM,LKED=
```

JOB 711

```
XXASMGFORT PROC DECK=11
XXASM EXEC PGM=ASMGASM,PARM='B,LO,ES,RL,UM,FX,CO=3,&DECK'
IEF653I SUBSTITUTION JCL - PGM=ASMGASM,PARM='B,LO,ES,RL,UM,FX,CO=3,'
XXSYSLIB DD DSN=MATH210.MACLIB,DISP=SHR
XX DD DSN=SYS1.MACLIB,DISP=SHR
XXSYSUT1 DD UNIT=SYSDA,SPACE=(3500,(400,50))
XXSYSUT2 DD UNIT=SYSDA,SPACE=(3500,(400,50))
XXSYSUT3 DD UNIT=SYSDA,SPACE=(3500,(400,50))
//SYSPRINT DD DUMMY
A/SYSPRINT DD SYSOUT=A
XXSYSPPUNCH DD SYSOUT=B
XXSYSLIN DD DSN=LOADSET,UNIT=SYSDA,SPACE=(80,(200,50)),
XX DISP=(MOD,PASS),DCB=(RECFM=F,LRECL=80,BLKSIZE=80)
//ASM.SYSIN DD *
```

```
00000100
00000200
00000300
00000400
00000500
00000600
00000700
00000800
00000900
X00001000
00001100
```

```
IEF236I ALLOC. FOR RAI9ADA9 ASM
IEF237I 251 ALLOCATED TO SYSLIB
IEF237I 150 ALLOCATED TO
IEF237I 156 ALLOCATED TO SYSUT1
IEF237I 157 ALLOCATED TO SYSUT2
IEF237I 155 ALLOCATED TO SYSUT3
IEF237I 300 ALLOCATED TO SYSPPUNCH
IEF237I 156 ALLOCATED TO SYSLIN
IEF237I 351 ALLOCATED TO SYSIN
IEF142I - STEP WAS EXECUTED - COND CODE 0000
IEF285I MATH210.MACLIB KEPT
IEF285I VOL SER NOS= LD3001. KEPT
IEF285I SYS1.MACLIB
IEF285I VOL SER NOS= SYSVOL.
IEF285I SYS77241.T013450.RF000.RAI9ADA9.R0009175 DELETED
IEF285I VOL SER NOS= WORK07.
IEF285I SYS77241.T013450.RF000.RAI9ADA9.R0009176 DELETED
IEF285I VOL SER NOS= LD3030.
IEF285I SYS77241.T013450.RF000.RAI9ADA9.R0009177 DELETED
IEF285I VOL SER NOS= LD3029.
IEF285I SYS77241.T013450.RF000.RAI9ADA9.LOADSET PASSED
IEF285I VOL SER NOS= WORK07.
```

```
IEF373I STEP /ASM / START 77242.2038
IEF374I STEP /ASM / STOP 77242.2039 CPU OMIN 06.22SEC MATN 230K LCS OK
ILU002I STEP /ASM / UNIT 251 4 EXCPS
ILU002I STEP /ASM / UNIT 150 26 EXCPS
ILU002I STEP /ASM / UNIT 156 0 EXCPS
ILU002I STEP /ASM / UNIT 157 0 EXCPS
ILU002I STEP /ASM / UNIT 155 0 EXCPS
ILU002I STEP /ASM / UNIT 300 0 EXCPS
ILU002I STEP /ASM / UNIT 156 39 EXCPS
ILU002I STEP /ASM / UNIT 351 519 EXCPS
ILU003I STEP /ASM / EXCPS: DISK 69: TAPE 0: UR 519: TP 0: TOTAL 588:
```

```
00001200
00001300
00001400
X00001500
00001600
```

```
XXFORT EXEC PGM=IEYFORT,PARM='LINECNT=60'
XXSYSPRINT DD SYSOUT=A
XXSYSPPUNCH DD SYSOUT=B
XXSYSLIN DD DSN=LOADSET,DISP=(MOD,PASS),
XX DCH=(RECFM=F,LRECL=80,BLKSIZE=80)
//FORT.SYSIN DD *
```

```
IEF236I ALLOC. FOR RAI9ADA9 FORT
IEF237I 388 ALLOCATED TO SYSPRINT
IEF237I 300 ALLOCATED TO SYSPPUNCH
IEF237I 156 ALLOCATED TO SYSLIN
IEF237I 355 ALLOCATED TO SYSIN
IEF142I - STEP WAS EXECUTED - COND CODE 0000
```

```

IEF285I SYS77241.T013450.RF000.RAI9ADA9.LOADSET PASSED
IEF285I VOL SER NOS= WORK07.
IEF373I STEP /FORT / START 77242.2039
IEF374I STEP /FORT / STOP 77242.2039 CPU OMIN 07.72SEC MAIN 80K LCS OK
ILU002I STEP /FORT / UNIT 388 204 EXCPS
ILU002I STEP /FORT / UNIT 3C0 0 EXCPS
ILU002I STEP /FORT / UNIT 156 117 EXCPS
ILU002I STEP /FORT / UNIT 355 130 EXCPS
ILU003I STEP /FORT / EXCPS: DISK 117: TAPE 0: UR 334: TP 0: TOTAL 451:
XXLKED EXEC PGM=IEWL,PARM=XREF,LET,LIST,COND=(4,LT,FORT) 00001700
XXSYSLIB DD DSN=SYS1.FORTLIB,DISP=SHR 00001800
XX DD DSN=SSPLIB,DISP=SHR 00001900
XXSYSLMOD DD DSN=&GOSET(MAIN),DISP=(NEW,PASS),UNIT=SYSDA, X00002000
XX SPACE=(TRK,(20,5,1)) 00002100
XXSYSPRINT DD SYSOUT=A 00002200
XXSYST1 DD UNIT=SYSDA,DISP=(,DELETE),SPACE=(CYL,(1,1)) 00002300
XXSYSLIN DD DSN=&LOADSET,DISP=(OLD,DELETE), X00002400
XX DCH=(RECFM=F,LRECL=80,BLKSIZE=80) 00002500
XX DD DDNAME=SYSLIN 00002600
IEF236I ALLOC. FOR RAI9ADA9 LKED
IEF237I 150 ALLOCATED TO SYSLIB
IEF237I 251 ALLOCATED TO
IEF237I 156 ALLOCATED TO SYSLMOD
IEF237I 388 ALLOCATED TO SYSPRINT
IEF237I 157 ALLOCATED TO SYST1
IEF237I 156 ALLOCATED TO SYSLIN
IEF142I - STEP WAS EXECUTED - COND CODE 0000
IEF285I SYS1.FORTLIB KEPT
IEF285I VOL SER NOS= SYSVOL. KEPT
IEF285I SSPLIB KEPT
IEF285I VOL SER NOS= LD3001.
IEF285I SYS77241.T013450.RF000.RAI9ADA9.GOSET PASSED
IEF285I VOL SER NOS= WORK07.
IEF285I SYS77241.T013450.RF000.RAI9ADA9.R0009184 DELETED
IEF285I VOL SER NOS= LD3030.
IEF285I SYS77241.T013450.RF000.RAI9ADA9.LOADSET DELETED
IEF285I VOL SER NOS= WORK07.
IEF373I STEP /LKED / START 77242.2039
IEF374I STEP /LKED / STOP 77242.2040 CPU OMIN 01.59SEC MAIN 96K LCS OK
ILU002I STEP /LKED / UNIT 150 84 EXCPS
ILU002I STEP /LKED / UNIT 251 14 EXCPS
ILU002I STEP /LKED / UNIT 156 27 EXCPS
ILU002I STEP /LKED / UNIT 388 5 EXCPS
ILU002I STEP /LKED / UNIT 157 18 EXCPS
ILU002I STEP /LKED / UNIT 156 157 EXCPS
ILU003I STEP /LKED / EXCPS: DISK 300: TAPE 0: UR 5: TP 0: TOTAL 305:
XXGO EXEC PGM=*.LKED.SYSLMOD,COND=((4,LT,FORT),(4,LT,LKED)) 00002700
XXSYSLIN DD DDNAME=ASMINPT 00002800
XXSYSPRINT DD SYSOUT=A 00002900
XXFT05F001 DD DDNAME=FORTINPT 00003000
XXFT06F001 DD SYSOUT=A 00003100
XXFT07F001 DD SYSOUT=B 00003200
//GO.FT08F001 DD DSN=&&T1,UNIT=SYSDA,VOL=SER=WORK05,
// DISP=(NEW,PASS),DCH=(LRECL=133,RECFM=FB,BLKSIZE=3990),
// SPACE=(3990,(40,1))
//GO.FORTINPT DD *
IEF236I ALLOC. FOR RAI9ADA9 GO
IEF237I 156 ALLOCATED TO PGM=*.DD
IEF237I 388 ALLOCATED TO SYSPRINT
IEF237I 356 ALLOCATED TO FT05F001
IEF237I 38C ALLOCATED TO FT06F001
IEF237I 3C0 ALLOCATED TO FT07F001

```

```

IEF237I 250 ALLOCATED TO FT08F001
IEF142I - STEP WAS EXECUTED - COND CODE 0000
IEF285I SYS77241.T013450.RF000.RAI9ADA9.GOSET PASSED
IEF285I VOL SER NOS= WORK07.
IEF285I SYS77241.T013450.RF000.RAI9ADA9.T1 PASSED
IEF285I VOL SER NOS= WORK05.
IEF373I STEP /GO / START 77242.2040
IEF374I STEP /GO / STOP 77242.2053 CPU 2MIN 33.10SEC MAIN 48K LCS OK
ILU002I STEP /GO / UNIT 156 0 EXCPS
ILU002I STEP /GO / UNIT 388 0 EXCPS
ILU002I STEP /GO / UNIT 356 3 EXCPS
ILU002I STEP /GO / UNIT 30C 3 EXCPS
ILU002I STEP /GO / UNIT 300 0 EXCPS
ILU002I STEP /GO / UNIT 250 34 EXCPS
ILU003I STEP /GO / EXCPS: DISK 341 TAPE 01 UR 61 TP 01 TOTAL 401

```

```

//S2 EXEC SORTD
XXSORT EXEC PGM=SORT,REGION=60K 00000010
XXSYSOUT DD SYSOUT=A 00000020
XXSORTLIB DD DSN=SYS1.SORTLIB,DISP=SHR 00000030
//SORTWK01 DD DSN=K&TEMP1,UNIT=SYSDA,VOL=SER=WORK05,
// DISP=(NEW,DELETE),SPACE=(CYL,(1))
//SORTWK02 DD DSN=K&TEMP2,UNIT=SYSDA,VOL=SER=WORK05,
// DISP=(NEW,DELETE),SPACE=(CYL,(1))
//SORTWK03 DD DSN=K&TEMP3,UNIT=SYSDA,VOL=SER=WORK05,
// DISP=(NEW,DELETE),SPACE=(CYL,(1))
//SORTIN DD DSN=K&T1,DISP=(OLD,DELETE)
//SORTOUT DD SYSOUT=A,DCB=BLKSIZE=133
//SYSIN DD *
//

```

```

IEF236I ALLOC. FOR RAI9ADA9 SORT S2
IEF237I 381 ALLOCATED TO SYSOUT
IEF237I 150 ALLOCATED TO SORTLIB
IEF237I 250 ALLOCATED TO SORTWK01
IEF237I 250 ALLOCATED TO SORTWK02
IEF237I 250 ALLOCATED TO SORTWK03
IEF237I 250 ALLOCATED TO SORTIN
IEF237I 382 ALLOCATED TO SORTOUT
IEF237I 359 ALLOCATED TO SYSIN

```

```

IEF142I - STEP WAS EXECUTED - COND CODE 0000
IEF285I SYS1.SORTLIB KEPT
IEF285I VOL SER NOS= SYSVOL
IEF285I SYS77241.T013450.RF000.RAI9ADA9.TEMP1 DELETED
IEF285I VOL SER NOS= WORK05.
IEF285I SYS77241.T013450.RF000.RAI9ADA9.TEMP2 DELETED
IEF285I VOL SER NOS= WORK05.
IEF285I SYS77241.T013450.RF000.RAI9ADA9.TEMP3 DELETED
IEF285I VOL SER NOS= WORK05.
IEF285I SYS77241.T013450.RF000.RAI9ADA9.T1 DELETED
IEF285I VOL SER NOS= WORK05.
IEF373I STEP /SORT / START 77242.2053
IEF374I STEP /SORT / STOP 77242.2054 CPU 0MIN 03.45SEC MAIN 230K LCS OK
ILU002I STEP /SORT / UNIT 301 10 EXCPS
ILU002I STEP /SORT / UNIT 150 0 EXCPS
ILU002I STEP /SORT / UNIT 250 24 EXCPS
ILU002I STEP /SORT / UNIT 250 0 EXCPS
ILU002I STEP /SORT / UNIT 250 0 EXCPS
ILU002I STEP /SORT / UNIT 250 36 EXCPS
ILU002I STEP /SORT / UNIT 382 1,000 EXCPS
ILU002I STEP /SORT / UNIT 359 2 EXCPS
ILU003I STEP /SORT / EXCPS: DISK 601 TAPE 01 UR 1,0121 TP 01 TOTAL 1,0721
IEF285I SYS77241.T013450.RF000.RAI9ADA9.GOSET DELETED
IEF285I VOL SER NOS= WORK07.

```

IEF375I JOH /KAIYADAY/ START 77242.2038
IEF376I JOH /KAIYADAY/ STOP 77242.2054 CPU 2MIN 52.08SEC


```

0001      DIMENSION INT(10),BND(10)
0002      INTEGER BR
0003      60 READ(5,101,END=50) DF,E,BND
0004      IDF=DF+.1
0005      WRITE(6,102) DF,E,(BND(I),I=1,IDF)
0006      II=IDF+1
0007      KK=DF/2.+.1
0008      ZF=0
0009      ITEMS=(DF+1)*F+.1
0010      ASSIGN 6 TO BR
0011      IF(MOD(IDF,2).EQ.1) ASSIGN 7 TO BR
0012      DO 20 J=1,1000
0013      DO 10 I=1,II
0014      10 INT(I)=0
0015      DO 9 I=1,ITEMS
0016      Y=0
0017      DO 8 K=1,KK
0018      8 Y=Y+REXP(U)
0019      GO TO BR,(6,7)
0020      6 Y=Y+Y
0021      GO TO 5
0022      7 Y=Y+Y+RNOR(0)**2
0023      DO 4 K=1,IDF
0024      IF(Y.GT.BND(K)) GO TO 4
0025      INT(K)=INT(K)+1
0026      GO TO 9
0027      4 CONTINUE
0028      INT(II)=INT(II)+1
0029      9 CONTINUE
0030      CHISQP=0
0031      CHISQL=0
0032      DO 15 I=1,II
0033      IF(INT(I).NE.0) GO TO 14
0034      ZF=1
0035      GO TO 15
0036      14 CHISQL=CHISQL+INT(I)*ALOG(INT(I)/E)
0037      15 CHISQP=CHISQP+((INT(I)-E)**2)/E
0038      IF(ZF) 18,18,17
0039      17 CHISQL=999
0040      PL=99
0041      DL=999
0042      IERRL=999
0043      ZF=0
0044      GO TO 19
0045      18 CHISQL=CHISQL+CHISQL
0046      CALL CDTR(CHISQL,DF,PL,DL,IERRL)
0047      19 CALL CDTR(CHISQP,DF,PP,DP,IERR)
0048      WRITE(8,100)CHISQP,PP,DP,IERR,CHISQL,PL,DL,IERRL,(INT(I),I=1,II)
0049      20 CONTINUE
0050      GO TO 60
0051      50 END FILE 8
0052      STOP
0053      100 FORMAT(
A DEN=',F6.4,' IERR=',I1,' CHISQL=',F7.4,' P=',F7.5,'
8,4,' IERR=',I1,' CELL FREQ',10I3)
0054      101 FORMAT(2F3.0,10F7.4)
0055      102 FORMAT(' DEGRFFS OF FREEDOM=',F3.0,/, ' EXPECTED FREQ=',F3.0,/,
1' BOUNDARY VALUES=',10F8.4)

```

PAGE 0002

20/39/15

DATE = 77242

MAIN

FORTRAN IV 6 LEVEL <1
0056 END

		SUBPROGRAMS CALLED							
SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION
IRCOM#	11C	REXP	120	RNOR	124	CUTR	128	ALOG	12C
		SCALAR MAP							
SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION
DF	16C	E	170	IDF	174	I	178	II	17C
KK	180	ZF	184	ITEMS	188	BR	18C	J	190
Y	194	K	198	CHISQP	19C	CHISQL	1A0	PL	1A4
DL	1A8	IEHRL	1AC	PP	1B0	UP	1B4	IEHR	1B8
		ARRAY MAP							
SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION
INT	1BC	RND	1E4						
		FORMAT STATEMENT MAP							
SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION
100	20C	101	278	102	284				

OPTIONS IN EFFECT ID,ENCDC, SOURCE,NOLIST,NODECK,LOAD,MAP
 OPTIONS IN EFFECT NAME = MAIN , LINECNT = 60
 STATISTICS SOURCE STATEMENTS = 56,PROGRAM SIZE = 1976
 STATISTICS NO DIAGNOSTICS GENERATED

```

C RNOR TOOTH FUNCTION
0001 FUNCTION RNORTH(K)
0002 DIMENSION C(45)
0003 DATA C/240FD2H5F,Z40FD2H5F,Z40FAA9AD,Z40F5A648,Z40F32496,
$ Z40EE2131,Z40E69C1A,Z40E198R5,Z40DA139E,Z40D28E87,Z40C887BE,
$ Z40C102A6,Z40H6FBDD,Z40ACF513,Z40A2EE4A,Z4098E780,Z40916269,
$ Z406758A0,Z407D54D6,Z40734E0D,Z406BC8F6,Z4061C22C,Z405A3D15,
$ Z405267FE,Z404B32E7,Z4043ADD0,Z403C2889,Z40372554,Z402FAV3D,
$ Z402A9CDB,Z40259973,Z4020960E,Z401E145C,Z401910F7,Z40168F45,
$ Z40140D93,Z401188E0,Z3FF0A2E4,Z3FC887BE,Z3FA06C98,Z3F785172,
$ Z3F785172,Z3F50364C,Z3F50364C,Z3F50364C/
0004 DATA I1/ZF8CJ5400/I2/ZFE79702F/
0005 IF(K.GT.I1)GO TO 3
0006 S=UNI(0)
0007 T=UNI(0)
0008 B=AINT(7.*(S+T)+37.*ABS(S-T))
0009 X=UNI(0)-UNI(0)
0010 RNORTH=.0625*(X+SIGN(B,X))
0011 RETURN
0012 3 IF(K.GT.I2)GO TO 5
0013 4 RNORTH=2.75*VNI(0)
0014 J=16.*ABS(RNORTH)+1.
0015 IF(J-14) 6,6,7
0016 6 P=(J+J-1)*.1497466E-2
0017 GO TO 8
0018 7 P=(89-J-J)*.698817E-3
0019 8 IF(UNI(0).GT.79.78846*(EXP(-.5*RNORTH*RNORTH)
$ -C(J)-P*(J-16.*ABS(RNORTH)))) GOTO4
0020 RETURN
0021 5 V=VNI(0)
0022 IF(V.EQ.0) GO TO 5
0023 X=SQRT(7.5625-2.*ALOG(ABS(V)))
0024 IF(UNI(0)*X.GT.2.75)GO TO 5
0025 RNORTH=SIGN(X,V)
0026 RETURN
0027 END

```

5180
5190
5200
5210
5220
5230
5240
5250
5260
5270
5280
5290
5300
5310
5320
5330
5340
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		SUBPROGRAMS CALLED							
SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION
UNI	D4	VNI	D8	EXP	DC	SQRT	E0	ALOG	E4
		EQUIVALENCE DATA MAP							
SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION
RNORTH	138								
		SCALAR MAP							
SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION
I1	13C	I2	140	K	144	S	148	T	14C
B	150	X	154	J	158	P	15C	V	160
		ARRAY MAP							
SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION
C	164								

```

*OPTIONS IN EFFECT* ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,MAP
*OPTIONS IN EFFECT* NAME = RNORTH , LINECNT = 60
*STATISTICS* SOURCE STATEMENTS = 27,PROGRAM SIZE = 1368
*STATISTICS* NO DIAGNOSTICS GENERATED
    
```

```

0001      C      REXP IOOTH FUNCTION
0002      FUNCTION REXPTH(K)
0003      DIMENSION C(65)
      DATA C/240F0000,Z40E10000,Z40D40000,Z40C70000,Z40R80000,
$ 240AF0000,Z40A50000,Z409B0000,Z40910000,Z40890000,Z40800000,
$ 240780000,Z40710000,Z406A0000,Z40640000,Z405E0000,Z40580000,
$ 240530000,Z404E0000,Z40490000,Z40440000,Z40400000,Z403C0000,
$ 240390000,Z40350000,Z40320000,Z402F0000,Z402C0000,Z40290000,
$ 240270000,Z40240000,Z40220000,Z40200000,Z401E0000,Z401C0000,
$ 2401A0000,Z40190000,Z40170000,Z40160000,Z40150000,Z40130000,
$ 240120000,Z40110000,Z40100000,Z3FF00000,Z3FE00000,Z3FD00000,
$ Z3FC00000,Z3FB00000,Z3FB00000,Z3FA00000,Z3F900000,Z3F900000,
$ Z3F800000,Z3F800000,Z3F700000,Z3F700000,Z3F600000,Z3F600000,
$ Z3F600000,Z3F500000,Z3F400000,Z3F400000,Z3F400000,Z3F400000/
      DATA I1/ZFB4FAA91/
      IF(K.GT.I1)GO TO 5
      U1=UNI(0)
1      IF(U1.GT..7917049) GO TO 3
      T=1.-1.239962*U1
      REXPTH=-ALOG(T)
      J=16.*REXPTH+1.
      IF(UNI(0)*(.0604*T+.0039).GT.T-C(J))GOTO1
      RETURN
3      REXPTH=19.20352*U1-15.20352
      J=16.*REXPTH+1.
      EX=EXP(-REXPTH)
      IF(UNI(0)*(.0604*EX+.0039).GT.EX-C(J))GOTO1
      RETURN
5      U1=UNI(0)
      IF(U1.EQ.0)GO TO 5
      REXPTH=4.-ALOG(U1)
      RETURN
      END
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		SUBPROGRAMS CALLED							
SYMBOL UNI	LOCATION HC	SYMBOL ALOG	LOCATION C0	SYMBOL EXP	LOCATION C4	SYMBOL	LOCATION	SYMBOL	LOCATION
		EQUIVALENCE DATA MAP							
SYMBOL REXPTH	LOCATION EB	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION
		SCALAR MAP							
SYMBOL II EX	LOCATION EC 100	SYMBOL K	LOCATION F0	SYMBOL UI	LOCATION F4	SYMBOL T	LOCATION F8	SYMBOL J	LOCATION FC
		ARRAY MAP							
SYMBOL C	LOCATION 104	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION	SYMBOL	LOCATION

OPTIONS IN EFFECT ID,EBCDIC,SOURCE,NOLIST,NODECK,LOAD,MAP
 OPTIONS IN EFFECT NAME = REXPTH , LINECNT = 60
 STATISTICS SOURCE STATEMENTS = 22,PROGRAM SIZE = 1076
 STATISTICS NO DIAGNOSTICS GENERATED
 STATISTICS NO DIAGNOSTICS THIS STEP

FRS-LEVEL LINKAGE EDITOR OPTIONS SPECIFIED NONE
DEFAULT OPTION(S) USED - SIZE=(92160,28672)
***MAIN DOES NOT EXIST BUT HAS BEEN ADDED TO DATA SET

DEGREES OF FREEDOM= 7.
EXPECTED FREQU=10.
BOUNDARY VALUES= 3.1040 4.2550 5.2870 6.3460 8.0920 9.0370 11.3390

```
*** SORT/MERGE 5734-SM1,VER 1,MOD 4, CONTROL STATEMENTS DATE= 77.242 ***
  SORT FIELDS=(20,7,F1,A,67,7,F1,A)
IGH036I - H = 97
IGH037I - G = 1116
IGH038I - NMAX APROX EQUAL 3000
IGH070I - FILE SIZE IS NOT SPECIFIED
IGH045I - END SORT PH
IGH049I - SKIP MERGE PH
IGH054I - RCD IN 1000,OUT 1000
IGH052I - END OF SORT/MERGE
```


CHISQPF	4.0000	P=0.22022	UEN=0.1152	IERR=0	CHISQPF	3.8859	P=0.20718	DEN=0.1134	IERR=0	CELL	FREQ	13
CHISQPF	4.0000	P=0.22022	UEN=0.1152	IERR=0	CHISQPF	3.9881	P=0.218A5	DEN=0.1150	IERR=0	CELL	FREQ	14
CHISQPF	4.0000	P=0.22022	UEN=0.1152	IERR=0	CHISQPF	4.0103	P=0.22142	DEN=0.1153	IERR=0	CELL	FREQ	15
CHISQPF	4.0000	P=0.22022	UEN=0.1152	IERR=0	CHISQPF	4.0744	P=0.22883	DEN=0.1162	IERR=0	CELL	FREQ	16
CHISQPF	4.0000	P=0.22022	UEN=0.1152	IERR=0	CHISQPF	4.11255	P=0.23327	DEN=0.1167	IERR=0	CELL	FREQ	17
CHISQPF	4.0000	P=0.22022	UEN=0.1152	IERR=0	CHISQPF	4.11590	P=0.23355	DEN=0.1167	IERR=0	CELL	FREQ	18
CHISQPF	4.0000	P=0.22022	UEN=0.1152	IERR=0	CHISQPF	4.1430	P=0.23684	DEN=0.1171	IERR=0	CELL	FREQ	19
CHISQPF	4.0000	P=0.22022	UEN=0.1152	IERR=0	CHISQPF	4.1430	P=0.23684	DEN=0.1171	IERR=0	CELL	FREQ	20
CHISQPF	4.0000	P=0.22022	UEN=0.1152	IERR=0	CHISQPF	4.1765	P=0.24076	DEN=0.1175	IERR=0	CELL	FREQ	21
CHISQPF	4.0000	P=0.22022	UEN=0.1152	IERR=0	CHISQPF	4.46660	P=0.27519	DEN=0.1202	IERR=0	CELL	FREQ	22
CHISQPF	4.0000	P=0.22022	UEN=0.1152	IERR=0	CHISQPF	4.4842	P=0.27739	DEN=0.1203	IERR=0	CELL	FREQ	23
CHISQPF	4.2000	P=0.24352	UEN=0.1177	IERR=0	CHISQPF	4.2107	P=0.24478	DEN=0.1179	IERR=0	CELL	FREQ	24
CHISQPF	4.2000	P=0.24352	UEN=0.1177	IERR=0	CHISQPF	4.2970	P=0.25500	DEN=0.1188	IERR=0	CELL	FREQ	25
CHISQPF	4.2000	P=0.24352	UEN=0.1177	IERR=0	CHISQPF	4.2970	P=0.25500	DEN=0.1188	IERR=0	CELL	FREQ	26
CHISQPF	4.4000	P=0.26728	UEN=0.1197	IERR=0	CHISQPF	4.3433	P=0.26951	DEN=0.1199	IERR=0	CELL	FREQ	27
CHISQPF	4.4000	P=0.26728	UEN=0.1197	IERR=0	CHISQPF	4.3433	P=0.26951	DEN=0.1199	IERR=0	CELL	FREQ	28
CHISQPF	4.4000	P=0.26728	UEN=0.1197	IERR=0	CHISQPF	4.3992	P=0.27118	DEN=0.1197	IERR=0	CELL	FREQ	29
CHISQPF	4.4000	P=0.26728	UEN=0.1197	IERR=0	CHISQPF	4.4223	P=0.27630	DEN=0.1190	IERR=0	CELL	FREQ	30
CHISQPF	4.4000	P=0.26728	UEN=0.1197	IERR=0	CHISQPF	4.3929	P=0.26641	DEN=0.1196	IERR=0	CELL	FREQ	31
CHISQPF	4.4000	P=0.26728	UEN=0.1197	IERR=0	CHISQPF	4.4334	P=0.27127	DEN=0.1199	IERR=0	CELL	FREQ	32
CHISQPF	4.4000	P=0.26728	UEN=0.1197	IERR=0	CHISQPF	4.4334	P=0.27127	DEN=0.1199	IERR=0	CELL	FREQ	33
CHISQPF	4.4000	P=0.26728	UEN=0.1197	IERR=0	CHISQPF	4.4949	P=0.27867	DEN=0.1204	IERR=0	CELL	FREQ	34
CHISQPF	4.4000	P=0.26728	UEN=0.1197	IERR=0	CHISQPF	4.5254	P=0.28235	DEN=0.1206	IERR=0	CELL	FREQ	35
CHISQPF	4.4000	P=0.26728	UEN=0.1197	IERR=0	CHISQPF	4.5660	P=0.28725	DEN=0.1208	IERR=0	CELL	FREQ	36
CHISQPF	4.4000	P=0.26728	UEN=0.1197	IERR=0	CHISQPF	4.5660	P=0.28725	DEN=0.1208	IERR=0	CELL	FREQ	37
CHISQPF	4.4000	P=0.26728	UEN=0.1197	IERR=0	CHISQPF	4.5660	P=0.28725	DEN=0.1208	IERR=0	CELL	FREQ	38
CHISQPF	4.4000	P=0.26728	UEN=0.1197	IERR=0	CHISQPF	4.6498	P=0.29734	DEN=0.1213	IERR=0	CELL	FREQ	39
CHISQPF	4.4000	P=0.26728	UEN=0.1197	IERR=0	CHISQPF	4.6498	P=0.29734	DEN=0.1213	IERR=0	CELL	FREQ	40
CHISQPF	4.4000	P=0.26728	UEN=0.1197	IERR=0	CHISQPF	4.6498	P=0.29734	DEN=0.1213	IERR=0	CELL	FREQ	41
CHISQPF	4.4000	P=0.26728	UEN=0.1197	IERR=0	CHISQPF	4.8789	P=0.32526	DEN=0.1220	IERR=0	CELL	FREQ	42
CHISQPF	4.6000	P=0.29136	UEN=0.1210	IERR=0	CHISQPF	4.3833	P=0.26527	DEN=0.1205	IERR=0	CELL	FREQ	43
CHISQPF	4.6000	P=0.29136	UEN=0.1210	IERR=0	CHISQPF	4.5596	P=0.28647	DEN=0.1208	IERR=0	CELL	FREQ	44
CHISQPF	4.6000	P=0.29136	UEN=0.1210	IERR=0	CHISQPF	4.6236	P=0.29421	DEN=0.1211	IERR=0	CELL	FREQ	45
CHISQPF	4.6000	P=0.29136	UEN=0.1210	IERR=0	CHISQPF	4.7704	P=0.31204	DEN=0.1217	IERR=0	CELL	FREQ	46
CHISQPF	4.6000	P=0.29136	UEN=0.1210	IERR=0	CHISQPF	4.7862	P=0.31396	DEN=0.1218	IERR=0	CELL	FREQ	47
CHISQPF	4.6000	P=0.29136	UEN=0.1210	IERR=0	CHISQPF	5.0609	P=0.34747	DEN=0.1220	IERR=0	CELL	FREQ	48
CHISQPF	4.8000	P=0.31564	UEN=0.1218	IERR=0	CHISQPF	4.3572	P=0.26216	DEN=0.1193	IERR=0	CELL	FREQ	49
CHISQPF	4.8000	P=0.31564	UEN=0.1218	IERR=0	CHISQPF	4.6059	P=0.32207	DEN=0.1210	IERR=0	CELL	FREQ	50
CHISQPF	4.8000	P=0.31564	UEN=0.1218	IERR=0	CHISQPF	4.6914	P=0.30243	DEN=0.1214	IERR=0	CELL	FREQ	51
CHISQPF	4.8000	P=0.31564	UEN=0.1218	IERR=0	CHISQPF	4.6914	P=0.30243	DEN=0.1214	IERR=0	CELL	FREQ	52
CHISQPF	4.8000	P=0.31564	UEN=0.1218	IERR=0	CHISQPF	4.7320	P=0.30736	DEN=0.1216	IERR=0	CELL	FREQ	53
CHISQPF	4.8000	P=0.31564	UEN=0.1218	IERR=0	CHISQPF	4.7823	P=0.31349	DEN=0.1217	IERR=0	CELL	FREQ	54
CHISQPF	4.8000	P=0.31564	UEN=0.1218	IERR=0	CHISQPF	4.8463	P=0.32129	DEN=0.1219	IERR=0	CELL	FREQ	55
CHISQPF	4.8000	P=0.31564	UEN=0.1218	IERR=0	CHISQPF	4.8463	P=0.32129	DEN=0.1219	IERR=0	CELL	FREQ	56
CHISQPF	4.8000	P=0.31564	UEN=0.1218	IERR=0	CHISQPF	4.8463	P=0.32129	DEN=0.1219	IERR=0	CELL	FREQ	57
CHISQPF	4.8000	P=0.31564	UEN=0.1218	IERR=0	CHISQPF	4.8463	P=0.32129	DEN=0.1219	IERR=0	CELL	FREQ	58
CHISQPF	4.8000	P=0.31564	UEN=0.1218	IERR=0	CHISQPF	4.9021	P=0.32810	DEN=0.1220	IERR=0	CELL	FREQ	59
CHISQPF	4.8000	P=0.31564	UEN=0.1218	IERR=0	CHISQPF	4.9484	P=0.33374	DEN=0.1220	IERR=0	CELL	FREQ	60
CHISQPF	4.8000	P=0.31564	UEN=0.1218	IERR=0	CHISQPF	4.9484	P=0.33374	DEN=0.1220	IERR=0	CELL	FREQ	61
CHISQPF	4.8000	P=0.31564	UEN=0.1218	IERR=0	CHISQPF	4.9707	P=0.33646	DEN=0.1220	IERR=0	CELL	FREQ	62
CHISQPF	4.8000	P=0.31564	UEN=0.1218	IERR=0	CHISQPF	4.9707	P=0.33646	DEN=0.1220	IERR=0	CELL	FREQ	63
CHISQPF	4.8000	P=0.31564	UEN=0.1218	IERR=0	CHISQPF	5.1693	P=0.36069	DEN=0.1219	IERR=0	CELL	FREQ	64
CHISQPF	4.8000	P=0.31564	UEN=0.1218	IERR=0	CHISQPF	5.2278	P=0.36781	DEN=0.1217	IERR=0	CELL	FREQ	65
CHISQPF	4.8000	P=0.31565	UEN=0.1214	IERR=0	CHISQPF	4.7609	P=0.33088	DEN=0.1217	IERR=0	CELL	FREQ	66
CHISQPF	4.8000	P=0.31565	UEN=0.1218	IERR=0	CHISQPF	5.2714	P=0.33312	DEN=0.1216	IERR=0	CELL	FREQ	67
CHISQPF	5.0000	P=0.34804	UEN=0.1220	IERR=0	CHISQPF	4.5295	P=0.30284	DEN=0.1206	IERR=0	CELL	FREQ	68
CHISQPF	5.0000	P=0.34804	UEN=0.1220	IERR=0	CHISQPF	4.8566	P=0.32254	DEN=0.1219	IERR=0	CELL	FREQ	69
CHISQPF	5.0000	P=0.34804	UEN=0.1220	IERR=0	CHISQPF	4.9430	P=0.33308	DEN=0.1220	IERR=0	CELL	FREQ	70

CHISQPP#	6.6000	F#0.52832	DEN#0.1098	IERR#0	CHISQ#	6.8851	P#0.55906	DEN#0.1058	IERR#0	CELL	FREQ	6
CHISQPP#	6.6000	F#0.52832	DEN#0.1098	IERR#0	CHISQ#	6.9973	P#0.57084	DEN#0.1042	IERR#0	CELL	FREQ	12
CHISQPP#	6.6000	F#0.52832	DEN#0.1098	IERR#0	CHISQ#	7.4389	P#0.61534	DEN#0.0973	IERR#0	CELL	FREQ	14
CHISQPP#	6.6000	F#0.52832	DEN#0.1098	IERR#0	CHISQ#	7.4723	P#0.61859	DEN#0.0959	IERR#0	CELL	FREQ	14
CHISQPP#	6.6000	F#0.52832	DEN#0.1098	IERR#0	CHISQ#	7.6248	P#0.63859	DEN#0.0949	IERR#0	CELL	FREQ	14
CHISQPP#	6.6000	F#0.55000	DEN#0.1070	IERR#0	CHISQ#	6.1076	P#0.64276	DEN#0.1157	IERR#0	CELL	FREQ	10
CHISQPP#	6.6000	F#0.55000	DEN#0.1070	IERR#0	CHISQ#	6.5963	P#0.63791	DEN#0.1098	IERR#0	CELL	FREQ	10
CHISQPP#	6.6000	F#0.55000	DEN#0.1070	IERR#0	CHISQ#	6.6369	P#0.63236	DEN#0.1093	IERR#0	CELL	FREQ	10
CHISQPP#	6.6000	F#0.55000	DEN#0.1070	IERR#0	CHISQ#	6.7710	P#0.64689	DEN#0.1074	IERR#0	CELL	FREQ	10
CHISQPP#	6.6000	F#0.55000	DEN#0.1070	IERR#0	CHISQ#	6.7710	P#0.64689	DEN#0.1074	IERR#0	CELL	FREQ	10
CHISQPP#	6.6000	F#0.55000	DEN#0.1070	IERR#0	CHISQ#	6.8355	P#0.65379	DEN#0.1065	IERR#0	CELL	FREQ	10
CHISQPP#	6.6000	F#0.55000	DEN#0.1070	IERR#0	CHISQ#	6.9192	P#0.65266	DEN#0.1053	IERR#0	CELL	FREQ	10
CHISQPP#	6.6000	F#0.55000	DEN#0.1070	IERR#0	CHISQ#	7.0102	P#0.65218	DEN#0.1040	IERR#0	CELL	FREQ	10
CHISQPP#	6.6000	F#0.55000	DEN#0.1070	IERR#0	CHISQ#	7.4365	P#0.61511	DEN#0.0974	IERR#0	CELL	FREQ	10
CHISQPP#	6.6000	F#0.55000	DEN#0.1070	IERR#0	CHISQ#	7.5538	P#0.62642	DEN#0.0999	IERR#0	CELL	FREQ	10
CHISQPP#	6.6000	F#0.55000	DEN#0.1070	IERR#0	CHISQ#	7.5538	P#0.62642	DEN#0.0999	IERR#0	CELL	FREQ	10
CHISQPP#	6.6000	F#0.55000	DEN#0.1070	IERR#0	CHISQ#	7.7007	P#0.64028	DEN#0.0931	IERR#0	CELL	FREQ	10
CHISQPP#	6.6000	F#0.55000	DEN#0.1070	IERR#0	CHISQ#	7.7007	P#0.64028	DEN#0.0931	IERR#0	CELL	FREQ	10
CHISQPP#	7.0000	F#0.57112	DEN#0.1041	IERR#0	CHISQ#	6.6946	P#0.63864	DEN#0.1085	IERR#0	CELL	FREQ	10
CHISQPP#	7.0000	F#0.57112	DEN#0.1041	IERR#0	CHISQ#	6.8412	P#0.65440	DEN#0.1064	IERR#0	CELL	FREQ	10
CHISQPP#	7.0000	F#0.57112	DEN#0.1041	IERR#0	CHISQ#	7.0510	P#0.67641	DEN#0.1034	IERR#0	CELL	FREQ	10
CHISQPP#	7.0000	F#0.57112	DEN#0.1041	IERR#0	CHISQ#	7.0677	P#0.67814	DEN#0.1031	IERR#0	CELL	FREQ	10
CHISQPP#	7.0000	F#0.57112	DEN#0.1041	IERR#0	CHISQ#	7.1013	P#0.68159	DEN#0.1026	IERR#0	CELL	FREQ	10
CHISQPP#	7.0000	F#0.57112	DEN#0.1041	IERR#0	CHISQ#	7.1013	P#0.68159	DEN#0.1026	IERR#0	CELL	FREQ	10
CHISQPP#	7.0000	F#0.57112	DEN#0.1041	IERR#0	CHISQ#	7.1653	P#0.68813	DEN#0.1016	IERR#0	CELL	FREQ	10
CHISQPP#	7.0000	F#0.57112	DEN#0.1041	IERR#0	CHISQ#	7.1699	P#0.68859	DEN#0.1015	IERR#0	CELL	FREQ	10
CHISQPP#	7.0000	F#0.57112	DEN#0.1041	IERR#0	CHISQ#	7.2105	P#0.69270	DEN#0.1009	IERR#0	CELL	FREQ	10
CHISQPP#	7.0000	F#0.57112	DEN#0.1041	IERR#0	CHISQ#	7.2689	P#0.69776	DEN#0.1001	IERR#0	CELL	FREQ	10
CHISQPP#	7.0000	F#0.57112	DEN#0.1041	IERR#0	CHISQ#	7.2677	P#0.69843	DEN#0.1000	IERR#0	CELL	FREQ	10
CHISQPP#	7.0000	F#0.57112	DEN#0.1041	IERR#0	CHISQ#	7.7191	P#0.64199	DEN#0.0925	IERR#0	CELL	FREQ	10
CHISQPP#	7.0000	F#0.57112	DEN#0.1041	IERR#0	CHISQ#	7.7359	P#0.64354	DEN#0.0925	IERR#0	CELL	FREQ	10
CHISQPP#	7.0000	F#0.57112	DEN#0.1041	IERR#0	CHISQ#	7.8213	P#0.65138	DEN#0.0911	IERR#0	CELL	FREQ	10
CHISQPP#	7.0000	F#0.57112	DEN#0.1041	IERR#0	CHISQ#	7.8213	P#0.65138	DEN#0.0911	IERR#0	CELL	FREQ	10
CHISQPP#	7.0000	F#0.57112	DEN#0.1041	IERR#0	CHISQ#	7.8953	P#0.65809	DEN#0.0899	IERR#0	CELL	FREQ	10
CHISQPP#	7.0000	F#0.57112	DEN#0.1041	IERR#0	CHISQ#	7.9920	P#0.66670	DEN#0.0883	IERR#0	CELL	FREQ	10
CHISQPP#	7.0000	F#0.57112	DEN#0.1041	IERR#0	CHISQ#	8.7162	P#0.72632	DEN#0.0764	IERR#0	CELL	FREQ	10
CHISQPP#	7.2000	F#0.59164	DEN#0.1011	IERR#0	CHISQ#	6.5306	P#0.52067	DEN#0.1107	IERR#0	CELL	FREQ	10
CHISQPP#	7.2000	F#0.59164	DEN#0.1011	IERR#0	CHISQ#	6.5306	P#0.52067	DEN#0.1107	IERR#0	CELL	FREQ	10
CHISQPP#	7.2000	F#0.59164	DEN#0.1011	IERR#0	CHISQ#	6.5586	P#0.52376	DEN#0.1103	IERR#0	CELL	FREQ	10
CHISQPP#	7.2000	F#0.59164	DEN#0.1011	IERR#0	CHISQ#	6.8949	P#0.60009	DEN#0.1057	IERR#0	CELL	FREQ	10
CHISQPP#	7.2000	F#0.59164	DEN#0.1011	IERR#0	CHISQ#	6.8949	P#0.60009	DEN#0.1057	IERR#0	CELL	FREQ	10
CHISQPP#	7.2000	F#0.59164	DEN#0.1011	IERR#0	CHISQ#	7.0193	P#0.67313	DEN#0.1038	IERR#0	CELL	FREQ	10
CHISQPP#	7.2000	F#0.59164	DEN#0.1011	IERR#0	CHISQ#	7.0335	P#0.67460	DEN#0.1036	IERR#0	CELL	FREQ	10
CHISQPP#	7.2000	F#0.59164	DEN#0.1011	IERR#0	CHISQ#	7.0918	P#0.68062	DEN#0.1027	IERR#0	CELL	FREQ	10
CHISQPP#	7.2000	F#0.59164	DEN#0.1011	IERR#0	CHISQ#	7.0918	P#0.68062	DEN#0.1027	IERR#0	CELL	FREQ	10
CHISQPP#	7.2000	F#0.59164	DEN#0.1011	IERR#0	CHISQ#	7.1940	P#0.69103	DEN#0.1012	IERR#0	CELL	FREQ	10
CHISQPP#	7.2000	F#0.59164	DEN#0.1011	IERR#0	CHISQ#	7.2179	P#0.69345	DEN#0.1008	IERR#0	CELL	FREQ	10
CHISQPP#	7.2000	F#0.59164	DEN#0.1011	IERR#0	CHISQ#	7.2443	P#0.69611	DEN#0.1004	IERR#0	CELL	FREQ	10
CHISQPP#	7.2000	F#0.59164	DEN#0.1011	IERR#0	CHISQ#	7.3880	P#0.61037	DEN#0.0981	IERR#0	CELL	FREQ	10
CHISQPP#	7.2000	F#0.59164	DEN#0.1011	IERR#0	CHISQ#	7.4343	P#0.61490	DEN#0.0974	IERR#0	CELL	FREQ	10
CHISQPP#	7.2000	F#0.59164	DEN#0.1011	IERR#0	CHISQ#	7.5002	P#0.62129	DEN#0.0963	IERR#0	CELL	FREQ	10
CHISQPP#	7.2000	F#0.59164	DEN#0.1011	IERR#0	CHISQ#	7.6024	P#0.63104	DEN#0.0947	IERR#0	CELL	FREQ	10
CHISQPP#	7.2000	F#0.59164	DEN#0.1011	IERR#0	CHISQ#	7.8621	P#0.65509	DEN#0.0905	IERR#0	CELL	FREQ	10
CHISQPP#	7.2000	F#0.59164	DEN#0.1011	IERR#0	CHISQ#	8.0216	P#0.66931	DEN#0.0878	IERR#0	CELL	FREQ	10
CHISQPP#	7.2000	F#0.59164	DEN#0.1011	IERR#0	CHISQ#	8.1460	P#0.68010	DEN#0.0858	IERR#0	CELL	FREQ	10
CHISQPP#	7.2000	F#0.59164	DEN#0.1011	IERR#0	CHISQ#	8.1460	P#0.68010	DEN#0.0858	IERR#0	CELL	FREQ	10

CHISQP	8.6000	P=0.71734	DEN=0.0783	IERR=0	CHISQL	9.3412	P=0.77092	DEN=0.0664	IERR=0	CELL	12
CHISQP	8.6000	P=0.71734	DEN=0.0783	IERR=0	CHISQL	9.4209	P=0.77617	DEN=0.0652	IERR=0	CELL	11
CHISQP	8.6000	P=0.71734	DEN=0.0783	IERR=0	CHISQL	10.1227	P=0.81828	DEN=0.0549	IERR=0	CELL	9
CHISQP	8.8000	P=0.73266	DEN=0.0750	IERR=0	CHISQL	7.8674	P=0.65557	DEN=0.0904	IERR=0	CELL	8
CHISQP	8.8000	P=0.73266	DEN=0.0750	IERR=0	CHISQL	8.2421	P=0.68827	DEN=0.0842	IERR=0	CELL	7
CHISQP	8.8000	P=0.73266	DEN=0.0750	IERR=0	CHISQL	8.2727	P=0.69083	DEN=0.0837	IERR=0	CELL	6
CHISQP	8.8000	P=0.73266	DEN=0.0750	IERR=0	CHISQL	8.5453	P=0.71303	DEN=0.0792	IERR=0	CELL	5
CHISQP	8.8000	P=0.73266	DEN=0.0750	IERR=0	CHISQL	8.9922	P=0.74678	DEN=0.0719	IERR=0	CELL	4
CHISQP	8.8000	P=0.73266	DEN=0.0750	IERR=0	CHISQL	8.9922	P=0.74678	DEN=0.0719	IERR=0	CELL	3
CHISQP	8.8000	P=0.73266	DEN=0.0750	IERR=0	CHISQL	9.0620	P=0.75177	DEN=0.0708	IERR=0	CELL	2
CHISQP	8.8000	P=0.73266	DEN=0.0750	IERR=0	CHISQL	9.1471	P=0.75773	DEN=0.0695	IERR=0	CELL	1
CHISQP	8.8000	P=0.73266	DEN=0.0750	IERR=0	CHISQL	9.2087	P=0.76198	DEN=0.0685	IERR=0	CELL	1
CHISQP	8.8000	P=0.73266	DEN=0.0750	IERR=0	CHISQL	9.2087	P=0.76198	DEN=0.0685	IERR=0	CELL	1
CHISQP	8.8000	P=0.73266	DEN=0.0750	IERR=0	CHISQL	9.2087	P=0.76198	DEN=0.0685	IERR=0	CELL	1
CHISQP	8.8000	P=0.73266	DEN=0.0750	IERR=0	CHISQL	9.4395	P=0.77737	DEN=0.0649	IERR=0	CELL	1
CHISQP	8.8000	P=0.73266	DEN=0.0750	IERR=0	CHISQL	9.5239	P=0.78533	DEN=0.0630	IERR=0	CELL	1
CHISQP	8.8000	P=0.73266	DEN=0.0750	IERR=0	CHISQL	9.5239	P=0.78533	DEN=0.0630	IERR=0	CELL	1
CHISQP	8.8000	P=0.73266	DEN=0.0750	IERR=0	CHISQL	9.5239	P=0.78533	DEN=0.0630	IERR=0	CELL	1
CHISQP	8.8000	P=0.73266	DEN=0.0750	IERR=0	CHISQL	10.0294	P=0.82406	DEN=0.0535	IERR=0	CELL	1
CHISQP	8.8000	P=0.73266	DEN=0.0750	IERR=0	CHISQL	10.4070	P=0.83334	DEN=0.0511	IERR=0	CELL	1
CHISQP	8.8000	P=0.73266	DEN=0.0750	IERR=0	CHISQL	11.8147	P=0.89318	DEN=0.0347	IERR=0	CELL	1
CHISQP	8.8000	P=0.73266	DEN=0.0750	IERR=0	CHISQL	11.8553	P=0.89458	DEN=0.0343	IERR=0	CELL	1
CHISQP	9.0000	P=0.74734	DEN=0.0718	IERR=0	CHISQL	8.0037	P=0.66774	DEN=0.0881	IERR=0	CELL	1
CHISQP	9.0000	P=0.74734	DEN=0.0718	IERR=0	CHISQL	8.4307	P=0.70385	DEN=0.0811	IERR=0	CELL	1
CHISQP	9.0000	P=0.74734	DEN=0.0718	IERR=0	CHISQL	8.4307	P=0.70385	DEN=0.0811	IERR=0	CELL	1
CHISQP	9.0000	P=0.74734	DEN=0.0718	IERR=0	CHISQL	8.4307	P=0.70385	DEN=0.0811	IERR=0	CELL	1
CHISQP	9.0000	P=0.74734	DEN=0.0718	IERR=0	CHISQL	8.4307	P=0.70385	DEN=0.0811	IERR=0	CELL	1
CHISQP	9.0000	P=0.74734	DEN=0.0718	IERR=0	CHISQL	8.5288	P=0.71172	DEN=0.0794	IERR=0	CELL	1
CHISQP	9.0000	P=0.74734	DEN=0.0718	IERR=0	CHISQL	8.8315	P=0.73503	DEN=0.0745	IERR=0	CELL	1
CHISQP	9.0000	P=0.74734	DEN=0.0718	IERR=0	CHISQL	9.1184	P=0.75573	DEN=0.0699	IERR=0	CELL	1
CHISQP	9.0000	P=0.74734	DEN=0.0718	IERR=0	CHISQL	9.2160	P=0.76248	DEN=0.0684	IERR=0	CELL	1
CHISQP	9.0000	P=0.74734	DEN=0.0718	IERR=0	CHISQL	9.2160	P=0.76248	DEN=0.0684	IERR=0	CELL	1
CHISQP	9.0000	P=0.74734	DEN=0.0718	IERR=0	CHISQL	9.3755	P=0.77319	DEN=0.0659	IERR=0	CELL	1
CHISQP	9.0000	P=0.74734	DEN=0.0718	IERR=0	CHISQL	9.4999	P=0.78127	DEN=0.0640	IERR=0	CELL	1
CHISQP	9.0000	P=0.74734	DEN=0.0718	IERR=0	CHISQL	9.5713	P=0.78580	DEN=0.0629	IERR=0	CELL	1
CHISQP	9.0000	P=0.74734	DEN=0.0718	IERR=0	CHISQL	9.8245	P=0.80126	DEN=0.0592	IERR=0	CELL	1
CHISQP	9.0000	P=0.74734	DEN=0.0718	IERR=0	CHISQL	10.0804	P=0.81482	DEN=0.0558	IERR=0	CELL	1
CHISQP	9.0000	P=0.74734	DEN=0.0718	IERR=0	CHISQL	11.5352	P=0.89729	DEN=0.0335	IERR=0	CELL	1
CHISQP	9.0000	P=0.74734	DEN=0.0718	IERR=0	CHISQL	11.9352	P=0.89729	DEN=0.0335	IERR=0	CELL	1
CHISQP	9.2000	P=0.76139	DEN=0.0686	IERR=0	CHISQL	8.1858	P=0.68351	DEN=0.0851	IERR=0	CELL	1
CHISQP	9.2000	P=0.76139	DEN=0.0686	IERR=0	CHISQL	8.7174	P=0.72641	DEN=0.0764	IERR=0	CELL	1
CHISQP	9.2000	P=0.76139	DEN=0.0686	IERR=0	CHISQL	9.0599	P=0.75162	DEN=0.0708	IERR=0	CELL	1
CHISQP	9.2000	P=0.76139	DEN=0.0686	IERR=0	CHISQL	9.1664	P=0.75907	DEN=0.0692	IERR=0	CELL	1
CHISQP	9.2000	P=0.76139	DEN=0.0686	IERR=0	CHISQL	9.3411	P=0.77092	DEN=0.0664	IERR=0	CELL	1
CHISQP	9.2000	P=0.76139	DEN=0.0686	IERR=0	CHISQL	9.3524	P=0.77166	DEN=0.0663	IERR=0	CELL	1
CHISQP	9.2000	P=0.76139	DEN=0.0686	IERR=0	CHISQL	9.3590	P=0.77210	DEN=0.0662	IERR=0	CELL	1
CHISQP	9.2000	P=0.76139	DEN=0.0686	IERR=0	CHISQL	9.6472	P=0.79054	DEN=0.0618	IERR=0	CELL	1
CHISQP	9.2000	P=0.76139	DEN=0.0686	IERR=0	CHISQL	9.7059	P=0.79414	DEN=0.0609	IERR=0	CELL	1
CHISQP	9.2000	P=0.76139	DEN=0.0686	IERR=0	CHISQL	9.7381	P=0.79609	DEN=0.0605	IERR=0	CELL	1
CHISQP	9.2000	P=0.76139	DEN=0.0686	IERR=0	CHISQL	9.7716	P=0.79811	DEN=0.0600	IERR=0	CELL	1
CHISQP	9.2000	P=0.76139	DEN=0.0686	IERR=0	CHISQL	9.8441	P=0.80242	DEN=0.0589	IERR=0	CELL	1
CHISQP	9.2000	P=0.76139	DEN=0.0686	IERR=0	CHISQL	9.9443	P=0.80836	DEN=0.0574	IERR=0	CELL	1
CHISQP	9.2000	P=0.76139	DEN=0.0686	IERR=0	CHISQL	10.1228	P=0.81827	DEN=0.0549	IERR=0	CELL	1
CHISQP	9.2000	P=0.76139	DEN=0.0686	IERR=0	CHISQL	10.5502	P=0.84052	DEN=0.0492	IERR=0	CELL	1
CHISQP	9.4000	P=0.77480	DEN=0.0655	IERR=0	CHISQL	8.4725	P=0.70723	DEN=0.0804	IERR=0	CELL	1
CHISQP	9.4000	P=0.77480	DEN=0.0655	IERR=0	CHISQL	9.1449	P=0.75758	DEN=0.0695	IERR=0	CELL	1
CHISQP	9.4000	P=0.77480	DEN=0.0655	IERR=0	CHISQL	9.1449	P=0.75758	DEN=0.0695	IERR=0	CELL	1
CHISQP	9.4000	P=0.77480	DEN=0.0655	IERR=0	CHISQL	9.1721	P=0.75947	DEN=0.0691	IERR=0	CELL	1
CHISQP	9.4000	P=0.77480	DEN=0.0655	IERR=0	CHISQL	9.2190	P=0.76269	DEN=0.0683	IERR=0	CELL	1
CHISQP	9.4000	P=0.77480	DEN=0.0655	IERR=0	CHISQL	9.5034	P=0.78150	DEN=0.0640	IERR=0	CELL	1
CHISQP	9.4000	P=0.77480	DEN=0.0655	IERR=0	CHISQL	9.5130	P=0.78211	DEN=0.0638	IERR=0	CELL	1
CHISQP	9.4000	P=0.77480	DEN=0.0655	IERR=0	CHISQL	9.5192	P=0.78251	DEN=0.0637	IERR=0	CELL	1

CHISQP=11.2000	P=0.86987	UFN=0.0413	IERR=0	CHISQL=10.4679	P=0.833643	UEN=0.0503	IERR=0	CELL FREQ	8	8	15	11	8	17	6
CHISQP=11.2000	P=0.86987	UFN=0.0413	IERR=0	CHISQL=10.5182	P=0.83894	UEN=0.0496	IERR=0	CELL FREQ	7	7	17	11	15	18	13
CHISQP=11.2000	P=0.86987	UFN=0.0413	IERR=0	CHISQL=10.5901	P=0.84247	UEN=0.0487	IERR=0	CELL FREQ	10	10	18	14	20	18	10
CHISQP=11.2000	P=0.86987	UFN=0.0413	IERR=0	CHISQL=10.6665	P=0.84616	UEN=0.0477	IERR=0	CELL FREQ	10	10	18	14	20	18	10
CHISQP=11.2000	P=0.86987	UFN=0.0413	IERR=0	CHISQL=11.0312	P=0.86274	UEN=0.0432	IERR=0	CELL FREQ	10	10	18	14	20	18	10
CHISQP=11.2000	P=0.86987	UFN=0.0413	IERR=0	CHISQL=11.8065	P=0.89290	UEN=0.0348	IERR=0	CELL FREQ	10	10	18	14	20	18	10
CHISQP=11.2000	P=0.86987	UFN=0.0413	IERR=0	CHISQL=12.2057	P=0.90600	UEN=0.0310	IERR=0	CELL FREQ	10	10	18	14	20	18	10
CHISQP=11.2000	P=0.86987	UFN=0.0413	IERR=0	CHISQL=12.2483	P=0.90732	UEN=0.0306	IERR=0	CELL FREQ	10	10	18	14	20	18	10
CHISQP=11.2000	P=0.86987	UFN=0.0413	IERR=0	CHISQL=12.5465	P=0.91604	UEN=0.0280	IERR=0	CELL FREQ	10	10	18	14	20	18	10
CHISQP=11.2000	P=0.86987	UFN=0.0413	IERR=0	CHISQL=12.6190	P=0.91805	UEN=0.0274	IERR=0	CELL FREQ	14	13	18	10	10	15	13
CHISQP=11.2000	P=0.86987	UFN=0.0413	IERR=0	CHISQL=12.9335	P=0.92623	UEN=0.0249	IERR=0	CELL FREQ	14	13	18	10	10	15	13
CHISQP=11.2000	P=0.86987	UFN=0.0413	IERR=0	CHISQL=12.9795	P=0.92739	UEN=0.0245	IERR=0	CELL FREQ	11	9	11	6	13	12	14
CHISQP=11.4000	P=0.87790	UFN=0.0390	IERR=0	CHISQL=13.7620	P=0.850666	UEN=0.0465	IERR=0	CELL FREQ	12	9	6	6	6	6	7
CHISQP=11.4000	P=0.87790	UFN=0.0390	IERR=0	CHISQL=13.9305	P=0.85832	UEN=0.0445	IERR=0	CELL FREQ	12	9	6	6	6	6	7
CHISQP=11.4000	P=0.87790	UFN=0.0390	IERR=0	CHISQL=11.5461	P=0.88349	UEN=0.0375	IERR=0	CELL FREQ	11	12	17	19	19	16	9
CHISQP=11.4000	P=0.87790	UFN=0.0390	IERR=0	CHISQL=11.5876	P=0.88504	UEN=0.0370	IERR=0	CELL FREQ	11	12	17	19	19	16	9
CHISQP=11.4000	P=0.87790	UFN=0.0390	IERR=0	CHISQL=11.6210	P=0.88627	UEN=0.0367	IERR=0	CELL FREQ	12	7	16	18	15	15	13
CHISQP=11.4000	P=0.87790	UFN=0.0390	IERR=0	CHISQL=11.7426	P=0.890665	UEN=0.0354	IERR=0	CELL FREQ	11	9	11	6	13	12	14
CHISQP=11.4000	P=0.87790	UFN=0.0390	IERR=0	CHISQL=12.0068	P=0.89967	UEN=0.0328	IERR=0	CELL FREQ	11	9	11	6	13	12	14
CHISQP=11.4000	P=0.87790	UFN=0.0390	IERR=0	CHISQL=22.0295	P=0.90041	UEN=0.0326	IERR=0	CELL FREQ	9	9	5	5	5	5	15
CHISQP=11.4000	P=0.87790	UFN=0.0390	IERR=0	CHISQL=22.2892	P=0.90856	UEN=0.0302	IERR=0	CELL FREQ	6	6	6	12	17	19	14
CHISQP=11.4000	P=0.87790	UFN=0.0390	IERR=0	CHISQL=22.3725	P=0.91104	UEN=0.0295	IERR=0	CELL FREQ	10	10	13	13	14	13	14
CHISQP=11.4000	P=0.87790	UFN=0.0390	IERR=0	CHISQL=22.5284	P=0.91553	UEN=0.0281	IERR=0	CELL FREQ	10	10	13	13	14	13	14
CHISQP=11.4000	P=0.87790	UFN=0.0390	IERR=0	CHISQL=13.1156	P=0.93066	UEN=0.0235	IERR=0	CELL FREQ	10	10	12	10	11	11	13
CHISQP=11.4000	P=0.87790	UFN=0.0390	IERR=0	CHISQL=13.9503	P=0.94793	UEN=0.0181	IERR=0	CELL FREQ	13	12	10	10	11	11	13
CHISQP=11.6000	P=0.88550	UFN=0.0369	IERR=0	CHISQL=13.2795	P=0.82672	UEN=0.0528	IERR=0	CELL FREQ	12	9	7	7	8	7	12
CHISQP=11.6000	P=0.88550	UFN=0.0369	IERR=0	CHISQL=13.2795	P=0.82672	UEN=0.0528	IERR=0	CELL FREQ	12	9	7	7	8	7	12
CHISQP=11.6000	P=0.88550	UFN=0.0369	IERR=J	CHISQL=10.5187	P=0.83897	UEN=0.0496	IERR=0	CELL FREQ	12	9	7	7	8	7	12
CHISQP=11.6000	P=0.88550	UFN=0.0369	IERR=0	CHISQL=11.0131	P=0.86195	UEN=0.0435	IERR=0	CELL FREQ	11	10	10	10	11	11	11
CHISQP=11.6000	P=0.88550	UFN=0.0369	IERR=U	CHISQL=11.2150	P=0.87049	UEN=0.0411	IERR=0	CELL FREQ	11	10	10	10	11	11	11
CHISQP=11.6000	P=0.88550	UFN=0.0369	IERR=U	CHISQL=11.3675	P=0.87663	UEN=0.0394	IERR=0	CELL FREQ	11	10	10	10	11	11	11
CHISQP=11.6000	P=0.88550	UFN=0.0369	IERR=U	CHISQL=11.7688	P=0.89158	UEN=0.0352	IERR=0	CELL FREQ	10	8	12	14	17	17	8
CHISQP=11.6000	P=0.88550	UFN=0.0369	IERR=U	CHISQL=11.8053	P=0.89285	UEN=0.0348	IERR=0	CELL FREQ	10	8	12	14	17	17	8
CHISQP=11.6000	P=0.88550	UFN=0.0369	IERR=0	CHISQL=11.8710	P=0.89512	UEN=0.0341	IERR=0	CELL FREQ	8	8	8	8	10	12	11
CHISQP=11.6000	P=0.88550	UFN=0.0369	IERR=0	CHISQL=13.3261	P=0.90967	UEN=0.0299	IERR=0	CELL FREQ	13	7	13	11	16	16	10
CHISQP=11.6000	P=0.88550	UFN=0.0369	IERR=0	CHISQL=12.8268	P=0.92356	UEN=0.0257	IERR=0	CELL FREQ	14	7	10	10	15	15	12
CHISQP=11.6000	P=0.88550	UFN=0.0369	IERR=0	CHISQL=13.9160	P=0.92582	UEN=0.0250	IERR=0	CELL FREQ	14	7	10	10	15	15	12
CHISQP=11.6000	P=0.88550	UFN=0.0369	IERR=0	CHISQL=13.1564	P=0.93161	UEN=0.0232	IERR=0	CELL FREQ	13	7	9	8	14	15	10
CHISQP=11.6000	P=0.88550	UFN=0.0369	IERR=0	CHISQL=14.3253	P=0.95431	UEN=0.0160	IERR=0	CELL FREQ	8	13	13	13	15	15	8
CHISQP=11.6000	P=0.88550	UFN=0.0369	IERR=0	CHISQL=14.4330	P=0.95601	UEN=0.0155	IERR=0	CELL FREQ	14	7	10	10	14	14	13
CHISQP=11.8000	P=0.89267	UFN=0.0348	IERR=0	CHISQL=10.8080	P=0.85279	UEN=0.0459	IERR=0	CELL FREQ	14	7	9	9	11	11	7
CHISQP=11.8000	P=0.89267	UFN=0.0348	IERR=0	CHISQL=10.8944	P=0.85571	UEN=0.0449	IERR=0	CELL FREQ	7	7	16	18	18	14	7
CHISQP=11.8000	P=0.89267	UFN=0.0348	IERR=0	CHISQL=11.1919	P=0.86953	UEN=0.0414	IERR=0	CELL FREQ	10	10	11	11	13	13	15
CHISQP=11.8000	P=0.89267	UFN=0.0348	IERR=0	CHISQL=11.3401	P=0.87554	UEN=0.0397	IERR=0	CELL FREQ	7	7	16	18	18	14	7
CHISQP=11.8000	P=0.89267	UFN=0.0348	IERR=0	CHISQL=11.6363	P=0.88683	UEN=0.0365	IERR=0	CELL FREQ	5	5	8	8	12	14	8
CHISQP=11.8000	P=0.89267	UFN=0.0348	IERR=0	CHISQL=11.7049	P=0.88931	UEN=0.0358	IERR=0	CELL FREQ	6	6	9	9	12	14	7
CHISQP=11.8000	P=0.89267	UFN=0.0348	IERR=0	CHISQL=11.7388	P=0.89052	UEN=0.0355	IERR=0	CELL FREQ	6	6	9	9	12	14	7
CHISQP=11.8000	P=0.89267	UFN=0.0348	IERR=0	CHISQL=11.7648	P=0.89144	UEN=0.0352	IERR=0	CELL FREQ	12	7	15	5	10	11	10
CHISQP=11.8000	P=0.89267	UFN=0.0348	IERR=0	CHISQL=12.1216	P=0.90337	UEN=0.0317	IERR=0	CELL FREQ	12	7	13	9	11	11	16
CHISQP=11.8000	P=0.89267	UFN=0.0348	IERR=0	CHISQL=12.4802	P=0.91417	UEN=0.0285	IERR=0	CELL FREQ	12	7	13	9	11	11	16
CHISQP=11.8000	P=0.89267	UFN=0.0348	IERR=0	CHISQL=12.5024	P=0.91480	UEN=0.0283	IERR=0	CELL FREQ	14	7	11	14	14	15	6
CHISQP=11.8000	P=0.89267	UFN=0.0348	IERR=0	CHISQL=12.8458	P=0.92405	UEN=0.0255	IERR=0	CELL FREQ	14	7	11	14	14	15	6
CHISQP=11.8000	P=0.89267	UFN=0.0348	IERR=0	CHISQL=12.8997	P=0.92541	UEN=0.0251	IERR=0	CELL FREQ	15	13	13	4	11	13	9
CHISQP=11.8000	P=0.89267	UFN=0.0348	IERR=0	CHISQL=13.3567	P=0.93612	UEN=0.0218	IERR=0	CELL FREQ	13	12	10	11	11	14	10
CHISQP=11.8000	P=0.89267	UFN=0.0348	IERR=0	CHISQL=14.3327	P=0.95443	UEN=0.0160	IERR=0	CELL FREQ	13	12	10	11	11	14	10
CHISQP=12.0000	P=0.89944	UFN=0.0329	IERR=0	CHISQL=10.7825	P=0.84787	UEN=0.0473	IERR=0	CELL FREQ	11	14	6	6	7	7	18
CHISQP=12.0000	P=0.89944	UFN=0.0329	IERR=0	CHISQL=11.0947	P=0.86546	UEN=0.0425	IERR=0	CELL FREQ	14	8	6	6	7	7	18
CHISQP=12.0000	P=0.89944	UFN=0.0329	IERR=0	CHISQL=11.1811	P=0.86909	UEN=0.0415	IERR=0	CELL FREQ	14	8	6	6	7	7	18
CHISQP=12.0000	P=0.89944	UFN=0.0329	IERR=0	CHISQL=11.3236	P=0.87489	UEN=0.0399	IERR=0	CELL FREQ	8	8	15	9	12	16	6
CHISQP=12.0000	P=0.89944	UFN=0.0329	IERR=0	CHISQL=11.4492	P=0.87981	UEN=0.0385	IERR=0	CELL FREQ	7	7	16	18	18	16	6
CHISQP=12.0000	P=0.89944	UFN=0.0329	IERR=0	CHISQL=11.9391	P=0.89742	UEN=0.0335	IERR=0	CELL FREQ	7	7	16	18	18	16	6
CHISQP=12.0000	P=0.89944	UFN=0.0329	IERR=0	CHISQL=12.1240	P=0.90345	UEN=0.0317	IERR=0	CELL FREQ	11	10	12	6	14	14	15

CHISQPPH	1220000	P=0.899444	UEN=0.0329	IERR=0	CHISQ	22.1440	P=0.90408	UEN=0.0315	IERR=0	CELL	FREQ	12	10	6	17	17	10	6	13	5
CHISQPPH	1220000	P=0.899444	UEN=0.0329	IERR=0	CHISQ	22.1440	P=0.90408	UEN=0.0315	IERR=0	CELL	FREQ	12	17	15	11	16	16	13	6	11
CHISQPPH	1220000	P=0.899444	UEN=0.0329	IERR=0	CHISQ	22.3062	P=0.90907	UEN=0.0302	IERR=0	CELL	FREQ	6	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.899444	UEN=0.0329	IERR=0	CHISQ	22.4083	P=0.91209	UEN=0.0292	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.899444	UEN=0.0329	IERR=0	CHISQ	22.4421	P=0.91451	UEN=0.0284	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.899444	UEN=0.0329	IERR=0	CHISQ	22.6453	P=0.91876	UEN=0.0271	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.899444	UEN=0.0329	IERR=0	CHISQ	22.7313	P=0.92107	UEN=0.0265	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.899444	UEN=0.0329	IERR=0	CHISQ	22.8245	P=0.92375	UEN=0.0214	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.899444	UEN=0.0329	IERR=0	CHISQ	22.8899	P=0.92524	UEN=0.0173	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.899444	UEN=0.0329	IERR=0	CHISQ	22.9165	P=0.92606	UEN=0.0066	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.899444	UEN=0.0310	IERR=0	CHISQ	22.9124	P=0.92695	UEN=0.0414	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.905833	UEN=0.0310	IERR=0	CHISQ	22.1924	P=0.89955	UEN=0.0414	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.905833	UEN=0.0310	IERR=0	CHISQ	22.2472	P=0.87181	UEN=0.0408	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.905833	UEN=0.0310	IERR=0	CHISQ	22.8201	P=0.89337	UEN=0.0346	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.905833	UEN=0.0310	IERR=0	CHISQ	22.8775	P=0.89534	UEN=0.0341	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.905833	UEN=0.0310	IERR=0	CHISQ	22.0233	P=0.90020	UEN=0.0327	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.905833	UEN=0.0310	IERR=0	CHISQ	22.1137	P=0.90312	UEN=0.0318	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.905833	UEN=0.0310	IERR=0	CHISQ	22.6889	P=0.94300	UEN=0.0196	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.905833	UEN=0.0310	IERR=0	CHISQ	22.7069	P=0.94335	UEN=0.0195	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.905833	UEN=0.0310	IERR=0	CHISQ	22.8396	P=0.96189	UEN=0.0135	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.911855	UEN=0.0292	IERR=0	CHISQ	22.3744	P=0.87690	UEN=0.0393	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.911855	UEN=0.0292	IERR=0	CHISQ	22.5294	P=0.88288	UEN=0.0378	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.911855	UEN=0.0292	IERR=0	CHISQ	22.5619	P=0.88408	UEN=0.037	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.911855	UEN=0.0292	IERR=0	CHISQ	22.0708	P=0.90174	UEN=0.0322	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.911855	UEN=0.0292	IERR=0	CHISQ	22.2880	P=0.90852	UEN=0.0307	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.911855	UEN=0.0292	IERR=0	CHISQ	22.3100	P=0.90918	UEN=0.0300	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.911855	UEN=0.0292	IERR=0	CHISQ	22.5086	P=0.91497	UEN=0.0283	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.911855	UEN=0.0292	IERR=0	CHISQ	22.6246	P=0.91820	UEN=0.0273	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.911855	UEN=0.0292	IERR=0	CHISQ	22.7267	P=0.92095	UEN=0.0265	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.911855	UEN=0.0292	IERR=0	CHISQ	22.0294	P=0.92860	UEN=0.0241	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.911855	UEN=0.0292	IERR=0	CHISQ	22.1426	P=0.93129	UEN=0.0233	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.911855	UEN=0.0292	IERR=0	CHISQ	22.5271	P=0.93974	UEN=0.0207	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.911855	UEN=0.0292	IERR=0	CHISQ	22.8764	P=0.94658	UEN=0.0185	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.917522	UEN=0.0275	IERR=0	CHISQ	22.5760	P=0.84179	UEN=0.0489	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.917522	UEN=0.0275	IERR=0	CHISQ	22.3021	P=0.87403	UEN=0.0401	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.917522	UEN=0.0275	IERR=0	CHISQ	22.8968	P=0.89599	UEN=0.0339	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.917522	UEN=0.0275	IERR=0	CHISQ	22.9670	P=0.89835	UEN=0.0332	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.917522	UEN=0.0275	IERR=0	CHISQ	22.1959	P=0.90570	UEN=0.0310	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.917522	UEN=0.0275	IERR=0	CHISQ	22.8969	P=0.92534	UEN=0.0251	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.917522	UEN=0.0275	IERR=0	CHISQ	22.0590	P=0.92932	UEN=0.0239	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.917522	UEN=0.0275	IERR=0	CHISQ	22.2684	P=0.93417	UEN=0.0224	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.917522	UEN=0.0275	IERR=0	CHISQ	22.3581	P=0.93615	UEN=0.0218	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.917522	UEN=0.0275	IERR=0	CHISQ	22.0236	P=0.94924	UEN=0.0177	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.92287	UEN=0.0259	IERR=0	CHISQ	22.4516	P=0.95629	UEN=0.0154	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.92287	UEN=0.0259	IERR=0	CHISQ	22.9209	P=0.89681	UEN=0.0337	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.92287	UEN=0.0259	IERR=0	CHISQ	22.4826	P=0.91423	UEN=0.0285	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.92287	UEN=0.0259	IERR=0	CHISQ	22.5683	P=0.91663	UEN=0.0278	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.92287	UEN=0.0259	IERR=0	CHISQ	22.9521	P=0.92672	UEN=0.0247	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.92287	UEN=0.0259	IERR=0	CHISQ	22.1380	P=0.93118	UEN=0.0233	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.92287	UEN=0.0259	IERR=0	CHISQ	22.2025	P=0.93267	UEN=0.0229	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.92287	UEN=0.0259	IERR=0	CHISQ	22.3489	P=0.93704	UEN=0.0215	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.92287	UEN=0.0259	IERR=0	CHISQ	22.8479	P=0.94605	UEN=0.0187	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.92287	UEN=0.0259	IERR=0	CHISQ	22.9883	P=0.94861	UEN=0.0179	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1220000	P=0.92287	UEN=0.0259	IERR=0	CHISQ	22.1389	P=0.95124	UEN=0.0170	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1330000	P=0.92789	UEN=0.0244	IERR=0	CHISQ	22.4351	P=0.95604	UEN=0.0154	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1330000	P=0.92789	UEN=0.0244	IERR=0	CHISQ	22.4764	P=0.91406	UEN=0.0286	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1330000	P=0.92789	UEN=0.0244	IERR=0	CHISQ	22.3343	P=0.93563	UEN=0.0230	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13
CHISQPPH	1330000	P=0.92789	UEN=0.0244	IERR=0	CHISQ	22.5950	P=0.94113	UEN=0.0202	IERR=0	CELL	FREQ	12	17	13	14	11	13	10	16	13

CHISQP=13.0000	P=0.92789	UFN=0.0244	IERN=0	CHISQL=13.9315	P=0.94759	UFN=0.0182	IERN=0	CELL	FREQ	9
CHISQP=13.0000	P=0.92789	UFN=0.0244	IERN=0	CHISQL=13.9315	P=0.94759	UFN=0.0182	IERN=0	CELL	FREQ	14
CHISQP=13.0000	P=0.92789	UFN=0.0244	IERN=0	CHISQL=13.9315	P=0.94759	UFN=0.0182	IERN=0	CELL	FREQ	15
CHISQP=13.0000	P=0.92789	UFN=0.0244	IERN=0	CHISQL=13.9315	P=0.94759	UFN=0.0182	IERN=0	CELL	FREQ	16
CHISQP=13.0000	P=0.92789	UFN=0.0244	IERN=0	CHISQL=13.9315	P=0.94759	UFN=0.0182	IERN=0	CELL	FREQ	17
CHISQP=13.0000	P=0.92789	UFN=0.0244	IERN=0	CHISQL=13.9315	P=0.94759	UFN=0.0182	IERN=0	CELL	FREQ	18
CHISQP=13.0000	P=0.92789	UFN=0.0244	IERN=0	CHISQL=13.9315	P=0.94759	UFN=0.0182	IERN=0	CELL	FREQ	19
CHISQP=13.0000	P=0.92789	UFN=0.0244	IERN=0	CHISQL=13.9315	P=0.94759	UFN=0.0182	IERN=0	CELL	FREQ	20
CHISQP=13.2000	P=0.93262	UFN=0.0229	IERN=0	CHISQL=12.3348	P=0.90993	UFN=0.0298	IERN=0	CELL	FREQ	3
CHISQP=13.2000	P=0.93262	UFN=0.0229	IERN=0	CHISQL=12.5425	P=0.91593	UFN=0.0280	IERN=0	CELL	FREQ	12
CHISQP=13.2000	P=0.93262	UFN=0.0229	IERN=0	CHISQL=12.5721	P=0.91675	UFN=0.0278	IERN=0	CELL	FREQ	13
CHISQP=13.2000	P=0.93262	UFN=0.0229	IERN=0	CHISQL=12.9559	P=0.92661	UFN=0.0247	IERN=0	CELL	FREQ	14
CHISQP=13.2000	P=0.93262	UFN=0.0229	IERN=0	CHISQL=13.2202	P=0.93308	UFN=0.0228	IERN=0	CELL	FREQ	15
CHISQP=13.2000	P=0.93262	UFN=0.0229	IERN=0	CHISQL=13.3206	P=0.93533	UFN=0.0221	IERN=0	CELL	FREQ	16
CHISQP=13.2000	P=0.93262	UFN=0.0229	IERN=0	CHISQL=13.5714	P=0.94065	UFN=0.0204	IERN=0	CELL	FREQ	17
CHISQP=13.2000	P=0.93262	UFN=0.0229	IERN=0	CHISQL=13.5849	P=0.94092	UFN=0.0203	IERN=0	CELL	FREQ	18
CHISQP=13.2000	P=0.93262	UFN=0.0229	IERN=0	CHISQL=13.7691	P=0.94456	UFN=0.0191	IERN=0	CELL	FREQ	19
CHISQP=13.2000	P=0.93262	UFN=0.0229	IERN=0	CHISQL=13.8176	P=0.94548	UFN=0.0189	IERN=0	CELL	FREQ	20
CHISQP=13.2000	P=0.93262	UFN=0.0229	IERN=0	CHISQL=13.8857	P=0.94675	UFN=0.0185	IERN=0	CELL	FREQ	3
CHISQP=13.2000	P=0.93262	UFN=0.0229	IERN=0	CHISQL=13.8857	P=0.94675	UFN=0.0185	IERN=0	CELL	FREQ	12
CHISQP=13.2000	P=0.93262	UFN=0.0229	IERN=0	CHISQL=13.9043	P=0.94709	UFN=0.0183	IERN=0	CELL	FREQ	13
CHISQP=13.2000	P=0.93262	UFN=0.0229	IERN=0	CHISQL=13.9480	P=0.94789	UFN=0.0181	IERN=0	CELL	FREQ	14
CHISQP=13.2000	P=0.93262	UFN=0.0229	IERN=0	CHISQL=14.1542	P=0.95149	UFN=0.0169	IERN=0	CELL	FREQ	15
CHISQP=13.2000	P=0.93262	UFN=0.0229	IERN=0	CHISQL=14.6496	P=0.95924	UFN=0.0144	IERN=0	CELL	FREQ	16
CHISQP=13.2000	P=0.93262	UFN=0.0229	IERN=0	CHISQL=14.7560	P=0.96074	UFN=0.0139	IERN=0	CELL	FREQ	17
CHISQP=13.4000	P=0.93706	UFN=0.0215	IERN=0	CHISQL=12.3370	P=0.91176	UFN=0.0293	IERN=0	CELL	FREQ	14
CHISQP=13.4000	P=0.93706	UFN=0.0215	IERN=0	CHISQL=12.8497	P=0.92414	UFN=0.0255	IERN=0	CELL	FREQ	15
CHISQP=13.4000	P=0.93706	UFN=0.0215	IERN=0	CHISQL=13.3283	P=0.93550	UFN=0.0220	IERN=0	CELL	FREQ	16
CHISQP=13.4000	P=0.93706	UFN=0.0215	IERN=0	CHISQL=13.4524	P=0.93818	UFN=0.0212	IERN=0	CELL	FREQ	17
CHISQP=13.4000	P=0.93706	UFN=0.0215	IERN=0	CHISQL=13.8877	P=0.96625	UFN=0.0133	IERN=0	CELL	FREQ	18
CHISQP=13.4000	P=0.93706	UFN=0.0215	IERN=0	CHISQL=13.8821	P=0.96592	UFN=0.0129	IERN=0	CELL	FREQ	19
CHISQP=13.4000	P=0.93706	UFN=0.0215	IERN=0	CHISQL=13.8293	P=0.96929	UFN=0.0041	IERN=0	CELL	FREQ	20
CHISQP=13.6000	P=0.94123	UFN=0.0202	IERN=0	CHISQL=12.7172	P=0.92069	UFN=0.0266	IERN=0	CELL	FREQ	3
CHISQP=13.6000	P=0.94123	UFN=0.0202	IERN=0	CHISQL=13.0454	P=0.92899	UFN=0.0240	IERN=0	CELL	FREQ	12
CHISQP=13.6000	P=0.94123	UFN=0.0202	IERN=0	CHISQL=13.1301	P=0.93100	UFN=0.0234	IERN=0	CELL	FREQ	13
CHISQP=13.6000	P=0.94123	UFN=0.0202	IERN=0	CHISQL=13.2545	P=0.93385	UFN=0.0225	IERN=0	CELL	FREQ	14
CHISQP=13.6000	P=0.94123	UFN=0.0202	IERN=0	CHISQL=13.4646	P=0.93843	UFN=0.0211	IERN=0	CELL	FREQ	15
CHISQP=13.6000	P=0.94123	UFN=0.0202	IERN=0	CHISQL=13.5668	P=0.94055	UFN=0.0204	IERN=0	CELL	FREQ	16
CHISQP=13.6000	P=0.94123	UFN=0.0202	IERN=0	CHISQL=13.7030	P=0.94328	UFN=0.0196	IERN=0	CELL	FREQ	17
CHISQP=13.6000	P=0.94123	UFN=0.0202	IERN=0	CHISQL=13.7575	P=0.94433	UFN=0.0192	IERN=0	CELL	FREQ	18
CHISQP=13.6000	P=0.94123	UFN=0.0202	IERN=0	CHISQL=14.3590	P=0.95485	UFN=0.0158	IERN=0	CELL	FREQ	19
CHISQP=13.8000	P=0.94514	UFN=0.0190	IERN=0	CHISQL=13.8560	P=0.96281	UFN=0.0134	IERN=0	CELL	FREQ	20
CHISQP=13.8000	P=0.94514	UFN=0.0190	IERN=0	CHISQL=13.8085	P=0.96281	UFN=0.0134	IERN=0	CELL	FREQ	3
CHISQP=13.8000	P=0.94514	UFN=0.0190	IERN=0	CHISQL=13.6296	P=0.94182	UFN=0.0200	IERN=0	CELL	FREQ	12
CHISQP=13.8000	P=0.94514	UFN=0.0190	IERN=0	CHISQL=13.7513	P=0.94421	UFN=0.0193	IERN=0	CELL	FREQ	13
CHISQP=13.8000	P=0.94514	UFN=0.0190	IERN=0	CHISQL=13.7911	P=0.94498	UFN=0.0190	IERN=0	CELL	FREQ	14
CHISQP=13.8000	P=0.94514	UFN=0.0190	IERN=0	CHISQL=13.9564	P=0.94804	UFN=0.0180	IERN=0	CELL	FREQ	15
CHISQP=13.8000	P=0.94514	UFN=0.0190	IERN=0	CHISQL=14.3206	P=0.95424	UFN=0.0160	IERN=0	CELL	FREQ	16
CHISQP=13.8000	P=0.94514	UFN=0.0190	IERN=0	CHISQL=14.5531	P=0.95783	UFN=0.0149	IERN=0	CELL	FREQ	17
CHISQP=13.8000	P=0.94514	UFN=0.0190	IERN=0	CHISQL=15.9404	P=0.96388	UFN=0.0129	IERN=0	CELL	FREQ	18
CHISQP=14.0000	P=0.94882	UFN=0.0178	IERN=0	CHISQL=15.7283	P=0.97228	UFN=0.0100	IERN=0	CELL	FREQ	19
CHISQP=14.0000	P=0.94882	UFN=0.0178	IERN=0	CHISQL=12.1918	P=0.90558	UFN=0.0311	IERN=0	CELL	FREQ	20
CHISQP=14.0000	P=0.94882	UFN=0.0178	IERN=0	CHISQL=13.1067	P=0.93045	UFN=0.0236	IERN=0	CELL	FREQ	3
CHISQP=14.0000	P=0.94882	UFN=0.0178	IERN=0	CHISQL=13.1571	P=0.93163	UFN=0.0232	IERN=0	CELL	FREQ	12
CHISQP=14.0000	P=0.94882	UFN=0.0178	IERN=0	CHISQL=13.3440	P=0.93584	UFN=0.0219	IERN=0	CELL	FREQ	13
CHISQP=14.0000	P=0.94882	UFN=0.0178	IERN=0	CHISQL=13.4548	P=0.93823	UFN=0.0212	IERN=0	CELL	FREQ	14
CHISQP=14.0000	P=0.94882	UFN=0.0178	IERN=0	CHISQL=13.8534	P=0.94615	UFN=0.0186	IERN=0	CELL	FREQ	15
CHISQP=14.0000	P=0.94882	UFN=0.0178	IERN=0	CHISQL=14.3027	P=0.95395	UFN=0.0161	IERN=0	CELL	FREQ	16
CHISQP=14.0000	P=0.94882	UFN=0.0178	IERN=0	CHISQL=14.3568	P=0.95481	UFN=0.0158	IERN=0	CELL	FREQ	17
CHISQP=14.0000	P=0.94882	UFN=0.0178	IERN=0	CHISQL=14.8283	P=0.96174	UFN=0.0136	IERN=0	CELL	FREQ	18
CHISQP=14.2000	P=0.95226	UFN=0.0167	IERN=0	CHISQL=17.8654	P=0.98741	UFN=0.0047	IERN=0	CELL	FREQ	19
CHISQP=14.2000	P=0.95226	UFN=0.0167	IERN=0	CHISQL=12.4145	P=0.91227	UFN=0.0291	IERN=0	CELL	FREQ	20
CHISQP=14.2000	P=0.95226	UFN=0.0167	IERN=0	CHISQL=12.9017	P=0.92566	UFN=0.0251	IERN=0	CELL	FREQ	3

CHISQP=14.2000	P=0.952226	VEN=0.0167	IERR=0	CHISQL=13.2025	P=0.93267	DEN=0.0229	IERR=0	CELL FREQ	8
CHISQP=14.2000	P=0.952226	VEN=0.0167	IERR=0	CHISQL=13.0489	P=0.94221	DEN=0.0199	IERR=0	CELL FREQ	7
CHISQP=14.2000	P=0.952226	VEN=0.0167	IERR=0	CHISQL=14.0425	P=0.94957	DEN=0.0175	IERR=0	CELL FREQ	13
CHISQP=14.2000	P=0.952226	VEN=0.0167	IERR=0	CHISQL=14.9082	P=0.96281	DEN=0.0132	IERR=0	CELL FREQ	17
CHISQP=14.2000	P=0.952226	VEN=0.0167	IERR=0	CHISQL=15.0410	P=0.96452	DEN=0.0126	IERR=0	CELL FREQ	13
CHISQP=14.2000	P=0.952226	VEN=0.0167	IERR=0	CHISQL=15.0259	P=0.97324	DEN=0.0097	IERR=0	CELL FREQ	12
CHISQP=14.4000	P=0.95549	VEN=0.0156	IERR=0	CHISQL=12.7012	P=0.92027	DEN=0.0267	IERR=0	CELL FREQ	11
CHISQP=14.4000	P=0.95549	VEN=0.0156	IERR=0	CHISQL=13.4947	P=0.94691	DEN=0.0184	IERR=0	CELL FREQ	12
CHISQP=14.4000	P=0.95549	VEN=0.0156	IERR=0	CHISQL=14.3548	P=0.95478	DEN=0.0159	IERR=0	CELL FREQ	13
CHISQP=14.4000	P=0.95549	VEN=0.0156	IERR=0	CHISQL=14.3548	P=0.95478	DEN=0.0159	IERR=0	CELL FREQ	10
CHISQP=14.4000	P=0.95549	VEN=0.0156	IERR=0	CHISQL=16.0235	P=0.97510	DEN=0.0091	IERR=0	CELL FREQ	11
CHISQP=14.4000	P=0.95549	VEN=0.0156	IERR=0	CHISQL=16.1942	P=0.97660	DEN=0.0085	IERR=0	CELL FREQ	14
CHISQP=14.4000	P=0.95549	VEN=0.0156	IERR=0	CHISQL=19.2645	P=0.99260	DEN=0.0028	IERR=0	CELL FREQ	15
CHISQP=14.6000	P=0.95852	VEN=0.0146	IERR=0	CHISQL=12.6771	P=0.91962	DEN=0.0269	IERR=0	CELL FREQ	12
CHISQP=14.6000	P=0.95852	VEN=0.0146	IERR=0	CHISQL=13.9019	P=0.94705	DEN=0.0184	IERR=0	CELL FREQ	14
CHISQP=14.6000	P=0.95852	VEN=0.0146	IERR=0	CHISQL=14.4232	P=0.95585	DEN=0.0155	IERR=0	CELL FREQ	15
CHISQP=14.6000	P=0.95852	VEN=0.0146	IERR=0	CHISQL=15.4786	P=0.97375	DEN=0.0095	IERR=0	CELL FREQ	14
CHISQP=14.6000	P=0.95852	VEN=0.0146	IERR=0	CHISQL=15.3229	P=0.97375	DEN=0.0095	IERR=0	CELL FREQ	11
CHISQP=14.6000	P=0.95852	VEN=0.0146	IERR=0	CHISQL=19.0478	P=0.99333	DEN=0.0026	IERR=0	CELL FREQ	11
CHISQP=14.8000	P=0.96135	VEN=0.0137	IERR=0	CHISQL=13.2364	P=0.93345	DEN=0.0226	IERR=0	CELL FREQ	13
CHISQP=14.8000	P=0.96135	VEN=0.0137	IERR=0	CHISQL=13.3913	P=0.93687	DEN=0.0216	IERR=0	CELL FREQ	5
CHISQP=14.8000	P=0.96135	VEN=0.0137	IERR=0	CHISQL=14.6274	P=0.95892	DEN=0.0145	IERR=0	CELL FREQ	8
CHISQP=14.8000	P=0.96135	VEN=0.0137	IERR=0	CHISQL=15.1589	P=0.96598	DEN=0.0122	IERR=0	CELL FREQ	14
CHISQP=14.8000	P=0.96135	VEN=0.0137	IERR=0	CHISQL=15.9266	P=0.97420	DEN=0.0094	IERR=0	CELL FREQ	11
CHISQP=14.8000	P=0.96135	VEN=0.0137	IERR=0	CHISQL=15.9368	P=0.97430	DEN=0.0093	IERR=0	CELL FREQ	3
CHISQP=14.8000	P=0.96135	VEN=0.0137	IERR=0	CHISQL=16.5343	P=0.97934	DEN=0.0076	IERR=0	CELL FREQ	10
CHISQP=15.0000	P=0.96400	VEN=0.0128	IERR=0	CHISQL=12.4241	P=0.91255	DEN=0.0290	IERR=0	CELL FREQ	13
CHISQP=15.0000	P=0.96400	VEN=0.0128	IERR=0	CHISQL=13.9476	P=0.94016	DEN=0.0205	IERR=0	CELL FREQ	10
CHISQP=15.0000	P=0.96400	VEN=0.0128	IERR=0	CHISQL=14.6378	P=0.95907	DEN=0.0145	IERR=0	CELL FREQ	7
CHISQP=15.0000	P=0.96400	VEN=0.0128	IERR=0	CHISQL=15.4306	P=0.96914	DEN=0.0111	IERR=0	CELL FREQ	10
CHISQP=15.0000	P=0.96400	VEN=0.0128	IERR=0	CHISQL=15.5261	P=0.97018	DEN=0.0107	IERR=0	CELL FREQ	17
CHISQP=15.0000	P=0.96400	VEN=0.0128	IERR=0	CHISQL=16.1236	P=0.97599	DEN=0.0088	IERR=0	CELL FREQ	13
CHISQP=15.0000	P=0.96400	VEN=0.0128	IERR=0	CHISQL=19.4378	P=0.99313	DEN=0.0026	IERR=0	CELL FREQ	11
CHISQP=15.2000	P=0.96648	VEN=0.0120	IERR=0	CHISQL=12.6747	P=0.91956	DEN=0.0269	IERR=0	CELL FREQ	17
CHISQP=15.2000	P=0.96648	VEN=0.0120	IERR=0	CHISQL=13.3868	P=0.93677	DEN=0.0216	IERR=0	CELL FREQ	8
CHISQP=15.2000	P=0.96648	VEN=0.0120	IERR=0	CHISQL=13.9465	P=0.94786	DEN=0.0181	IERR=0	CELL FREQ	5
CHISQP=15.2000	P=0.96648	VEN=0.0120	IERR=0	CHISQL=14.0442	P=0.94960	DEN=0.0175	IERR=0	CELL FREQ	19
CHISQP=15.2000	P=0.96648	VEN=0.0120	IERR=0	CHISQL=14.2383	P=0.95290	DEN=0.0165	IERR=0	CELL FREQ	14
CHISQP=15.2000	P=0.96648	VEN=0.0120	IERR=0	CHISQL=15.2695	P=0.96730	DEN=0.0117	IERR=0	CELL FREQ	18
CHISQP=15.2000	P=0.96648	VEN=0.0120	IERR=0	CHISQL=15.6402	P=0.97139	DEN=0.0103	IERR=0	CELL FREQ	10
CHISQP=15.2000	P=0.96648	VEN=0.0120	IERR=0	CHISQL=16.3057	P=0.97753	DEN=0.0082	IERR=0	CELL FREQ	15
CHISQP=15.2000	P=0.96648	VEN=0.0120	IERR=0	CHISQL=16.6795	P=0.98042	DEN=0.0072	IERR=0	CELL FREQ	10
CHISQP=15.2000	P=0.96648	VEN=0.0120	IERR=0	CHISQL=*****	P=*****	DEN=*****	IERR=*	CELL FREQ	10
CHISQP=15.4000	P=0.96880	VEN=0.0112	IERR=0	CHISQL=13.9122	P=0.94723	DEN=0.0183	IERR=0	CELL FREQ	11
CHISQP=15.4000	P=0.96880	VEN=0.0112	IERR=0	CHISQL=14.8396	P=0.96189	DEN=0.0135	IERR=0	CELL FREQ	18
CHISQP=15.4000	P=0.96880	VEN=0.0112	IERR=0	CHISQL=14.8846	P=0.96249	DEN=0.0133	IERR=0	CELL FREQ	15
CHISQP=15.4000	P=0.96880	VEN=0.0112	IERR=0	CHISQL=15.3167	P=0.96785	DEN=0.0115	IERR=0	CELL FREQ	10
CHISQP=15.4000	P=0.96880	VEN=0.0112	IERR=0	CHISQL=16.0710	P=0.97552	DEN=0.0089	IERR=0	CELL FREQ	10
CHISQP=15.4000	P=0.96880	VEN=0.0112	IERR=0	CHISQL=16.1350	P=0.97609	DEN=0.0087	IERR=0	CELL FREQ	17
CHISQP=15.4000	P=0.96880	VEN=0.0112	IERR=0	CHISQL=17.5109	P=0.98562	DEN=0.0054	IERR=0	CELL FREQ	10
CHISQP=15.4000	P=0.96880	VEN=0.0112	IERR=0	CHISQL=17.9449	P=0.98778	DEN=0.0046	IERR=0	CELL FREQ	6
CHISQP=15.4000	P=0.96880	VEN=0.0112	IERR=0	CHISQL=18.0974	P=0.98846	DEN=0.0044	IERR=0	CELL FREQ	13
CHISQP=15.4000	P=0.96880	VEN=0.0112	IERR=0	CHISQL=18.1196	P=0.98856	DEN=0.0043	IERR=0	CELL FREQ	12
CHISQP=15.6000	P=0.97097	VEN=0.0105	IERR=0	CHISQL=14.1227	P=0.95096	DEN=0.0171	IERR=0	CELL FREQ	18
CHISQP=15.6000	P=0.97097	VEN=0.0105	IERR=0	CHISQL=14.6658	P=0.95947	DEN=0.0143	IERR=0	CELL FREQ	10
CHISQP=15.6000	P=0.97097	VEN=0.0105	IERR=0	CHISQL=15.6542	P=0.97153	DEN=0.0103	IERR=0	CELL FREQ	8
CHISQP=15.6000	P=0.97097	VEN=0.0105	IERR=0	CHISQL=15.9049	P=0.97304	DEN=0.0098	IERR=0	CELL FREQ	9
CHISQP=15.6000	P=0.97097	VEN=0.0105	IERR=0	CHISQL=16.7422	P=0.98086	DEN=0.0071	IERR=0	CELL FREQ	15
CHISQP=15.8000	P=0.97299	VEN=0.0098	IERR=0	CHISQL=14.7340	P=0.96044	DEN=0.0140	IERR=0	CELL FREQ	8
CHISQP=15.8000	P=0.97299	VEN=0.0098	IERR=0	CHISQL=17.5859	P=0.98602	DEN=0.0052	IERR=0	CELL FREQ	8
CHISQP=15.8000	P=0.97299	VEN=0.0098	IERR=0	CHISQL=18.1359	P=0.98863	DEN=0.0043	IERR=0	CELL FREQ	11
CHISQP=16.0000	P=0.97488	VEN=0.0091	IERR=0	CHISQL=15.7089	P=0.97209	DEN=0.0101	IERR=0	CELL FREQ	10

CHISQP=16.0000	P=0.974888	DEN=0.0091	IERR=0	CHISQL=16.3054	P=0.97753	DEN=0.0082	IERR=0	CELL FREQ	7
CHISQP=16.0000	P=0.974888	DEN=0.0091	IERR=0	CHISQL=17.1715	P=0.98368	DEN=0.0061	IERR=0	CELL FREQ	4
CHISQP=16.0000	P=0.974888	DEN=0.0091	IERR=0	CHISQL=18.3041	P=0.98933	DEN=0.0040	IERR=0	CELL FREQ	12
CHISQP=16.0000	P=0.974888	DEN=0.0091	IERR=0	CHISQL=18.3464	P=0.98950	DEN=0.0040	IERR=0	CELL FREQ	14
CHISQP=16.0000	P=0.974888	DEN=0.0091	IERR=0	CHISQL=18.9243	P=0.99157	DEN=0.0032	IERR=0	CELL FREQ	16
CHISQP=16.0000	P=0.974888	DEN=0.0091	IERR=0	CHISQL=19.1162	P=0.99217	DEN=0.0030	IERR=0	CELL FREQ	15
CHISQP=16.2000	P=0.976655	DEN=0.0085	IERR=0	CHISQL=14.4270	P=0.95591	DEN=0.0155	IERR=0	CELL FREQ	10
CHISQP=16.2000	P=0.976655	DEN=0.0085	IERR=0	CHISQL=15.5356	P=0.97029	DEN=0.0107	IERR=0	CELL FREQ	15
CHISQP=16.4000	P=0.978330	DEN=0.0080	IERR=0	CHISQL=15.3534	P=0.96827	DEN=0.0114	IERR=0	CELL FREQ	16
CHISQP=16.4000	P=0.978330	DEN=0.0080	IERR=0	CHISQL=16.8705	P=0.97367	DEN=0.0096	IERR=0	CELL FREQ	15
CHISQP=16.4000	P=0.978330	DEN=0.0080	IERR=0	CHISQL=16.3541	P=0.97793	DEN=0.0091	IERR=0	CELL FREQ	14
CHISQP=16.4000	P=0.978330	DEN=0.0080	IERR=0	CHISQL=16.6040	P=0.97986	DEN=0.0074	IERR=0	CELL FREQ	12
CHISQP=16.4000	P=0.978330	DEN=0.0080	IERR=0	CHISQL=16.7365	P=0.98082	DEN=0.0071	IERR=0	CELL FREQ	12
CHISQP=16.4000	P=0.978330	DEN=0.0080	IERR=0	CHISQL=17.0566	P=0.98296	DEN=0.0063	IERR=0	CELL FREQ	13
CHISQP=16.4000	P=0.978330	DEN=0.0080	IERR=0	CHISQL=17.1070	P=0.98328	DEN=0.0062	IERR=0	CELL FREQ	12
CHISQP=16.4000	P=0.978330	DEN=0.0080	IERR=0	CHISQL=17.2357	P=0.98406	DEN=0.0059	IERR=0	CELL FREQ	9
CHISQP=16.4000	P=0.978330	DEN=0.0080	IERR=0	CHISQL=18.3803	P=0.98963	DEN=0.0039	IERR=0	CELL FREQ	15
CHISQP=16.4000	P=0.978330	DEN=0.0080	IERR=0	CHISQL=19.3926	P=0.99296	DEN=0.0027	IERR=0	CELL FREQ	11
CHISQP=16.4000	P=0.978330	DEN=0.0080	IERR=0	CHISQL=20.5154	P=0.99544	DEN=0.0018	IERR=0	CELL FREQ	12
CHISQP=16.4000	P=0.978330	DEN=0.0080	IERR=0	CHISQL=20.6418	P=0.99566	DEN=0.0017	IERR=0	CELL FREQ	11
CHISQP=16.6000	P=0.979833	DEN=0.0074	IERR=0	CHISQL=15.6203	P=0.97118	DEN=0.0104	IERR=0	CELL FREQ	14
CHISQP=16.6000	P=0.979833	DEN=0.0074	IERR=0	CHISQL=16.6196	P=0.97998	DEN=0.0074	IERR=0	CELL FREQ	14
CHISQP=16.6000	P=0.979833	DEN=0.0074	IERR=0	CHISQL=18.9916	P=0.99179	DEN=0.0031	IERR=0	CELL FREQ	16
CHISQP=16.8000	P=0.98127	DEN=0.0069	IERR=0	CHISQL=15.4014	P=0.96882	DEN=0.0112	IERR=0	CELL FREQ	12
CHISQP=16.8000	P=0.98127	DEN=0.0069	IERR=0	CHISQL=15.5564	P=0.97051	DEN=0.0106	IERR=0	CELL FREQ	10
CHISQP=16.8000	P=0.98127	DEN=0.0069	IERR=0	CHISQL=15.5564	P=0.97051	DEN=0.0106	IERR=0	CELL FREQ	10
CHISQP=16.8000	P=0.98127	DEN=0.0069	IERR=0	CHISQL=15.5564	P=0.97051	DEN=0.0106	IERR=0	CELL FREQ	10
CHISQP=16.8000	P=0.98127	DEN=0.0069	IERR=0	CHISQL=15.8206	P=0.97319	DEN=0.0097	IERR=0	CELL FREQ	10
CHISQP=16.8000	P=0.98127	DEN=0.0069	IERR=0	CHISQL=17.1901	P=0.98379	DEN=0.0060	IERR=0	CELL FREQ	8
CHISQP=16.8000	P=0.98127	DEN=0.0069	IERR=0	CHISQL=17.5131	P=0.98563	DEN=0.0054	IERR=0	CELL FREQ	8
CHISQP=16.8000	P=0.98127	DEN=0.0069	IERR=0	CHISQL=17.6954	P=0.98658	DEN=0.0050	IERR=0	CELL FREQ	8
CHISQP=16.8000	P=0.98127	DEN=0.0069	IERR=0	CHISQL=17.8351	P=0.98827	DEN=0.0048	IERR=0	CELL FREQ	8
CHISQP=16.8000	P=0.98127	DEN=0.0069	IERR=0	CHISQL=19.1167	P=0.99217	DEN=0.0030	IERR=0	CELL FREQ	10
CHISQP=16.8000	P=0.98127	DEN=0.0069	IERR=0	CHISQL=19.2021	P=0.99242	DEN=0.0029	IERR=0	CELL FREQ	10
CHISQP=17.0000	P=0.98260	DEN=0.0064	IERR=0	CHISQL=16.2856	P=0.97737	DEN=0.0083	IERR=0	CELL FREQ	17
CHISQP=17.0000	P=0.98260	DEN=0.0064	IERR=0	CHISQL=17.7627	P=0.98691	DEN=0.0049	IERR=0	CELL FREQ	13
CHISQP=17.0000	P=0.98260	DEN=0.0064	IERR=0	CHISQL=21.4398	P=0.99683	DEN=0.0013	IERR=0	CELL FREQ	13
CHISQP=17.2000	P=0.98385	DEN=0.0060	IERR=0	CHISQL=15.1530	P=0.96591	DEN=0.0122	IERR=0	CELL FREQ	14
CHISQP=17.2000	P=0.98385	DEN=0.0060	IERR=0	CHISQL=16.7126	P=0.98065	DEN=0.0071	IERR=0	CELL FREQ	13
CHISQP=17.2000	P=0.98385	DEN=0.0060	IERR=0	CHISQL=18.2039	P=0.98892	DEN=0.0042	IERR=0	CELL FREQ	13
CHISQP=17.2000	P=0.98385	DEN=0.0060	IERR=0	CHISQL=18.5371	P=0.99023	DEN=0.0037	IERR=0	CELL FREQ	11
CHISQP=17.4000	P=0.98501	DEN=0.0056	IERR=0	CHISQL=14.9761	P=0.96369	DEN=0.0129	IERR=0	CELL FREQ	11
CHISQP=17.4000	P=0.98501	DEN=0.0056	IERR=0	CHISQL=19.0934	P=0.99210	DEN=0.0030	IERR=0	CELL FREQ	11
CHISQP=17.6000	P=0.98609	DEN=0.0052	IERR=0	CHISQL=15.5354	P=0.97078	DEN=0.0107	IERR=0	CELL FREQ	11
CHISQP=17.6000	P=0.98609	DEN=0.0052	IERR=0	CHISQL=16.1734	P=0.97642	DEN=0.0086	IERR=0	CELL FREQ	13
CHISQP=17.6000	P=0.98609	DEN=0.0052	IERR=0	CHISQL=16.4964	P=0.97905	DEN=0.0077	IERR=0	CELL FREQ	13
CHISQP=17.6000	P=0.98609	DEN=0.0052	IERR=0	CHISQL=16.7454	P=0.98089	DEN=0.0071	IERR=0	CELL FREQ	12
CHISQP=17.6000	P=0.98609	DEN=0.0052	IERR=0	CHISQL=17.0700	P=0.98305	DEN=0.0063	IERR=0	CELL FREQ	10
CHISQP=17.6000	P=0.98609	DEN=0.0052	IERR=0	CHISQL=17.4586	P=0.98533	DEN=0.0055	IERR=0	CELL FREQ	10
CHISQP=17.6000	P=0.98609	DEN=0.0052	IERR=0	CHISQL=17.5279	P=0.98571	DEN=0.0053	IERR=0	CELL FREQ	9
CHISQP=17.6000	P=0.98609	DEN=0.0052	IERR=0	CHISQL=18.4166	P=0.98977	DEN=0.0039	IERR=0	CELL FREQ	6
CHISQP=17.8000	P=0.98709	DEN=0.0048	IERR=0	CHISQL=18.1137	P=0.98853	DEN=0.0043	IERR=0	CELL FREQ	5
CHISQP=18.0000	P=0.98803	DEN=0.0045	IERR=0	CHISQL=17.9960	P=0.98801	DEN=0.0045	IERR=0	CELL FREQ	5
CHISQP=18.0000	P=0.98803	DEN=0.0045	IERR=0	CHISQL=19.1369	P=0.99223	DEN=0.0030	IERR=0	CELL FREQ	12
CHISQP=18.0000	P=0.98803	DEN=0.0045	IERR=0	CHISQL=19.4335	P=0.99307	DEN=0.0027	IERR=0	CELL FREQ	12
CHISQP=18.0000	P=0.98803	DEN=0.0045	IERR=0	CHISQL=19.5912	P=0.99348	DEN=0.0025	IERR=0	CELL FREQ	13
CHISQP=18.0000	P=0.98803	DEN=0.0045	IERR=0	CHISQL=19.6313	P=0.99358	DEN=0.0025	IERR=0	CELL FREQ	13
CHISQP=18.0000	P=0.98803	DEN=0.0045	IERR=0	CHISQL=20.9730	P=0.99437	DEN=0.0022	IERR=0	CELL FREQ	12
CHISQP=18.2000	P=0.98890	DEN=0.0042	IERR=0	CHISQL=16.1350	P=0.97609	DEN=0.0087	IERR=0	CELL FREQ	13
CHISQP=18.2000	P=0.98890	DEN=0.0042	IERR=0	CHISQL=17.4486	P=0.98528	DEN=0.0055	IERR=0	CELL FREQ	13
CHISQP=18.2000	P=0.98890	DEN=0.0042	IERR=0	CHISQL=19.0725	P=0.99204	DEN=0.0030	IERR=0	CELL FREQ	13

CHISQP=26.8000	P=0.99964	DEN=0.0001	IERR=0	CHISQL=22.4138	P=0.99785	DEN=0.0009	IERR=0	CELL FREQ	12	7	11	6	24	6	9	5
CHISQP=26.8000	P=0.99964	DEN=0.0001	IERR=0	CHISQL=23.7090	P=0.99872	DEN=0.0005	IERR=0	CELL FREQ	5	6	14	11	23	6	10	5
CHISQP=27.4000	P=0.99972	DEN=0.0001	IERR=0	CHISQL=25.1525	P=0.99929	DEN=0.0003	IERR=0	CELL FREQ	7	4	8	12	16	3	6	22
CHISQP=27.4000	P=0.99972	DEN=0.0001	IERR=0	CHISQL=26.4496	P=0.99958	DEN=0.0002	IERR=0	CELL FREQ	12	14	12	6	22	5	5	6
CHISQP=27.8000	P=0.99976	DEN=0.0001	IERR=0	CHISQL=21.9559	P=0.99741	DEN=0.0010	IERR=0	CELL FREQ	8	9	5	8	25	7	7	11
CHISQP=28.8000	P=0.99984	DEN=0.0001	IERR=0	CHISQL=25.1958	P=0.99930	DEN=0.0003	IERR=0	CELL FREQ	7	8	3	14	24	8	6	13
CHISQP=30.4000	P=0.99992	DEN=0.0000	IERR=0	CHISQL=28.5806	P=0.99983	DEN=0.0001	IERR=0	CELL FREQ	8	3	11	9	24	3	10	12
CHISQP=37.6000	P=1.00000	DEN=0.0000	IERR=0	CHISQL=36.6049	P=0.99999	DEN=0.0000	IERR=0	CELL FREQ	18	3	9	8	23	3	12	4

APPENDIX E

STATISTICAL PACKAGE FOR THE SOCIAL SCIENCES

08/23/77

PAGE 1

SPSS FOR OS/360, VERSION F, RELEASE 7.1, JULY 11, 1977

DEFAULT SPACE ALLOCATION..	ALLOWS FOR..	100 TRANSFORMATIONS
WORKSPACE 70000 BYTES		400 RECODE VALUES + LAG VARIABLES
TRANSSPACE 10000 BYTES		1600 IF/COMPUTE OPERATIONS

RUN NAME	CHI-SQUARE TEST CHECK JOB222
VARIABLE LIST	CATEGORY, FREQ
INPUT FORMAT	FREELIEN
N OF CASES	5
WEIGHT	FREQ
NPAR TESTS	CHI-SQUARE = CATEGORY/

GIVEN 1 VARIABLES, WORKSPACE ALLOWS FOR 4373 CASES

READ INPUT DATA

CHI-SQUARE TEST CHECK JOB022
FILE PROGRAM (CREATION DATE = 08/23/77)

--- CHI-SQUARE TEST

CATEGORY

VALUE	1:	2:	3:	4:	5:
COUNT	4:	5:	5:	5:	5:
EXPECTED	5.00	5.00	5.00	5.00	5.00
CHI-SQUARE	0.800				
D.F.	0.4				
SIGNIFICANCE	0.938				

APPROVAL SHEET

The dissertation submitted by Adam J. Miller II has been read and approved by the following committee:

Dr. Jack A. Kavanagh, Director
Associate Professor, Foundations, Loyola

Dr. Samuel T. Mayo
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The final copies have been examined by the director of the dissertation and the signature which appears below verifies the fact that any necessary changes have been incorporated and that the dissertation is now given final approval by the Committee with reference to content and form.

The dissertation is therefore accepted in partial fulfillment of the requirements for the degree of Doctor of Education.

Date

April 23, 1979

Director

Jack A. Kavanagh