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A Quantitative Study of the Stresses Developed in the Apical Fibers of the Periodontal Ligament Using Geometic Analogues

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A QUANTITATIVE STUDY OF THE STRESSES DEVELOPED IN THE APICAL FIBERS OF THE PERIODONTAL LIGAMENT USING GEOMETRIC ANALOGUES

by

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LIFE

James A. Evans was born in Rapid City, South Dakota on June 7, 1933. He graduated from Rapid City High School in May 1951 and entered the University of Nebraska in September of the same year. He graduated in June 1956 with a Bachelor of Science degree in Education.

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The author was married in June 1956 and is the father of three children.

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CHAPTER I

INTRODUCTION

The periodontal ligament is the medium through which stresses are transmitted to the alveolar process when forces are applied to the crowns of teeth. How these stresses are distributed to the walls of the alveolus is not fully understood, however it is known that when forces exceed the capillary pressure, cellular activity is increased. Osteoblastic activity occurs in areas of tensile stress and osteoclastic activity in areas of compressive stress. In addition to the stresses already mentioned there is a third -a shearing stress- which influences cellular activity. This investigation will attempt to study quantitatively the distribution of these three stresses in the apical region of the root surface of a geometric analogue.

The pressure created on the root depends on magnitude, duration, and point of application of a force to the crown. The actual application of the force is indeterminate; the determining factor is the subsequent root pressure distributed as stress through the periodontal ligament. For a given magnitude of force, the stresses in the periodontal ligament would vary in accordance with tooth size, root morphology, and width of the periodontal ligament.

Orthodontic literature contains numerous references where attempts have been made to analyze the biomechanics of tooth movement. Early investigators approached the problem from a strictly mechanical viewpoint. Current emphasis acknowledges the mechanical concept but goes a step further and fits it to a biologic environment. Thus, it may be said that the current concept is biophysical.

An ideal approach to this problem would be to measure the pressures along the entire length of the periodontal ligament. Obviously, this cannot be done on human teeth without subjecting the patient to prohibitive surgical trauma. Studies on models, however, have been numerous. They range from sophisticated mathematical models to match sticks embedded in plaster. These studies have yielded some information, yet this information is not wholly reliable because it has not recognized the elasticity of the periodontal ligament. A significant physical study of stresses developed about a tooth would be more meaningful if an elastic medium having the physical characteristics of the periodontal ligament were used. Using such an elastic medium would make it possible to analyze quantitatively the stresses on various portions of the root surface when a force is applied to the crown.

Purpose

The purpose of this investigation is to study quantitatively the stresses that develop in that portion of a geometric analogue which correspond to the APICAL FIBERS of the periodontal ligament when specific force systems are applied to the portion representing the crown of the tooth.

Review of the Literature

Harris (1863) was the first to propose the theory that tooth movement is the result of bone resorption on the pressure side of the root surface and apposition on the tension side. Talbott (1888) and Guilford (1898) supported this contention which was later substantiated histologically by many investigators.

Kingsley (1877) stated that there is a bending of the alveolar bone during orthodontic treatment followed by its subsequent rearrangement. This was also the belief of Farrar (1888), however this theory was short-lived.

Walkhoff (1900) made the generalization that bone was displaced during active orthodontic treatment and is rearranged during retention. He based his theory on Wolff's Law of the Transformation of Bone.

Angle (1907) explained that the strong attachment of a tooth to its alveolar process is due to the inelastic fibers of the periodontal ligament originating on the surface of the cancellous bone of the alveolar process. He believed that when a force is applied to the crown of a tooth the fibers are compressed on the pressure side and those on the opposite side are stressed in tension. Angle was one of the first to mention the stresses in the periodontal ligament that result from orthodontic forces, however, he said little about their distribution or quantitation.

Case (1921) refers to a relationship of power, stress and movement. Without mentioning specific forces he points out that an excess of power will cause tipping during orthodontic tooth movement.

These investigators based their theories primarily on clinical experience making no mention of specific force magnitudes and their resultant stresses on the alveolar periodontal environment. Force magnitudes were described by other investigators as being light, medium, or heavy.

Sandstedt (1904, 1905), studying the effects of orthodontic tooth movement histologically, observed that light and heavy forces were responsible for deposition of bone on the tension side of the alveolus, the trabeculae of the new bone being arranged in the direction of the applied force. Heavy forces resulted in what Sandstedt called "undermining resorption." It is significant to note that this phenomenon was not again described until Schwarz (1932) made similar observations. In one of the earliest biomechanical explanations of tooth movement Sandstedt described the tooth as a "double-armed lever."

Oppenheim (1911) studied various types of tooth movements using light, medium, and heavy forces. His observations, based on histologic data, differed from those of Sandstedt, however agreed with those of Angle in that he believed the tooth to be a one-armed lever with the fulcrum near the apex. He also observed pathological changes in the pulp leading him to conclude that physiologic tooth movement from orthodontic appliances is not possible regardless of the magnitude of the force.

Schwarz (1928) also confirmed Case's "stake in ground" explanation of tooth movement only he used a stick match in soft plaster. He carried this work further by determining the exact center of tipping after assigning values to the various forces and dimensions. He concluded the height of force application was irrelevant providing the direction of force remained the same. He reported (1932) the relationship of specific force increments to tooth movement and tissue response, concluding from histologic data that orthodontic forces should not exceed capillary pressure (approximately 23 Gm./sq.cm.).

Klein (1923, 1928) found the average measurement of the periodontal ligament to be 0.25 millimeters in patients between twenty and twenty-five years of age and 0.23 millimeters in patients between forty and fifty years of age. Others (Kronfeld 1931, Preissecker 1931, Coolidge 1937) found these measurements to be reliable.

A major source of disagreement was whether or not the periodontal space decreases in width with age. Klein and Jozat believed that it did decrease. Coolidge, however, pointed out that Black, Boedecker, Haupl and Lang, Noyes and Thomas, and Lenhossek opposed this theory.

Kellner (1928), in a comparison of functioning teeth with those not in function, observed that function tends to stimulate a reinforcement of spongy bone and lack of function leads to atrophy. It increases the width of the periodontal ligament and when not in function the ligament becomes a mass of loose connective tissue with few, if any, fiber bundles.

Kronfeld (1931) stated that "the distribution of the fibers on the surface of the tooth corresponds to the direction of the force which may be determined by studying the occlusion.

Gabel (1934), studying the periodontal ligament, arrived at three basic assumptions; the ligament is homogeneous, it is isotropic (without grain) and it is incompressible. His conclusions, although far from accurate, indicated considerable analytical thinking and an attempt to correlate tooth movement with tissue biology. He further stated, "when a force is applied to a fixed body, the latter reacts with an equal force or stress as a result of being strained. The strain manifests itself as a change in volume, in shape, or in both- usually the latter."

Stuteville (1937) stated that the magnitude of the force is not as important as the distance through which the force acts.

This distance is the "radius of action" according to Adler (1948). Stuteville found areas of root resorption in all cases in which the force was active through a greater distance than the width of the periodontal ligament.

Boedecker (1942), comparing the effects of light and excessive forces in the pressure and tension sides of a tooth, noted that light forces produce mild pressure but do not completely close the periodontal space on the pressure side and thereby permit biologic resorption of the alveolar surface. Conversely, rapid movement may result in eventual undermining resorption of bone and cementum.

Moyers (1950) states that, "the physiologic response of the periodontal ligament to induced pressures and strains is a controlling or limiting factor in effecting tooth movement and in maintaining the teeth in their new positions.

Storey and Smith (1952), tipping mandibular canines into extraction sites, observed that tipping occurred about a fulcrum approximately at the apical one-third of the root. They found that the force on the crown is not as important as the resulting pressure at the interfaces between the tooth, periodontal ligament, and the bone. They used high and low force magnitude helical expansion springs and observed that the tooth either moved or did not move when forces were applied to its crown. Movement was predicated by the force magnitude. Where the force magnitude was low, movement occurred as a result of direct root resorption. Where the force magnitude was high, there was no movement until undermining resorption prepared the alveolar trough.

Shroff (1953), studying the effects of the forces of occlusion, postulated that these forces are transferred to the alveolar bone either through "stress tension" in the collagenous fibers of the periodontal ligament or through pressure due to the high fluid content of the ligament. He also pointed out that pressure would not be dissipated as rapidly in the highly viscous periodontal ligament as in other materials because of the complex network of fibers and cells.

Parfitt (1958) pointed out that the following properties of tissues that contain the teeth are of interest: tensile strength, elastic range, compressive strength, fluid content and response to transient forces. He mentioned that tissues fastening the teeth to their sockets provide both a cushioning effect and tensile strength.

Jarabak (1960) suggested specific values for the terms light, medium and excessive orthodontic forces. A light orthodontic force should not exceed 120 grams. Medium forces range from 140-160 grams. Histologic studies indicate that forces beyond 180-210 grams are excessive for the normal cellular response and decrease the rate of tooth movement (Storey and Smith 1952, Begg 1954, Reitan 1957, and Jarabak 1960).

Rashevsky (1960) assessed stresses from a biophysical aspect and concluded that stresses may be built into the cells of tissues as early as the primary divisions of the cells. He surmised that concentrations of protoplasm may produce non-uniform cells resulting in tissues with "built-in" mechanical stresses.

Stallard (1964) appears to support the contentions of Rashevsky. According to Stallard, there is no evidence whether or not previous orthodontic tooth movements inhibit tooth mobility, tooth migration, or periodontal degeneration from occurring in later years. Discussing retention, he stated that "well-treated dentitions left unretained will relapse to their former forms because of the inertia of the periodontium."

Burstone (1961) believes that successful orthodontics depends upon a tissue response to the applied forces. His belief is predicated on the length, diameter and contour of the root, the nature of the periodontal ligament, and the site of force application.

To define precisely tooth moving force systems, the following characteristics must be considered according to Burstone; force magnitude, direction of the force, the point of force application, the distance the force is active, and the uniformity of the force within this distance.

Jarabak and Fizzell (1963), approaching the subject of forces used in orthodontics from a biophysical aspect, see the tooth as a "biologic lever" to which the orthodontist attaches appliances to produce force systems for purposes of realigning the teeth. They determined approximate anchorage "values" for each tooth based upon the relative resistance of the tooth to movement. The resistance was attributed to mechanical and physiologic constituents. They believe that the number of collagenous fibers is the mechanical factor and this number is proportional to the number of roots and the root size. The physiologic factors are the capability of the alveolar bone to remodel and the periodontal ligament to repair. They state that the magnitude of the force is not as important as how it is distributed in stresses within the periodontal ligament. They divide orthodontic forces into four categories- "threshold forces are at the lowest biologic stratum of producing stimuli causing remodeling of alveolar bone,""optimal forces are those that will catalyze cellular activity to cause an accelerated breakdown of bone on the pressure side and a simultaneous build-up on the tension side," "maximal force is one that can be applied to the tooth short of causing strangulation of physiologic activity in the periodontal space on the pressure side," "excessive forces are those crushing the periodontal ligament and temporarily destroying its physiologic processes."

The preceding investigations have been primarily concerned with the biologic make-up of the periodontium and the manner in which it responds to the infinite variety of forces to which it is subjected.

Rateitschak and Herzog-Specht (1965), using extracted teeth which have been moved orthodontically, observed that tooth mobility increases 100 percent during active treatment, however, decreases during retention but the teeth did not in any case return to pretreatment stability.

Tweedle and Bundy (1965) found that tooth movement took place more rapidly under the influence of heat. There was more movement of the teeth on the heated side than on the opposite side or on the control animals. They also observed there was greater movement if the heat was by induction rather than by diathermy.

Fish (1917) discussed orthodontic tooth movements from a mathematical viewpoint. He described an equal and opposite resisting force acting through what he termed the "center of resistance." He studied the magnitudes of forces and explained the significance of moments of force and of couples.

Carothers (1949) studied the elastic properties of bone using a tensile machine. He determined the modulus of elasticity of a variety of samples from different bones in the human body. His study indicated that bone follows Hooke's Law over a considerable range with a reasonable degree of accuracy.

King and Lawton (1950), investigating the elasticity of the tissues of the body, noted that soft tissues having a high percentage of collagen could be compared to rubber-like elastomers. They defined elastomers as substances exhibiting long range elastic behavior. He found a close resemblance in the stress-strain diagrams and the modulus of elasticity. He also observed that the thermal behavior and X-ray defraction patterns were similar.

Haack (1963) illustrated the importance of a basic understanding of mechanics to the orthodontist. He analyzed different force systems that are commonly used during orthodontic treatment. For example, he analyzed the forces necessary for lingual translation of maxillary incisors and also, the forces associated with a face bow attached to maxillary molars. Haack stated that, "a concept of equilibrium is the foundation necessary to study forces and their actions."

Weinstein, Haack, Morris, Snyder, and Attaway (1963) proposed a theory regarding the "equilibrium position" of teeth. They state that a tooth in this position will not be moved by the natural forces acting upon it. If a tooth is in the equilibrium position, the average distributed force at any point in the periodontal ligament is zero over any reasonably extended period of time.

Bauer and Lang (1928) challenged the conclusions of Schwarz on the basis that he had made no allowance for the lack of dimensional uniformity of the roots of teeth. They designed a physical facsimile of a tooth having a conical root for their investigation. The root was two-thirds of the total length of the tooth and the force was applied perpendicular to the long axis of the tooth at the extreme end of the crown.

Fickel (1930) constructed a mathematical model assuming the root to be a paraboloid curve. The center of rotation of his model compared favorably with that of Bauer and Lang and he also concluded that the model designed by Schwarz was inadequate.

Synge (1933) suggested a mechanical theory of equilibrium of the tooth in its environment. He made the following assumptions concerning the periodontal ligament: it is not necessarily of uniform thickness, homogeneous, isotrophic, incompressible, has finite rigidity, and has atmospheric pressure on the free edges. He stated that the pressure in this incompressible medium accounts for the tightness ot a tooth and that this pressure takes the place of three ot the ordinary components of stress. He found that displacement from a finite force varies as the cube of the membrane thickness.

In another experiment in 1933, Synge developed a mathematical theory of the periodontal ligament using a two-dimensional model designed to have a ligament of a constant thickness, a root that was conical and had a regular surface.

Zak (1935) built models of teeth two to five millimeters thick and encased in a celluloid alveolus to analyze the mechanics of tooth movement. He also analyzed celluloid teeth within metal alveoli. The double refraction of light through the celluloid enabled Zak to visualize the mechanical stresses and thereby determine the center of rotation as the tooth tipped. He concluded that this method gave a qualitative evaluation of the stresses of the tooth and alveolus.

Hay (1939) investigated the displacement of a point on the incisal edge of a tooth mathematically. His work differed from that of Synge in that he assumed the periodontal ligament to be compressible to the same extent as water. He also divided the root surface into areas of "push" and "pull." In another study Hay evaluated the "push and pull" zones during ischemia. He suggested that forces used in orthodontics should be "3 Gm. per $cm.^2$ or a little more."

Mühlemann (1951), using the macroperiodontometer for anterior teeth and the microperiodontometer for posterior teeth, concluded that movement takes place in two phases and that the initial mobility of an erupting tooth was always greater than that of erupted teeth. He described the first or initial phase as one in which the slope gradient was greater, indicating more movement than in the second phase. "Forces up to 50 or 100 Gm. created the initial phase of movement." The forces in secondary phase ranged from 100 Gm. to approximately 1500-1750 Gm. depending on the tooth.

Mühlemann, Savdir, Sanik, and Rateitschak (1965) stated that "tooth mobility depends on a quantitative factor (the surface area of tooth attachment by periodontal ligament fibers) and qualitative factors (the structural and biophysical properties of the periodontal membrane and the alveolar bone supporting the teeth)." They mention that mobility may vary from hour to hour and is greater during pregnancy, during marginal infection, and following traumatization by orthodontic forces.

They further define initial tooth mobility as an intra-alveolar displacement of the tooth whereby the periodontal fibers are preparing for functional action in areas of tension. This initial mobility is response to horizontal forces of 80 to 120 Gm. to the crown. Secondary tooth mobility (from forces of approximately 500 Gm.) will also cause an elastic deformation of the periodontal bone.

Renfroe (1951) infers that the periodontal fibers in different areas of the root's surface were being subjected to different types of stress as a result of orthodontic forces. He mentioned, for example, that only fifty percent of the periodontal ligament fibers would be resisting movement when a tooth having a round root is moved bodily.

Manly (1955), using the vibration technique to measure tooth mobility on a model, observed that "the ratio of force to motion is an index of the effective spring constant."

Hirt and Mühlemann (1965), measuring the mobility of teeth at various times of the day, found it to be greater after the patient has been awake for a few hours, with the greatest mobility between the hours of 8 P.M. and 6 A.M.

Jarabak and Fizzell (1963) employed integral calculus in conjunction with mathematical models in developing the concept of projected root surface area. This concept subsequently led to the theory of "effective root surface area" which is that portion of the total root surface area directly involved in resisting tooth movement. Assuming the distribution of stress to be uniform throughout each individual fiber, they proposed a formula for a tooth being translated. This formula, ($S= \frac{Fm}{Ar}$), states that the stress (S) is equal to the translating force (F_m) divided by the projected root surface area (A_r). When considering a tipping movement, ($S= \frac{Ma}{I}$), the stress (S) is equal to the product of the magnitude of the tipping couple (M) and the distance from centroid to the most remote apical fiber (a) divided by the moment of inertia (I).

They propose the theory of "superpositioning" to better evaluate the actual stresses resulting from orthodontic forces knowing that it is difficult to create true tipping or true translation. This theory of superpositioning states that "the simultaneous application of a translating force and a tipping couple causes additional stresses." The preceding formulae were combined as shown:

$S_{min.} = \frac{Fm}{Ar} - \frac{Fmla}{I}$ to determine the stress at the most remote apical fiber

$S_{max.} = \frac{Fm}{Ar} + \frac{Fmlg}{I}$ to determine the stress at the most remote cervical fiber

Letter equivalents:

Fm = translating force (Gm.)

A_r = projected root area (in.)^2

1 = distance from the point of force application to centroid

a = distance from centroid to the extreme apical fiber

g = distance from centroid to the extreme cervical fiber

I = moment of inertia

Rudd, O'Leary and Stumpf (1963) measured horizontal tooth mobility on sixty-one subjects between the ages of seventeen and twenty-four using a dial indicator attached to a clutch. Studying mobility of all of the teeth, they found that the mandibular first premolars have the least mobility.

Dempster and Duddles (1964) contrasted two static systems of force. The first system was a "force couple on the crown." The second system resulted from "oblique or transverse forces to the crown." They concluded that "the force vectors acting on different parts of the roots attack them at specific angulations, at particular regions, and with varying magnitudes." They also determined that the resultant forces at the apex or alveolar margins may be nearly equal to the force applied to the crown.

Geigel (1965) constructed an anatomical three-dimensional model of a tooth in a qualitative evaluation of tooth movements. He applied various force systems of different magnitudes to the crown of the model tooth and studied analytically the influence of each. He calculated mathematically the apparent center of tipping for the different magnitudes of the force systems. He concluded that the center of tipping was always between centroid and the apex when a single force was applied. The center of tipping moved closer to the apex as the point of force application was lowered (cervically) on the crown, and the center of tipping moved upward or closer to centroid as the magnitude of the force was increased.

CHAPTER II

MATERIALS AND METHODOLOGY

Patient Measurements

The deflection of the mandibular right canine teeth were taken at 20 Gm. increments from 0 to 500 Gm. The force deflection characteristics were graphed and used to select a synthetic material to simulate the periodontal ligament.

The measurements of deflection of the mandibular right canine tooth were taken on six children, three boys and three girls. Their ages ranged from eleven to fifteen years. The children had the following characteristics:

(1) no previous orthodontic treatment;

(2) a mandibular right canine without mesial or distal contact;

(3) a mandibular right canine without caries or restorations;

(4) no indications of bruxism.

Children of this age group were selected because the majority of orthodontic patients are within this range and the widths of the periodontal ligaments would be similar and nearly uniform at this age. The synthetic material selected to simulate the periodontal ligament in the analogue will also be of uniform thickness.

Construction of the Measuring Apparatus

An acrylic resin crown was made for the mandibular right canine. The mesial and distal sides of the crown were reduced in width to prevent interference during labial or lingual movement. Each acrylic resin crown had a rigid 3/4 inch metal loop (.040 inch diameter Truchrome Wire) attached to the labial surface (Figure 1). The loop was placed perpendicular to the long axis of the tooth and radial to the arch. It extended labially from the cervical one-third of the acrylic resin crown.

An acrylic resin interocclusal clutch was constructed for each patient (Appendix I). A horizontal table of clear acrylic resin was attached to each clutch. This table extended approximately two inches outside of the patient's mouth. A small dial indicator was mounted beneath the table (Figure 2), with the measuring tip resting near the center of the labial surface of the crown just above or incisal to the metal loop (Figure 3). The acrylic resin clutch and crown made on the plaster models was

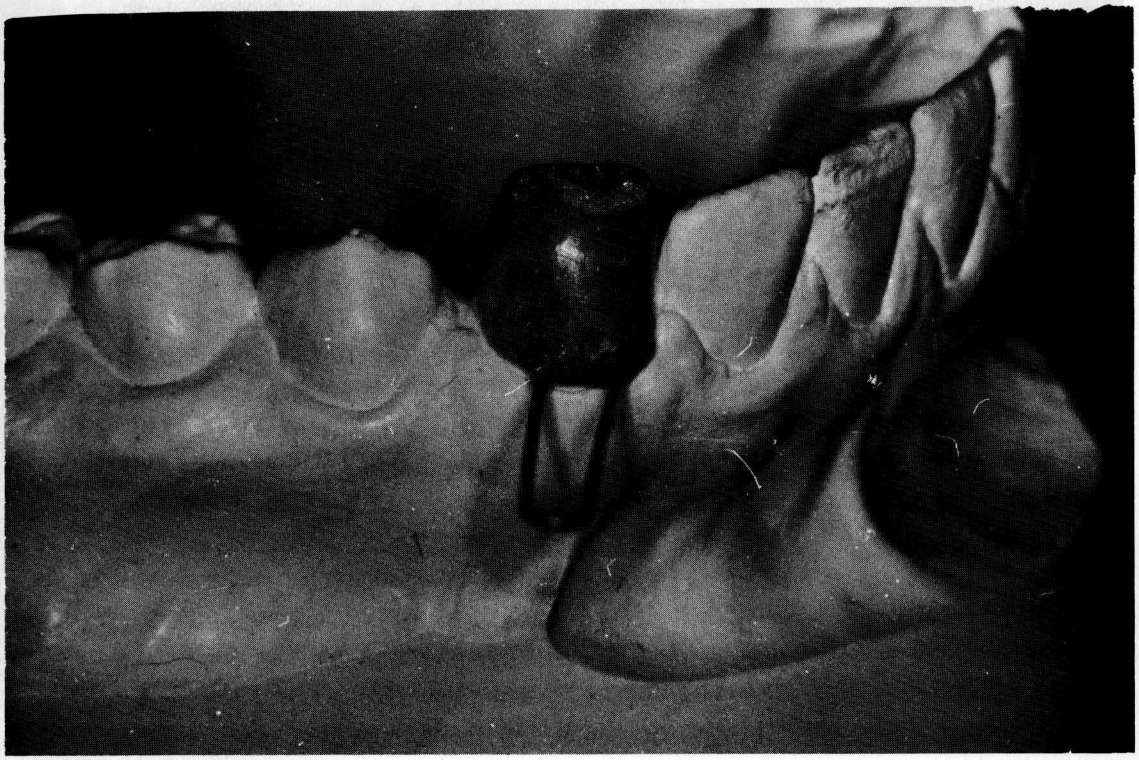


FIGURE 1

POSITION OF THE ACRYLIC RESIN CROWN

ON THE MANDIBULAR RIGHT CANINE

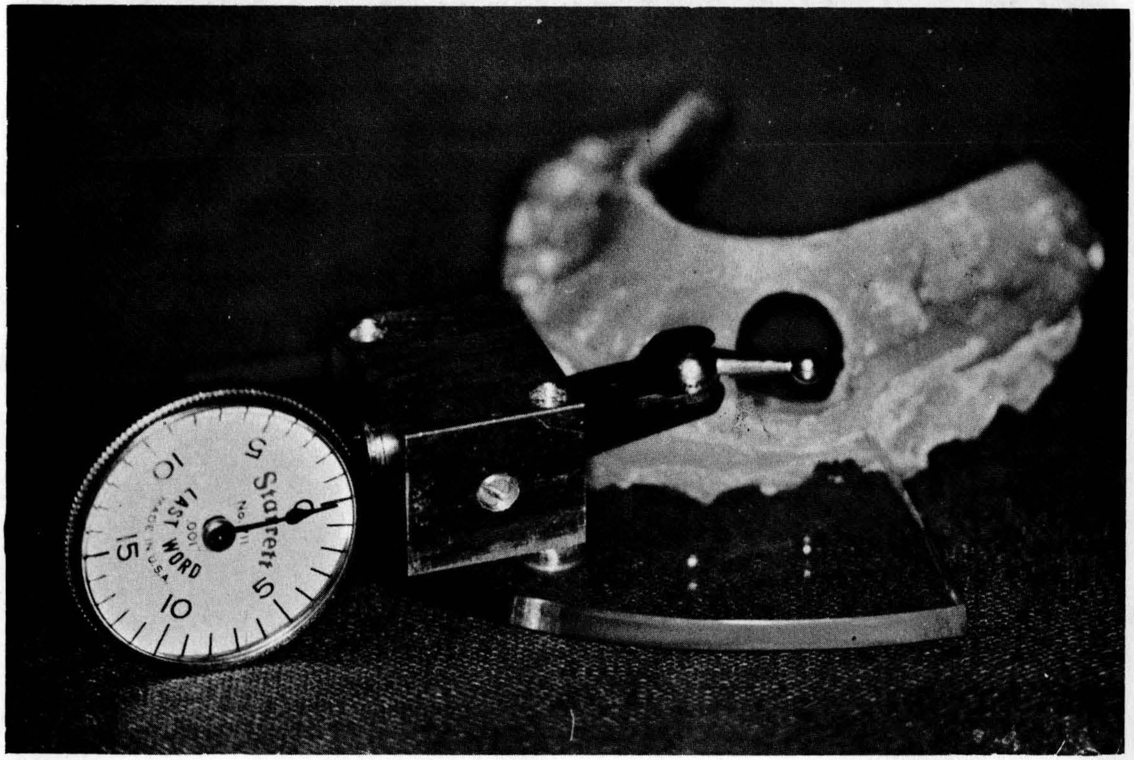


FIGURE 2

DIAL INDICATOR ATTACHED TO THE CLUTCH

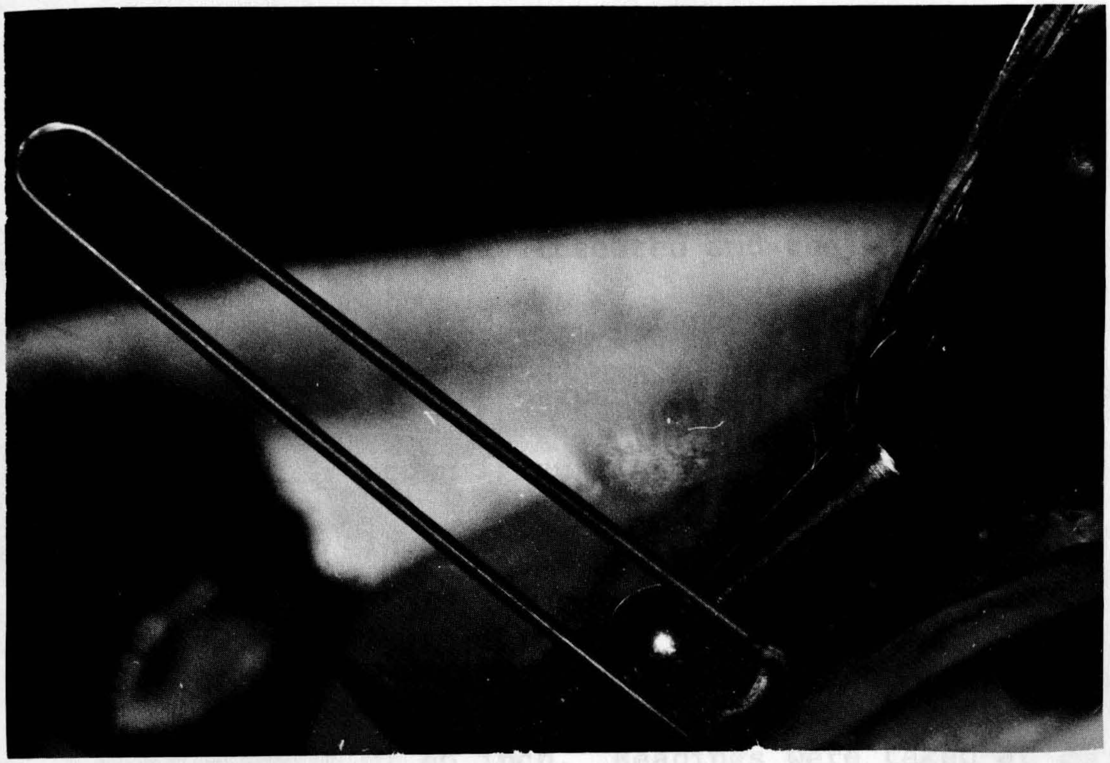


FIGURE 3

POSITION OF THE MEASURING TIP

OF THE DIAL INDICATOR

fitted to the mandibular teeth and any necessary adjustments were made before measurements were attempted.

Procedure for Measuring the Deflection of the Mandibular Right Canine Teeth

After the acrylic resin crown was cemented with zinc oxyphosphate cement the clutch was inserted and the patient was instructed to bite firmly into it. A force gage was used to apply the pulling force to the perpendicular wire loop extending from the acrylic resin crown (Figure 4). The direction of pull was collinear with the legs of the wire loop or perpendicular to the labial surface of the crown. The movement of the tooth measured as a deflection on the dial indicator was recorded to the nearest thousandth of an inch. Readings were taken at 20 Gm. increments from 0 to 500 Gm. as previously mentioned. The force at each succeeding increment was applied for two seconds before taking the reading of deflection.

Four series of measurements were made on each patient -- two series for pulling forces and two for pushing forces. The pushing force was applied in a lingual direction on the acrylic resin crown and at a point midway between the legs of the wire

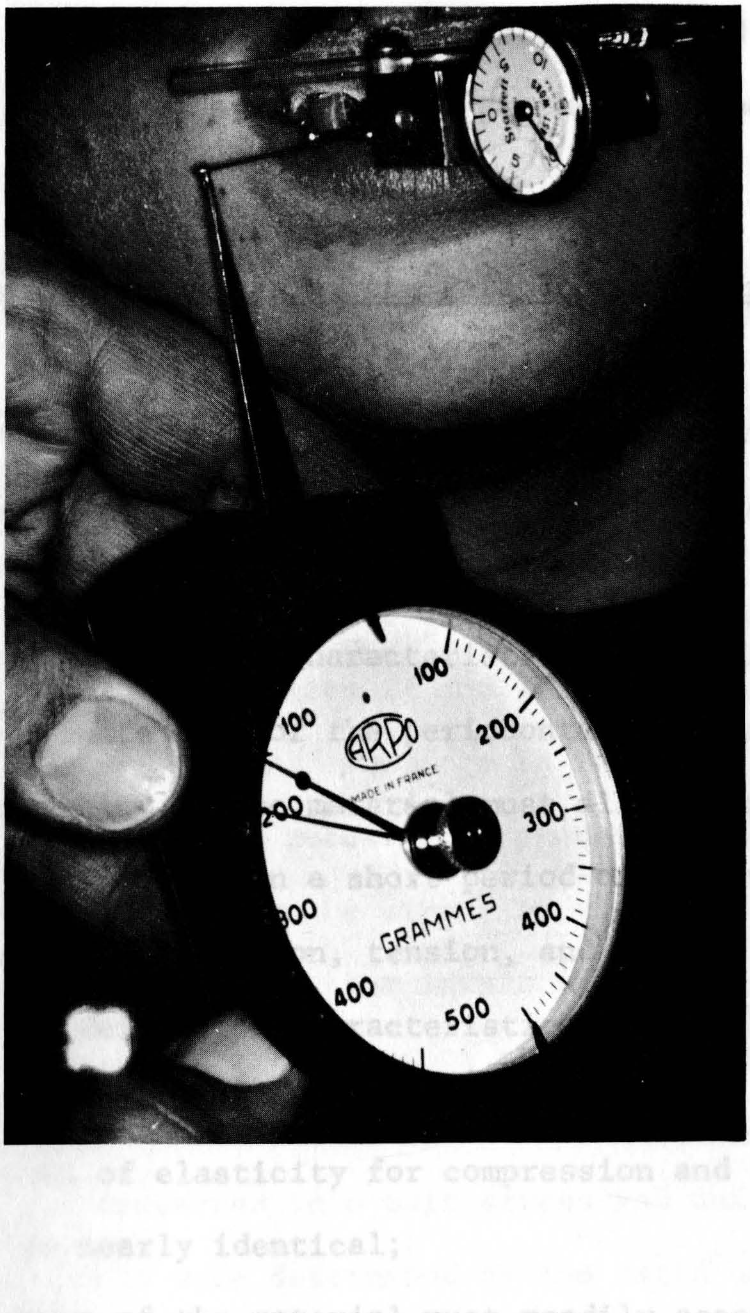


FIGURE 4

FORCE BEING APPLIED TO THE WIRE LOOP

loop. The clutch was removed for ten minutes between each series of measurements. Each series of measurements consisted of ten pairs of readings and the data were then graphed and analyzed for each patient.

Selecting a Material Simulating the Periodontal Ligament

One of the most important parts of this study was to find a suitable material to simulate the periodontal ligament. A search was conducted to find a material having the following characteristics:

(1) the force deflection characteristic of the material had to approximate that of the periodontal ligament;

(2) the elasticity of the material must allow nearly 100 percent return within a short period of time when stressed in compression, tension, and shear;

(3) the force deflection characteristics must be linear and nearly identical in compression and tension;

(4) the moduli of elasticity for compression and tension should be nearly identical;

(5) the surface of the material must readily accept an adhesive without becoming saturated or having its

molecular characteristics changed.

The following four adhesives were examined: epoxy-resins, Dupont Ducote, three plastic cements, and Elmer's glue. Each adhesive was applied uniformly to a piece of 1/8 inch clear plastic (Rohm and Haas). After two minutes a sample of foam was applied to each adhesive and allowed to dry for twenty-four hours. The specimens were removed and examined for structural change and saturation. Elmer's glue, a product of Borden Chemical Company, was selected.

Thirty-two tests were concluded on different types and thicknesses of material (Appendices II and III) but a 3/16 inch foam rubber material was chosen. Durafoam is the brand name of this material which is a latex rubber foam produced by U.S. Rubber Company. It was found to be linear in all three stresses over a range of from 0 to 400 Gm. per square inch.

The moduli of elasticity (E) and of rigidity (G) were computed for each test. The individual force-deflection characteristics were first converted into unit stress and unit strain and graphed. The moduli were determined by the ratio of unit stress to unit strain.

$$E \text{ or } G = \frac{\text{unit stress}}{\text{unit strain}} = \frac{(s)}{(e)}$$

$$\text{Unit Stress } (s) = \frac{\text{Force } (F)}{\text{per unit area } (A)}$$

$$\text{Unit Strain } (e) = \frac{\text{Deflection}}{\text{Original Gage Length (O.G.L.)}}$$

The least squares formula $\left(\frac{\Sigma x^2}{\Sigma xy}\right)$ was used to determine the modulus of elasticity in compression, tension, and shear for each individual test. The moduli were averaged and the average value for E was 14.972 psi and for G was 4.275 psi.

Construction of a Two-Dimensional Analogue

Two two-dimensional models were made from .75 inch, clear, pre-shrunk, Rohm and Haas plexiglass (Figures 5,6, and 7). Each model consisted of two outer rectangular pieces representing the alveolar bone with a longer inner piece sandwiched between to simulate the tooth. The longer inner rectangle representing the tooth was approximately ten times as large as a normal mandibular canine tooth. The outer pieces representing the alveolar process were separated by six spacers with center holes through which bolts were passed to assemble the model. The length of each spacer was 1.117 inch. This length is .007 inch less than the sum of the width of the .75 inch center rectangle and two

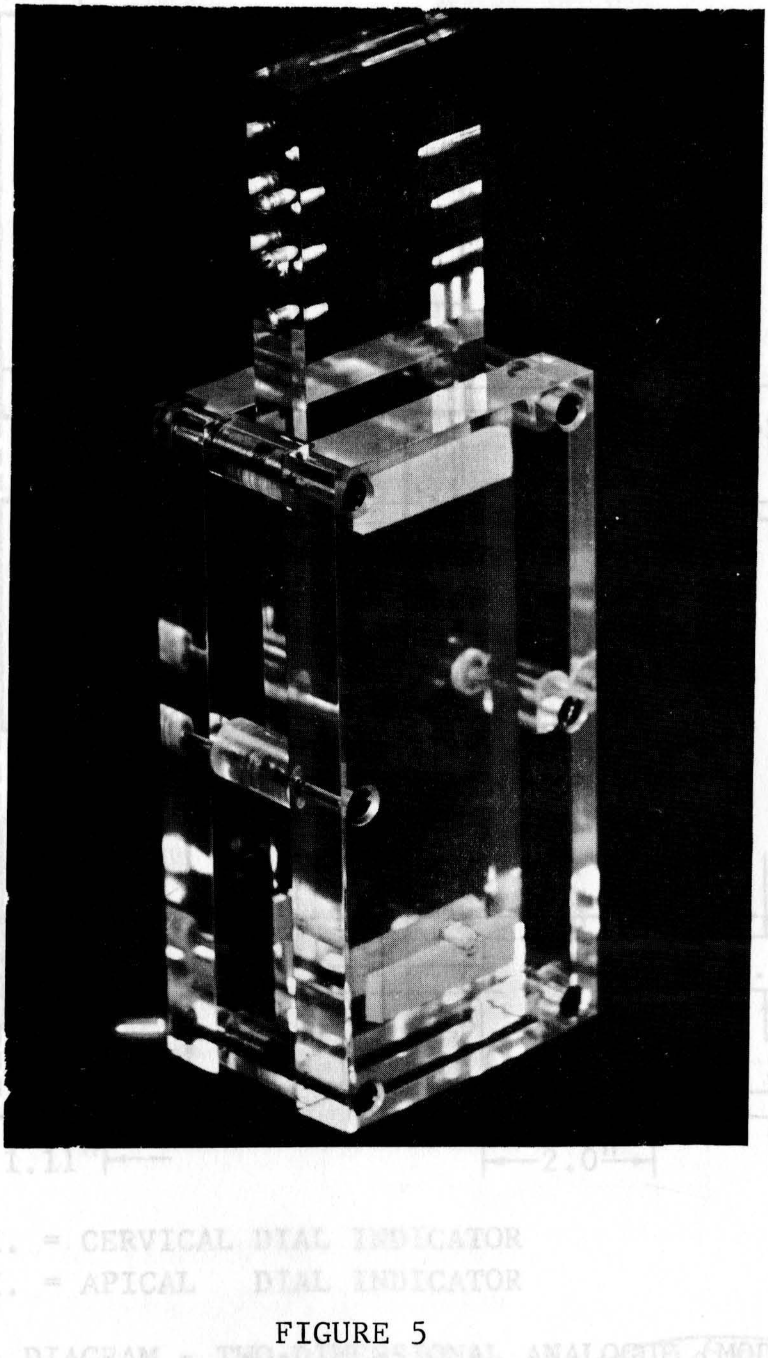
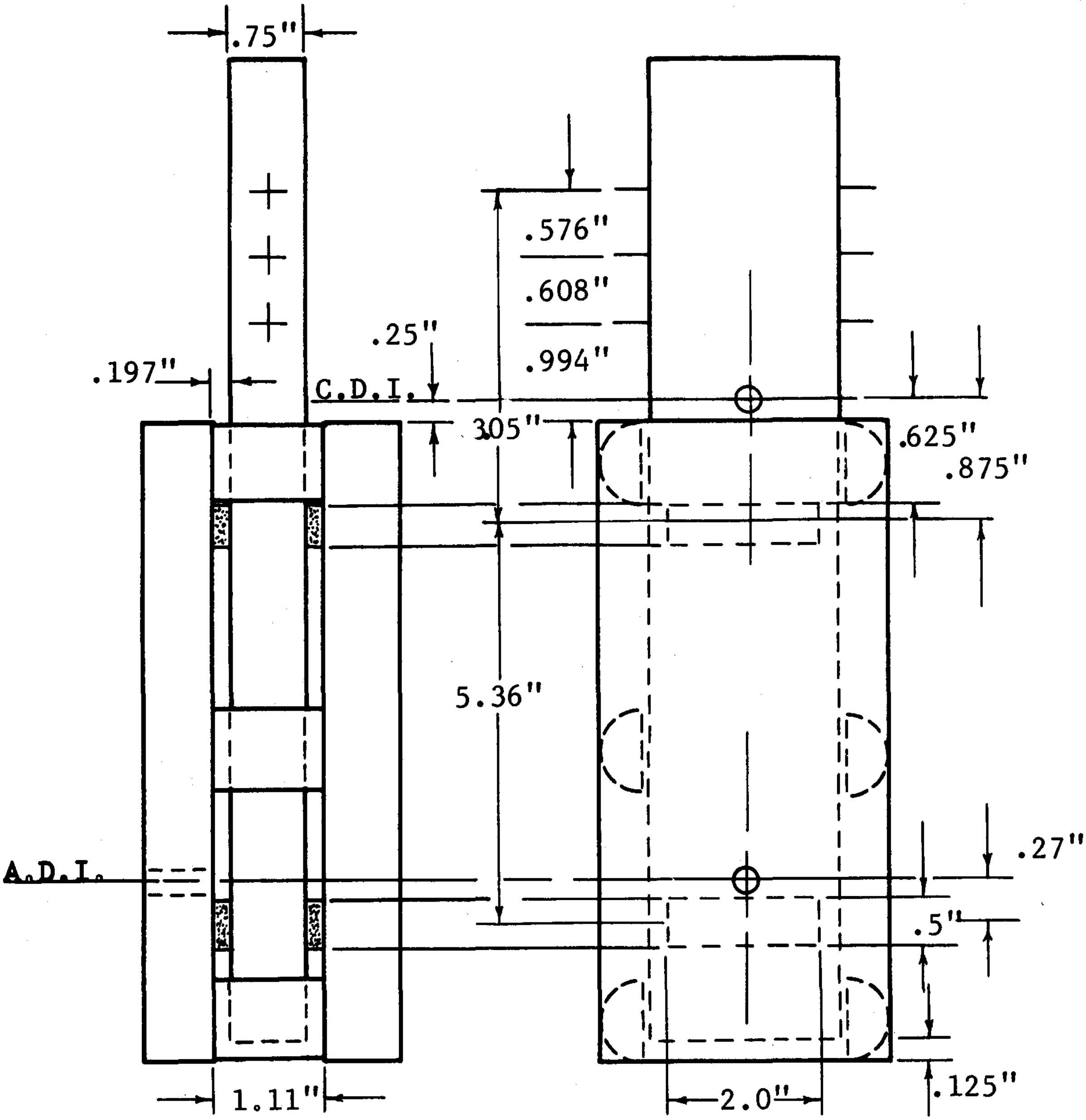


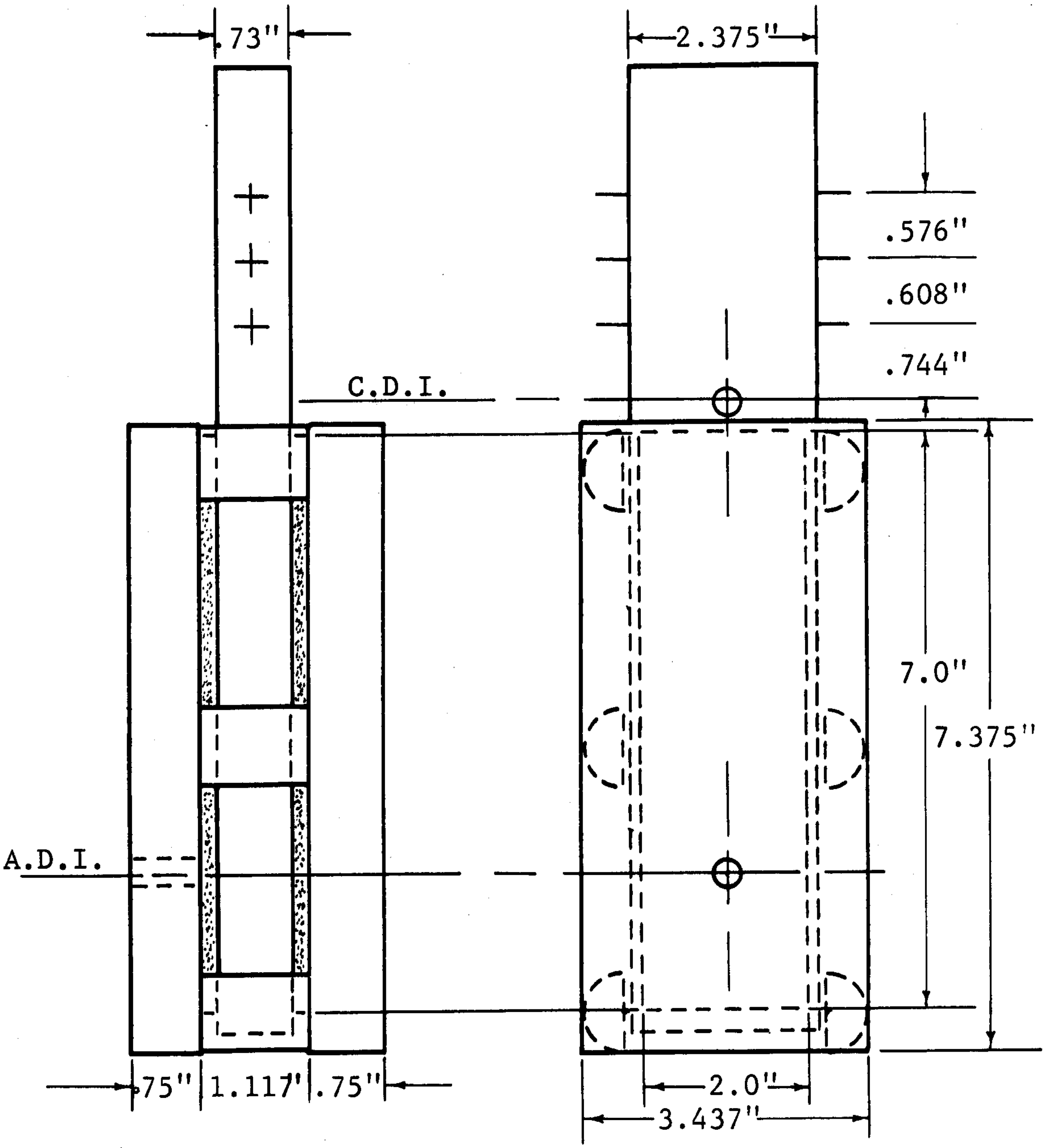
FIGURE 5

TWO-DIMENSIONAL ANALOGUE (MODEL I)



C.D.I. = CERVICAL DIAL INDICATOR
A.D.I. = APICAL DIAL INDICATOR

FIGURE 6 -- DIAGRAM - TWO-DIMENSIONAL ANALOGUE (MODEL I)



C.D.I. = CERVICAL DIAL INDICATOR
A.D.I. = APICAL DIAL INDICATOR

FIGURE 7 -- DIAGRAM - TWO-DIMENSIONAL ANALOGUE (MODEL II)

pieces of .197 inch Durafoam, therefore creating a minimal pressure on the simulated periodontal ligament following cementation.

Holes were drilled and threaded and screws were inserted at measured distances in the crown portion of the tooth to serve as points of force application. A hole was also drilled in the apical region of one of the rectangular blocks representing the alveolus to allow the sensing tip of the apical dial indicator to pass through and contact the root of the model during compression and tension tests. During shearing tests the model was rotated 90 degrees and a shorter sensing tip was used to contact the model.

Construction of a Three-Dimensional Analogue

A three-dimensional model of the tooth was made from a cylindrical rod of clear, pre-shrunk acrylic (Rohm and Haas) having a diameter of 2.375 inches (Figures 8,9, and 10). The root portion of the model was machined on a lathe to a truncated cone. The cylindrical crown portion had three diametral holes drilled through it at chosen positions to serve as points of application for the forces.

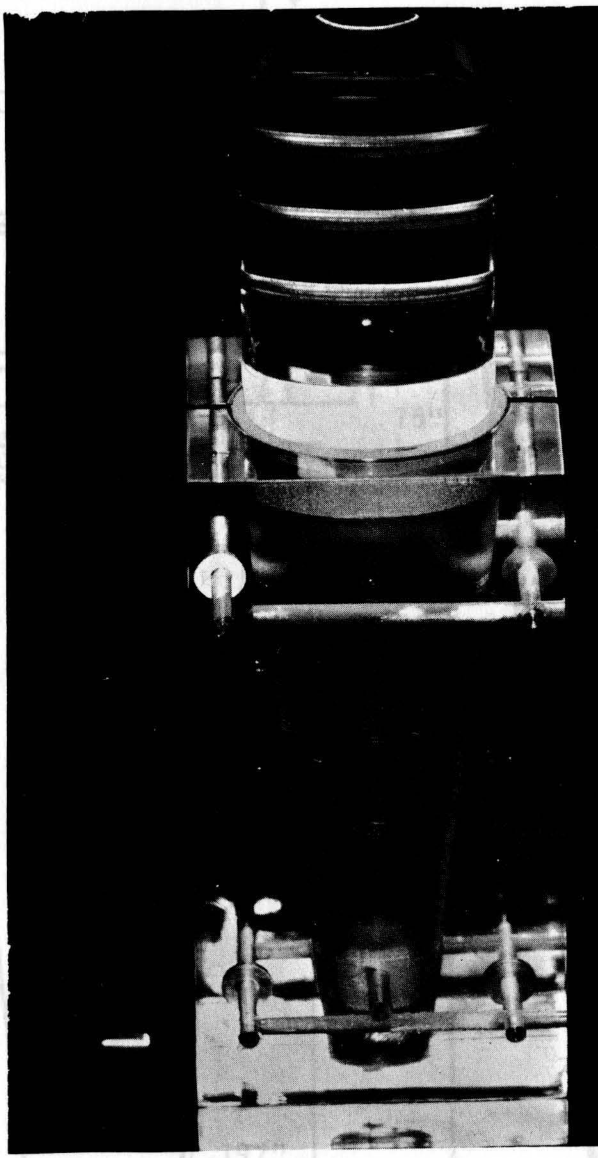


FIGURE 8

THREE-DIMENSIONAL ANALOGUE (MODEL III)

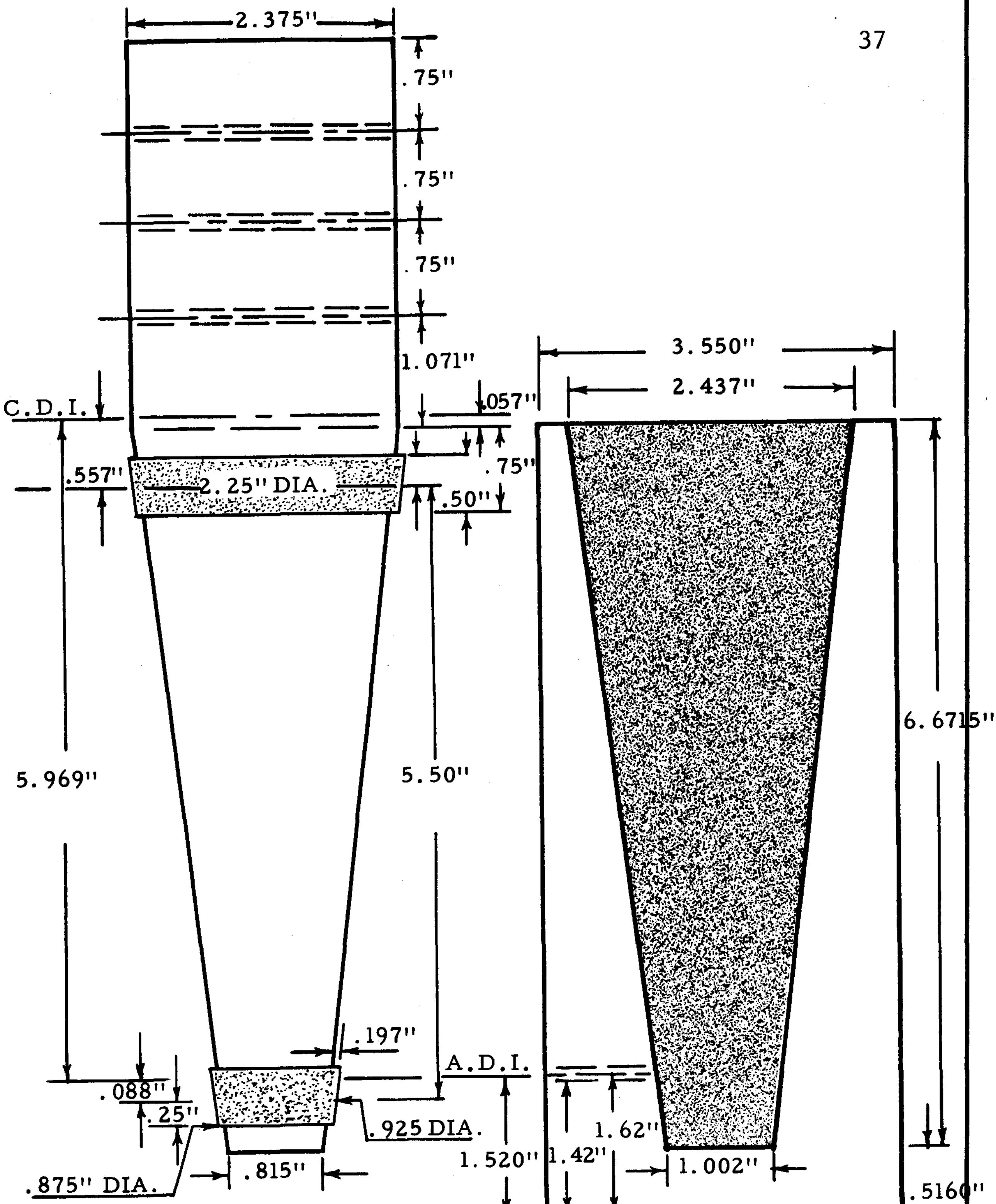


FIGURE 8A-DIAGRAM--THREE-DIMENSIONAL ANALOGUE

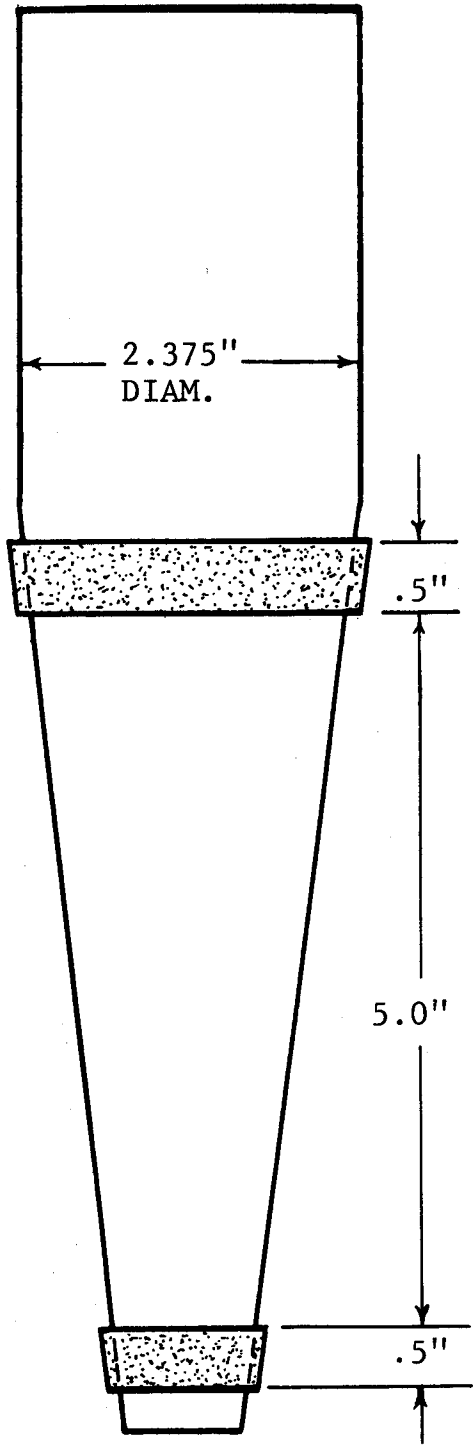


FIGURE 9--DIAGRAM - THREE-DIMENSIONAL ANALOGUE (MODEL III)

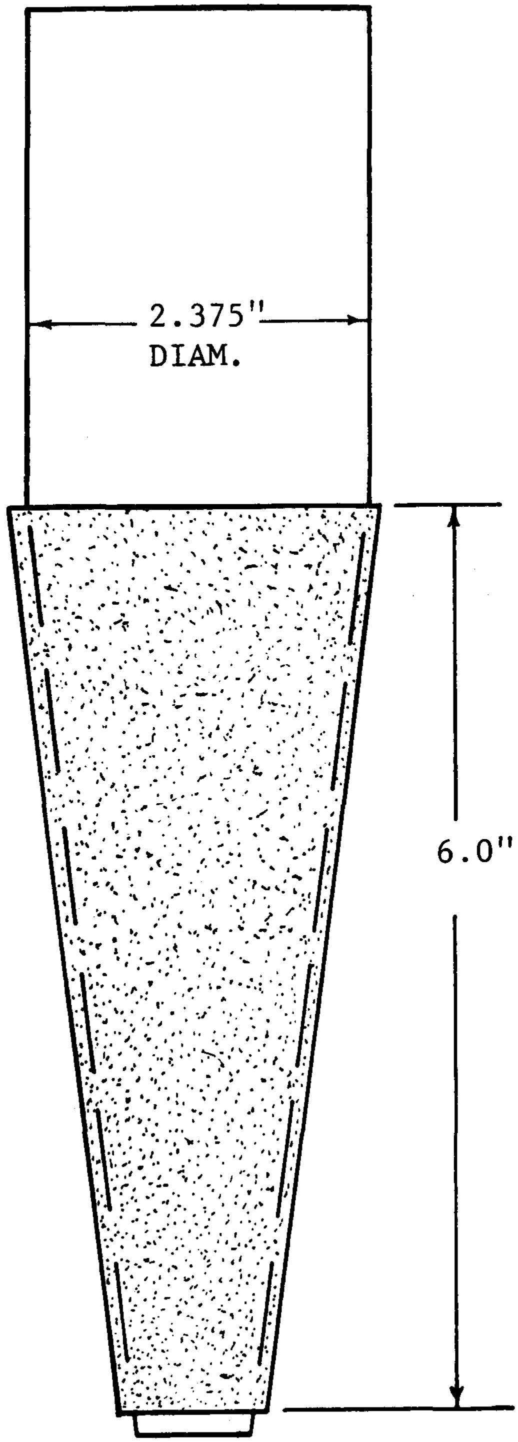


FIGURE 10--DIAGRAM - THREE-DIMENSIONAL ANALOGUE (MODEL IV)

The model alveolus was made from a block of the same clear acrylic material. The block was bored out to accept the truncated cone. Its diameter, however, was .375 inch greater than the diameter of the root portion of the model at corresponding levels. This clearance was necessary to provide space for the Durafoam simulating the periodontal ligament. As in the two-dimensional model, a hole was drilled in the apical region to allow the sensing tip of the apical deflection gage to pass through the "alveolus" and contact the root surface of the model.

The acrylic "alveolus" was sectioned into equal halves to facilitate the cementing of the Durafoam "periodontal ligament." Four alignment holes were drilled through the outer portion of the model alveolus before it was sectioned. Brass rods pass through these alignment holes when the model is assembled. .079 inch brass washers were used as spacers approximately returning the model alveolus to its original size. This was necessary because the .082 inch saw blade removed a portion of the acrylic model during its sectioning.

Cutting of the Durafoam for Each Analogue

The deflection of two different shapes of Durafoam was measured on the two-dimensional models. It was decided that two configurations would be used also on the three-dimensional model. The analogues will be referred to throughout this paper as shown below according to the configuration of the Durafoam which they contain.

(1) Model I is the two-dimensional model with two .5 inch by two inch pieces of Durafoam in both the apical and cervical regions (Figure 6).

(2) Model II is the two-dimensional model containing a two inch by seven inch piece of Durafoam on each side of the root of the model tooth (Figure 7).

(3) Model III is the three-dimensional model with a .5 inch band of Durafoam around both the apical and cervical regions of the root of the model tooth (truncated cone) (Figure 9).

(4) Model IV is the three-dimensional model with a six inch piece of Durafoam surrounding the truncated or root portion of the model (Figure 10).

Assembling the Two-Dimensional Models and the Measuring Apparatus

The two-dimensional model was fixed securely in the measuring apparatus (Appendix IV). Four angle irons were attached to the apparatus to help stabilize the model--two at the cervical and two at the apical portion of the rectangular "alveolus." The model was lowered through the hole in the horizontal platform and between the angle irons which were tightened to their respective surfaces with wood screws. Bolts were passed through holes in each angle iron in a horizontal direction or parallel to the platform. The holes were drilled so that the bolts would lie flush with the outer surface of the plastic rectangular "alveolus." As the nuts were tightened the heads of the bolts pulled the angle iron jaws together fastening the model securely to the measuring apparatus.

Two dial indicators were used to measure medial-lateral movements of the two-dimensional model tooth. The upper or "cervical" dial indicator was attached to the superior surface of the horizontal platform. The deflection arm contacted the crown of the model tooth .25 inch above the platform. This indicator was to the right of the model as one faces the apparatus and was graduated in hundredths of a millimeter.

The lower or apical dial indicator (graduated in thousandths of a millimeter) was mounted on the left side of the base. The sensing tip contacted the model tooth 1.4 inches above the base. As previously mentioned a .050 inch diameter hole was drilled in one plastic wall representing the alveolus. This allowed the sensing point to contact the inner plastic block representing the tooth when deflections were measured during compression and tension.

After the model was securely fastened to the apparatus, forces for deflection were applied at the top, middle, and lower posts. A "Y" shaped yoke was designed to attach bilaterally at the desired post height when the simulated periodontal ligament was being stressed in compression or tension. The "Y" shaped yoke was made of .012 inch orthodontic ligature wire. A double strand of the same ligature wire was attached to one post only when the model was rotated 90 degrees and the simulated ligament was stressed in shear.

The height of the pulley was adjusted to correspond with the height of the post being measured. The ligature extended outward from the post, perpendicular to the long axis of the model tooth, and passed over the pulley and downward suspending the weight tray.

DEFLECTION--When the Simulated Periodontal Ligament is Stressed in Compression and Tension

Individual forces of a known magnitude and direction were applied to produce tipping within the limits of the model alveolus. The weights used on Model I ranged from .5 to 1.5 pounds and on Model II from .125 to 1.5 pounds. "Apical" and "cervical" deflections were recorded 2.5 minutes after the weight trays were placed allowing the material time to respond to the respective force.

Three sets of data were record3d and averaged for each weight increment for each post. The data were recorded at each increment for the upper post, center post, and lower post in that order, and the procedure was then repeated twice. The pulley height was changed for each succeeding post as previously mentioned.

DEFLECTION--When the Simulated Periodontal Ligament is Stressed in Shear

The model was removed from the measuring apparatus, rotated 90 degrees, and secured as previously described. A shorter sensing tip was placed on the apical dial indicator to maintain contact with the apical or root portion of the model. The forces were applied in much the same manner as described above, however, the orthodontic ligature wire could now be attached to one post at the chosen height.

Force Analysis - Model I

The actual resisting force at the center of the apical Durafoam strip was determined in the following manner (Figure 11).

Letter Equivalents for Figure 11:

F_a = the actual resisting force at the center of the apical Durafoam strip

F_f = the actual resisting force at the center of the cervical Durafoam strip

F_m = the applied force to the crown of the model

Letter Equivalents for Figure 12:

A = the deflection of the cervical dial indicator (C.D.I.)

A'= the deflection at the center of the cervical Durafoam strip

B = the deflection of the root dial indicator (R.D.I.)

B'= the deflection at the center of the apical Durafoam strip

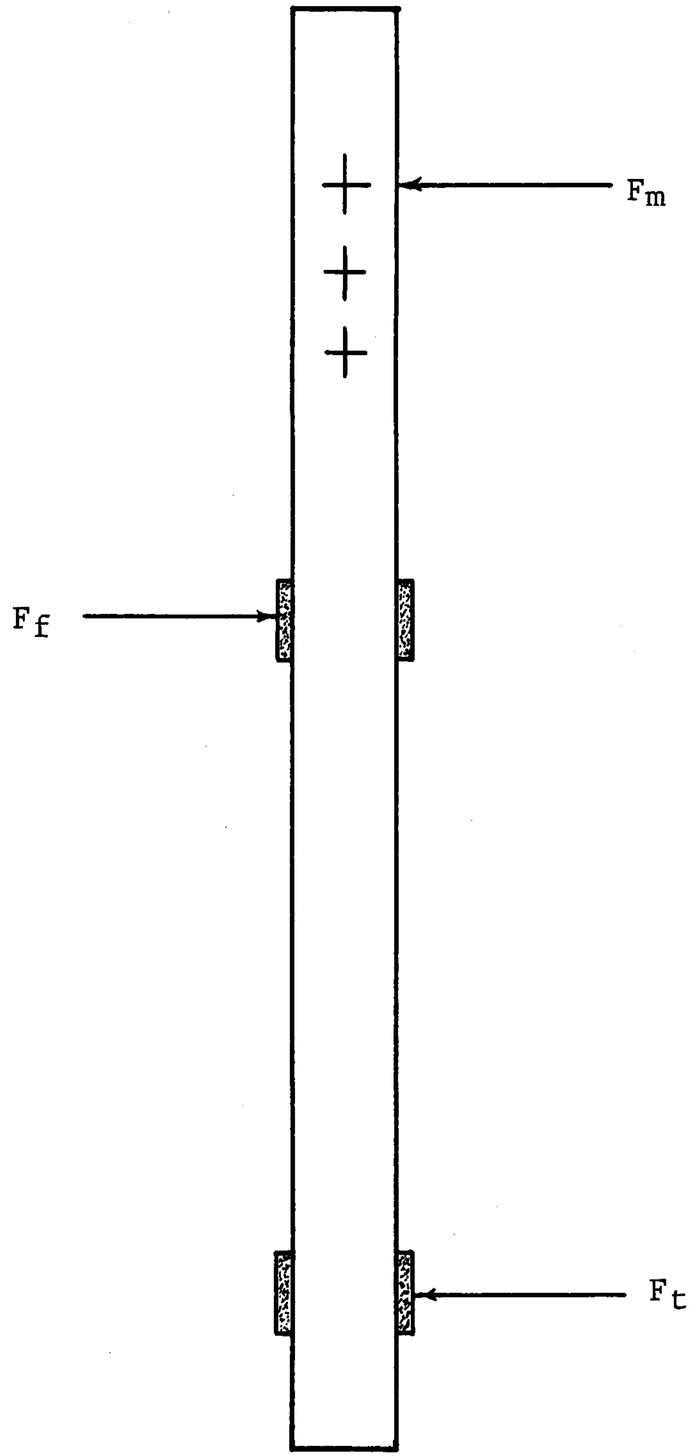


FIGURE 11- VECTOR DIAGRAM OF RESISTING FORCES (MODEL I)

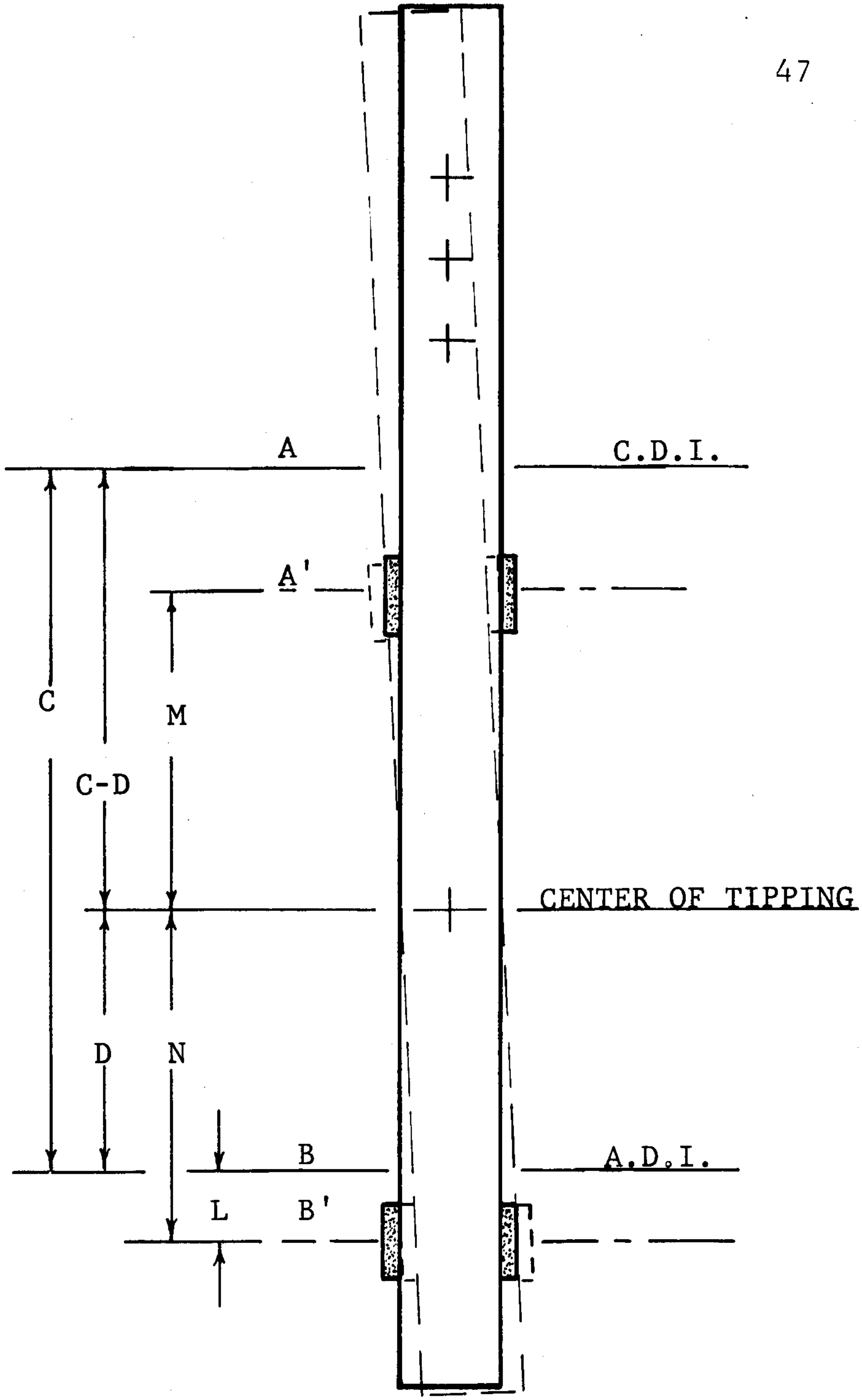


FIGURE 12 - MODEL I (TIPPING)

C = the distance between the dial indicators

D = the distance to the center of tipping from the apical dial indicator

L = the distance from the apical dial indicator to the center of the apical Durafoam strip

M = the distance from the cervical dial indicator to the center of the cervical Durafoam strip

N = the distance from the center of tipping to the center of the apical Durafoam strip

First, the center of tipping was calculated using the following ratio:

$$\frac{B}{D} = \frac{A}{(C-D)}$$

The deflection at the center of the apical Durafoam strip (B') was calculated using the following ratio (Figure 12):

$$\frac{B'}{F} = \frac{B}{D}$$

The formula for Young's modulus of elasticity was used to determine the actual resisting force (F_a) at the center of the apical Durafoam strip. This resisting force was assumed to be at the center of the apical Durafoam strip.

$$\text{Modulus of Elasticity (E)} = \frac{\text{unit stress (s)}}{\text{unit strain (e)}}$$

$$E = \frac{s\ (\text{psi})}{e\ (\text{in./in.})}$$

$$s = \frac{\text{Force (lbs.)}}{\text{Unit Area (in.)}^2} = \frac{F_a}{A_r}$$

$$e = \frac{\text{deflection (in.)}}{\text{Original Gage Length (in.)}}$$

$$E = \frac{\frac{F_a}{A_r}}{\frac{\text{defl.}}{\text{O.G.L.}}} \times (2)$$

$$F_a = \frac{E \times A_r \times \text{defl.}}{\text{O.G.L.}} (\text{lbs.}) \times (2)$$

The equations of equilibrium were used to find the theoretical resisting force (F_t) for Model I (Figure 13).

Letter Equivalents for Figure 13:

F_f is the theoretical cervical resisting force

F_m is the known force applied to the crown at the respective post being measured

F_t is the theoretical apical resisting force

g is the distance from the respective post to the center of the cervical Durafoam

h is the distance from the center of the cervical Durafoam to the center of the apical Durafoam

P is the point of rotation when calculating F_t

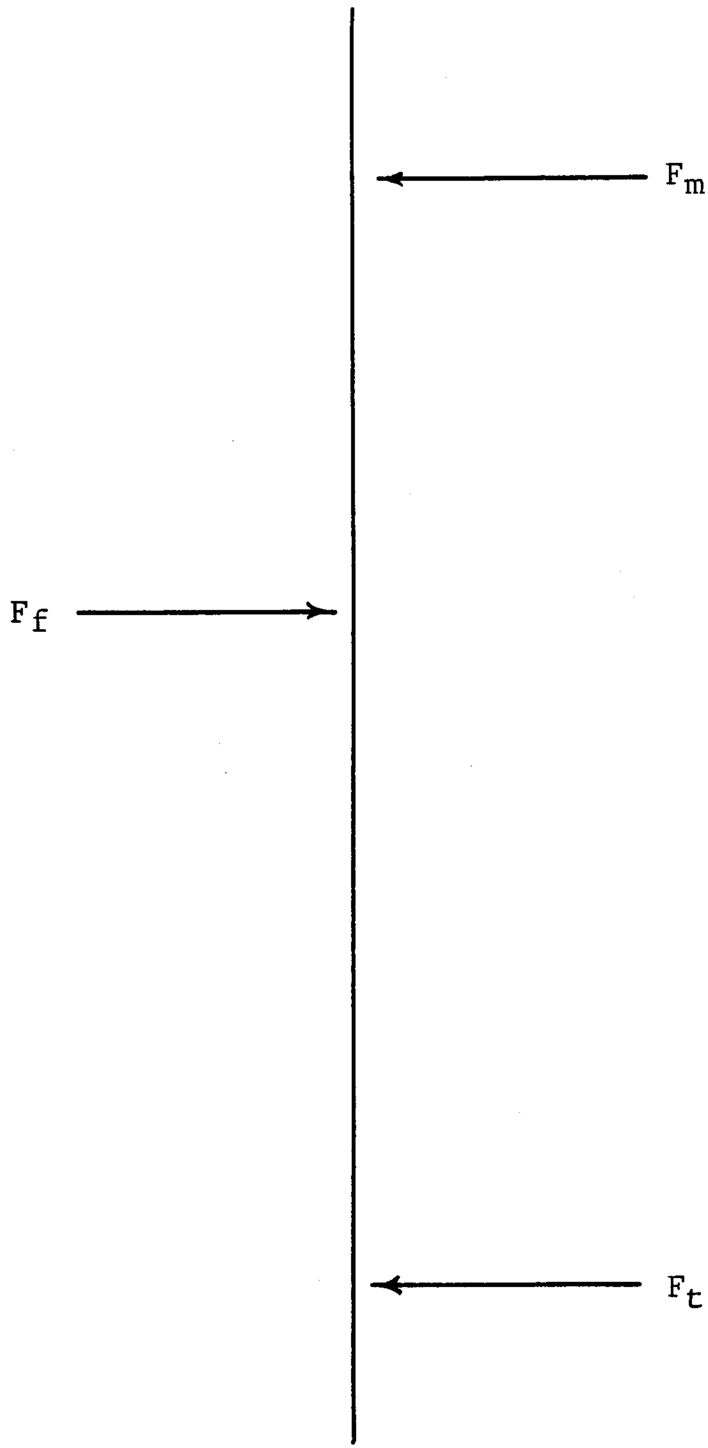


FIGURE 13 -- FREE BODY DIAGRAM

Equations of Equilibrium-

$$F_h = -F_m - F_t + F_f = 0$$

$$F_v = 0$$

$$M_p = 0$$

The sum of the moments about point P

$$M_p = -F_t (h) - F_m (g) = 0$$

$$F_t (h) = -F_m (g)$$

$$F_t = \frac{-F_m (g)}{h}$$

The actual apical resisting force (Fa) was calculated using Young's modulus of elasticity (E). The theoretical apical resisting force (F_t) was calculated using the equations of equilibrium. The apical resisting force has been calculated by two distinctly different methods. A comparison of these results will be made in Chapter IV to test the reliability of the methods.

The same procedure was used to analyze the data obtained when the Durafoam strips were stressed in shear except that the modulus of rigidity (G) replaced the modulus of elasticity (E).

Stress Analysis - Model II

The center of tipping in Model II was calculated in the same manner as for Model I.

$$\frac{B}{D} = \frac{A}{C-D}$$

The actual extreme fiber stress of the Durafoam in Model II was determined using Young's modulus of elasticity (E), the measured deflection (defl.) and the original gage length (O.G.L.).

$$\text{Young's modulus (E)} = \frac{s}{e} \qquad \text{unit strain (e)} = \frac{\text{defl.}_e}{\text{O.G.L.}}$$

Actual extreme fiber stress (s_a) = E x e

Equivalents: defl.= deflection as calculated for the extreme apical fiber

E = 14.972 constant

O.G.L. = .197 measured original gage length of the Durafoam

The theoretical extreme fiber stress (s_t) in Model II was calculated using a formula proposed by Jarabak and Fizzell in their textbook (Technique and Treatment with the Light Wire Appliance; Light Differential Forces in Clinical Orthodontics, St. Louis, C.V. Mosby, 1963). The derivation of their formula was based on formulae for beams.

$$\text{Theoretical extreme fiber stress } (s_t) = \frac{Fm}{Ar} - \frac{Fmla}{I}$$

Equivalents:

a = is the distance from centroid to the most remote apical fiber

A_r= is the surface area of the root to which the Durafoam is attached

F_m= is the applied force to the crown of the model

I = is the moment of inertia of that area of the root to which the Durafoam is attached (around axis through centroid)

l = the distance from the point of force application to centroid

The actual extreme fiber stress (s_a) and the theoretical extreme fiber stress (s_t) have been calculated by two entirely different methods. The actual extreme fiber stress calculation was based upon Young's modulus of elasticity while the theoretical extreme fiber stress was based upon formulae for beams. The results will be compared in Chapter IV.

The shearing stress was analyzed in the same way, however, the modulus of rigidity (G) was substituted for E.

Assembling the Three-Dimensional Analogue

Model III and Model IV differed only in the amount of Durafoam used to simulate the periodontal ligament. Both models were assembled using the three-dimensional analogue. Model III contained two 1/2 inch bands of Durafoam - one in the apical and one in the cervical region. The most remote fibers of Durafoam bands were six inches apart. Model IV contained one continuous six inch band of Durafoam material.

The Durafoam was first glued to the root portion of the truncated cone and then to the alveolus in both cases. The larger six inch piece was accurately measured and cut with beveled edges to assure close approximation of the six inch seam. The root portion of the model was rolled onto the large band because after numerous attempts this method was found to be the most successful. The glue was allowed to dry eight hours before the simulated periodontal ligament was cemented to the alveolar portion of the model.

The cementation to the plastic alveolus was done in two steps. The section of the alveolus without the sensing tip hole was cemented first. The truncated cone was cemented so that parallel holes in the crown were perpendicular to the sensing arm of the dial indicators. After the glue had dried the opposing half of the alveolus was placed in position and Durafoam was marked opposite the sensing tip of the apical dial indicator. The Durafoam in this area was removed allowing the sensing tip to make direct contact with the surface of the model in the apical region. The Durafoam was then cemented to the opposing one-half of alveolar portion of the model. The alignment rods were placed in position with the brass spacers separating the two halves of the alveolus. Eight hours were allowed for drying before the model was attached to the measuring apparatus.

The data were collected following a procedure similar to the method used on Models I and II with two exceptions. Heavier weights were used and the model was not repositioned for measurements of deflection during shearing stress. All forms of stress were developed simultaneously.

Force Analysis - Model III

The center of tipping was calculated in the same manner for Model III as for Models I and II. The actual resisting force (F_a) for Model III was calculated using the numerical integration of the stresses at five degree intervals as shown below (Table I).

$$F_a = 4\int_0^{\frac{\pi}{2}} \frac{Ghr\ \Delta x}{t} \sqrt{\frac{E^2}{G^2} \cos^2\beta + \sin^2\beta\, d\beta}$$

Equivalents:

$d\beta$ = angular change around the apical band of Durafoam

Δx = the deflection of the analogue root

E = modulus of elasticity

F_a = total or actual resisting force (lbs.)

G = modulus of rigidity

h = height of the Durafoam (in.)

r = radius of the Durafoam (in.)

t = original gage length (.197 in.)

The theoretical resisting force (F_t) for Model III was calculated in the same manner as for Model I, using the equations of equilibrium.

The results of the calculations of the resisting force from the two diverse methods will be discussed in Chapter IV.

Stress Analysis - Model IV

The center of tipping for Model IV was calculated using the same ratio that was used for the previous three models.

TABLE I

NUMERICAL INTEGRATION
(5 Degree Increments)

INCR.	Cos. B	Sin. B	$Cos.^2 B$	$Sin.^2 B$	$\frac{E^2}{G^2}$ Cos. B	$\frac{\text{Column 4}}{\text{Column 5}}$	$\sqrt{\ }$	$\Delta B \sqrt{\ }$
0	1.000	.000	1.000	.000	13.265	13.265	3.627	.316
5	.996	.087	.992	.007	12.172	12.129	3.479	.306
10	.984	.173	.969	.030	11.895	11.925	3.452	.301
15	.965	.258	.932	.066	11.442	11.509	3.391	.295
20	.939	.342	.883	.116	10.830	10.947	3.307	.288
25	.906	.422	.821	.178	10.073	10.252	3.185	.278
30	.866	.500	.749	.250	9.197	9.447	3.173	.268
35	.819	.537	.671	.289	8.231	8.520	2.918	.254
40	.766	.642	.586	.413	7.196	7.609	2.758	.240
45	.707	.707	.499	.499	6.131	6.681	2.573	.224
50	.642	.766	.413	.586	5.068	5.654	2.377	.207
55	.537	.819	.289	.671	3.544	4.215	2.053	.179
60	.500	.866	.250	.749	3.066	3.816	1.956	.170
65	.422	.906	.178	.821	2.189	3.010	1.734	.151
70	.342	.939	.116	.883	1.433	2.321	1.523	.132
75	.258	.965	.066	.932	.820	1.753	1.324	.115
80	.173	.984	.030	.969	.369	1.338	1.156	.100
85	.087	.996	.007	.992	.093	1.085	1.088	.094
90	.000	1.000	.000	1.000	.000	1.000	1.000	.087
								4.011

$$\frac{B}{D} = \frac{A}{C-D}$$

The actual extreme fiber stress (s_a) for Model IV was determined using Young's modulus of elasticity (E).

$$E = \frac{s_a}{e}$$

$$s_a = E \times e$$

Equivalents:

s_a = actual stress at the most extreme apical fiber

E = Young's modulus of elasticity (14.972 psi)

e = unit strain = $\dfrac{\text{defl. at extreme apical fiber}}{\text{original gage length}}$

The deflection at the most remote apical fiber had to be determined before the above formula could be used. This deflection was determined using the following ratio (Figure 10).

$$\frac{D}{\text{defl.}_I} = \frac{D + .338}{\text{defl.}_e}$$

$$\text{defl.}_e = \frac{(D + .338) \times \text{defl.}_I}{D}$$

Equivalents:

D = the distance from the center of tipping to the apical indicator

D + .338 = the distance from the center of tipping to the most extreme apical fiber

$defl._I$= the measured deflection of the apical dial indicator

$defl._e$= the calculated deflection at the extreme apical fiber

The theoretical extreme fiber stress for Model IV was calculated using the same formula that was used for Model II.

Theoretical extreme fiber stress (s_t) = $\frac{F_m}{A_r} - \frac{F_m l a}{I}$

Equivalents:

F_m = is the applied force to the crown of the model (lbs.)

A_r = is the projected root area (sq.in.)- this projected area is considerably smaller than the actual root surface area

l = is the distance from the point of force application to centroid

a = is the distance from centroid to the most extreme apical fiber

I = is the moment of inertia of the projected root area

The actual extreme fiber stress for Model IV was calculated using Young's modulus of elasticity. The theoretical extreme fiber stress was calculated using a formula proposed by Jarabak and Fizzell. The concept of projected root surface area was also considered in determining the theoretical extreme fiber stress. The results of these different methods of calculating stress will be compared in Chapter IV.

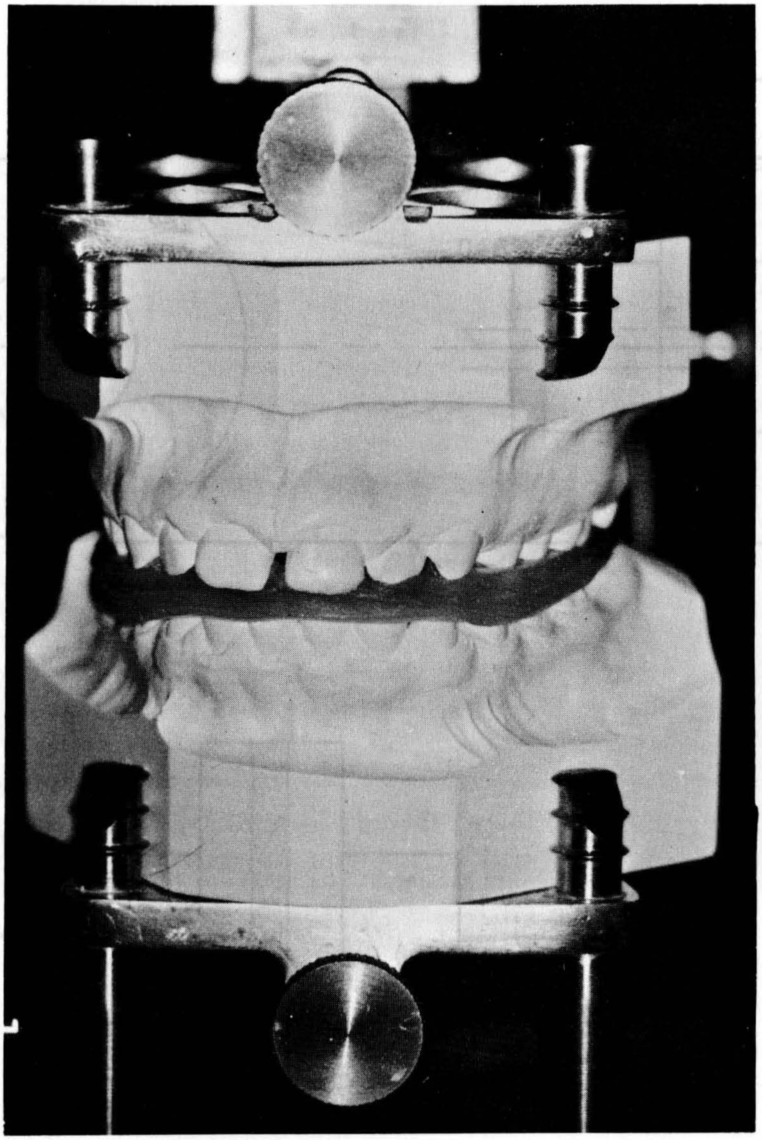


FIGURE 14

IMPRINTING A NEGATIVE OF THE OCCLUSAL SURFACES

OF THE MAXILLARY TEETH

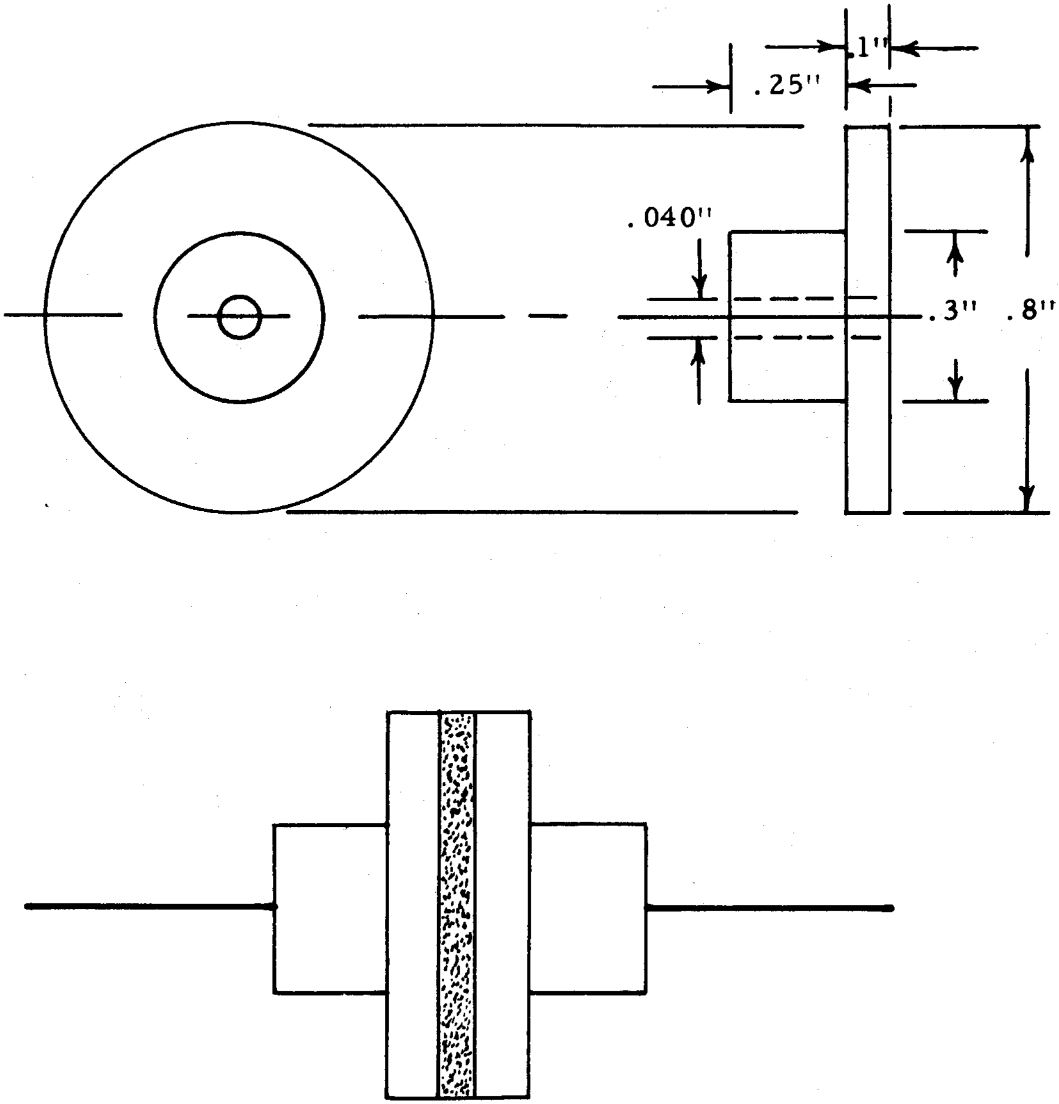


Figure 15

Diagram--Machined Plastic Hubs

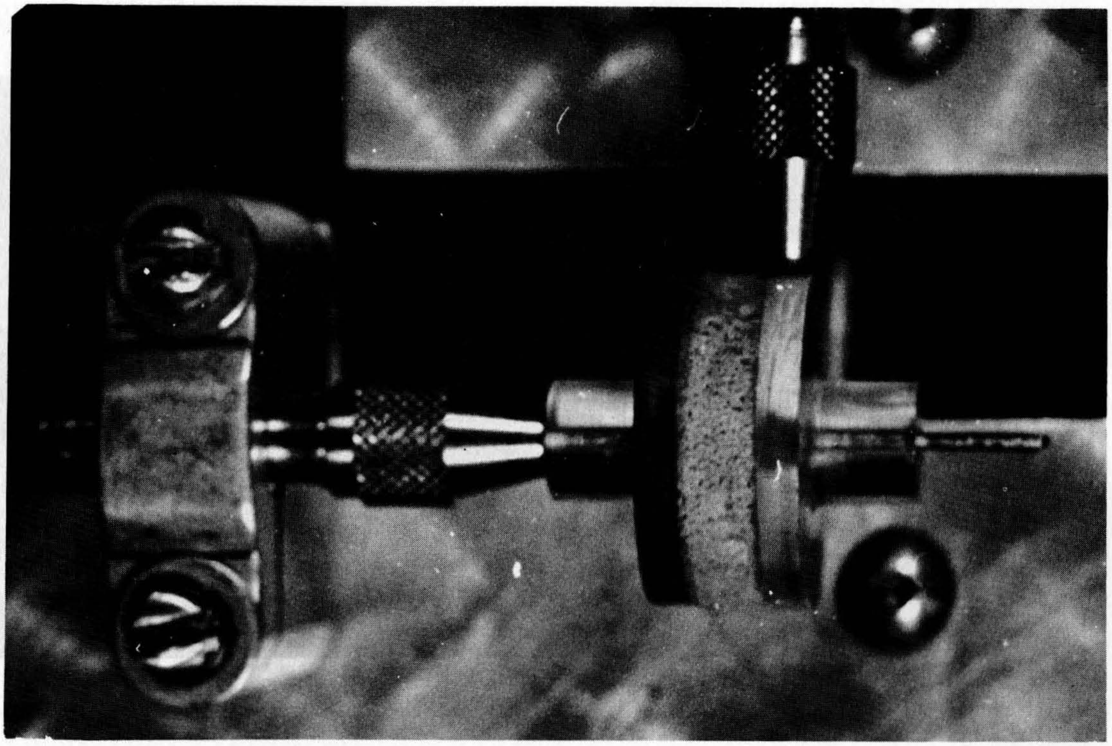


FIGURE 16

PIN VISES OF THE TESTING MACHINE

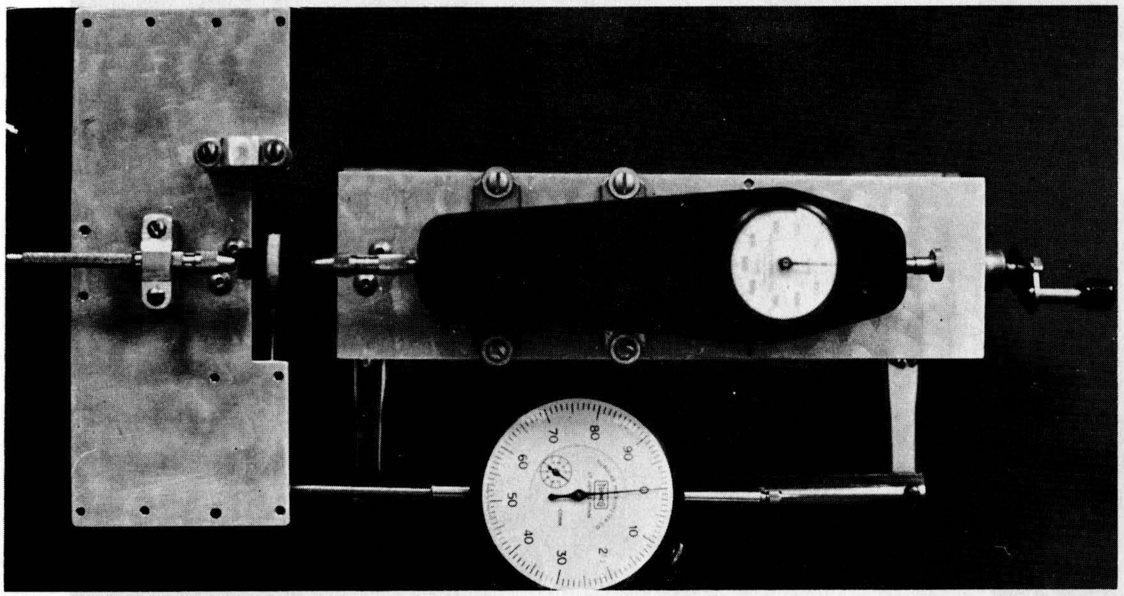


FIGURE 17

TESTING MACHINE

FIGURE 18

COMPONENT PARTS OF MEASURING APPARATUS

CHAPTER III

RESULTS

The results will be reported in the order in which the data were gathered. First, the results of the measurements taken on the six patients will be given; second, the results of the tests exploring material simulating the periodontal ligament; and third, the results of the tests using the two and three-dimensional analogues.

Patient Measurements

Two series of deflection measurements were taken for each patient; one of pushing, the other of pulling. The deflections were averaged at each 20 Gm. increment and the average values at each one pound increment were plotted against deflection (Figure 19).

The graphs of force versus deflection for the six patients indicated a characteristic increase in deflection as the force was increased. The deflection increased quite rapidly from 0 to

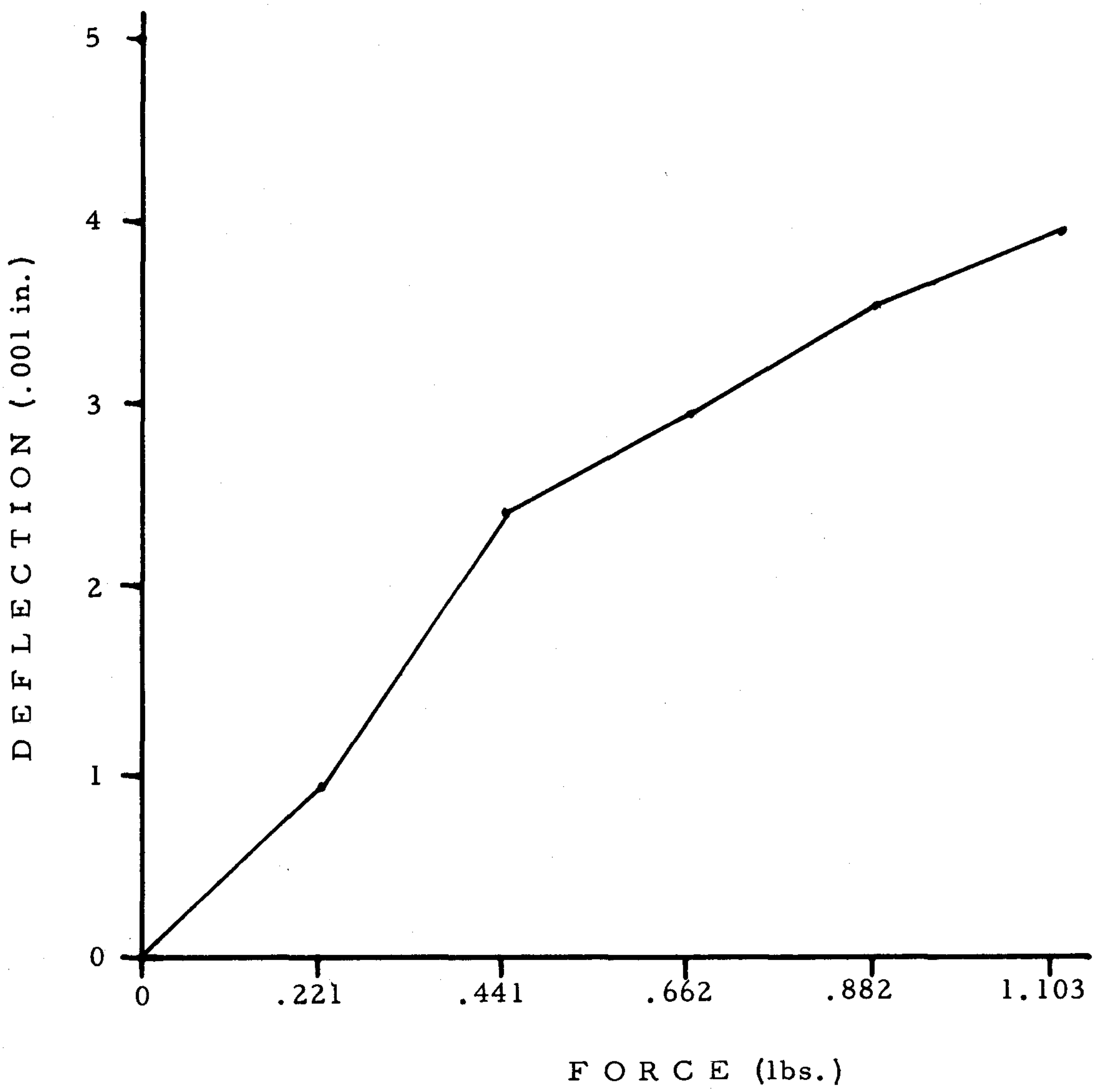


FIGURE 19--AVERAGE FORCE DEFLECTION CHARACTERISTIC FOR THE SIX PATIENTS

approximately 200 Gm. At a point between 180 and 200 Gm. the characteristic began to appear more level although there was still a gradual increase.

Selecting a Material Simulating the Periodontal Ligament

These force-deflection characteristics were compared to similar force-deflection characteristics obtained from measurements taken on several synthetic materials.

Durafoam, a product of the U.S. Rubber Company, was selected to simulate the periodontal ligament from the thirty-two tests because the force-deflection characteristics up to a certain force magnitude were similar to those of the periodontal ligament. Samples of Durafoam, 3/16 inch in thickness, yielded force-deflection characteristics that most closely approximated those of the periodontal ligament.

The majority of the materials tested responded in a non-linear manner. The responses of Durafoam and another material, Polyurathane, were the most nearly similar materials in respect to linearity and to their force-deflection characteristics. The 3/16 inch Durafoam maintained the linearity through a greater range of forces than the Polyurathane. It began to lose linearity at approximately .011 inches of deflection or 200 Gm. of force on a .5 square inch sample in compression.

The force-deflection characteristics of 3/16 inch Durafoam were nearly identical for stressing in compression and tension. The characteristic for shear was linear, however the modulus of rigidity (G) was approximately one-fourth of the modulus of elasticity (E).

The measurements for the 3/16 inch Durafoam were analyzed using Fisher's Analysis of Variance and the results obtained are shown in Table II. The sources of variation were samples, forces, stresses, and operators. The usual 2, 3, and 4 factor interactions were calculated. The mean squares of non-significant sources were combined and resulted in an estimate of experimental error (.000313). The remaining sources of variation were tested against the estimated experimental error to determine their significance. The standard error was plus and minus .0176 millimeters. The ninety-nine percent confidence limits were .0513 millimeters. This was between five and six times the least count of the measuring instruments which was .01 millimeters. This amount of experimental error was reasonable and gave confidence in the

TABLE II

ANALYSIS OF VARIANCE
(MEASUREMENTS OF .1875 INCH DURAFOAM)

SOURCES	D.F.	S.S.	M.S.	V.R.	SIGNIFICANCE	
SAMPLES (S)	2	.148242	.074121	25.3	6.23 (1%)	*
FORCES (F)	2	2.161092	1.080546	3452.0	6.23 (1%)	***
STRESSING (S')	2	7.498229	3.749114	1197.8	6.23 (1%)	***
OPERATORS (O)	1	.008513	.888513	27.4	8.53 (1%)	*
S x F	4	.008903	.002225	7.10	4.77 (1%)	*
S x S'	4	.158152	.039538	126.0	4.77 (1%)	**
S x O	2	.012636	.006318	20.18	6.23 (1%)	*
F x S'	4	1.009598	.252399	806.	4.77 (1%)	**
F x O	2	.002058	.001029	3.28		N.S.
S' x O	2	.004551	.002275	7.26	6.23 (1%)	*
S x F x S'	8	.009428	.001178	3.76	2.59 (5%)	*
S x F x O	4	.001385	.000346	1.105		N.S.
F x S' x O	4	.001346	.000336	1.07		N.S.
S x O x S'	4	.004141	.001035	3.30	3.01 (5%)	*
S x F x S' x O	8	.002281	.000285	.9105		N.S.
TOTAL	53	11.030555				

Standard Deviation of Error = .0176 mm.
ESTIMATE OF EXPERIMENTAL ERROR = .000313
99% CONFIDENCE LIMITS are $\pm$ (2.92 x .0176) = $\pm$ .0513 mm.

D.F. = Degrees of Freedom
S.S. = Sums of Squares
M.S. = Means of Squares
V.S. = Variation Ratio

*** = Highly Significant Variance Ratio
N. S. = Non-significant Variance Ratio

reliability of the measurements.

Measurements of Deflection During Tests Using the Two and Three-Dimensional Analogues

Single forces of known magnitudes were applied to that portion of the analogue representing the crown of the tooth. These forces of different magnitudes but constant direction were applied at the three levels previously described. The crown portion of all four models tipped in the direction of the applied force and the root portion of the models tipped in the opposite direction. The measured deflections varied according to the magnitude of the force and to the height of the post to which the force was applied. The direction of all forces was constant. The forces were applied parallel to the platform of the measuring apparatus or perpendicular to the long axis of the model.

Model I

Forces were applied at half pound increments from zero to one and one-half pounds to the crown portion of Model I. These forces were applied at all three post heights beginning with the upper post and ending with the lower post. The increase in the measured deflection during measurements of compression and tension was nearly proportional to the increase in force increment (Table III). The characteristic of the actual resisting forces (F_a) of the Durafoam for the measured deflections is compared with that of the proportional theoretical resisting forces (F_t) in Figures 20 and 21. The resisting force was assumed to be in the center of the Durafoam strips. The linear theoretical resisting forces were calculated using the equations of equilibrium and are represented on the graph by the different lines with post height as the parameter. The actual resisting forces were calculated with respect to the measured deflections using Young's modulus of elasticity (E) for the Durafoam.

The actual resisting forces of the Durafoam in compression and tension became less linear as the point of force application was lowered and as the force magnitude was increased beyond 200 Gm. The actual resisting forces of the Durafoam were less linear during shearing stress than during compression and tensile stress (Table IV). The Durafoam showed the greatest degree of linearity during all three types of stressing when the force was applied to the upper post. In general, the deflections and the actual resisting forces (F_a) decreased as the point of force

TABLE III

MEASUREMENTS IN COMPRESSION AND TENSION--MODEL I

FORCE (lbs.)	CENTER OF TIPPING DISTANCE ABOVE ADI (in.)	DEFLECTION AT CENTER OF APICAL FOAM (in.)	ACTUAL APICAL RESISTING FORCE (F_a)	THEORETICAL APICAL RESISTING FORCE (lbs.) (F_t)
		UPPER POST		
0	0	0	0	0
.5	1.374	.00189	.28837	.28458
1.0	1.138	.0037	.57005	.56916
1.5	1.114	.00561	.85369	.85374
		CENTER POST		
0	0	0	0	0
.5	1.225	.00145	.22100	.23089
1.0	1.007	.00300	.45621	.46178
1.5	1.017	.00455	.69305	.69267
		LOWER POST		
0	0	0	0	0
.5	.835	.00937	.14243	.17421
1.0	.876	.01837	.27926	.34843
1.5	.846	.03438	.48723	.52264

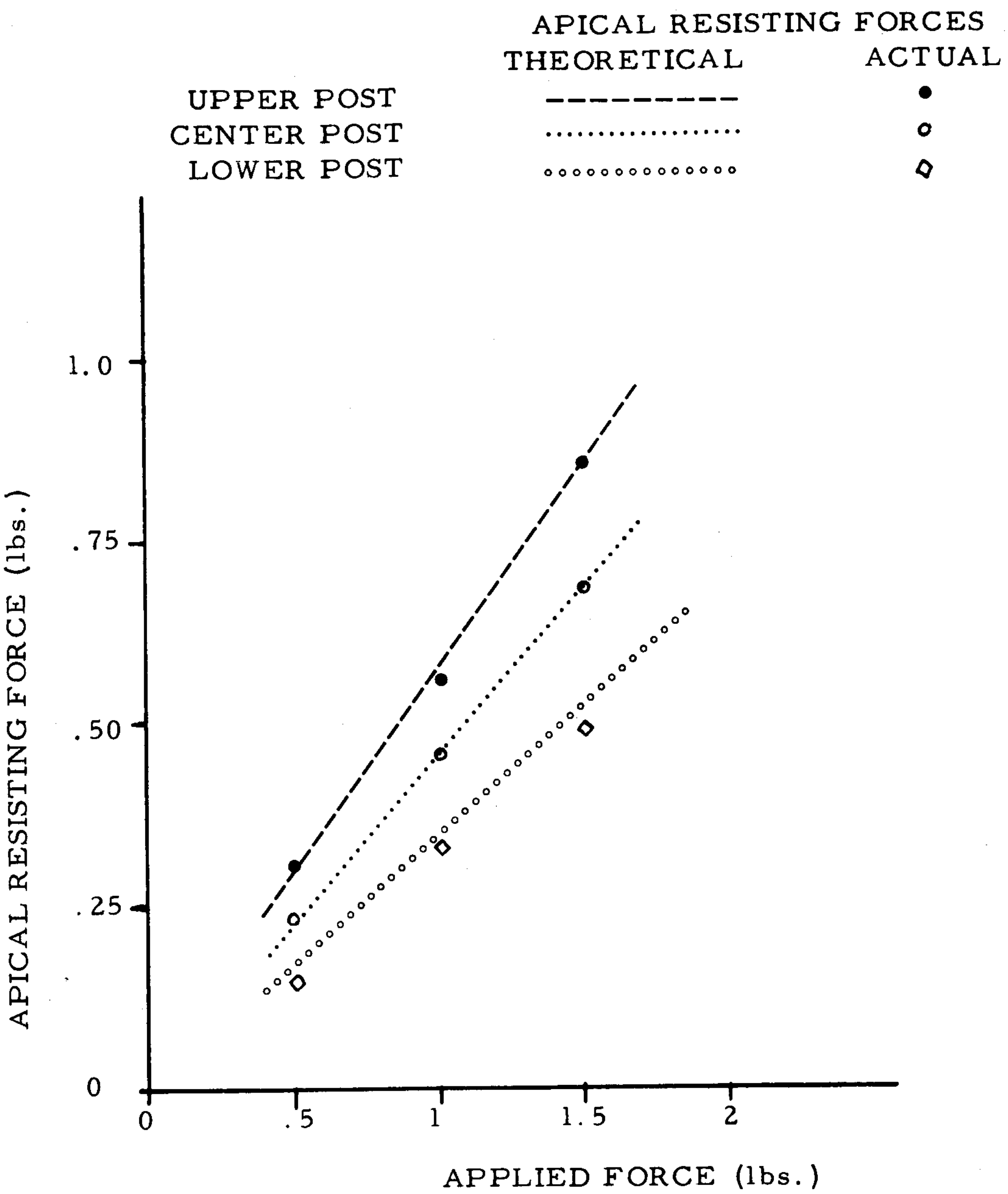


FIGURE 20--MEASUREMENTS IN COMPRESSION AND TENSION FOR MODEL I

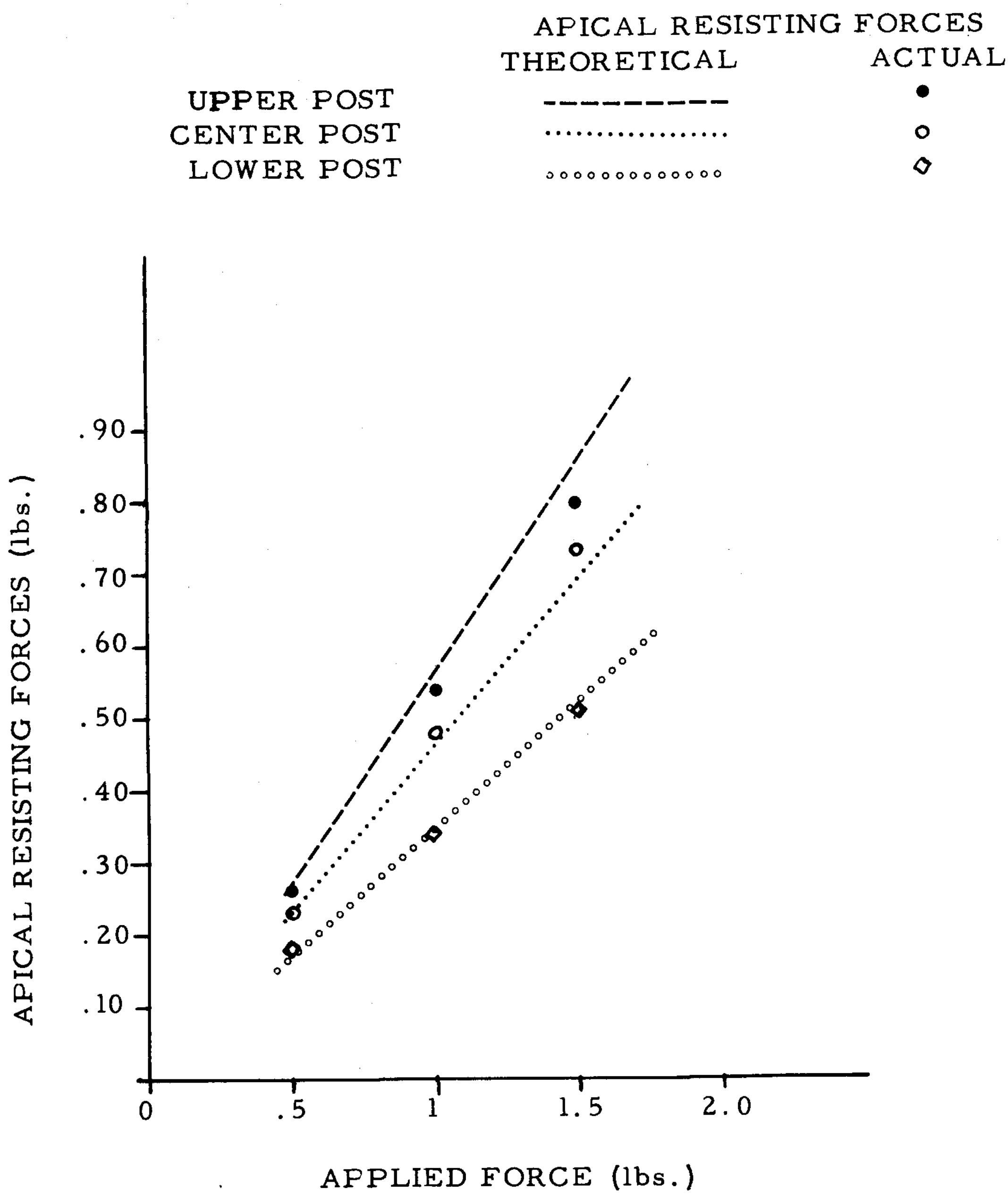


FIGURE21--MEASUREMENTS IN SHEAR--MODEL I

TABLE IV

MEASUREMENTS IN SHEAR--MODEL I

FORCE (lbs.)	CENTER OF TIPPING DISTANCE ABOVE ADI (in.)	DEFLECTION AT CENTER OF APICAL FOAM (in.)	ACTUAL APICAL RESISTING FORCE (F_a)	THEORETICAL APICAL RESISTING FORCE (lbs.) (F_t)
		UPPER POST		
0	0	0	0	0
.5	1.063	.00616	.26745	.28458
1.0	1.010	.01245	.54049	.56916
1.5	1.064	.01848	.80223	.85374
		CENTER POST		
0	0	0	0	0
.5	1.090	.00557	.24191	.23089
1.0	1.090	.01114	.48385	.46178
1.5	.959	.01716	.74491	.69267
		LOWER POST		
0	0	0	0	0
.5	.851	.00320	.18229	.17421
1.0	.770	.00793	.34448	.34843
1.5	.831	.01204	.51963	.52264

application was changed from the upper to the lower posts.

The center of tipping remained quite constant for each post level of applied force. It stayed within a .2 inch to .3 inch range at each post level. The height of the center of tipping above the apical dial indicator decreased as the height of force application decreased.

Model II

Two sets of measurements of deflection were taken on Model II. One, when the Durafoam was stressed in compression and tension and the other when stressed in shear (Tables V and VI). These measurements were recorded at .5, 1, 2, and 3 pound levels of force application during compressive and tensile stressing. The following three calculations were based upon the recorded measurements: center of tipping, the deflection at the extreme apical fiber, and the extreme fiber stress.

The actual extreme fiber stress (S_a) was calculated using the formula for Young's modulus of elasticity (E) with respect to the calculated deflection at the extreme apical fiber. The extreme fiber stress (s_t), theoretically computed, was determined using the formula suggested by Jarabak and Fizzell based upon

TABLE V

MEASUREMENTS IN COMPRESSION AND TENSION--MODEL II

FORCE (lbs.)	CENTER OF TIPPING DISTANCE ABOVE ADI (in.)	DEFLECTION AT EXTREME APICAL FIBER (in.)	ACTUAL EXTREME FIBER STRESS (psi) (S_a)	THEORETICAL EXTREME FIBER STRESS (psi) (S_t)
		UPPER POST		
0	0	0	0	0
.5	1.424	.00104	.08092	.07312
1	1.306	.00183	.14017	.14624
2	1.429	.00365	.27806	.29036
3	1.519	.00595	.45261	.43772
		CENTER POST		
0	0	0	0	0
.5	1.297	.00807	.06194	.06419
1	1.325	.00158	.12037	.12838
2	1.382	.00355	.27020	.25676
3	1.489	.00485	.36911	.38552
		LOWER POST		
0	0	0	0	0
.5	1.517	.00769	.05864	.05423
1	1.245	.00144	.11072	.10946
2	1.258	.00292	.22215	.21892
3	1.272	.00404	.30739	.32838

TABLE VI

MEASUREMENTS IN SHEAR--MODEL II

FORCE (lbs.)	CENTER OF TIPPING DISTANCE ABOVE ADI (in.)	DEFLECTION AT EXTREME APICAL FIBER (in.)	ACTUAL EXTREME FIBER STRESS(psi) (S_a)	THEORETICAL EXTREME FIBER STRESS (psi) (S_t)
		UPPER POST		
0	0	0	0	0
1	1.448	.00829	.14119	.14501
2	1.439	.01714	.29120	.29002
3	1.420	.02639	.44792	.43690
4	1.442	.03361	.57107	.58304
		CENTER POST		
0	0	0	0	0
1	1.374	.00734	.12581	.12811
2	1.385	.01538	.26143	.25622
3	1.398	.02169	.36992	.38433
4	1.369	.02965	.50435	.51244
		LOWER POST		
0	0	0	0	0
1	1.260	.00681	.11574	.10907
2	1.278	.01323	.22535	.21904
3	1.317	.01910	.32512	.32812
4	1.246	.02479	.42237	.43808

formulae for beams. The linearity of the actual compressive and tensile stress was compared to the linear theoretical stress (Figures 22 and 23).

Deflections were recorded at 1, 2, 3, and 4 pound increments when the Durafoam was stressed in shear. The linearity of the actual shearing stress at the extreme apical fiber was compared to the theoretical stress in the same manner as was done for the compressive and tensile stresses. In general, the comparison of actual stresses to theoretical stresses for Model II was similar to the comparison of actual forces to the estimated theoretical forces for Model I with two exceptions. The measured deflection was smaller for each respective increment of applied force on Model II and the center of tipping during shearing stress for Model II was more erratic.

Model III

Forces were applied to the crown portion of the model tooth at quarter pound increments from 0 to 1.5 pounds. Measurements of deflection were obtained for each increment and the actual resisting force was calculated using the numerical integration explained in Chapter II (Table VII). The resisting force was

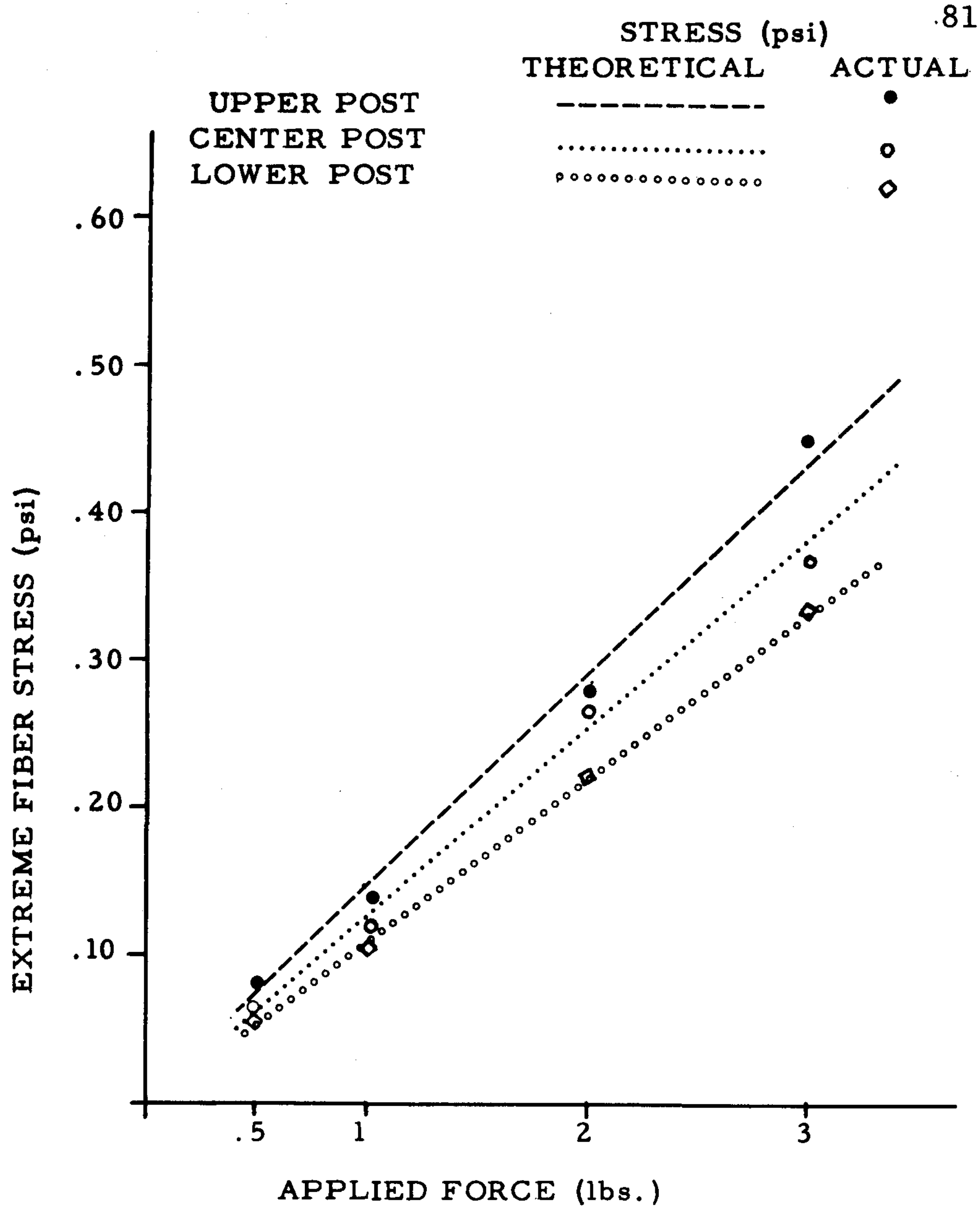


FIGURE 22--MEASUREMENTS IN COMPRESSION AND TENSION FOR MODEL II

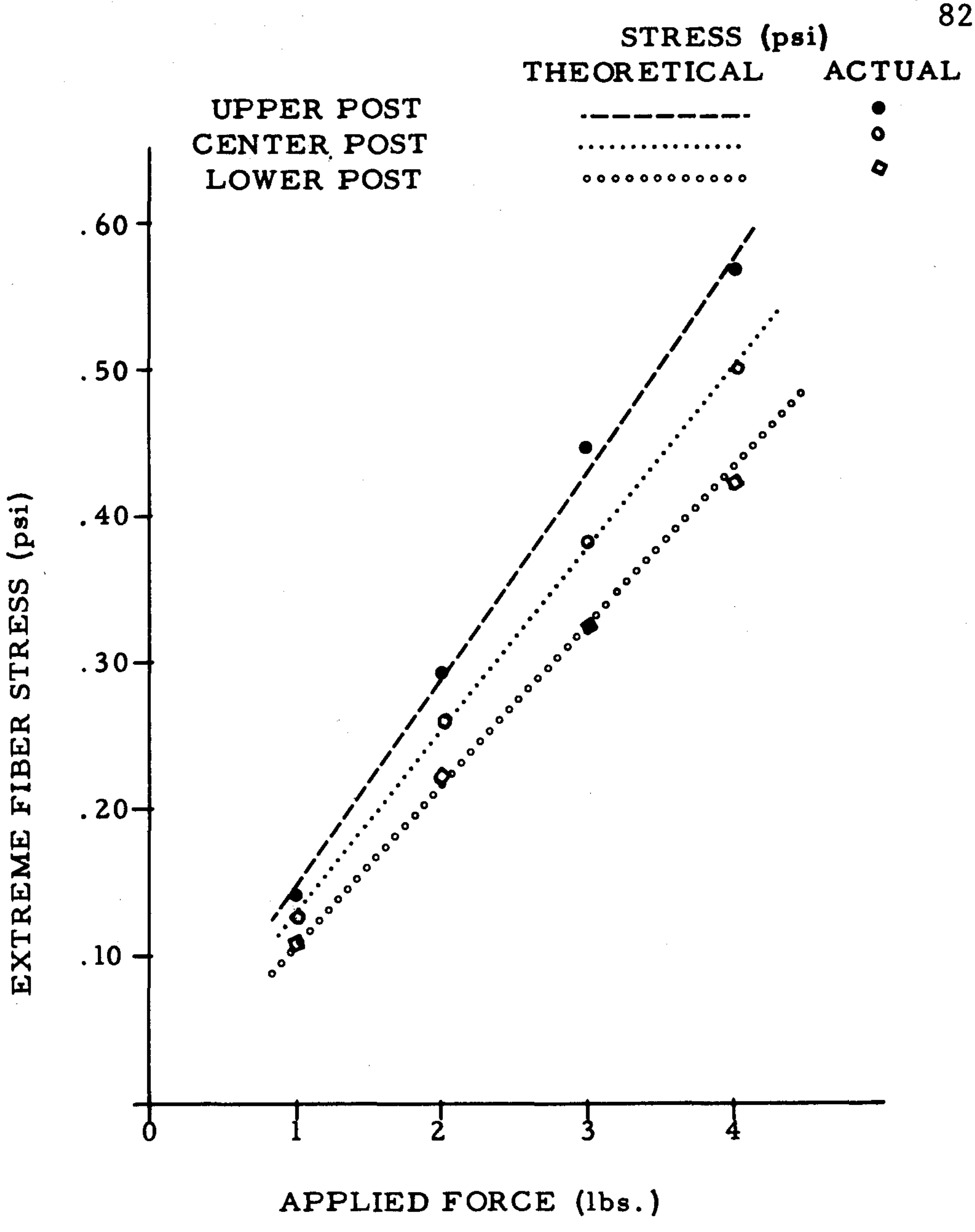


FIGURE 23--MEASUREMENTS IN SHEAR--MODEL II

TABLE VII

MEASUREMENTS FOR MODEL III

FORCE (lbs.)	CENTER OF TIPPING DISTANCE ABOVE ADI(in.)	DEFLECTION AT CENTER OF APICAL FOAM (in.)	ACTUAL APICAL RESISTING FORCE (F_a)	THEORETICAL APICAL RESISTING FORCE (lbs.) (F_t)
		UPPER POST		
0	0	0	0	0
.25	2.267	.00192	.15476	.14227
.5	2.261	.00388	.31285	.28454
.75	2.368	.00494	.39780	.42681
1.0	2.173	.00671	.54090	.56909
1.25	2.306	.00935	.75359	.71136
1.5	2.303	.01181	.95108	.85363
		CENTER POST		
0	0	0	0	0
.25	2.111	.00155	.12547	.10818
.5	2.099	.00291	.23449	.21636
.75	2.284	.00421	.33907	.32454
1.0	2.096	.00598	.48222	.43272
1.25	2.121	.00815	.65697	.54090
1.5	2.118	.00975	.78577	.64909
		LOWER POST		
0	0	0	0	0
.25	1.790	.00115	.09311	.07419
.5	1.688	.00161	.13006	.14838
.75	1.843	.00251	.20259	.22257
1.0	1.671	.00356	.28696	.29676
1.25	1.663	.00501	.40384	.37095
1.5	1.762	.00682	.54914	.44514

assumed to be in the center of the Durafoam all the way around the band. As previously mentioned, all three types of stress were developed simultaneously in Models III and IV.

The linearity of the actual resisting force was compared to that of the theoretical resisting force in Figure 24. As with Model I, the theoretical resisting force was calculated using the equations of equilibrium. The actual resisting force did not compare as favorably with the theoretical resisting force as it did for Model I. The height of the center of tipping above the apical dial indicator followed the pattern described for Model I, however.

Model IV

A similar procedure was followed in gathering data for Model IV, however, the applied forces were much greater. Measurements of deflection were recorded in one pound increments from 0 to 7 pounds at each of the described heights (Table VIII). The actual and theoretical stresses were calculated for the extreme apical fibers and a comparison was made of their linearity (Figure 25). The linearity of the actual extreme fiber stress (s_a) compared favorably to that of the theoretical extreme

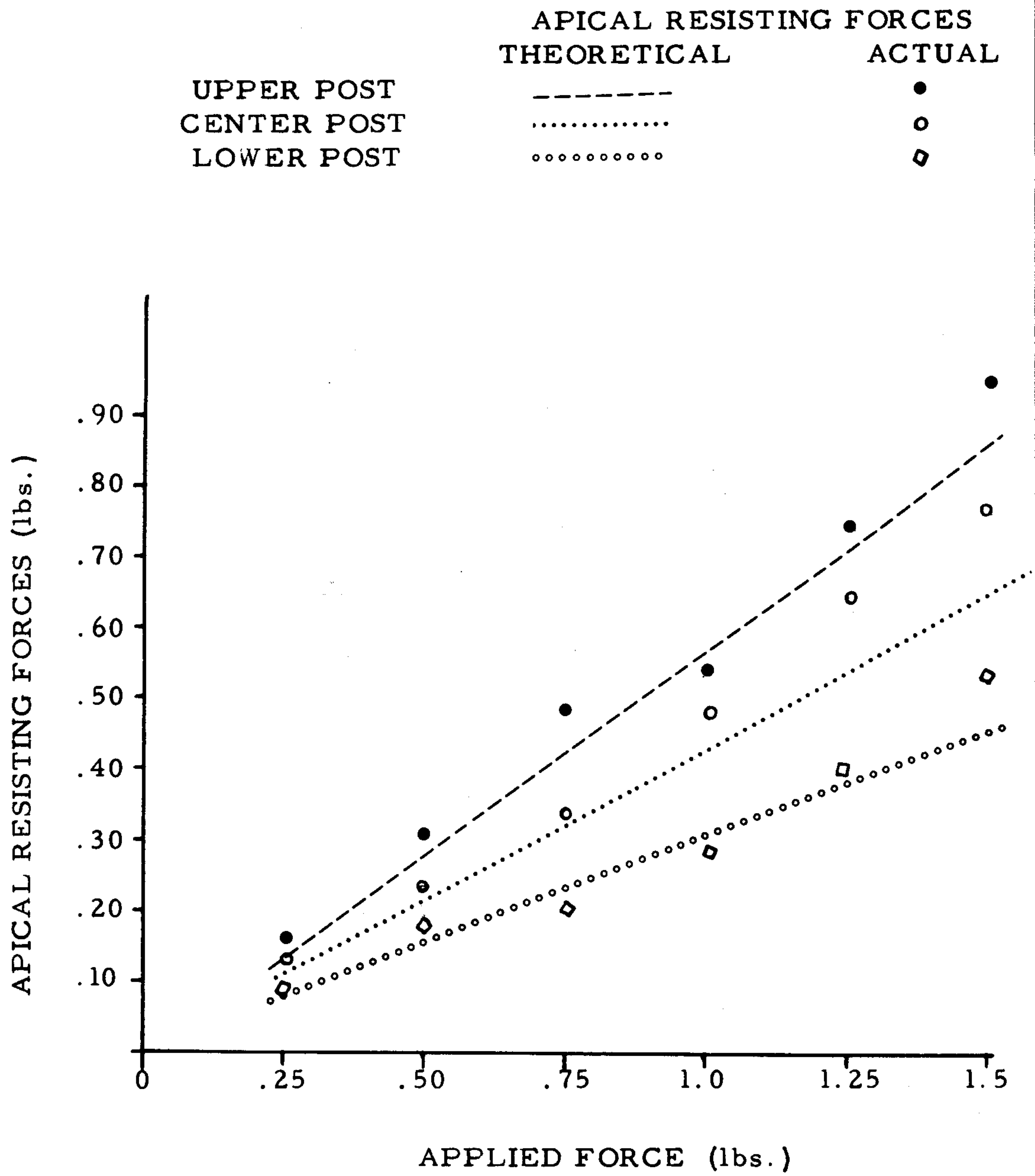


FIGURE 24--MEASUREMENTS FOR MODEL III

TABLE VIII

MEASUREMENTS FOR MODEL IV

FORCE (lbs.)	CENTER OF TIPPING DISTANCE ABOVE ADI (in.)	DEFLECTION AT EXTREME APICAL FIBER (in.)	ACTUAL EXTREME FIBER STRESS (psi) (S_a)	THEORETICAL EXTREME FIBER STRESS (psi) (S_t)
		UPPER POST		
0	0	0	0	0
1	2.082	.00353	.53709	.59114
2	2.074	.00609	.92623	1.18229
3	2.566	.00103	1.56710	1.77344
4	2.771	.01458	2.21707	2.36459
5	2.806	.01793	2.72551	2.95573
6	2.777	.02361	3.58897	3.54688
7	2.823	.03019	4.59021	4.13803
		CENTER POST		
0	0	0	0	0
1	2.496	.00260	.39592	.49033
2	2.463	.00591	.89902	.98067
3	2.499	.00821	1.24870	1.47100
4	2.394	.01104	1.67812	1.96134
5	2.702	.01680	2.25549	2.45167
6	2.543	.02013	3.06044	2.94201
7	2.574	.02397	3.64349	3.43234
		LOWER POST		
0	0	0	0	0
1	2.182	.00174	.26506	.38952
2	2.022	.00355	.54109	.77904
3	2.182	.00646	.98337	1.16856
4	2.187	.00890	1.35284	1.55808
5	2.023	.01309	1.99033	1.94761
6	2.434	.01474	2.24095	2.33713
7	2.509	.01778	2.70398	2.72665

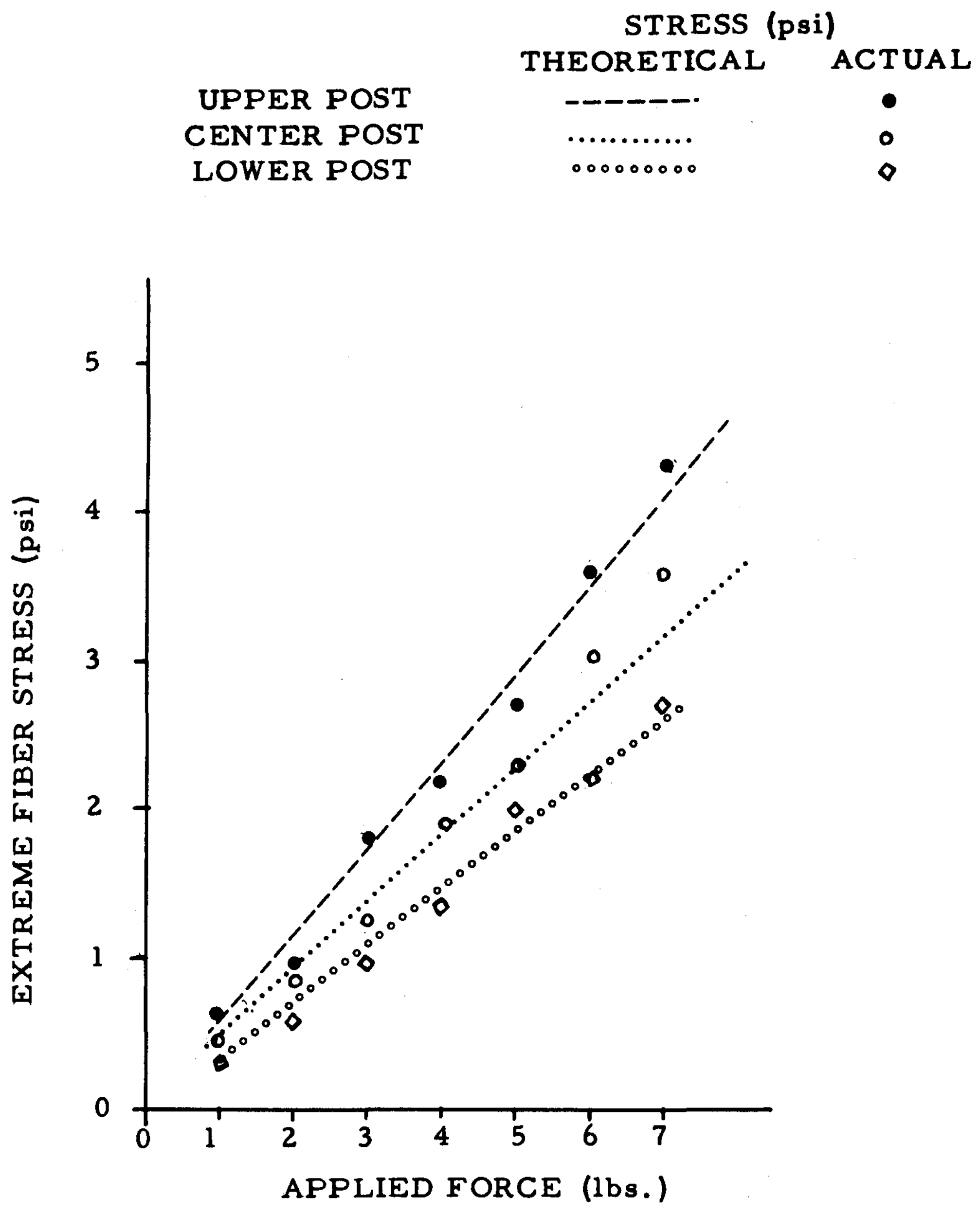


FIGURE 25--MEASUREMENTS FOR MODEL IV

fiber stress (s_t) when the applied forces were at the five pound level or below. The measured deflections at the higher force increments exceeded the linearity of the Durafoam. The actual and theoretical stresses for Model IV were calculated in the same manner as for Model II using Young's modulus of elasticity (E) and the Jarabak and Fizzell formula, respectively.

Reducing the height of force application affected Model IV the same as the three predecessors, by generally lowering the center of tipping and by a decrease in the deflection. The height of the center of tipping varied through a greater range for Model IV, however.

CHAPTER IV

DISCUSSION

One of the more challenging of the many unsolved problems in orthodontics is that of calculating the stresses developed in the fibers of the periodontal ligament when specific orthodontic forces are applied to the crowns of teeth. Having knowledge of the quantitative relation between applied forces and resulting stresses would enable the orthodontist to develop force systems which would move teeth effectively with little pain to the patient and minimal damage to the teeth and their periodontal environment. It would be impractical to attempt to develop and test proposed methods of quantitating fiber stresses in patients without having first tested the proposed methods using models or animals. This research was planned to develop and test methods of calculating stresses in the apical fibers of the periodontal ligament using models.

Numerous investigators, Sandstedt (1904, 1905), Oppenheim (1911), Schwarz (1928), Kellner (1928), Kronfeld (1931), Bodecker (1942), Storey and Smith (1952), and Begg (1954), have qualitated changes in alveolar bone, cementum, and dentin histologically and roentgenographically. It must be emphasized that all of these studies were made over a varying period of time during which numerous force systems may have been applied to the crowns of the teeth. In many cases these forces were described as being "light" or "heavy," and the resulting deflections were described as "slight movement" or "moderate tipping" et cetera. Schwarz (1928) was among the first to measure the applied forces while simultaneously qualitating their effects on bone and cementum. In the midst of these qualitative expressions the genuine search for knowledge could be seen but the results were of little assistance in planning this research.

In order to accurately quantitate the stresses that develop in the fibers of the periodontal ligament, the study must be done over a short period of time. It is understood that as soon as a force is applied to the crown of a tooth and the tooth is displaced, there are stresses set up immediately in the periodontal ligament. If this applied force is dissipated when the tooth moves through .003 inches, for example, biologic changes in the periodontal ligament will relieve the fiber stresses allowing them to return to their original values. This fact makes it imperative that a study of stresses within the periodontal ligament be made immediately after the force is applied. It is equally important that numerical measurements be made in place of visual observations.

Mühlemann (1951) recognized the importance of taking simultaneous measurements of the applied force and the resulting deflection of a tooth. His work disclosed two phases of tooth movement that may be observed as a gradually increasing force being applied to the crown of a tooth. This work was helpful in planning a study of the force-deflection characteristics of canine teeth in a select group of young patients. It was important to this research to know the shape of this characteristic in order to select a synthetic elastic material that could be used to simulate the periodontal ligament in the models that were to be constructed. While the average characteristic obtained from the group of young patients selected for this project had about the same general shape as the published curve in Mühlemann's work, the "knee" of the curve in his work was found at a deflection of .0024 inches. When it was noticed that the point of measurement was much closer to the gingival margin in this latter work, the smaller deflection appears reasonable. Of course this smaller deflection should logically require higher forces since the moment arm between force application and centroid was shorter in the teeth of the young patients. In addition to the obvious physical differences there may have been biologic differences to help explain the discrepancy in absolute values of force and deflection.

Qualitative studies using various kinds of models have been described by many authors including Hanau (1917), Fish (1917), Schwarz (1928), Bauer and Lang (1928), Fickel (1930), Synge (1933), Hay (1939), Haack and Weinstein (1963), and Dempster and Duddles (1964) who have shown models that were designed for mathematical studies for demonstrations and for educational purposes. The most helpful models were those discussed by Haack and Weinstein and by Dempster and Duddles who emphasized measuring important quantities and using the concepts of equilibrium when analyzing the forces developed in the periodontal ligament. Earlier authors such as Hanau and Fish had listed important engineering formulas and relations that were applicable to the problem of analyzing force systems that developed around teeth but these formulas and concepts were applied only rarely to research work that was published in the following years.

Those workers who have used materials to simulate the periodontal ligament in their models appear to have made no effort to select a material having physical properties or characteristics similar to those of the live periodontal ligament. The men who used mathematical models simply defined some of the characteristics that they believed this ligament possessed and then carried out their analyses under these assumptions. The more recent workers who made demonstration models used elastic foam sponge or the compliance of a series of force gages that supported the model of the tooth that was being studied. While these substitutes form a sharp contrast with the plaster used by Schwarz and Fickel, they were not selected to have any predetermined force-deflection characteristics.

It was the plan of this research to build upon the foundation established by these earlier workers but to apply the principles of analytical mechanics more extensively. At the beginning of this discussion, it was indicated that the aim was to calculate the <u>stresses</u> in the periodontal ligament but in most earlier work with models the investigators had been concerned with <u>forces</u>.

There has existed the seemingly unrecognized necessity for relating the three quantities- stress, force, and deflection. The material selected for the periodontal ligament analogue had to possess a force-deflection characteristic that was linear through the initial phase of deflection so that it would lend itself to mathematical analysis.

In engineering texts, the relation between the three fundamental quantities- stress, force, and deflection is established through the parameter known as Young's Modulus of Elasticity. To be specific, this modulus is applicable only when tensile and compressive stresses are being considered and the modulus of rigidity is applicable when shearing stresses are being considered. It is through the use of these parameters that the ultimate calculation of stresses within the periodontal ligament is to be accomplished. Synge (1933) had defined the terms stress and deflection or strain as if expecting to use these in his analysis but his summary did not mention these properties and it is not known how he used them. Manly (1955) used the term "spring constant" in some of his writing, showing that he recognized the existance of a relation between force and deflection but he did not include the idea of stress. Burstone (1962) mentioned the stress-strain response as being the most important and the least understood response of a tooth to force. When the material had been selected for use as the periodontal ligament analogue for this research, the first step was to determine accurately the moduli of elasticity and of rigidity. The introduction of these two parameters into studies of the periodontal ligament through the use of models is perhaps the most significant contribution of this study. The general plan was to study a series of models having increasing complexity and requiring more complex methods of analysis until the final three-dimensional model having the full periodontal ligament analogue had been constructed, tested, and analyzed. The belief was that the results of this final analysis might be applied eventually to the study of the periodontal ligament of a single rooted tooth in either animals or human subjects.

The data obtained from Model I were used to determine whether equations of equilibrium could be used to calculate or theorize the amount of resisting force which would be encountered in the apical fibers when specific forces were applied to the crown of a tooth. The apical forces were calculated theoretically and were then compared to the actual apical resisting forces obtained in the investigation. The values compared favorably within the linear range of the Durafoam.

Model II was used to calculate the stress in the apical fibers using the method suggested by Jarabak and Fizzell. The theoretical results obtained from this method, which is based upon formulae for beams, were compared to those obtained from the formulae for stresses which are based upon the moduli of elasticity and rigidity and measurements of deflections. The comparison indicated that the suggested method was accurate and could be used with assurance. The validity of this method was rewarding from another standpoint. As explained in Chapter II, the formula for stress in the apical fibers incorporates the area being stressed. The area of the Durafoam used in Model II was substituted into the equation for this term. This area, which was considered in one plane of space in the two-dimensional model, would be comparable to the projected surface area of a three-dimensional object as will be described in the discussion of Model IV.

The calculations using the data from Model III indicate that the equations of equilibrium are applicable when considering objects in three planes of space. The theoretical values of force determined using the equations of equilibrium compared favorably with actual values of force calculated using the numerical integration discussed in Chapter II. This would seem to substantiate the use of the equations of equilibrium in predicting the resistance that could be expected in the apical fibers when three types of stressing were developed simultaneously.

Using the numerical integration to determine the actual resisting force also afforded the opportunity to determine what percentage of this force could be attributed to the three principal stresses considered in this investigation. The calculations indicated that approximately seventy-eight percent of the resistance could be attributed to the combined compression and tensile stresses. The remaining twenty-two percent of the resistance was due to shearing stress.

This information is interesting because when a tooth is rotated the correcting force must overcome primarily the resistance of the periodontal ligament to shearing stress which may be only a fraction of stresses in tension and compression. This is significant to the orthodontist because it is apparent that less force would be required to rotate a tooth than to tip it, however, it should be pointed out that during rotation nearly every fiber is stressed in shear and there would be a greater tendency for the tooth to return or relapse. The preceding explanation of the tendency of a tooth to relapse is more realistic than that offered by Stallard (1964) which attributed the return to the "inertia of the periodontium." It also is more feasible than the theory offered by Rashevsky in which he proposed that cells have "built-in" mechanical stresses.

The data obtained from the models in this study indicate that the ratio of tensile stress to shearing stress is approximately three and one-half to one. The exact stress ratio in the periodontal ligament is unknown because the complex structure of this organ cannot be, or perhaps stated more accurately, was not duplicated in the synthetic facsimile of the periodontal ligament used in this investigation. There is good reason to believe that the ratio approximates the above value. The fact that resistance due to shearing stress is appreciably less than resistance due to tension may account for the rotation of a tooth when it has an elastic attached to either the buccal or the lingual surface.

Renfroe (1951) stated that only fifty percent of the fibers need be considered in assessing the resistance of a tooth to bodily movement when the root is round or circular in cross-section. His reasoning seems to be based only on the recognition of tensile and compressive stresses and the algebraic sum of the force vectors of all the periodontal fibers in a given direction. This is not realistic as has been shown by the analysis of Model III.

Model IV, like Model II, was used to determine the validity of the formula suggested by Jarabak and Fizzell based on the formulae for beams except that Model IV developed the three types of stress simultaneously. The projected root surface area of the model was calculated and used in determining the theoretical stress in the extreme apical fiber. The extreme stress was calculated using the modulus of elasticity because the highest stress could now be attributed to compression and tension. A comparison was made at each value of force and the method appeared to be satisfactory when applied to a three-dimensional model. These findings tend to validate the theory of projected root surface area.

It was pointed out in Chapter II that it was necessary to calculate the center of tipping in evaluating each of the four models. In each case this was calculated as a distance above the apical dial indicator. The results showed a definite pattern that supported the general contentions of Sandstedt (1904, 1905) and Kronfeld (1931) who stated that the center of tipping would be near the center of the root. The results concurred with those of Geigel (1964) in that the center of tipping was always between centroid and the apex and also that it moved closer to the apex as the point of force application was lowered on the crown. The results agreed with the conclusion by Geigel but contradicted that of Schwarz (1928) when it was shown that the center of tipping is directly influenced by the magnitude of the applied force. As the force was increased the center of tipping moved upward (cervically) or closer to centroid.

CHAPTER V

SUMMARY AND CONCLUSIONS

A. Summary:

This investigation was conducted to study quantitatively the stresses developed in the apical fibers of a synthetic periodontal ligament using geometric analogues. Basically, the problem was three-fold. The first phase of the problem involved patients and construction of a clutch to obtain force deflection characteristics of mandibular right canine teeth. The second phase of the problem involved designing and constructing the geometric models. The third phase involved the gathering and evaluating of data through a series of tests using the models.

Two and three-dimensional geometric models were constructed, however, before they could be assembled it was necessary to find a suitable synthetic material to simulate the periodontal ligament of these models. Force deflection characteristics were obtained for the mandibular right canine of six patients as described in Chapter II. These characteristics served as the criteria for the selection of the material to simulate the periodontal ligament. Extensive tests were conducted on thirty-two materials and the force deflection characteristics of these materials were compared to those of the live periodontal ligament. After selecting ,187 inch Durafoam, a product of U.S. Rubber Company, the moduli of elasticity and rigidity of the material were determined and served as constants for many of the calculations during the remainder of the study.

Models I and II permitted studying the three stresses individually and were considered to be two-dimensional models and were evaluated in one plane of space. Models III and IV developed the three types of stress simultaneously and were approached as three-dimensional models and analyzed in three planes of space. The difference between Models I and II and between III and IV was in the shape of the Durafoam used to simulate the periodontal ligament. A similar series of tests were conducted using each of the four models. Models I and II or the two-dimensional models were tested, first, in compression and tension, and second, in shear. Only one series of tests was conducted on each of the three-dimensional models because the three types of stress were developed simultaneously.

The theoretical force for Models I and III and the theoretical stress for Models II and IV were calculated using theories proposed by Jarabak and Fizzell. These theoretical force or stress values were compared to the actual resisting forces or actual stresses to test the validity of the theories used in their determination. The center of tipping was also calculated for each model at each force increment. The data were evaluated and the following conclusions were made.

B. Conclusions:

1. The force-deflection characteristics of mandibular right canine teeth are similar in patients between eleven and fifteen years of age.
2. The patient measurements indicated that there is an increase in the resistance to tooth movement when the force applied to the crown reaches a magnitude of approximately 180 to 200 Gm.
3. The equations of equilibrium provide a sound basis upon which the theoretical resisting force may be calculated for two and three-dimensional models.

4. Both the formula and the concept of projected root surface area as suggested by Jarabak and Fizzell appear valid when used to determine the stress in the extreme apical fiber of the periodontal ligament of the geometric analogues studied.
5. The data recorded from the tests using the geometric models indicate that the force resisting tipping or translation (bodily movement) is approximately 78 percent due to compression and tensile stresses or three and one-half times as great as that due to shearing stress.
6. The center of tipping is located between centroid and the apex and is lowered as the point of force application is lowered on the crown of a tooth. It is closer to centroid if the magnitude of the force applied to the crown is increased.
7. The properties known as the modulus of elasticity (E) and modulus of rigidity (G) may play an important part in future studies attempting to quantitate the stresses developed in the periodontal ligament.

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GLOSSARY

ALVEOLUS: The bone housing the root portion of the teeth (alveolar bone).

ANALOGUE: That which is analogous to something. It refers to something similar in function but different in origin and structure.

ANALOGUE, Two-Dimensional: One in which the material being stressed is regarded as lying in a single plane of space and having two significant dimensions.

ANALOGUE, Three-Dimensional: One in which the material being stressed is recognized as lying in three planes of space and having three significant dimensions.

APICAL REGION: That area of the tooth root near the apical foramen.

BODY: A quantity of inert matter, the particles of which move little or not at all in relation to one another.

CENTER OF MASS: That point in a (three-dimensional) body about which the sum of the moments of all the individual masses constituting the body is zero.

CENTER OF ROTATION: A fixed point about which a body rotates.

CENTROID: A point around which the area is balanced. A line passing through this point is the axis around which the first moment of area is zero.

CENTROID OF EFFECTIVE ROOT AREA: The point around which the area is balanced. A line passing through this point perpendicular to the long axis of the projected root area, bisects the projected root area.

CERVICAL REGION: That area of the tooth root immediately apical to the cervical line.

COLLINEAR FORCE SYSTEM: A coplanar force system in which the lines of action lie on the same straight line.

COPLANAR FORCE SYSTEM: A force system in which the lines of action of the forces lie in the same plane.

COUPLE: A system composed of two forces of equal magnitude and opposite direction having lines of action which are parallel but which do not coincide.

DURATION OF FORCE: That period of time in which a force is active.

EFFECTIVE ROOT PRESSURE: The net motivating force on a tooth divided by the Effective Root Surface Area of that tooth.

EFFECTIVE ROOT SURFACE AREA: That portion of the total surface area of the root that is involved directly in resisting the movement of a tooth in a specified direction.

ELASTICITY, Young's Modulus of: A constant of the stretch coefficient or the relationship of stress to strain.

EQUILIBRIUM, The Conditions of:

a. the sum of the forces in an "x" direction must equal zero.
b. the sum of the forces in a "y" direction must equal zero.
c. the sum of the forces in a "z" direction must equal zero.
d. the sum of the moments about a given point must equal zero.

FORCE: Any action of one body against another which alters or tends to alter a body's state of rest or of uniform motion in a straight line.

FORCE, Deflection Characteristic: A graph relating the applied force and the resulting deflection of some member.

FORCE SYSTEM: A combination of two or more forces acting on a body.

FREE-BODY DIAGRAM: A diagram illustrating all of the external forces acting on a body.

FULCRUM: A fixed point against which a lever pushes. Some authors have used this term to mean center of rotation or center of tipping.

LABIAL-LINGUAL DEFLECTION: A deflection in a labial or a lingual direction.

LINE OF ACTION: A line of indefinite length of which the force vector is a segment.

MAGNITUDE OF FORCE: Measureable quality of amount of force.

MOMENT OF FORCE: The product of the magnitude of a force, and the perpendicular distance from the line of action of the force to a point which is the center of rotation induced by that force.

PARALLEL FORCE SYSTEM: A force system in which the lines of action are parallel.

POINT OF APPLICATION: The point on a body at which a force is applied. The point of application of a given force acting on a rigid body may be transferred to any other point on the line of action without altering the effect of the force.

PRESSURE: A force per unit area and therefore pressure may be measured in the same units as is unit stress (force per unit area).

PROJECTED ROOT AREA: The area of the projection of the root of a tooth that is made on a screen that is in a plane parallel to the long axis of the tooth when the rays of light are parallel. For pure translation in the distal direction, the projection that is considered is the one that occurs when the screen is in a buccal-lingual plane and the light source is in a mesial position directed distally.

RIGIDITY, Modulus of: A constant of the stretch coefficient or the relationship of stress to strain when considering shearing stress.

SHEAR: A stress resulting from forces being applied to a body in a manner which tends to cause adjacent segments to slide over each other.

STRAIN: Change in volume and/or shape of a body due to applied forces. The three simplest strains are: (a) longitudinal- change in length per unit length; (b) volume- change in volume per unit volume; (c) shear- angular deformation without change in volume.

STRESS: The system of forces in equilibrium producing or tending to produce strain in a body or part of a body. Some writers regard stress as the force applied to deform the body and others as the equal and opposite forces with which the body resists. In all cases the stress is measured as force per unit area.

STRESSES: Internal forces considered in relationship to the area in which they develop.

UNIT STRAIN: The deflection or deformation per unit of original gage length.

UNIT STRESS: The force per unit area expressed in pounds per square inch. Unit stress may be for compression, tension, or shearing stress.

VECTOR: An arrow that is drawn to represent a vector quantity.

APPENDIX I

CONSTRUCTION OF INTEROCCLUSAL CLUTCH

Maxillary and mandibular plaster study models were mounted on a Galetti articulator using a centric occlusion wax bite to orient them. A covering of autopolymerizing acrylic resin was placed over the occlusal and incisal surfaces of the mandibular teeth. The acrylic resin did not extend beyond the greatest convexity of the crowns of the teeth (no undercuts were engaged). A horizontal table of clear acrylic resin was attached to mandibular portion of the clutch just above the incisal edge of the anterior teeth.

The mandibular portion of the clutch with attached table was seated on the mandibular model. A horseshoe-shaped quantity of freshly mixed autopolymerizing acrylic resin was placed on the superior surface of the clutch. The articulator was closed, imprinting a negative of the occlusal surfaces of the maxillary teeth into the fresh acrylic (Figure 15). After the acrylic resin had polymerized, the clutch was removed from the model and the acrylic resin surrounding the mandibular right canine was removed to provide clearance for the cap-like acrylic crown.

APPENDIX II

TESTS OF DIFFERENT TYPES AND THICKNESSES OF MATERIAL

<u>Five Kinds of Polyurethane of Four Thicknesses Were Tested</u>

1.	1/8"	Polyurethane	foam	-1 lb.	density
2.	"	"	"	-2 lb.	"
3.	"	"	"	-3 lb.	"
4.	"	"	"	-4 lb.	"
5.	"	"	"	-5 lb.	"
6.	3/16"	"	"	-1 lb.	"
7.	"	"	"	-2 lb.	"
8.	"	"	"	-3 lb.	"
9.	"	"	"	-4 lb.	"
10.	"	"	"	-5 lb.	"
11.	3/32"	"	"	-1 lb.	"
12.	"	"	"	-2 lb.	"
13.	"	"	"	-3 lb.	"
14.	"	"	"	-4 lb.	"
15.	"	"	"	-5 lb.	"
16.	1/2"	"	"	-1 lb.	"
17.	"	"	"	-2 lb.	"
18.	"	"	"	-3 lb.	"
19.	"	"	"	-4 lb.	"
20.	"	"	"	-5 lb.	"

<u>Other Materials Tested</u>

21. 3/32" Durafoam
22. 1/8" "
23. 3/16" "
24. 1/8" Rubber Sponge
25. 3/16" " "
26. 3/32" " "
27. 1/8" Ensolite, type AH
28. 3/16" "
29. 1/8" Ethane
30. 1/8" Cellutite, soft
31. 1/8" " , medium
32. 1/8" Ensolite, type M

APPENDIX III

METHOD OF TESTING SYNTHETIC MATERIALS

A random sample was cut from each specimen to be tested. They were circular in shape and had an area of .5 sq. in. This sample was glued between two machined plastic hubs (Figure 16). Each hub had a .040 inch piece of Truchrome wire extending from it perpendicular to the surface to which the foam was glued. This rigid wire extension permitted the hubs to be attached to the adjustable pin vises of the testing machine (Figures 17 and 18). During the gluing procedure the hubs were placed in the testing machine and a force of 20 Gm. was applied during the eight hour period in which the cement was allowed to dry.

Each material was tested twice in compression, tension, and shear by two operators. The measurements of deflection were recorded to the nearest .01 mm. at 50 Gm. increments of force from 0 to 500 Gm. Each force was applied for two seconds before the reading of deflection was taken. A ten minute interval was allowed between the testing of each stress on each sample of material.

A correction factor was applied to each reading of deflection to allow for the compliance of the force gage. The corrected deflections were plotted against the applied forces after being averaged at each force increment. The linearity of the force deflection characteristics was then compared with that of the periodontal ligament of the mandibular canine tooth. The readings were analyzed using Fisher's Analysis of Variance to determine their significance and the magnitude of experimental error.

APPENDIX IV

MEASURING APPARATUS

The measuring apparatus was constructed with .75 inch plywood (Figure 19). From the front the apparatus resembled an open-faced wooden box with a high back wall. The base also extended beyond the depth of the box. Large triangular braces stabilized the back wall. Long wood screws (2.5 inches) and glue were used to assemble the apparatus.

The upper surface of the horizontal platform was designed to lie flush with the top of the plastic alveolus. A hole was cut in the platform to permit the assembled model to be lowered in an upright position onto the base of the apparatus.

Adjustable pulleys were attached at both ends of the horizontal platform. Each pulley was attached to a hinged arm permitting height adjustment. The steel brackets to which the arms were hinged were fastened to the apparatus with wood screws.

APPROVAL SHEET

The thesis submitted by Dr. James A. Evans has been read and approved by three members of the department of Oral Biology.

The final copies have been examined by the director of the thesis whose signature appears below verifying the fact that the necessary changes have been incorporated and that the thesis has been given final approval in reference to content, form, and mechanical accuracy.

May 24 - 1966

Date

Joseph R Jarabak

Advisor