2018

Listen, Share, Play: Lessons from Preschool for Problem Solving

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Lessons from Preschool for Problem Solving

by Peter Tingley

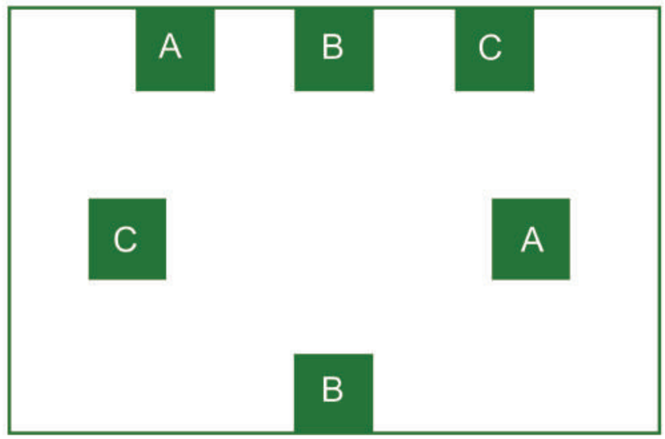
The main goal of our Math Teachers' Circle at Loyola University Chicago is to engage teachers in open-ended, interesting problem solving. In this article, I will talk about the problems I use to introduce what that means. I've used these a number of times with teachers, pre-service teachers, math majors, and even professors. But first I'm going to discuss a bit of philosophy.

I'm trying to engage teachers with good problems, to help them develop (and hopefully then teach) good problem-solving strategies and mindsets. But first, what is a good problem? For me it is a problem where when people first look at it, *they do not know what to do*. Solving it involves some exploration, which means doing things to see what will happen, not expecting them to lead directly to the answer. You'll see more what I mean when I introduce the problems.

Even more importantly though, what is a good problem-solving mindset? I've been led more and more to the language I hear from my 5-year-old. Her preschool has been doing a unit called I Can Problem Solve (ICPS). Well, in that context, it really means "Don't get in fights with the other 5-year-olds," but that is a kind of problem solving. Anyway, I'm talking about the following advice:

- ***Listen to the question.*** It has things to say, and they might be interesting. If you are always the one talking, you will miss out!
- ***Be willing to compromise with the problem.*** Maybe you won't solve it as stated. Maybe you should do an easier problem first. Maybe you'll have an idea for a similar problem that works out better, and you can learn from that. You'll have more fun if you can be flexible!
- ***Sometimes the problem wins, and that is OK!*** It isn't about who wins, its about how much fun you have together!
- ***You can keep playing even after the problem is solved!***

I usually talk about these things after first having people do the following problem, which I first learned about from Joshua Zucker:



Make paths connecting each pair of same-letter boxes so that none of the paths cross each other or leave the large box.

I ask them to do the problem individually, and allow only 3-5 minutes. My students once called this the "think outside the box but stay inside the box" question.

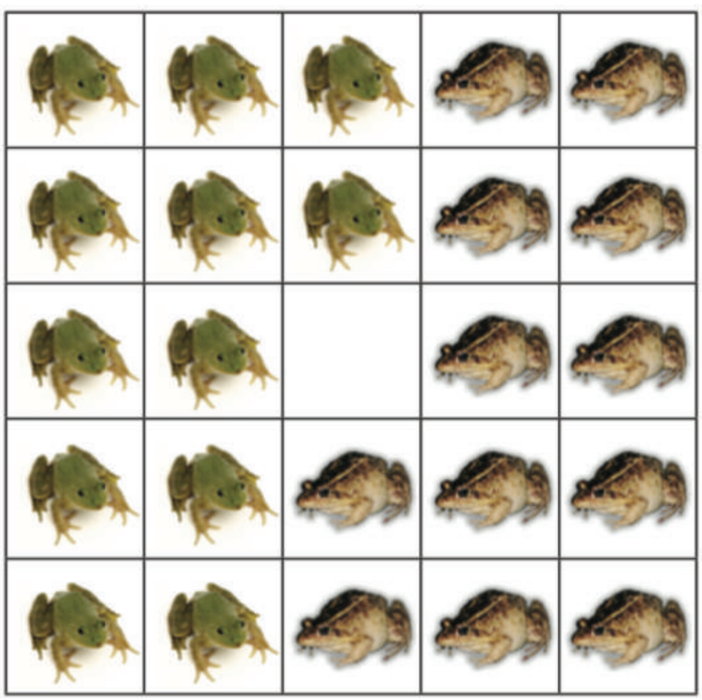
Tingley leads the Southwest Chicago MTC in a warm-up problem to start a conversation about strategies for problem solving.

Handprint photos by Tory Byrne at FreeImages.com.

I love this because none of the “formulaic” problem-solving techniques help at all. Draw a picture? Already done. Organize information? It already looks pretty organized.

But if you just start trying things and *listen to the problem* by thinking about what goes wrong, you will definitely solve it. If you *compromise* by simplifying somehow, say by first deleting the Cs, you will solve it (there are actually many interesting ways to simplify this question). In the end, if you’ve really understood things, you should be able to see what happens if, for example, I add a D box, or even an E and an F. That is, it is worthwhile to *keep playing,* even if you have a solution.

Then I do the frog and toad problem below. I got this from the summer 2013 MTCircular magazine, where (with slightly altered wording) it was the Problem Circle problem, also contributed by Joshua Zucker:



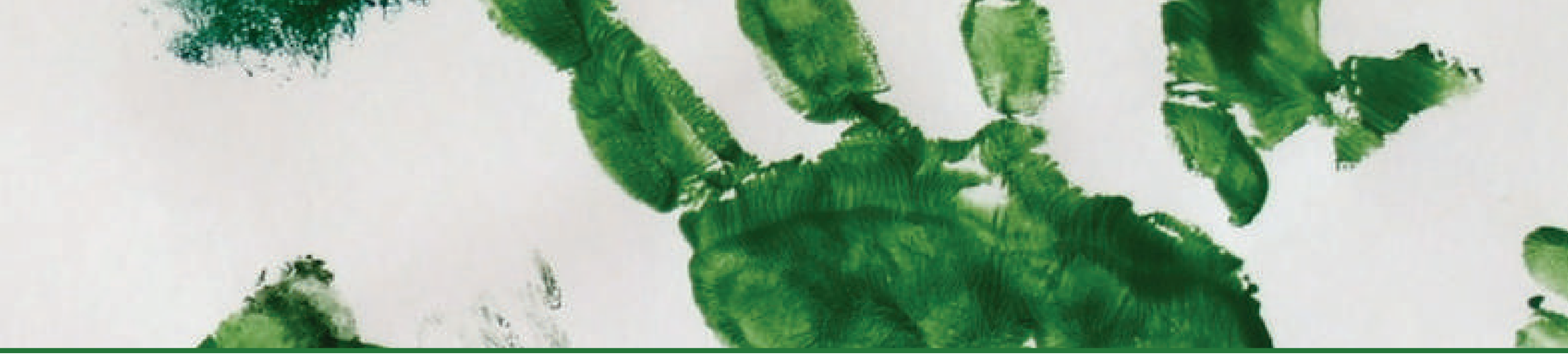
In this game, frogs can move right or down. Toads can move left or up. A frog or toad can slide into an adjacent empty space. They can also jump over exactly one of the other kind of animal to land in an empty space. They want to swap places, so that every place that now has a frog will have a toad, and vice versa. Can it be done? How?

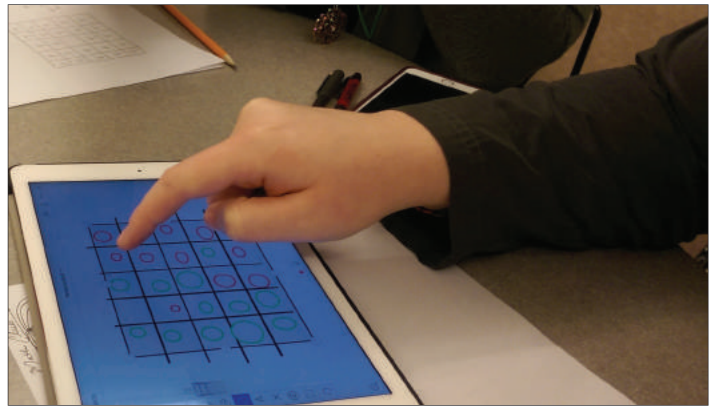
I have teachers do this in groups of 4, and plan on it taking an hour or so. Note that this is clearly not from the curriculum. That is a point I make: The solution in a sense doesn’t matter. It is experiencing finding the solution that is worthwhile. It isn’t immediately clear to anyone what to do (or even if it is possible), but you can start doing things. Very quickly, though, you see it is too complicated. You need to do a simpler problem! Most people choose to do a 3-by-3 version of the question, but this step takes longer then you might think.

From there, there are three quite distinct ways to solve the problem, each of which gives insight into problem solving:

Method 1: Try and fail to do the 3-by-3 case a bunch of times. Keep a record of the stuck positions. Notice that most of these have three frogs at the bottom or three toads at the top, which is bad (not quite all have this property). Conclude: Moving up or down is dangerous...which you should have known! The frogs and toads mostly need to go left-right, not up-down! Systematically go left-right as much as possible. Now you’ll solve it.

Method 2: Solve the 3-by-3 case by trial and error, then organize your solution so you can describe it nicely in words, at which point it should generalize. This can be problematic, because there are solutions that don’t generalize well. But that just gives a chance to talk about the value in looking for better solutions once a problem is solved. It also gives a way to understand what a better solution is: It is one that is easier to explain and generalize.





Method 3: Be more creative about generalizing the problem! For example, it makes sense for a 3-by-5 rectangle, or 1-by-3! A 1-by-3 rectangle is easy. A 1-by-5 rectangle is a bit harder, but you'll get it. And now you can notice, in the original 5-by-5 question, you can start by flipping the middle row using the 1-by-5 case! Then move one animal up or down and flip another row. Now you'll get it...

Method 3 is my favorite, and I push some people towards it a bit. But I mostly try to let people go their own course. Because actually my favorite thing is that there are so many nice solutions, and that you can have great discussions about them.

There are extensions for people who finish early (not many do, in my experience). You can ask how many moves it takes (I know an answer for my solution, but not a proof that all solutions will have the same number of steps, although I think they do). You can do different rectangles. You can even think about what to do if one of the lengths is even (so the initial hole has to be a bit off-center). I don't know the full answer to that either. So, there is plenty left to explore!

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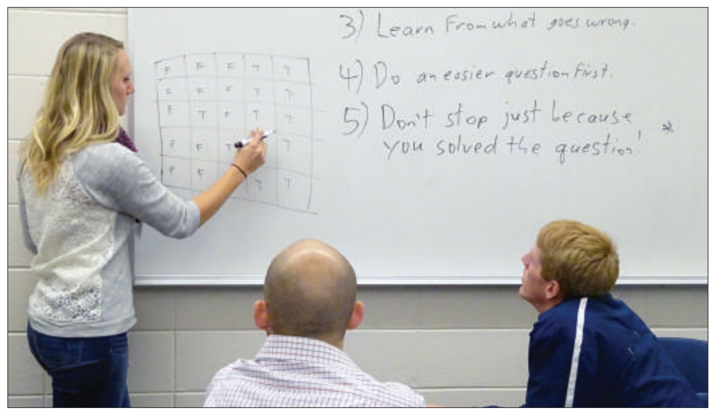

Southwest Chicago MTC members make good use of problem-solving strategies in the Frogs and Toads problem.