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## Conformal Gravity Holography in Four Dimensions

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We formulate four-dimensional conformal gravity with (anti-)de Sitter boundary conditions that are weaker than Starobinsky boundary conditions, allowing for an asymptotically subleading Rindler term concurrent with a recent model for gravity at large distances. We prove the consistency of the variational principle and derive the holographic response functions. One of them is the conformal gravity version of the Brown–York stress tensor, the other is a “partially massless response”. The on shell action and response functions are finite and do not require holographic renormalization. Finally, we discuss phenomenologically interesting examples, including the most general spherically symmetric solutions and rotating black hole solutions with partially massless hair.

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Conformal gravity (CG) in four dimensions is a recurrent topic in theoretical physics, as it provides a possible resolution to some of the problematic issues with Einstein gravity, the established theory of the gravitational interaction, though it usually introduces new ones.

For instance, like other higher-derivative theories, CG is renormalizable [1,2], but has ghosts [3], whereas Einstein gravity is ghost free, but 2-loop nonrenormalizable [4]. See [5–8] for important early work on CG. Later, CG was studied phenomenologically by Mannheim in an attempt to explain galactic rotation curves without dark matter [9–12] and emerges theoretically from twistor string theory [13] or as a counter term in the anti-de Sitter/conformal field theory (AdS/CFT) correspondence [14,15]. More recently, 't Hooft has studied CG in a quantum gravity context [16] and Maldacena has shown how Einstein gravity can emerge from CG upon imposing suitable boundary conditions that eliminate ghosts [17].

Physical theories in general require boundary conditions as part of their definition. In many cases, “natural” boundary conditions—the rapid falloff of all fields near a boundary or in an asymptotic region—are the appropriate choice, but this is not the case in gravitational theories, since the metric should be nonzero. A prime example is gravity in AdS, where the boundary conditions define the behavior of the dual field theory that lives on the conformal boundary of spacetime. Gravity in de Sitter (dS) requires similar boundary conditions; they were provided for four-dimensional Einstein gravity by Starobinsky [18]. (See also [19,20] for a more recent discussion of future boundary conditions and conserved charges in dS.) These boundary conditions played a crucial role in Maldacena’s reduction from CG to solutions of Einstein gravity [17].

It is, however, not clear that the Starobinsky boundary conditions are the most general or phenomenologically interesting ones for CG. Experience with three-dimensional

(3D) CG [21] suggests that a weaker set of boundary conditions should be possible also in four dimensions. Finding such boundary conditions is interesting for purely theoretical reasons and also phenomenologically. Indeed, the CG analogue of the Schwarzschild solution, the spherically symmetric Mannheim–Kazanas–Riegert (MKR) solution [9,22], does not obey the Starobinsky boundary conditions. A related motivation is to investigate whether it is true that CG provides an example of a theory that allows a nontrivial Rindler term, as suggested in the discussion of an effective model for gravity at large distances [23]. The difficulty does not lie in showing that the CG equations of motion (EOM) permit a Rindler term (they do), but in determining a set of boundary conditions consistent with such a term.

The main purpose of our Letter is to establish the consistency of a set of (A)dS boundary conditions for CG, weaker than the ones proposed by Starobinsky, that are compatible with the existence of an asymptotic Rindler term, the MKR solution, and other solutions with a condensate of partially massless gravitons.

Before starting, we review the most salient features of CG. A distinguishing property of CG is that the theory depends only on (Lorentz) angles but not on distances. This means that the theory is invariant under local Weyl rescalings of the metric,

$$g_{\mu\nu} \rightarrow \tilde{g}_{\mu\nu} = e^{2\omega} g_{\mu\nu}, \quad (1)$$

where the Weyl factor  $\omega$  is allowed to depend on the coordinates. The bulk action of CG,

$$I_{\text{CG}} = \alpha_{\text{CG}} \int d^4x \sqrt{|g|} g_{\alpha\mu} g^{\beta\nu} g^{\gamma\lambda} g^{\delta\tau} C^\alpha_{\beta\gamma\delta} C^\mu_{\nu\lambda\tau}, \quad (2)$$

is manifestly invariant under Weyl rescalings Eq. (1), since the Weyl tensor  $C^\alpha_{\beta\gamma\delta}$  is Weyl invariant, and the Weyl factor

coming from the square root of the determinant of the metric is precisely canceled by the Weyl factor coming from the metric terms. The dimensionless coupling constant  $\alpha_{\text{CG}}$  is the only free parameter in the CG action. In most of what follows, we set  $\alpha_{\text{CG}} = 1$  to reduce clutter. The EOM of CG are fourth order and require the vanishing of the Bach tensor,

$$\left(\nabla^\delta \nabla_\gamma + \frac{1}{2} R^\delta{}_\gamma\right) C^\gamma{}_{\alpha\delta\beta} = 0. \quad (3)$$

There are two especially simple classes of solutions to the EOM: conformally flat metrics ( $C^\gamma{}_{\alpha\delta\beta} = 0$ ) and Einstein metrics ( $R_{\alpha\beta} \propto g_{\alpha\beta}$ ) both have vanishing Bach tensor. Therefore, solutions of Einstein gravity are a subset of the broader class of solutions of CG.

The most general spherically symmetric solution of CG is given by the line element [22]

$$ds^2 = -k(r)dt^2 + \frac{dr^2}{k(r)} + r^2 d\Omega_{S^2}^2, \quad (4)$$

where  $d\Omega_{S^2}^2$  is the line element of the round 2 sphere and

$$k(r) = \sqrt{1 - 12aM} - \frac{2M}{r} - \Lambda r^2 + 2ar. \quad (5)$$

For  $a = 0$ , the solution reduces to Schwarzschild-(A)dS. It is noteworthy that for  $aM \ll 1$ , the solution Eqs. (4) and (5) corresponds to the one presented in [23], derived from an effective model for gravity at large distances. Phenomenologically relevant numbers (in Planck units) are  $\Lambda \approx 10^{-123}$ ,  $a \approx 10^{-61}$ ,  $M \approx 10^{38} M_\odot$ , where  $M_\odot = 1$  for the Sun, so that indeed  $aM \approx 10^{-23} M_\odot \ll 1$  for all black holes or galaxies in our Universe.

We propose now boundary conditions that admit the MKR solution Eqs. (4) and (5). This requires the introduction of a length scale  $\ell$ , which in Einstein gravity would be related to the cosmological constant as  $\Lambda = 3\sigma/\ell^2$  (with  $\sigma = -1$  for AdS and  $\sigma = +1$  for dS). Then our asymptotic ( $0 < \rho \ll \ell$ ) line element reads

$$ds^2 = \frac{\ell^2}{\rho^2} (-\sigma d\rho^2 + \gamma_{ij} dx^i dx^j). \quad (6)$$

For simplicity, we partially fix the gauge and use Gaussian coordinates. Close to the conformal boundary at  $\rho = 0$ , the 3D metric has the following asymptotic expansion:

$$\gamma_{ij} = \gamma_{ij}^{(0)} + \frac{\rho}{\ell} \gamma_{ij}^{(1)} + \frac{\rho^2}{\ell^2} \gamma_{ij}^{(2)} + \frac{\rho^3}{\ell^3} \gamma_{ij}^{(3)} + \dots \quad (7)$$

The boundary metric  $\gamma^{(0)}$  is required to be invertible. All the coefficient matrices  $\gamma^{(n)}$  are allowed to depend on the boundary coordinates  $x^i$ , but not on the ‘‘holographic’’ coordinate  $\rho$ .

As part of the specification of our boundary conditions, we fix the leading and first-order terms in Eq. (7) on  $\partial\mathcal{M}$  up to a local Weyl rescaling

$$\delta\gamma_{ij}^{(0)}|_{\partial\mathcal{M}} = 2\lambda\gamma_{ij}^{(0)}, \quad \delta\gamma_{ij}^{(1)}|_{\partial\mathcal{M}} = \lambda\gamma_{ij}^{(1)}, \quad (8)$$

where  $\lambda$  is a regular function on  $\partial\mathcal{M}$ , while the subleading terms at second and higher order are allowed to vary freely,  $\delta\gamma^{(n)}|_{\partial\mathcal{M}} \neq 0$  for  $n \geq 2$ . An essential difference to the Starobinsky boundary conditions is the presence of a subleading term  $\gamma_{ij}^{(1)}$ . This term is absent in [18] because the EOM for Einstein gravity force it to vanish. By contrast, the EOM [Eq. (3)] do not give any conditions on  $\gamma_{ij}^{(1)}$ , analogous to 3D CG [21].

To check the consistency of the boundary conditions Eqs. (6)–(8), we consider first the on shell action and then the variational principle. On general grounds, one might expect the bulk action Eq. (2) to be supplemented by two kinds of boundary terms: a ‘‘Gibbons–Hawking–York’’ boundary term [24,25] that produces the desired boundary value problem (for instance, a Dirichlet boundary value problem), and holographic counterterms [26–31] that guarantee that the action is stationary for all variations that preserve our boundary conditions.

We claim that no such boundary terms are required for CG, so that the full action is just the bulk action Eq. (2)

$$\Gamma_{\text{CG}} = I_{\text{CG}} = \int_{\mathcal{M}} d^4x \sqrt{|g|} C^\lambda{}_{\mu\nu} C^\lambda{}_{\mu\sigma\nu}. \quad (9)$$

The first piece of evidence that no counterterms might be needed comes from the calculation of the on-shell action. It is straightforward to show that the on shell action for any metric behaving like Eqs. (6) and (7), evaluated on a compact region  $\rho_c \leq \rho$ , remains finite as  $\rho_c \rightarrow 0$ . A related piece of evidence was provided in [32], where it was shown that the free energy derived from the on shell action Eq. (9) is consistent with the Arnowitt–Deser–Misner mass and Wald’s definition of the entropy [33]. The fact that the on shell action yields the correct free energy suggests that any boundary terms added to the action Eq. (9) should vanish on shell. The simplest possibility is that these terms are identically zero [34].

A more stringent check of our claim is obtained by proving the consistency of the variational principle and the finiteness of the holographic response functions. To this end, we first rewrite the Weyl-squared action Eq. (9) as

$$\begin{aligned} \Gamma_{\text{CG}} = & \int_{\mathcal{M}} d^4x \sqrt{|g|} \left( 2R^{\mu\nu} R_{\mu\nu} - \frac{2}{3} R^2 \right) + 32\pi^2 \chi(\mathcal{M}) \\ & + \int_{\partial\mathcal{M}} d^3x \sqrt{|\gamma|} \left( -8\sigma \mathcal{G}^{ij} K_{ij} + \frac{4}{3} K^3 - 4K K^{ij} K_{ij} \right. \\ & \left. + \frac{8}{3} K^{ij} K_j{}^k K_{ki} \right). \end{aligned} \quad (10)$$

The action has been separated into a topological part—the Euler characteristic  $\chi(\mathcal{M})$ —and a Ricci-squared action, with the boundary terms in the last two lines canceling similar terms that appear in the Euler characteristic for spacetimes with (conformal) boundary; see [35]. Here and in all subsequent expressions, calligraphic letters indicate quantities intrinsic to the 3D surface  $\partial\mathcal{M}$ . Thus,  $\mathcal{G}^{ij}$  is the 3D Einstein tensor for the metric  $\gamma_{ij}$ . The extrinsic curvature is defined as  $K_{ij} = -(\sigma/2)\mathcal{L}_n\gamma_{ij}$ , where  $\mathcal{L}_n$  is

$$\begin{aligned} \pi^{ij} = & \frac{\sigma}{4}(\gamma^{ij}K^{kl} - \gamma^{kl}K^{ij})f_{kl} + \frac{\sigma}{4}f^\rho{}_\rho(\gamma^{ij}K - K^{ij}) - \frac{1}{2}\gamma^{ij}\mathcal{D}_k(n_\rho f^{k\rho}) + \frac{1}{2}\mathcal{D}^i(n_\rho f^{\rho j}) - \frac{1}{4}(\gamma^{ik}\gamma^{jl} - \gamma^{ij}\gamma^{kl})\mathcal{L}_n f_{kl} \\ & + \sigma[2K\mathcal{R}^{ij} - 4K^{ik}\mathcal{R}_k{}^j + 2\gamma^{ij}K_{kl}\mathcal{R}^{kl} - \gamma^{ij}K\mathcal{R} + 2\mathcal{D}^2K^{ij} - 4\mathcal{D}^i\mathcal{D}_kK^{kj} + 2\mathcal{D}^i\mathcal{D}^jK + 2\gamma^{ij}(\mathcal{D}_k\mathcal{D}_lK^{kl} - \mathcal{D}^k\mathcal{D}_kK)] \\ & + \frac{2}{3}\gamma^{ij}K_m{}^kK^{lm}K_{kl} - 4K^{ik}K^{jl}K_{kl} + 2K^{ij}K^{kl}K_{kl} + \frac{1}{3}\gamma^{ij}K^3 - 2K^{ij}K^2 - \gamma^{ij}KK^{kl}K_{kl} + 4KK^i{}_kK^{jk} + i \leftrightarrow j. \end{aligned} \quad (12)$$

The tensor  $f_{\mu\nu}$ , which appears in a convenient auxiliary field formulation of the action, is proportional to the four-dimensional Schouten tensor,  $f_{\mu\nu} = -4(R_{\mu\nu} - \frac{1}{6}g_{\mu\nu}R)$ . The momentum  $\Pi^{ij}$  reads

$$\begin{aligned} \Pi^{ij} = & -8\sigma\mathcal{G}^{ij} - \sigma(f^{ij} - \gamma^{ij}f^k{}_k) \\ & + 4\gamma^{ij}(K^2 - K^{kl}K_{kl}) - 8KK^{ij} + 8K^i{}_kK^{kj}. \end{aligned} \quad (13)$$

It is noteworthy that we allow the boundary metric and the extrinsic curvature to vary independently in Eq. (11).

Let us check now the variational principle. Evaluating Eq. (11) on a compact region  $\rho_c \leq \rho$ , applying the EOM, and making use of the asymptotic expansion Eq. (7) with Eqs. (12) and (13) yields

$$\delta\Gamma_{\text{CG}}|_{\text{EOM}} = \int_{\partial\mathcal{M}} d^3x \sqrt{|\gamma^{(0)}|} (\tau^{ij}\delta\gamma_{ij}^{(0)} + P^{ij}\delta\gamma_{ij}^{(1)}). \quad (14)$$

The tensors  $\tau^{ij}$  and  $P^{ij}$  are finite as  $\rho_c \rightarrow 0$ . As we will show below, they satisfy the trace conditions ( $\psi_{ij}^{(1)} := \gamma_{ij}^{(1)} - \frac{1}{3}\gamma_{ij}^{(0)}\gamma_k{}^k$ )

$$\gamma_{ij}^{(0)}\tau^{ij} + \frac{1}{2}\psi_{ij}^{(1)}P^{ij} = 0, \quad \gamma_{ij}^{(0)}P^{ij} = 0, \quad (15)$$

so that the first variation of the action vanishes on shell when the boundary conditions Eq. (8) are satisfied. Therefore, the action Eq. (9) and our proposed boundary conditions constitute a well-defined variational principle.

The quantities  $\tau^{ij}$  and  $P^{ij}$  appearing in Eq. (14) are the holographic response functions conjugate to the sources  $\gamma_{ij}^{(0)}$  and  $\gamma_{ij}^{(1)}$ , respectively. We evaluate now the first of these functions, which is proportional to the usual Brown–York stress tensor. It is useful to introduce the electric  $E_{ij}$  and magnetic  $B_{ijk}$  parts of the Weyl tensor.

$$E_{ij} = n_\alpha n^\beta C^\alpha{}_{i\beta j}, \quad B_{ijk} = n_\alpha C^\alpha{}_{ijk}. \quad (16)$$

the Lie derivative along the outward- or future-pointing unit vector  $n^\mu$  normal to  $\partial\mathcal{M}$ .

In this formulation, the first variation of the action is given by

$$\delta\Gamma_{\text{CG}} = \text{EOM} + \int_{\partial\mathcal{M}} d^3x \sqrt{|\gamma|} (\pi^{ij}\delta\gamma_{ij} + \Pi^{ij}\delta K_{ij}). \quad (11)$$

The momentum  $\pi^{ij}$  reads

For metrics that satisfy the boundary conditions Eqs. (6) and (7), their asymptotic expansions are given by

$$E_{ij} = E_{ij}^{(2)} + \frac{\rho}{\ell} E_{ij}^{(3)} + \dots, \quad (17)$$

$$B_{ijk} = \frac{\ell}{\rho} B_{ijk}^{(1)} + B_{ijk}^{(2)} + \dots, \quad (18)$$

with

$$B_{ijk}^{(1)} = \frac{1}{2\ell} \left( \mathcal{D}_j \psi_{ik}^{(1)} - \frac{1}{2} \gamma_{ij}^{(0)} \mathcal{D}^l \psi_{kl}^{(1)} \right) - j \leftrightarrow k, \quad (19)$$

$$E_{ij}^{(2)} = -\frac{1}{2\ell^2} \psi_{ij}^{(2)} + \frac{\sigma}{2} \left( \mathcal{R}_{ij}^{(0)} - \frac{1}{3} \gamma_{ij}^{(0)} \mathcal{R}^{(0)} \right) + \frac{1}{8\ell^2} \gamma^{(1)} \psi_{ij}^{(1)}, \quad (20)$$

$$\begin{aligned} E_{ij}^{(3)} = & -\frac{3}{4\ell^2} \psi_{ij}^{(3)} - \frac{1}{12\ell^2} \gamma_{ij}^{(0)} \psi_{kl}^{(1)} \psi_{kl}^{(2)} - \frac{1}{16\ell^2} \psi_{ij}^{(1)} \psi_{kl}^{(1)} \psi_{kl}^{(1)} \\ & - \frac{\sigma}{12} \left( \mathcal{R}^{(0)} \psi_{ij}^{(1)} - \gamma_{ij}^{(0)} \mathcal{R}_{kl}^{(0)} \psi_{kl}^{(1)} + \gamma_{ij}^{(0)} \mathcal{D}_l \mathcal{D}_k \psi_{kl}^{(1)} \right) \\ & + \frac{3}{2} \mathcal{D}_k \mathcal{D}^k \psi_{ij}^{(1)} - 3 \mathcal{D}_k \mathcal{D}_i \psi_j^{(1)k} \Big) + \frac{1}{24\ell^2} E_{ij}^\gamma + i \leftrightarrow j, \end{aligned} \quad (21)$$

$$\begin{aligned} E_{ij}^\gamma := & \gamma_{(1)} \left( 3\psi_{ij}^{(2)} + \frac{1}{2} \gamma_{ij}^{(0)} \psi_{kl}^{(1)} \psi_{kl}^{(1)} - \gamma_{(1)} \psi_{ij}^{(1)} \right) + 5\gamma_{(2)} \psi_{ij}^{(1)} \\ & - \sigma \ell^2 \left( \mathcal{D}_j \mathcal{D}_i \gamma_{(1)} - \frac{1}{3} \gamma_{ij}^{(0)} \mathcal{D}^k \mathcal{D}_k \gamma_{(1)} \right). \end{aligned} \quad (22)$$

In these expressions, the coefficient matrices have been split into trace and trace-free parts as  $\gamma_{ij}^{(n)} = \frac{1}{3}\gamma_{ij}^{(0)}\gamma^{(n)} + \psi_{ij}^{(n)}$ . Then the (finite) result for  $\tau_{ij}$  (as

$\rho_c \rightarrow 0$ ) is

$$\begin{aligned} \tau_{ij} = & \sigma \left[ \frac{2}{\ell} \left( E_{ij}^{(3)} + \frac{1}{3} E_{ij}^{(2)} \gamma^{(1)} \right) - \frac{4}{\ell} E_{ik}^{(2)} \psi_j^{(1)k} \right. \\ & + \frac{1}{\ell} \gamma_{ij}^{(0)} E_{kl}^{(2)} \psi_{(1)}^{kl} + \frac{1}{2\ell^3} \psi_{ij}^{(1)} \psi_{kl}^{(1)} \psi_{(1)}^{kl} \\ & \left. - \frac{1}{\ell^3} \psi_{kl}^{(1)} \left( \psi_i^{(1)k} \psi_j^{(1)l} - \frac{1}{3} \gamma_{ij}^{(0)} \psi_m^{(1)k} \psi_{(1)}^{lm} \right) \right] \\ & - 4\mathcal{D}^k B_{ijk}^{(1)} + i \leftrightarrow j. \end{aligned} \quad (23)$$

We call the function  $P^{ij}$  the ‘‘partially massless response’’ (PMR). This name is justified, since it is sourced by the term  $\gamma_{ij}^{(1)}$  in the metric. The latter, when plugged into the linearized CG EOM around an (A)dS background, exhibits partial masslessness in the sense of Deser, Nepomechie, and Waldron [36,37]. This is expected from the corresponding behavior in 3D [21] and also on general grounds, since the Weyl invariance Eq. (1) is nothing but the nonlinear completion of the gauge enhancement at the linearized level due to partial masslessness; see, for instance, the recent discussion in [38,39]. (Note that such a nonperturbative completion of partial masslessness is not generic to higher derivative theories [40].) Calculating the PMR yields

$$P_{ij} = -\frac{4\sigma}{\ell} E_{ij}^{(2)}, \quad (24)$$

as  $\rho_c \rightarrow 0$ . Like  $\tau_{ij}$ , the PMR is finite and does not require holographic renormalization.

Given the expressions Eqs. (23) and (24) for the response functions, the trace conditions Eq. (15) follow from tracelessness of the electric and magnetic parts of the Weyl tensor, which give identities  $\gamma_{(0)}^{ij} E_{ij}^{(3)} = \psi_{(1)}^{ij} E_{ij}^{(2)}$  and  $\gamma_{(0)}^{ij} E_{ij}^{(2)} = \gamma_{(0)}^{ij} B_{ijk}^{(1)} = 0$ . Note that for Starobinsky boundary conditions the Brown-York stress tensor is traceless, but in general, only the PMR is traceless.

To summarize, we have shown the consistency of the boundary conditions Eqs. (6)–(8) by demonstrating that the variational principle is well-defined for the action Eq. (9) and by proving finiteness of all 0- and 1-point functions. This is our main result.

Conserved charges may be computed from the currents

$$J^i = (2\tau^i_j + 2P^{ik} \gamma_{kj}^{(1)}) \xi^j, \quad (25)$$

where  $\xi^j$  is a boundary diffeomorphism associated with an asymptotic symmetry of the theory. (For now, we consider only the AdS case  $\sigma = -1$ , so that the conformal boundary  $\partial\mathcal{M}$  is a timelike surface.) Given a constant-time surface  $\mathcal{C}$  in  $\partial\mathcal{M}$ , the charge is

$$Q[\xi] = \int_{\mathcal{C}} d^2x \sqrt{h} u_i J^i, \quad (26)$$

where  $h$  is the metric on  $\mathcal{C}$  and  $u^i$  is the future-pointing unit vector normal to  $\mathcal{C}$ . The combination of  $\tau_{ij}$ ,  $P_{ij}$ , and  $\gamma_{ij}^{(1)}$  appearing in  $J^i$  is precisely the modified stress tensor of Hollands *et al.* [41]. Thus, the charges Eq. (26) are expected to generate the asymptotic symmetries. The covariant divergence of the modified stress tensor satisfies

$$\mathcal{D}_i (2\tau^{ij} + 2P^i_k \gamma^{(1)kj}) = P^{ik} \mathcal{D}_j \gamma_{ik}^{(1)}. \quad (27)$$

This ensures that the difference in charges computed on surfaces  $\mathcal{C}_1, \mathcal{C}_2$  that bound a region  $\mathcal{V} \subset \partial\mathcal{M}$  is given by

$$\Delta Q[\xi] = \int_{\mathcal{V}} d^3x \sqrt{|\gamma^{(0)}|} (\tau^{ij} \mathcal{L}_{\xi} \gamma_{ij}^{(0)} + P^{ij} \mathcal{L}_{\xi} \gamma_{ij}^{(1)}), \quad (28)$$

which vanishes for asymptotic symmetries.

We apply now our formulas to three pertinent examples. As a first special case, consider solutions that obey Starobinsky boundary conditions,  $\gamma_{ij}^{(1)} = 0$ . This includes the asymptotically (A)dS solutions of Einstein gravity with a cosmological constant. Then the EOM imply  $E_{ij}^{(2)} = 0$ , so the PMR vanishes and the Brown–York stress tensor simplifies to

$$\tau_{ij} = \frac{4\sigma}{\ell} E_{ij}^{(3)}. \quad (29)$$

This recovers the traceless and conserved stress tensor of Einstein gravity [30], in agreement with Maldacena’s analysis [17] and with earlier work by Deser and Tekin [42].

A more interesting example is provided by the MKR solution Eqs. (4) and (5). Setting  $\sigma = -1$  for concreteness, and defining the traceless matrix  $p^i_j = \text{diag}(1, -\frac{1}{2}, -\frac{1}{2})^i_j$  and constants  $a_M = (1 - \sqrt{1 - 12aM})/6$  and  $m = M/\ell^2$  yields  $\tau^i_j = -8(m/\ell^2) p^i_j + 8(aa_M/\ell^2) \text{diag}(1, -1, -1)^i_j$  and  $P^i_j = 8(a_M/\ell^2) p^i_j$ . For vanishing Rindler acceleration,  $a = a_M = 0$ , the previous Einstein case is recovered. For nonvanishing Rindler acceleration,  $a \neq 0$ , the PMR is linear and the trace of the Brown-York stress tensor quadratic in the Rindler parameter  $a$  when  $aM \ll 1$ . Thus, the Rindler parameter in the MKR solution can be interpreted as coming from a partially massless graviton condensate. The conserved charge associated with the Killing vector  $\partial_t$  may be computed using Eq. (26). If we normalize the action such that  $\alpha_{\text{CG}} = (1/64\pi)$ , we obtain  $Q[\partial_t] = m - aa_M$ . The entropy, obtained using Wald’s approach or from the on shell action, is  $S = A_h/(4\ell^2)$ , where  $A_h = 4\pi r_h^2$  is the area of the horizon  $k(r_h) = 0$ . Remarkably, the entropy obeys an area law despite the fact that CG is a higher-derivative theory.

As a third example, we consider rotating black hole solutions in AdS with Rindler hair parametrized by a Rindler acceleration  $\mu$  and rotation parameter  $\tilde{a}$ , but with a vanishing mass parameter; see Eq. (7) of [43]. Interestingly, we find that the absence of a mass parameter leads to a vanishing PMR,  $P_{ij} = 0$ . This shows that a nonzero Rindler term in the asymptotic expansion Eq. (7),  $\gamma_{ij}^{(1)} \neq 0$ , is necessary but not sufficient for a nonzero PMR. Evaluation of the Brown–York stress tensor leads to a conserved energy,  $E = -\tilde{a}^2\mu/[\ell^2(1 - \tilde{a}^2/\ell^2)^2]$ , and conserved angular momentum,  $J = E\ell^2/\tilde{a}$ , both linear in the Rindler parameter  $\mu$ .

Finally, it is possible to make a Legendre transformation of the action Eq. (9) that exchanges the role of the PMR and its source, namely by adding a Weyl invariant boundary term

$$\tilde{\Gamma}_{\text{CG}} = \Gamma_{\text{CG}} + 8 \int_{\partial\mathcal{M}} d^3x \sqrt{|\gamma|} K^{ij} E_{ij}. \quad (30)$$

This action is also finite on shell. Its first variation yields

$$\delta\tilde{\Gamma}_{\text{CG}} = \int_{\partial\mathcal{M}} d^3x \sqrt{|\gamma|} (\tilde{\tau}^{ij} \delta\gamma_{ij}^{(0)} + \tilde{P}^{ij} \delta E_{ij}^{(2)}), \quad (31)$$

with finite response functions.

$$\tilde{\tau}_{ij} = \tau_{ij} + \frac{2\sigma}{\ell} E_{(2)}^{kl} \psi_{kl}^{(1)} \gamma_{ij}^{(0)} + \frac{8\sigma}{3\ell} E_{ij}^{(2)} \gamma^{(1)} - \frac{4\sigma}{\ell} (E_{ik}^{(2)} \psi_j^{(1)k} + E_{jk}^{(2)} \psi_i^{(1)k}), \quad (32)$$

$$\tilde{P}_{ij} = \frac{4\sigma}{\ell} \gamma_{ij}^{(1)}. \quad (33)$$

The Brown–York stress tensor has zero trace,  $\tilde{\tau}^i_i = 0$ .

To summarize, the results of this Letter provide the basis for CG holography in four dimensions and show the viability of the MKR solution and other solutions with an asymptotic Rindler term. Possible next steps are the determination of the asymptotic symmetry group, calculation of higher  $n$ -point functions, and applications of our results to additional solutions of CG.

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