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Revisiting U.S. Stock Market Returns: Individual Retirement Accounts

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REVISITING U.S. STOCK MARKET RETURNS:

INDIVIDUAL RETIREMENT ACCOUNTS

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Returns of Individual Retirement Accounts

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ABSTRACT

Numerous studies have estimated U.S. stock market returns measured by various indexes such as the S&P 500 Index over certain periods. The purpose of this paper is twofold: first we calculate, under certain scenarios, the final total accumulation of a representative individual who invests a certain amount of funds per month during a long investment horizon of say 30 or 40 years. Second, we evaluate the performance of such an investment plan of defined monthly contributions. This evaluation is based on a benefit target and working backwards we compute the necessary monthly contributions. In our calculations we use actual monthly returns of the S&P 500 Index instead of averages obtained from a large sample. We calculate that accumulations of gradual investments over 30 or 40 years are skewed to the right and we also compute the probability that a given percentage of contributions will be sufficient to finance certain retirement benefits.

INTRODUCTION

Extensive academic work reported in Campbell, Lo and MacKinlay (1997), in investment books such as Siegel (2002) and data sources such as Ibbotson Associates (2003) document certain stylized facts about stock returns. Among them, the following ones are often cited:

First, annualized nominal total returns of a well-diversified portfolio in the long run are significantly higher for stocks than either bonds or cash. By total returns we mean both capital gains and dividends. Even when dividends are excluded, returns of a well-diversified portfolio over a long-term investment horizon are significantly higher for stocks than either bonds or cash.

Second, annualized real total returns of a well-diversified portfolio in the long run are also significantly higher for stocks than either bonds or cash. By long run we usually mean a period of at least 30 years. Another way to express the same idea is to say that investing in a well-diversified portfolio of stocks is a good hedge against inflation over a long-term investment horizon. However, for shorter periods such as 10 years, stocks need not outperform inflation, as was the case during the 1970s.

Third, these nominal and real total returns from a well-diversified portfolio of stocks are quite stable over major sub-periods. For example, Siegel (2002, p.13) reports annual total nominal and total real stock market returns for 1802-2001 and also for major sub-periods such as 1802-1870, 1871-1925 and 1926-2001 to illustrate that the average sample returns of these three sub-periods do not vary much from the average of the population. The stability, however, of these results depends on very large sample of over 50 years. For shorter periods of 30 or 40 years sample averages are less stable that is they fluctuate more.

Fourth, stocks are riskier than either bonds or cash in the short-run. As the investment horizon increases, equity risk decreases. Both academic researchers and investment advisors have carefully defined various measures of risk and size of investment horizon and independent of the specific concept of risk for holding periods up to ten years, stocks are riskier than either bonds or cash.

Fifth, annual stock returns (nominal, real, total) appear to be normally distributed as the sample size increases. Exact normality cannot be established statistically because annual returns of a stock index such as the S&P 500 exhibit fat tails. This near normality of returns over long period samples and the log-normality behavior of asset prices play a very important role in modern finance and financial engineering and empirical evidence both for and against the log-normality of asset prices is presented in Campbell, Lo and MacKinlay (1997).

Sixth, there is evidence that stock returns are mean reverting, i.e. high returns are followed on average by lower ones and vice- versa. Poterba and Summers (1988) have initially presented such evidence and a large literature has emerged since then studying this issue. There is also evidence that stock returns may be predictable over the long run. This idea of predictability has been vigorously debated in academic journals. Barberis (2000), Viceira and Campbell (2002), Constantinides (2002) and Siegel (2002) offer detailed discussion of the idea of predictability of returns, citing earlier studies by Campbell and Shiller (1988a, 1988b), Fama and French (1988), Shiller (1984) and others. A consensus is being formed that such predictability is not inconsistent with the theory of market efficiency. However, no rules have been discovered to allow investors to time the market, i.e. to exit before declines and reenter prior to major advances.

Seventh, in conclusion, a well-diversified portfolio of stocks should constitute the largest proportion of all portfolios with long horizons. This conclusion is subject to numerous qualifications within the framework of modern portfolio theory as elaborated in Viceira and Campbell (2002).

Many more stylized facts have been discovered concerning large, medium and small company stocks as well as value versus growth stocks. Furthermore, the importance of global investing has been emphasized as a tool to enhance diversification.

PURPOSE OF THIS STUDY

The purpose of this paper is twofold: first we calculate, under certain scenarios, the final total accumulation of a representative individual who invests a certain amount of funds per month during a long investment horizon of say 30 or

40 years. Second, we evaluate the performance of such an investment plan of defined monthly contributions. This evaluation is based on a benefit target and working backwards we compute the necessary monthly contributions.. The stylized facts reported in the introduction are not directly relevant to an individual investor who makes monthly contributions for an investment horizon of 30 to 40 years.

The data in this paper are those described and reported in Ibbotson Associates (2003). In particular we use data reporting the growth of $1 invested in large company stocks as measured by the S&P 500 Index, long-term government bonds and Treasury Bills from December 31, 1925 to December 31, 2002. Ibbotson Associates (2003) offer a detailed description of these investment classes as well as data about inflation measured by the Consumer Price Index for All Urban Consumers, not seasonally adjusted and anchored on December 31, 1925, that is, $1 on December 31, 1925 had the same purchasing power as $10.09 on December 31, 2002.

When we say that $1 is invested in large company stocks, Ibbotson Associates explain that this is represented by the total return (capital appreciation and reinvestment of dividends) of the S&P 500 index with several modifications that they describe in detail. With such modifications explained, Ibbotson Associates provide monthly returns for the various wealth indices. In all our calculations, Tables and Figures in this paper a monthly return is computed as the return between the value of the index at the last trading day of the month and its value the last trading day of the previous month. For example, the January 1926 return is the return between December 31, 1925 and January 31, 1926. If there was no trading on either date, it is understood that the return is calculated from the last day of the prior month to the last trading day of the current month.

Ibbotson Associates report the phenomenal growth of $1 invested in large company stocks as measured by the S&P 500 Index. An initial $1 invested in the S&P 500 Index on December 31, 1925 would have grown to $1,775.34 by the end of 2002. This is the total nominal return that translates into an annual nominal total growth rate of 10.2%. During the same period a $1 investment in government bonds would have grown to only $59.70. In this paper we argue that the evidence of this phenomenal growth of $1 invested in stocks over a long investment horizon is accurate only for investors who have all their wealth available to them at an early age and can enjoy the remarkable benefits of compounding over a very long investment horizon beginning in 1926 and ending in 2002, that is, 77 years. Obviously, such long horizons are not relevant to most individuals who have shorter investment horizons of 30 to 40 years. Also, most individual investors build their portfolios over their investment horizons by making periodic contributions, as is discussed next, because they do not have large sums to invest initially.

DEFINED CONTRIBUTION PRIVATE INDIVIDUAL ACCOUNTS

Realistically, the overwhelming majority of investors begin around the age of 25 years old, investing a small percentage of their monthly income for retirement purposes, often matched by their employers and continue to do so monthly over their working lifetime of 30 or 40 years. The initial contributions have a very long period to compound but subsequent contributions have shorter periods of compounding. To complicate matters, the initial monthly investments may be small but as wages and salaries increase because of productivity gains and adjustments for inflation, contributions towards the last third or fourth of the investment horizon may be larger than earlier ones but are not invested for a sufficiently long period to benefit from the long-term acceleration of compounding.

Under this rather realistic scenario of actual investing, how can one interpret the phenomenal growth of $1 to $1,775.34? Is it reasonable to take the approximate 10% average annual total growth of large stocks during the period 1926-2002 and apply it to a stream of monthly contributions over an investment horizon of 30 or 40 years?

The answer is no. Let us explain. When we calculate the accumulation of $1 invested over a 30- or 40-year investment horizon growing at a given term structure of monthly returns or growing by the average of the returns over the same horizon, we obtain the same final amount. For example, the growth of $1 to

$1,775.34 is the result of letting the initial investment grow or decline at the actual annual rate of growth of the S&P 500 Index over a period of 77 years. The average rate of growth over the same period is calculated by computing the average annual rate x that satisfies the equation $\$1(1+x)^{77} = \$1,775.34$. The solution is x = 10.2%. Thus, by construction the initial investment of $1 grows to the same accumulation either growing at the actual term structure of returns or by the average geometric return over the same period.

The situation is different when contributions are made on a monthly basis for two reasons. First, although returns are stable over large samples as indicated above in one of our stylized facts, when investment horizons are 20 or 30 or even 40 years long, average returns do not remain stable. Figure 1 illustrates the time series of 40-year investment horizon average returns. These average returns are obtained by forming a rolling window of generations of investors beginning with the first generation from January 1, 1926 to December 31, 1965 and ending with the generation whose investment horizon begins on January 1, 1963 and ends up on December 31, 2002. The total number of these generations is 444, that is, 37 years times 12 generations per year. The average returns for these 444 generations of investors, each one having a 40 year investment horizon, range according to Figure 1 from a low of about 8% to a high of 13.6% per investment horizon. This Figure confirms that the average geometric returns are not very stable, even for investment horizons of 40 years. Obviously, as investment horizons decrease to, say 30 or 20 or even 10 years, the range of these average returns becomes relatively larger.

The second reason is also critical. Even if the average returns over all investment horizons were almost the same, that is, average returns were totally stable, we need to emphasize that such stability does not imply that the actual term structure of monthly returns is the same across samples of different generations of investors. By term structure of monthly returns we define the actual sequence of monthly returns of the S&P 500 Index during, say a period of 480 months, corresponding to an investment horizon of 40 years. When $1 is invested initially with no further contributions, the actual term structure of monthly returns and its average, yield the same terminal wealth. Put differently, the term structure of returns for an initial investment with no further contributions can be substituted mathematically by its geometric average. However, if the investor makes monthly contributions over an investment horizon of 40 years, it does not necessarily follow that the accumulated wealth of these contributions growing by the actual term structure of monthly returns and the accumulated wealth of monthly returns growing at the average geometric rate of these monthly returns are the same. The actual term structure of returns plays an important role in the determination of the total accumulation at the end of a period of, say 40 years.

For an illustration, consider Figure 2. The horizontal axis denotes 444 successive generations of investors each contributing $1 monthly for a 40-year period. The first generation begins investing on January 1, 1926 and stops on December 31, 1965. The second generation starts and ends a month later and the very last generation in our sample begins investing on January 1, 1963 and ends on December 31, 2002. Note that Ibbotson Associates reports monthly returns with no reference to the first and last trading day of each month, so when we identify each generation as beginning on January 1 what we mean is the first trading day on January.

Figure 2 illustrates the time series of accumulated wealth of 444 overlapping generations, each contributing $1 monthly for 40 years, computed in two different ways: first, by using the 40-year geometric average return that is specific for each generation, or second, by using the actual term structure of returns during the 40-year period (also specific to each generation). The two accumulations coincide only in few years while for the majority of generations they diverge with a bias for higher accumulations obtained from the average rate of growth.

Note that accumulations calculated by the actual term structure of returns are more stable than those calculated by the geometric average return. This can be explained as follows: at any point in time, the younger generation begins and ends a month later than the one immediately before it. Computing the accumulation by using the term structure of returns is much more stable since the two nearby generations overlap over 478 out of 480 returns. However, when the geometric average is computed, it may be the case that two nearby generations have slightly different means, again because of the long overlap of 478 identical monthly returns, but applying two slightly different means to a long sequence of monthly contributions causes greater variability to the final accumulation. Thus, one need be careful in computing final accumulations by using a constant geometric sample average. These deviations in the final accumulated wealth do not occur in the case of an investment that consists of only one initial contribution.

INSERT FIGURE 1 ABOUT HERE

INSERT FIGURE 2 ABOUT HERE

In view of the above two remarks, in what follows, we consider actual monthly returns of the S&P 500 Index as reported by Ibbotson Associates. These returns are total, i.e. they include dividends and capital appreciation. They are either nominal or real depending upon the proposed scenario. Suppose that an investor contributes a given amount, say $1 per month over a period of 30 or 40 years. We consider several scenarios about the $1 monthly contribution. For example this contribution can stay the same during the 40-year period or it can grow by the inflation rate or it can grow by both the inflation and the productivity rate. Under these scenarios the question we want to answer is: what is the final amount accumulated?

The way the problem is defined reminds one of the contribution defined pension funding. Extensive details of this framework with all the institutional background, transactions costs, tax advantages, asset allocation, risk management, manager selection, computation of returns and other related issues are presented in Logue and Rader (1998). However, our methodology is much broader and can be used by any investor who makes periodic contributions over a certain investment horizon.

To clarify once again, we are focusing on periodic contributions that grow with a specific term structure of returns. We wish to illustrate that the stylized facts of returns over long-term horizons of only an initial contribution do no translate directly to individual retirement accumulations. Periodic contributions growing with a specific term structure of returns over horizons of 30 or 40 years is a much more complicated problem than the growth of a one-time initial contribution. The need for such a revision is twofold: first, individuals have shorter investment horizons and are thus, subject to unstable average returns that are generation dependent. Second, the generation specific term structure of monthly returns plays an important role in the final amount of total accumulation. This total accumulation, more often than not, is different from the accumulation computed from a geometric mean of monthly returns. There is no unique correspondence between a term structure of monthly returns and its geometric average. Consider 480 monthly returns and their geometric average. This average is unique, yet the

480 monthly returns can form 480! permutations of sequences of monthly term structures, each generating a different final accumulation.

Actually our problem needs further modifications to approximate reality. For example, does the average investor have a 30 or even a 40-year investment horizon? Anecdotal evidence suggests that individuals may spend the period they are 25 to 35 years old to forming a family and saving for a house down payment. They actually may dissave during the time they are 35 to 45 years old as they accumulate mortgage and other debts and may only have a 20-year investment horizon between 45 and 65 or 50 and 70. This simple story does not account for major college expenses, leaving even a shorter period for investing. Whatever scenario one wishes to develop, our methodology can still be applied. The two key concepts that are critical involve, first the fact that shorter horizons influence the stability of sample return averages and, second, the total accumulation of periodic contributions growing by a specific term structure of monthly returns is dependent on such a term structure of returns and cannot automatically be approximated by a geometric average of returns over the sample period.

INSERT TABLE 1 ABOUT HERE

INSERT TABLE 2 ABOUT HERE

COMPUTATIONAL RESULTS

Consider an individual who is 25 years old and plans to contribute $1 per month for 40 years. The monthly contributions are invested in a fund of large stocks indexed to the S&P 500 Index. Allocating a certain percent of the monthly contribution to large stocks and the rest in bonds will also be considered later. Initially, we let the allocation be 100% in large stocks. Later in this paper we consider two other allocations: only bonds and 50% bonds and 50% stocks.

Beginning with January 1, 1926 we compute the total accumulation at the end of 480 monthly contributions, i.e. 12 monthly contributions per year for 40 years. The calculations are carried out both in real and nominal terms. By real terms we mean the actual nominal monthly returns of the S&P 500 Index as reported by Ibbotson Associates (2003) adjusted by the monthly inflation rate. For these two broad categories of real and nominal accumulations we compute certain subcategories. We apply the real monthly returns to various scenarios of monthly contributions in columns (2) to (7). Columns 8 to 13 apply nominal monthly returns, again to various scenarios of monthly contributions.

To be more specific, consider column (2) of Table 1. An individual who contributes monthly $1 for 480 months beginning January 1, 1926 and ending December 31, 1965 would have contributed $480 nominal dollars growing at real monthly rates towards a total accumulation of $4,972.27 dollars. Column (2) lists the various amounts accumulated for the first generation of every year between

1926 and 1962. Note that although each generation contributes the same amount of $480 the range of accumulations is very wide with the lowest being $1,558.93 and the highest being $4,972.27, among the sample reported. Observe that to keep Table 1 short it only reports the 40-year investment accumulation for the generation beginning on January 1 and ending on December 31, as an illustration. If we consider all 444 generations (37 years of 12 generations per year) the lowest and highest accumulations among all these generations are $4,974.12 and $1,273.20, reported in the lowest section of Table 1, column (2).

This calculation demonstrates how sensitive accumulations are to the 40-year investment horizon. According to column (2), the luckiest generation among the ones listed in column 1 was that beginning on January 1, 1926 and retiring on December 31, 1965, while the least fortunate was that beginning in January 1, 1942 and retiring in December 31, 1982.

In column (3) we compute the real accumulation of $1 by allowing annual adjustment to the monthly contribution equivalent to the growth of productivity. We assume that the real wage productivity is 2% per year that is very close to the actual one for the U.S. economy. Inflation and/or productivity adjustments are made in the beginning of the year and do not change monthly. Thus, an investor in a given generation begins by investing $1 for 12 months and then contributes $1.02 for another 12 months and after that $1.0404 for another 12 months and so on, with last year's monthly contribution increasing by 2% annually. Under such a scenario the real dollar total contribution over 480 months is $731.44 with column (3) reporting the total real accumulation for various generations. Again, we note at the bottom of this column that the highest accumulation for the sample January generations is $6,444.84 and the lowest is 1,938.77. The equivalent numbers among all 444 generations are 6,446.40 and 1,592.28 respectively, reported in column (3) of the lowest box.

When one considers a contribution of $731.44 growing to only 1,938.77 over a 40-year investment horizon, i.e. 1:2.7, it is hard to reconcile such results with those reported in Ibbotson Associates where the final accumulation is 1:1775. The shorter horizon of 40 years instead of 77 in Ibbotson Associates and the specific term structure of returns during a generation who invested between January 1942 and December 1981 make it difficult to get the huge accumulation of 1:1775. Furthermore, an individual investor may wonder how come $1 grows to 1775 over 77 years and a contribution of $731.44 over 40 years grows to only 1,938.77.

In column (4) we report the accumulation over 480 months of the equivalent of $1 in real terms. The next column shows the amount accumulated by generation taking into account the real growth of real monthly contributions. In columns (5) and (6) we repeat the same experiment as in (4) and (5) by letting the real equivalent of $1 contribution to grow by a 2% productivity. At the bottom of these columns we report the minimum and maximum accumulations for both the January generations and the entire sample of 444 generations. For example, the most successful generation according to column 5 contributed $1501.03 in real terms and accumulated $9,091.56.

Results in nominal terms are presented in the second half of Table 1 under different scenarios. The accumulation after 40 years of a $1 per month contribution, growing at the monthly nominal rates of growth of the index is reported in column (8). If the monthly contribution grows by 2% at the beginning of every year, to reach a total contribution of $731.44, then column (9) reports the corresponding total accumulation in current dollars per generation. Columns (10) and (12) report contributions of $1 adjusted for inflation assumed to be 3% per year and also adjusted for productivity growth. Note that in most cases, among the January generations of Table 1, the luckiest generation is the January 1, 1960 to December 31, 1999, and the highest ratio of total contribution to total accumulation is 1:13, that is a contribution of $2,558.3 growing to a total accumulation of $33,277.34.

Table 2 repeats exactly the same calculations with a shorter horizon of 30 years. As in Table 1 we consider 3 scenarios for the monthly contributions, that is, $1 contribution for 360 months or beginning with $1 and adjusting it annually for inflation at the annual rate of 3%, and finally, adding to the inflation adjusted monthly contribution a productivity increase, also adjusted annually at the 2% rate. Table 2 presents both amounts contributed and corresponding accumulations and as in Table 1, the ratio of contributions to accumulations ranges from 1:1 to about 1:10.

Recall that Tables 1 and 2 select the generation beginning on January 1 for every year to report the contributions and accumulations. If we consider all 444 generations, the tables would end up being too long. Figures 3 and 4 show the time series accumulations for all 444 generations for one typical scenario. The scenario chosen is the one where the monthly contribution is adjusted annually by 3% for inflation and 2% for labor productivity. Figure 3 shows the accumulation of each generation that has invested for 40 years with its contributions growing at the nominal monthly rates of growth of the S&P 500 index.

Another way to represent the same data is to put it in bar Figures. This is done in Figures 5 and 6. The remarkable characteristic of these two Figures is that the distribution of total accumulations of either a 40 or 30 year investment horizon do not follow the familiar pattern of the normal distribution.

INSERT FIGURE 3 ABOUT HERE

INSERT FIGURE 4 ABOUT HERE

INSERT FIGURE 5 ABOUT HERE

INSERT FIGURE 6 ABOUT HERE

Three surprising conclusions can be drawn from the above calculations. First, similar real or nominal contributions yield drastically dissimilar accumulations due only to the beginning and end of the investment horizon of successive generations and the corresponding term structure of returns. In other words, we obtain wide ranges of accumulations depending on the beginning and end of the investment horizon. Put it another way, total accumulations are dependent on the generation vintage and the performance of the corresponding returns. A $1 invested over very long horizons as in Siegel (2002) is less affected than monthly investments over 20 to 40 years by the particular return characteristics of specific periods.

Second, the distribution of these accumulations is not normal. Actually it is significantly different from normal with a bias to be skewed to the right. For example, Figure 5 illustrates that most of the final accumulations of various generations investing over 40 years range between $10,000 and $16,000. The right hand tail of accumulations between 17,000 to a maximum of $33,277 has very low frequency.

Third, even with very long investment horizons, the resulting accumulated wealth is not as phenomenal as the amount of $1,775.34 reported by Ibbotson Associates. The ratio of the amounts contributed under various scenarios and the corresponding total accumulation never exceeds 1:15 at the highest. Furthermore these returns are with a 100% allocation to large stocks. In other words the phenomenal returns reported by Ibbotson Associates are not replicated for the ordinary person who saves and invests over a shorter time horizon, even if such horizons are 40 years long.

Fourth, investing 100% in the S&P 500 Index continues to outperform investing 100% in bonds or 50% in stocks and 50% in bonds. Tables 3, 4 and 5 illustrate the impact of allocation.

INSERT TABLE 3 ABOUT HERE

INSERT TABLE 4 ABOUT HERE

INSERT TABLE 5 ABOUT HERE

CONTRIBUTIONS AND ACCUMULATIONS RELATIVE TO MEDIAN INCOMES

Next, we wish to make few more calculations to evaluate the effectiveness of defined contribution investment strategies. Suppose that the representative investor sets as a goal during a 20-year retirement horizon to secure as income an amount equal of 60% of his/her pre-retirement annual income. Obviously, our calculations can be revised for a higher or lower percent and also for longer or shorter investment horizons as well as shorter or longer retirement horizons. Logue and Rader (1998) describe that a representative defined benefit retirement plan usually offers the retiree 1.5% of some a measure of final pay per year worked. Thus if an employee has worked for 40 year then 40 x 1.5 = 60% of final pay is expected to be received, possibly adjusted for inflation over a retirement horizon of, say 20 years. The question of a representative investor who contributes a certain amount per month over 20 to 40 years invested in a certain class of investments is the following: what is the necessary annual contribution as a percent of the representative employee's annual income that needs to be invested to contribute to the achievement of the 60% rule?

Actually, the problem can be formulated as a more general question: Consider the nominal median family income in the U.S. over the last 77 years. Choose also an investment horizon. Such a horizon can range between 20 to 40 years but as in the previous sections we choose to work with two cases: 40 and 30 year investment horizons. As earlier, for the 40-year horizon, our data generate 444 generations while when we use a 30-year horizon we end up with 564 generations. In contrast to hypothetical average returns we use as earlier the actual monthly returns. The actual term structure of such returns offers much greater realism to our calculations since returns are path-dependent, that is are influenced by the specific evolution of the macroeconomy. If a representative investor upon retirement can convert his/her accumulated wealth into a 20-year retirement horizon annuity paying a certain percent of his/her final year median income with a 5% rate, assuming 3% inflation adjustment and 2% real interest, then the problem can be stated as follows: what is the probability that a certain percentage monthly contribution can generate sufficient final accumulation to guarantee a certain percentage of the employee's final year's pay.

Calculations not reported here, consider both the time series of the nominal U.S. median family income from 1926 to 2002 and also the required accumulation needed to guarantee at the 5% annual interest a certain percent of the final pay for

20 years. Using such calculations we compute the probabilities reported in Table 3.

These probabilities are computed as follows. We take the annual nominal median income at the beginning of every year and let the typical investor contribute a certain percent of (1/12) x nominal median income, per month, growing at the specific term structure of the corresponding monthly returns. If the annual contribution is, say 4% of a given year's annual nominal median income, then the monthly contribution is .04 x (1/12) x median income. Thus, for a given percent contribution we calculate, using the specific term structure of monthly returns, the final investment accumulation per generation and then compare for how many of these 444 generations with a 40-year investment horizon, this contribution is equal to or greater than the value of the annuity needed to finance a certain percent of final pay.

For example, looking at Table 3, if individuals choose a 40-year horizon with 480 monthly distributions, adjusted annually as the median income changes, a 5% annual contribution of the nominal median income allows only 52.7% of the 444 generations to achieve a sufficient accumulation to guarantee an annual retirement income equal to 50% of the final pay during the 20 year retirement horizon. To achieve a 60% of final pay annually for 20 yeas almost certainly, i.e. with probability 96.2%, investors must contribute 10% of their annual nominal income.

Finally, if funds are invested in either only bonds or 50% bonds and 50% in stocks, Tables 4, and 5 report the various probabilities. These Tables show that investing only in stocks increases the probability to achieve one's retirement goals.

SIMULATIONS

Thus far, all calculations involved actual past monthly returns during the 1926-2002 period. From these 924 monthly returns we computed the accumulations of hundreds of overlapping generations making investments for either 30 or 40 years. Since these accumulations do not conform to a normal distribution, one cannot use statistical inference based on normality to infer mean and standard deviations for such accumulations. Figures 5 and 6 illustrate that the accumulations under certain scenarios are skewed to the right and raise concerns about statistical inference. Thus far we have performed various calculations using actual data that give a precise description of the past but do not tell us what to expect in the future under general conditions.

Using the actual 924 actual monthly returns of the S&P500 Index we wish to generate a much larger sample, say of 10,000 monthly data. This translates into 833.33 years (10,000 divided by 12). Since the average rate of annual inflation is about 3%, using nominal returns, nominal contributions and nominal accumulations over such a long period is too cumbersome. Thus we choose to work with real returns, real contributions and real accumulations.

Once we have converted the original 924 actual nominal monthly returns into real returns we follow two sampling procedures. First, we select from these 924 real monthly returns with replacement until we generate 10,000 observations, assuming that the original returns are statistically independent. Second, we choose at random blocks of 12 returns with replacement until we generate 10,000 observations, assuming that returns are serially correlated up to a certain order. MacKinnon (2002) and Ruiz and Pascual (2002) describe in detail bootstapping techniques.

With these two very large samples of 10,000 real monthly returns, we next compute the accumulated real contribution under a very simple assumption that the initial $1 monthly contribution grows in real terms by 2% annually adjusted. As in Table 1, column (3), the total amount contributed in real dollars, allowing for a 2% annual productivity growth is $731.44. All our generations contribute the same amount but since the monthly returns are different the accumulations vary. For each generation we also index its median income to $100 per month or $1200 per year, also adjusted by 2% annually for productivity growth. From the 10,000 monthly observations we obtain 9520 overlapping generations and compute for the two sampling methods the probability of a certain percentage of annual contribution being sufficient to finance a certain percent of final pay under certain assumptions. Tables 6 and 7 present such results, with Table 6 giving the results for individual sampling with replacement and Table 7 presenting the results when we follow a block sampling of 12 monthly returns. Observe that block sampling improves the probabilities because on average there are many more blocks of positive returns than negative.

INSERT TABLE 6 ABOUT HERE

INSERT TABLE 7 ABOUT HERE

CONCLUSIONS

There is an exhaustive literature in finance that reports both theoretical and empirical findings about long-term investing in various categories of assets. Several books such as Ibbotson Associates (2003) and Siegel (2002) have summarized these academic findings. However, it is not a straightforward exercise to translate all these reported findings into a retirement investment strategy. This is so because most individuals have much shorter investment horizons that range between 20 to 40 years instead of 70 or 100 or 200 years. This limitation seriously diminishes the power of continuous compounding.

But there is a second reason that makes the translation difficult. Most individual investors invest periodically, instead of a lump sum at the beginning of the investment horizon. Thus, the early contributions have longer time to compound but later contributions do not grow to very much since they have shorter period to grow. The calculations of periodic contributions are not complex but not all investors are skilled to perform them.

The third reason is the stability of average returns over very long periods. However, when instead of investing over 70 or 100 years, one invests over 40, both the average and term structure of monthly returns need not be stable. Realistically, each generation, like vintage wine, is characterized by monthly returns that are path dependent, meaning that each nearby return is correlated with the previous one reflecting the state of the economy that does not act totally at random. This means that individual investors live, invest and consume during their generation and cannot transport their investments at other times that might be more favorable, nor do they have such long lives that can patiently wait for good times to follow difficult ones.

In this paper we illustrate by performing various calculations based on alternative investment scenarios how much wealth can be accumulated over a 40 and 30-year investment horizon via monthly contributions invested at historical monthly returns for the S&P500 Index. These calculations yield accumulations that are not as spectacular as those reported by Ibbotson Associates. We find that the total amount contributed to the amount accumulated seldom exceeds 1:15. This finding is difficult to interpret unless one sets certain retirement goals. To do this we use the median nominal family income and calculate the probability of achieving certain retirement goals subject to certain defined contributions as a percentage of the median family income. The results show that these probabilities are rather low for low contributions. In other words, our calculations show that the average U.S. retiree should reconsider both his/her retirement goals and investment contributions. Individual retirement accounts certainly can play an important role in retirement financial planning but we also need to moderate our expectations about their significance.

The calculations performed confirm once again that investing in stocks versus either all bonds or 50% in stocks and 50% in bonds yield much better accumulations. The calculations also show that final accumulations are highly skewed to the right. This means that a very small percent of generations do remarkably well. This is to be expected because of the empirical fact that stock returns occasionally are very high causing individuals who are few years away from retirement to experience very high returns for large sums of accumulated funds.

Simple simulations confirm the actual results of both the shape of the accumulated contributions being skewed to the right and the fact that the median and mean accumulations do not converge. Also the probabilities of achieving certain percentages of final pay depending on the percent of contribution continue to remain low under simulations as with the actual data.

In our calculations we have ignored transactions costs, taxes and the possibility that the investor lacks the discipline to adhere to his or her 40-year investment plan. We have not also attempted to embed personal retirement accounts into a comprehensive retirement plan supplemented by social security as is done in

“Strengthening Social Security and Creating Personal Wealth for All Americans” (available at www.csss.gov). These and other real world complications, such as sickness, loss of employment, family problems, all contribute to lowering accumulations and thus lowering probabilities of achieving one’s retirement goals. However, even under ideal conditions and strict discipline so that monthly contributions are made consistently over a 40 year horizon, this study illustrates that unless an investor is prepared to contribute about 10% of his or her current income, he or she could not expect with sufficiently high probability to have as retirement income an amount at least 60% of their final pre-retirement earnings. Obviously, employer contributions and social security payments may reduce the need to invest 10% percent of the investor’s annual income.

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Table 1. Total Accumulation from Monthly Contributions for 40-years (various scenarios) Invested 100% in S&P 500 Index

1		2	3	4	5	6	7	8	9	10	11	12	13
		Contributions Earning S&P 500 Real Returns						**Contributions Earning S&P 500 Nominal Returns**					
From:	To:	No growth	2% Productivity	Contribution Inflation Adjusted		Inflation and 2% Productivity Adjusted		No growth	2% Productivity	Contribution Inflation Adjusted		Inflation and 2% Productivity Adjusted	
		$480 invested	$731.44 invested	Contribution	Accumulation	Contribution	Accumulation	$480 invested	$731.44 invested	Contribution	Accumulation	Contribution	Accumulation
January 1, 1926	December 31, 1965	$ 4,972.27	$ 6,444.84	$ 569.84	$ 4,823.46	$ 923.07	$ 6,487.36	$ 9,607.73	$ 12,123.78	$ 569.84	$ 8,841.18	$ 923.07	$ 11,459.57
January 1, 1927	December 31, 1966	$ 4,032.11	$ 5,220.65	$ 588.43	$ 3,974.34	$ 954.85	$ 5,360.20	$ 8,083.39	$ 10,152.69	$ 588.43	$ 7,521.52	$ 954.85	$ 9,738.63
January 1, 1928	December 31, 1967	$ 4,576.83	$ 5,902.14	$ 612.03	$ 4,619.44	$ 994.66	$ 6,225.74	$ 9,473.70	$ 11,814.23	$ 612.03	$ 8,984.73	$ 994.66	$ 11,586.43
January 1, 1929	December 31, 1968	$ 4,648.53	$ 5,949.05	$ 630.23	$ 4,753.57	$ 1,025.78	$ 6,376.03	$ 10,083.03	$ 12,446.05	$ 630.23	$ 9,648.64	$ 1,025.78	$ 12,348.05
January 1, 1930	December 31, 1969	$ 3,883.17	$ 4,918.84	$ 642.46	$ 3,977.88	$ 1,047.49	$ 5,296.12	$ 8,936.79	$ 10,892.72	$ 642.46	$ 8,534.14	$ 1,047.49	$ 10,810.26
January 1, 1931	December 31, 1970	$ 3,686.10	$ 4,629.61	$ 699.95	$ 4,043.36	$ 1,143.34	$ 5,357.06	$ 8,938.50	$ 10,770.85	$ 699.95	$ 9,093.32	$ 1,143.34	$ 11,420.88
January 1, 1932	December 31, 1971	$ 3,833.30	$ 4,788.59	$ 794.20	$ 4,705.41	$ 1,298.94	$ 6,218.52	$ 9,544.12	$ 11,408.65	$ 794.20	$ 10,801.15	$ 1,298.94	$ 13,492.90
January 1, 1933	December 31, 1972	$ 3,964.01	$ 4,975.32	$ 910.98	$ 5,593.32	$ 1,490.72	$ 7,431.85	$ 9,950.41	$ 11,933.36	$ 910.98	$ 12,877.64	$ 1,490.72	$ 16,163.02
January 1, 1934	December 31, 1973	$ 2,868.99	$ 3,601.26	$ 934.82	$ 4,154.65	$ 1,530.34	$ 5,520.77	$ 7,600.65	$ 9,116.77	$ 934.82	$ 10,086.68	$ 1,530.34	$ 12,661.17
January 1, 1935	December 31, 1974	$ 1,753.24	$ 2,199.88	$ 947.96	$ 2,568.22	$ 1,554.12	$ 3,419.88	$ 5,067.87	$ 6,069.17	$ 947.96	$ 6,778.37	$ 1,554.12	$ 8,505.19
January 1, 1936	December 31, 1975	$ 2,095.33	$ 2,629.49	$ 954.94	$ 3,082.09	$ 1,569.04	$ 4,118.33	$ 6,296.47	$ 7,530.20	$ 954.94	$ 8,416.60	$ 1,569.04	$ 10,562.25
January 1, 1937	December 31, 1976	$ 2,361.28	$ 2,947.91	$ 980.32	$ 3,523.83	$ 1,614.44	$ 4,701.53	$ 7,276.83	$ 8,643.29	$ 980.32	$ 9,823.87	$ 1,614.44	$ 12,264.47
January 1, 1938	December 31, 1977	$ 1,957.53	$ 2,432.03	$ 989.07	$ 2,913.39	$ 1,632.99	$ 3,887.10	$ 6,303.82	$ 7,433.61	$ 989.07	$ 8,433.92	$ 1,632.99	$ 10,478.29
January 1, 1939	December 31, 1978	$ 1,788.94	$ 2,221.79	$ 1,060.68	$ 2,846.74	$ 1,755.86	$ 3,815.83	$ 6,076.62	$ 7,147.46	$ 1,060.68	$ 8,643.93	$ 1,755.86	$ 10,735.39
January 1, 1940	December 31, 1979	$ 1,758.70	$ 2,181.37	$ 1,115.64	$ 2,929.80	$ 1,852.94	$ 3,944.78	$ 6,528.78	$ 7,653.83	$ 1,115.64	$ 9,675.37	$ 1,852.94	$ 12,000.67
January 1, 1941	December 31, 1980	$ 1,936.05	$ 2,401.23	$ 1,162.65	$ 3,348.77	$ 1,939.47	$ 4,537.47	$ 7,739.31	$ 9,057.03	$ 1,162.65	$ 11,858.71	$ 1,939.47	$ 14,708.06
January 1, 1942	December 31, 1981	$ 1,558.93	$ 1,938.77	$ 1,118.46	$ 2,595.82	$ 1,875.00	$ 3,550.54	$ 6,454.63	$ 7,562.52	$ 1,118.46	$ 9,481.27	$ 1,875.00	$ 11,791.42
January 1, 1943	December 31, 1982	$ 1,639.07	$ 2,057.95	$ 1,080.19	$ 2,669.43	$ 1,819.86	$ 3,715.89	$ 6,657.90	$ 7,853.41	$ 1,080.19	$ 9,473.32	$ 1,819.86	$ 11,894.28
January 1, 1944	December 31, 1983	$ 1,771.29	$ 2,234.83	$ 1,103.49	$ 2,951.28	$ 1,866.99	$ 4,151.63	$ 7,136.74	$ 8,439.26	$ 1,103.49	$ 10,300.12	$ 1,866.99	$ 12,995.60
January 1, 1945	December 31, 1984	$ 1,663.38	$ 2,109.18	$ 1,138.61	$ 2,870.33	$ 1,933.29	$ 4,079.41	$ 6,636.01	$ 7,870.05	$ 1,138.61	$ 9,846.15	$ 1,933.29	$ 12,485.01
January 1, 1946	December 31, 1985	$ 1,966.91	$ 2,500.07	$ 1,172.50	$ 3,493.23	$ 1,996.56	$ 4,997.33	$ 7,778.66	$ 9,233.44	$ 1,172.50	$ 11,818.04	$ 1,996.56	$ 15,021.49
January 1, 1947	December 31, 1986	$ 2,142.61	$ 2,726.17	$ 1,042.58	$ 3,369.20	$ 1,779.32	$ 4,839.52	$ 8,233.42	$ 9,767.56	$ 1,042.58	$ 11,018.11	$ 1,779.32	$ 14,019.01
January 1, 1948	December 31, 1987	$ 1,954.34	$ 2,503.28	$ 1,003.06	$ 2,961.60	$ 1,715.73	$ 4,295.32	$ 7,552.45	$ 8,990.00	$ 1,003.06	$ 9,626.45	$ 1,715.73	$ 12,330.35
January 1, 1949	December 31, 1988	$ 1,957.27	$ 2,533.92	$ 1,023.53	$ 3,046.83	$ 1,754.48	$ 4,483.28	$ 7,620.30	$ 9,121.98	$ 1,023.53	$ 9,802.28	$ 1,754.48	$ 12,690.24
January 1, 1950	December 31, 1989	$ 2,182.03	$ 2,863.09	$ 1,092.99	$ 3,675.12	$ 1,876.81	$ 5,486.20	$ 8,495.96	$ 10,268.15	$ 1,092.99	$ 11,645.93	$ 1,876.81	$ 15,280.90
January 1, 1951	December 31, 1990	$ 1,797.22	$ 2,383.74	$ 1,084.05	$ 3,034.52	$ 1,864.21	$ 4,578.82	$ 7,086.41	$ 8,634.50	$ 1,084.05	$ 9,619.64	$ 1,864.21	$ 12,759.04
January 1, 1952	December 31, 1991	$ 2,075.50	$ 2,777.44	$ 1,073.84	$ 3,490.27	$ 1,849.33	$ 5,314.39	$ 8,120.60	$ 9,949.27	$ 1,073.84	$ 10,815.55	$ 1,849.33	$ 14,473.39
January 1, 1953	December 31, 1992	$ 1,988.79	$ 2,683.46	$ 1,115.38	$ 3,492.02	$ 1,922.76	$ 5,356.61	$ 7,708.91	$ 9,494.35	$ 1,115.38	$ 10,569.03	$ 1,922.76	$ 14,260.69
January 1, 1954	December 31, 1993	$ 1,945.19	$ 2,649.29	$ 1,160.74	$ 3,585.87	$ 2,001.92	$ 5,538.67	$ 7,422.68	$ 9,207.96	$ 1,160.74	$ 10,554.61	$ 2,001.92	$ 14,375.07
January 1, 1955	December 31, 1994	$ 1,773.20	$ 2,432.69	$ 1,220.90	$ 3,463.47	$ 2,105.64	$ 5,372.78	$ 6,651.02	$ 8,301.02	$ 1,220.90	$ 9,928.17	$ 2,105.64	$ 13,620.66
January 1, 1956	December 31, 1995	$ 2,239.86	$ 3,076.35	$ 1,272.57	$ 4,549.67	$ 2,193.70	$ 7,049.30	$ 8,316.27	$ 10,386.25	$ 1,272.57	$ 12,838.93	$ 2,193.70	$ 17,633.73
January 1, 1957	December 31, 1996	$ 2,536.70	$ 3,478.84	$ 1,293.34	$ 5,202.73	$ 2,227.66	$ 8,029.05	$ 9,397.21	$ 11,719.41	$ 1,293.34	$ 14,602.10	$ 2,227.66	$ 20,025.39
January 1, 1958	December 31, 1997	$ 3,144.22	$ 4,308.76	$ 1,311.19	$ 6,507.21	$ 2,255.92	$ 9,998.55	$ 11,413.68	$ 14,232.62	$ 1,311.19	$ 17,864.38	$ 2,255.92	$ 24,480.92
January 1, 1959	December 31, 1998	$ 3,749.54	$ 5,135.53	$ 1,343.83	$ 7,927.24	$ 2,308.64	$ 12,119.92	$ 13,301.91	$ 16,599.43	$ 1,343.83	$ 21,265.99	$ 2,308.64	$ 29,126.80
January 1, 1960	December 31, 1999	$ 4,209.77	$ 5,741.88	$ 1,379.77	$ 9,055.93	$ 2,366.08	$ 13,736.93	$ 14,837.75	$ 18,464.74	$ 1,379.77	$ 24,169.14	$ 2,366.08	$ 32,965.30
January 1, 1961	December 31, 2000	$ 3,517.36	$ 4,780.80	$ 1,416.88	$ 7,717.14	$ 2,425.11	$ 11,617.30	$ 12,344.32	$ 15,337.75	$ 1,416.88	$ 20,570.01	$ 2,425.11	$ 27,958.78
January 1, 1962	December 31, 2001	$ 2,920.81	$ 3,950.69	$ 1,466.10	$ 6,576.32	$ 2,504.14	$ 9,822.97	$ 10,049.32	$ 12,445.04	$ 1,466.10	$ 17,237.17	$ 2,504.14	$ 23,315.05
	Min	$ 1,558.93	$ 1,938.77	$ 569.84	$ 2,568.22	$ 923.07	$ 3,419.88	$ 5,067.87	$ 6,069.17	$ 569.84	$ 6,778.37	$ 923.07	$ 8,505.19
	Max	$ 4,972.27	$ 6,444.84	$ 1,466.10	$ 9,055.93	$ 2,504.14	$ 13,736.93	$ 14,837.75	$ 18,464.74	$ 1,466.10	$ 24,169.14	$ 2,504.14	$ 32,965.30
	Average	$ 2,671.63	$ 3,467.86	$ 1,040.76	$ 4,164.69	$ 1,755.98	$ 5,968.46	$ 8,397.95	$ 10,218.17	$ 1,040.76	$ 11,434.22	$ 1,755.98	$ 14,930.77
	Median	$ 2,142.61	$ 2,777.44	$ 1,080.19	$ 3,523.83	$ 1,849.33	$ 5,356.61	$ 7,778.66	$ 9,494.35	$ 1,080.19	$ 9,928.17	$ 1,849.33	$ 12,759.04
	St. Dev.	$ 1,027.40	$ 1,350.22	$ 235.92	$ 1,591.11	$ 425.78	$ 2,504.80	$ 2,070.96	$ 2,691.59	$ 235.92	$ 3,934.93	$ 425.78	$ 5,648.62
		Statistics for the whole sample of 444 generations:											
	Min	$ 1,273.20	$ 1,592.28	$ 569.84	$ 2,083.17	$ 923.07	$ 2,879.06	$ 4,758.53	$ 5,698.40	$ 569.84	$ 6,237.48	$ 923.07	$ 7,822.52
	Max	$ 4,974.12	$ 6,446.40	$ 1,501.03	$ 9,091.56	$ 2,558.30	$ 13,763.67	$ 14,861.13	$ 18,483.09	$ 1,501.03	$ 24,422.37	$ 2,558.30	$ 33,277.34
	Average	$ 2,628.30	$ 3,412.78	$ 1,053.49	$ 4,157.93	$ 1,778.22	$ 5,966.81	$ 8,365.90	$ 10,174.20	$ 1,053.49	$ 11,491.18	$ 1,778.22	$ 15,009.95
	Median	$ 2,160.28	$ 2,817.87	$ 1,086.51	$ 3,551.91	$ 1,836.86	$ 5,327.06	$ 7,805.14	$ 9,523.27	$ 1,086.51	$ 10,309.37	$ 1,836.86	$ 13,390.83
	St. Dev.	$ 955.64	$ 1,256.77	$ 231.49	$ 1,523.24	$ 417.77	$ 2,408.03	$ 1,983.58	$ 2,574.33	$ 231.49	$ 3,781.38	$ 417.77	$ 5,437.41

Table 2. Total Accumulation from Monthly Contributions for 30-years (various scenarios) Invested 100% in S&P 500 Index

1		2	3	4	5	6	7	8	9	10	11	12	13
		Contributions Earning S&P 500 Real Returns						Contributions Earning S&P 500 Nominal Returns					
From:	To:	No growth	2% Productivity	Contribution Inflation Adjusted		Inflation and 2% Productivity Adjusted		No growth	2% Productivity	Contribution Inflation Adjusted		Inflation and 2% Productivity Adjusted	
		$360 invested	$491.26 invested	Contribution	Accumulation	Contribution	Accumulation	$360 invested	$491.26 invested	Contribution	Accumulation	Contribution	Accumulation
January 1, 1926	December 31, 1955	$ 1,979.32	$ 2,506.48	$ 371.67	$ 1,862.14	$ 525.48	$ 2,414.38	$ 3,283.63	$ 4,086.92	$ 371.67	$ 2,964.65	$ 525.48	$ 3,754.24
January 1, 1927	December 31, 1956	$ 1,910.63	$ 2,417.06	$ 383.69	$ 1,824.30	$ 544.09	$ 2,371.66	$ 3,274.60	$ 4,057.50	$ 383.69	$ 2,988.15	$ 544.09	$ 3,780.20
January 1, 1928	December 31, 1957	$ 1,562.90	$ 1,970.13	$ 399.28	$ 1,527.98	$ 567.76	$ 1,986.01	$ 2,764.59	$ 3,402.61	$ 399.28	$ 2,571.78	$ 567.76	$ 3,241.88
January 1, 1929	December 31, 1958	$ 2,113.11	$ 2,645.35	$ 411.33	$ 2,094.69	$ 586.35	$ 2,711.86	$ 3,803.95	$ 4,636.84	$ 411.33	$ 3,573.01	$ 586.35	$ 4,472.96
January 1, 1930	December 31, 1959	$ 2,260.66	$ 2,801.46	$ 418.86	$ 2,244.44	$ 598.30	$ 2,883.16	$ 4,129.97	$ 4,972.42	$ 418.86	$ 3,872.10	$ 598.30	$ 4,798.38
January 1, 1931	December 31, 1960	$ 2,155.63	$ 2,647.63	$ 455.23	$ 2,288.84	$ 651.26	$ 2,921.58	$ 3,992.73	$ 4,752.96	$ 455.23	$ 3,987.11	$ 651.26	$ 4,897.12
January 1, 1932	December 31, 1961	$ 2,567.13	$ 3,133.95	$ 515.11	$ 3,044.79	$ 737.26	$ 3,869.69	$ 4,758.63	$ 5,618.70	$ 515.11	$ 5,282.88	$ 737.26	$ 6,449.36
January 1, 1933	December 31, 1962	$ 2,077.19	$ 2,545.41	$ 589.34	$ 2,826.33	$ 842.97	$ 3,604.83	$ 3,803.17	$ 4,503.33	$ 589.34	$ 4,823.20	$ 842.97	$ 5,910.40
January 1, 1934	December 31, 1963	$ 2,310.84	$ 2,828.90	$ 602.47	$ 3,219.78	$ 860.61	$ 4,096.87	$ 4,177.26	$ 4,944.90	$ 602.47	$ 5,427.16	$ 860.61	$ 6,643.86
January 1, 1935	December 31, 1964	$ 2,470.87	$ 3,015.57	$ 606.28	$ 3,461.09	$ 864.85	$ 4,385.43	$ 4,406.69	$ 5,203.15	$ 606.28	$ 5,756.10	$ 864.85	$ 7,025.04
January 1, 1936	December 31, 1965	$ 2,528.96	$ 3,077.08	$ 604.08	$ 3,528.41	$ 860.44	$ 4,451.62	$ 4,480.40	$ 5,277.06	$ 604.08	$ 5,830.50	$ 860.44	$ 7,094.21
January 1, 1937	December 31, 1966	$ 2,093.84	$ 2,528.73	$ 612.80	$ 2,943.41	$ 871.58	$ 3,684.50	$ 3,759.32	$ 4,395.16	$ 612.80	$ 4,929.54	$ 871.58	$ 5,953.40
January 1, 1938	December 31, 1967	$ 2,394.10	$ 2,869.19	$ 610.16	$ 3,327.19	$ 866.66	$ 4,132.46	$ 4,347.50	$ 5,042.55	$ 610.16	$ 5,633.19	$ 866.66	$ 6,751.07
January 1, 1939	December 31, 1968	$ 2,360.86	$ 2,817.32	$ 645.18	$ 3,468.72	$ 915.17	$ 4,285.86	$ 4,360.85	$ 5,038.76	$ 645.18	$ 5,983.38	$ 915.17	$ 7,140.23
January 1, 1940	December 31, 1969	$ 1,904.44	$ 2,260.79	$ 667.74	$ 2,892.84	$ 945.99	$ 3,552.69	$ 3,614.84	$ 4,158.02	$ 667.74	$ 5,146.00	$ 945.99	$ 6,108.90
January 1, 1941	December 31, 1970	$ 1,746.57	$ 2,066.27	$ 682.41	$ 2,718.18	$ 965.87	$ 3,324.95	$ 3,362.72	$ 3,858.02	$ 682.41	$ 4,930.88	$ 965.87	$ 5,833.44
January 1, 1942	December 31, 1971	$ 1,760.23	$ 2,079.11	$ 641.98	$ 2,594.01	$ 908.03	$ 3,164.67	$ 3,346.64	$ 3,839.10	$ 641.98	$ 4,680.98	$ 908.03	$ 5,529.00
January 1, 1943	December 31, 1972	$ 1,800.31	$ 2,133.13	$ 605.50	$ 2,534.79	$ 856.22	$ 3,098.01	$ 3,370.30	$ 3,883.99	$ 605.50	$ 4,530.23	$ 856.22	$ 5,367.74
January 1, 1944	December 31, 1973	$ 1,282.40	$ 1,520.58	$ 605.38	$ 1,815.09	$ 856.31	$ 2,219.86	$ 2,510.44	$ 2,896.69	$ 605.38	$ 3,402.26	$ 856.31	$ 4,034.16
January 1, 1945	December 31, 1974	$ 768.31	$ 913.48	$ 613.94	$ 1,111.03	$ 869.45	$ 1,365.04	$ 1,614.20	$ 1,867.29	$ 613.94	$ 2,239.64	$ 869.45	$ 2,662.83
January 1, 1946	December 31, 1975	$ 912.57	$ 1,086.65	$ 623.91	$ 1,346.29	$ 885.23	$ 1,661.49	$ 1,964.54	$ 2,274.85	$ 623.91	$ 2,784.22	$ 885.23	$ 3,315.35
January 1, 1947	December 31, 1976	$ 1,000.45	$ 1,191.91	$ 548.86	$ 1,297.90	$ 780.48	$ 1,607.72	$ 2,173.67	$ 2,515.88	$ 548.86	$ 2,707.49	$ 780.48	$ 3,225.62
January 1, 1948	December 31, 1977	$ 784.58	$ 940.89	$ 523.01	$ 973.30	$ 746.02	$ 1,219.80	$ 1,758.38	$ 2,042.59	$ 523.01	$ 2,079.65	$ 746.02	$ 2,493.99
January 1, 1949	December 31, 1978	$ 684.04	$ 829.26	$ 529.59	$ 864.00	$ 758.20	$ 1,102.25	$ 1,618.76	$ 1,891.76	$ 529.59	$ 1,925.03	$ 758.20	$ 2,333.71
January 1, 1950	December 31, 1979	$ 632.35	$ 777.84	$ 563.70	$ 861.09	$ 810.41	$ 1,122.11	$ 1,624.96	$ 1,918.01	$ 563.70	$ 2,053.49	$ 810.41	$ 2,525.96
January 1, 1951	December 31, 1980	$ 671.79	$ 836.92	$ 560.50	$ 920.11	$ 809.83	$ 1,222.20	$ 1,853.36	$ 2,206.46	$ 560.50	$ 2,317.89	$ 809.83	$ 2,887.64
January 1, 1952	December 31, 1981	$ 535.49	$ 674.94	$ 558.64	$ 736.52	$ 811.70	$ 997.11	$ 1,547.82	$ 1,854.25	$ 558.64	$ 1,901.22	$ 811.70	$ 2,396.34
January 1, 1953	December 31, 1982	$ 576.94	$ 736.65	$ 585.03	$ 842.49	$ 854.47	$ 1,163.82	$ 1,660.36	$ 2,002.44	$ 585.03	$ 2,108.87	$ 854.47	$ 2,692.66
January 1, 1954	December 31, 1983	$ 623.37	$ 805.50	$ 613.72	$ 964.61	$ 900.06	$ 1,352.14	$ 1,781.45	$ 2,165.40	$ 613.72	$ 2,350.26	$ 900.06	$ 3,036.93
January 1, 1955	December 31, 1984	$ 591.35	$ 772.07	$ 651.22	$ 981.18	$ 958.20	$ 1,391.57	$ 1,676.43	$ 2,052.09	$ 651.22	$ 2,329.21	$ 958.20	$ 3,041.04
January 1, 1956	December 31, 1985	$ 715.33	$ 937.66	$ 684.86	$ 1,248.62	$ 1,010.14	$ 1,778.84	$ 2,019.52	$ 2,476.28	$ 684.86	$ 2,913.70	$ 1,010.14	$ 3,819.31
January 1, 1957	December 31, 1986	$ 799.95	$ 1,049.23	$ 701.64	$ 1,421.74	$ 1,036.46	$ 2,025.44	$ 2,199.71	$ 2,695.45	$ 701.64	$ 3,203.90	$ 1,036.46	$ 4,202.28
January 1, 1958	December 31, 1987	$ 764.31	$ 1,003.65	$ 717.03	$ 1,380.93	$ 1,060.30	$ 1,966.25	$ 2,109.85	$ 2,586.65	$ 717.03	$ 3,102.61	$ 1,060.30	$ 4,074.18
January 1, 1959	December 31, 1988	$ 810.26	$ 1,066.57	$ 741.48	$ 1,511.96	$ 1,097.19	$ 2,154.08	$ 2,237.64	$ 2,748.19	$ 741.48	$ 3,374.61	$ 1,097.19	$ 4,442.11
January 1, 1960	December 31, 1989	$ 976.13	$ 1,283.03	$ 769.03	$ 1,872.68	$ 1,138.36	$ 2,660.05	$ 2,716.28	$ 3,330.09	$ 769.03	$ 4,191.03	$ 1,138.36	$ 5,508.74
January 1, 1961	December 31, 1990	$ 851.57	$ 1,118.58	$ 798.28	$ 1,684.98	$ 1,181.84	$ 2,387.47	$ 2,411.13	$ 2,954.29	$ 798.28	$ 3,822.75	$ 1,181.84	$ 5,021.33
January 1, 1962	December 31, 1991	$ 1,038.11	$ 1,359.74	$ 835.34	$ 2,127.11	$ 1,236.49	$ 3,000.82	$ 2,912.42	$ 3,559.43	$ 835.34	$ 4,771.21	$ 1,236.49	$ 6,249.48
January 1, 1963	December 31, 1992	$ 1,040.82	$ 1,360.11	$ 868.69	$ 2,199.60	$ 1,285.02	$ 3,088.82	$ 2,875.48	$ 3,509.93	$ 868.69	$ 4,862.69	$ 1,285.02	$ 6,354.42
January 1, 1964	December 31, 1993	$ 1,071.75	$ 1,395.46	$ 898.62	$ 2,319.92	$ 1,327.86	$ 3,239.13	$ 2,913.87	$ 3,548.85	$ 898.62	$ 5,060.97	$ 1,327.86	$ 6,589.56
January 1, 1965	December 31, 1994	$ 1,023.99	$ 1,326.40	$ 932.81	$ 2,273.46	$ 1,376.32	$ 3,153.22	$ 2,744.25	$ 3,329.06	$ 932.81	$ 4,905.42	$ 1,376.32	$ 6,354.12
January 1, 1966	December 31, 1995	$ 1,328.58	$ 1,709.94	$ 960.53	$ 2,995.47	$ 1,414.65	$ 4,122.10	$ 3,516.40	$ 4,244.49	$ 960.53	$ 6,414.99	$ 1,414.65	$ 8,256.38
January 1, 1967	December 31, 1996	$ 1,528.84	$ 1,954.54	$ 974.58	$ 3,450.35	$ 1,432.42	$ 4,706.89	$ 4,010.71	$ 4,818.54	$ 974.58	$ 7,378.38	$ 1,432.42	$ 9,432.87
January 1, 1968	December 31, 1997	$ 1,936.46	$ 2,456.29	$ 990.70	$ 4,375.72	$ 1,452.77	$ 5,909.79	$ 4,962.43	$ 5,929.13	$ 990.70	$ 9,222.45	$ 1,452.77	$ 11,698.15
January 1, 1969	December 31, 1998	$ 2,366.86	$ 2,975.82	$ 989.27	$ 5,256.35	$ 1,446.94	$ 7,021.52	$ 5,931.09	$ 7,040.64	$ 989.27	$ 10,935.58	$ 1,446.94	$ 13,749.12
January 1, 1970	December 31, 1999	$ 2,684.68	$ 3,345.02	$ 973.59	$ 5,781.12	$ 1,420.17	$ 7,635.56	$ 6,653.54	$ 7,845.78	$ 973.59	$ 12,012.48	$ 1,420.17	$ 14,967.11
January 1, 1971	December 31, 2000	$ 2,247.79	$ 2,780.82	$ 963.14	$ 4,746.45	$ 1,401.28	$ 6,206.52	$ 5,499.68	$ 6,458.38	$ 963.14	$ 9,856.02	$ 1,401.28	$ 12,193.24
January 1, 1972	December 31, 2001	$ 1,868.61	$ 2,293.85	$ 971.71	$ 3,942.42	$ 1,410.04	$ 5,106.19	$ 4,451.09	$ 5,197.86	$ 971.71	$ 8,053.94	$ 1,410.04	$ 9,886.75
	Min	$ 535.49	$ 674.94	$ 371.67	$ 736.52	$ 525.48	$ 997.11	$ 1,547.82	$ 1,854.25	$ 371.67	$ 1,901.22	$ 525.48	$ 2,333.71
	Max	$ 2,684.68	$ 3,345.02	$ 990.70	$ 5,781.12	$ 1,452.77	$ 7,635.56	$ 6,653.54	$ 7,845.78	$ 990.70	$ 12,012.48	$ 1,452.77	$ 14,967.11
	Average	$ 1,483.62	$ 1,838.11	$ 665.22	$ 2,299.17	$ 963.64	$ 3,009.17	$ 3,185.57	$ 3,792.06	$ 665.22	$ 4,503.02	$ 963.64	$ 5,615.44
	Median	$ 1,428.71	$ 1,832.24	$ 613.83	$ 2,163.35	$ 878.41	$ 2,902.37	$ 3,094.23	$ 3,699.26	$ 613.83	$ 4,089.07	$ 878.41	$ 5,194.54
	St. Dev.	$ 698.21	$ 843.01	$ 174.50	$ 1,206.01	$ 264.80	$ 1,567.81	$ 1,250.58	$ 1,466.85	$ 174.50	$ 2,338.41	$ 264.80	$ 2,921.81
		Statistics for the whole sample of 564 generations:											
	Min	$ 443.67	$ 563.49	$ 371.67	$ 629.61	$ 525.48	$ 862.84	$ 1,298.76	$ 1,561.99	$ 371.67	$ 1,620.60	$ 525.48	$ 2,058.30
	Max	$ 2,684.68	$ 3,345.02	$ 992.01	$ 5,781.12	$ 1,453.29	$ 7,635.56	$ 6,660.11	$ 7,845.78	$ 992.01	$ 12,012.48	$ 1,453.29	$ 14,967.11
	Average	$ 1,477.18	$ 1,829.31	$ 678.25	$ 2,333.61	$ 982.67	$ 3,053.59	$ 3,199.24	$ 3,804.25	$ 678.25	$ 4,593.97	$ 982.67	$ 5,724.75
	Median	$ 1,454.95	$ 1,783.02	$ 623.45	$ 2,136.49	$ 887.91	$ 2,859.55	$ 3,084.90	$ 3,633.92	$ 623.45	$ 4,260.97	$ 887.91	$ 5,328.07
	St. Dev.	$ 668.53	$ 806.65	$ 176.04	$ 1,179.58	$ 266.27	$ 1,533.13	$ 1,203.64	$ 1,410.94	$ 176.04	$ 2,297.85	$ 266.27	$ 2,868.57

Table 3. Assuming a 40-year investment horizon and a 20-year retirement period, this Table lists the probability that a certain percentage of annual contributions will be sufficient to finance a certain percent of final pay (contributions invested 100% in S&P 500 Index)

		Percent of income contributed									
		1%	**2%**	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**	**10%**
Percent of final pay	50.0%	0.0%	0.0%	12.2%	24.8%	52.7%	67.3%	80.0%	93.0%	99.1%	99.8%
	55.0%	0.0%	0.0%	8.6%	16.7%	38.1%	60.1%	69.8%	85.1%	94.4%	99.3%
	60.0%	0.0%	0.0%	6.1%	14.9%	27.3%	52.7%	66.4%	75.0%	87.8%	96.2%
	65.0%	0.0%	0.0%	3.4%	12.4%	20.5%	40.3%	59.5%	67.6%	79.1%	91.0%
	70.0%	0.0%	0.0%	0.7%	10.1%	16.0%	29.7%	52.7%	64.6%	71.2%	82.9%
	75.0%	0.0%	0.0%	0.0%	8.1%	14.9%	24.8%	41.9%	58.6%	67.3%	75.0%
	80.0%	0.0%	0.0%	0.0%	6.1%	12.8%	17.8%	31.5%	52.7%	64.0%	68.7%
	85.0%	0.0%	0.0%	0.0%	4.5%	11.5%	15.8%	26.1%	43.5%	57.7%	66.4%
	90.0%	0.0%	0.0%	0.0%	1.6%	9.2%	14.9%	22.3%	34.0%	52.7%	62.2%
	95.0%	0.0%	0.0%	0.0%	0.2%	7.7%	13.1%	17.3%	28.4%	44.4%	57.2%
	100.0%	0.0%	0.0%	0.0%	0.0%	6.1%	12.2%	15.5%	24.8%	36.0%	52.7%

Table 4. Assuming a 40-year investment horizon and a 20-year retirement period, this Table lists the probability that a certain percentage of annual contribution will be sufficient to finance a percent of final pay (contributions invested 100% in Government Bonds)

		Percent of income contributed									
		1%	**2%**	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**	**10%**
Percent of final pay	**50%**	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.8%	17.6%	24.1%	27.9%
	55%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	4.1%	20.3%	24.5%
	60%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.5%	7.4%	21.8%
	65%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.1%	11.3%
	70%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.2%
	75%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.5%
	80%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	85%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	90%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	95%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
	100%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%

Table 5. Assuming a 40-year investment horizon and a 20-year retirement period, this Table lists the probability that certain percentage of annual contribution will be sufficient to finance a certain percent of final pay (contributions invested 50% in S&P 500 Index and 50% in Government Bonds)

		Percent of income contributed									
		1%	**2%**	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**	**10%**
Percent of final pay	**50%**	0.0%	0.0%	0.0%	3.6%	15.8%	22.1%	29.5%	34.2%	40.3%	45.5%
	55%	0.0%	0.0%	0.0%	0.0%	12.6%	19.4%	24.5%	30.4%	34.5%	41.4%
	60%	0.0%	0.0%	0.0%	0.0%	7.4%	15.8%	21.4%	28.2%	31.1%	34.9%
	65%	0.0%	0.0%	0.0%	0.0%	0.5%	13.3%	19.1%	23.0%	29.3%	32.0%
	70%	0.0%	0.0%	0.0%	0.0%	0.0%	9.0%	15.8%	20.3%	25.7%	30.2%
	75%	0.0%	0.0%	0.0%	0.0%	0.0%	3.6%	13.3%	18.0%	22.1%	28.2%
	80%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	10.1%	15.8%	19.6%	23.9%
	85%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	6.5%	13.5%	17.8%	21.4%
	90%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.7%	10.8%	15.8%	19.4%
	95%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	7.9%	14.0%	17.6%
	100%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	3.6%	11.9%	15.8%

Table 6. Assuming a 40-year investment horizon and a 20-year retirement period, this Table lists the probability that a certain percentage of annual contribution will be sufficient to finance a certain percent of final pay (9520 generations generated by individual boostrapping) (Contributions invested in 100% S&P 500 Index)

		Percent of income contributed									
		1%	**2%**	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**	**10%**
Percent of final pay	**50%**	2.4%	6.0%	16.6%	30.8%	39.9%	48.3%	58.7%	65.7%	70.0%	74.5%
	55%	2.0%	5.1%	13.1%	25.7%	36.6%	43.8%	52.2%	60.7%	66.7%	70.3%
	60%	1.8%	4.2%	10.4%	21.2%	32.7%	39.9%	46.7%	55.6%	62.4%	67.3%
	65%	1.8%	3.5%	8.6%	17.4%	28.9%	37.2%	43.2%	49.7%	57.9%	63.7%
	70%	1.6%	3.1%	7.2%	14.8%	24.9%	34.2%	39.9%	45.9%	52.9%	59.9%
	75%	1.4%	2.9%	6.0%	12.5%	21.2%	30.8%	37.6%	42.7%	48.3%	55.6%
	80%	1.3%	2.7%	5.4%	10.4%	18.1%	27.7%	35.1%	39.9%	45.1%	50.9%
	85%	1.2%	2.6%	4.9%	9.0%	15.9%	24.4%	32.1%	37.8%	42.4%	47.1%
	90%	1.1%	2.6%	4.2%	7.9%	13.7%	21.2%	29.5%	35.8%	39.9%	44.5%
	95%	1.0%	2.6%	3.8%	6.8%	12.1%	18.7%	26.6%	33.3%	38.1%	42.1%
	100%	0.9%	2.4%	3.3%	6.0%	10.4%	16.6%	23.9%	30.8%	36.2%	39.9%

Table 7. Assuming a 40-year investment horizon and a 20-year retirement period, this Table lists the probability that certain percentage of annual contribution will be sufficient to finance a percent of final pay

(9520 generations generated by block boostrapping)

(Contributions invested in 100% S&P 500 Index)

		Percent of income contributed									
		1%	**2%**	**3%**	**4%**	**5%**	**6%**	**7%**	**8%**	**9%**	**10%**
	50%	3.3%	18.2%	35.2%	50.2%	59.6%	66.7%	71.5%	76.1%	79.7%	82.3%
	55%	2.8%	14.9%	29.0%	45.7%	55.8%	63.0%	68.6%	72.7%	76.8%	80.0%
Percent	**60%**	2.1%	12.6%	25.4%	41.1%	52.1%	59.6%	65.7%	70.1%	73.9%	77.4%
of final pay	**65%**	1.5%	10.5%	23.0%	36.7%	48.4%	56.4%	62.5%	67.5%	71.2%	74.7%
	70%	1.1%	8.8%	20.2%	31.9%	44.7%	53.7%	59.6%	65.0%	68.9%	72.1%
	75%	0.7%	7.4%	18.2%	27.9%	41.1%	50.2%	56.8%	62.1%	66.7%	70.1%
	80%	0.3%	6.2%	16.0%	25.4%	37.7%	47.2%	54.5%	59.6%	64.3%	68.1%
	85%	0.2%	5.0%	14.1%	23.6%	33.9%	44.1%	51.6%	57.2%	61.9%	66.0%
	90%	0.1%	4.2%	12.6%	21.4%	30.2%	41.1%	48.9%	55.1%	59.6%	63.8%
	95%	0.1%	3.7%	11.1%	19.8%	27.3%	38.4%	46.3%	52.6%	57.4%	61.6%
	100%	0.1%	3.3%	9.7%	18.2%	25.4%	35.2%	43.7%	50.2%	55.5%	59.6%

Figure 1. Time Series of Average Annual Returns for a 40 year invetment horizon beginning with Jan 1, 1926 to Dec 31, 1955; Feb 1, 1926 to Jan 31, 1956 and continuing to Jan 1, 1963 to Dec 31, 2002 (Invested 100% in S&P 500 Index)

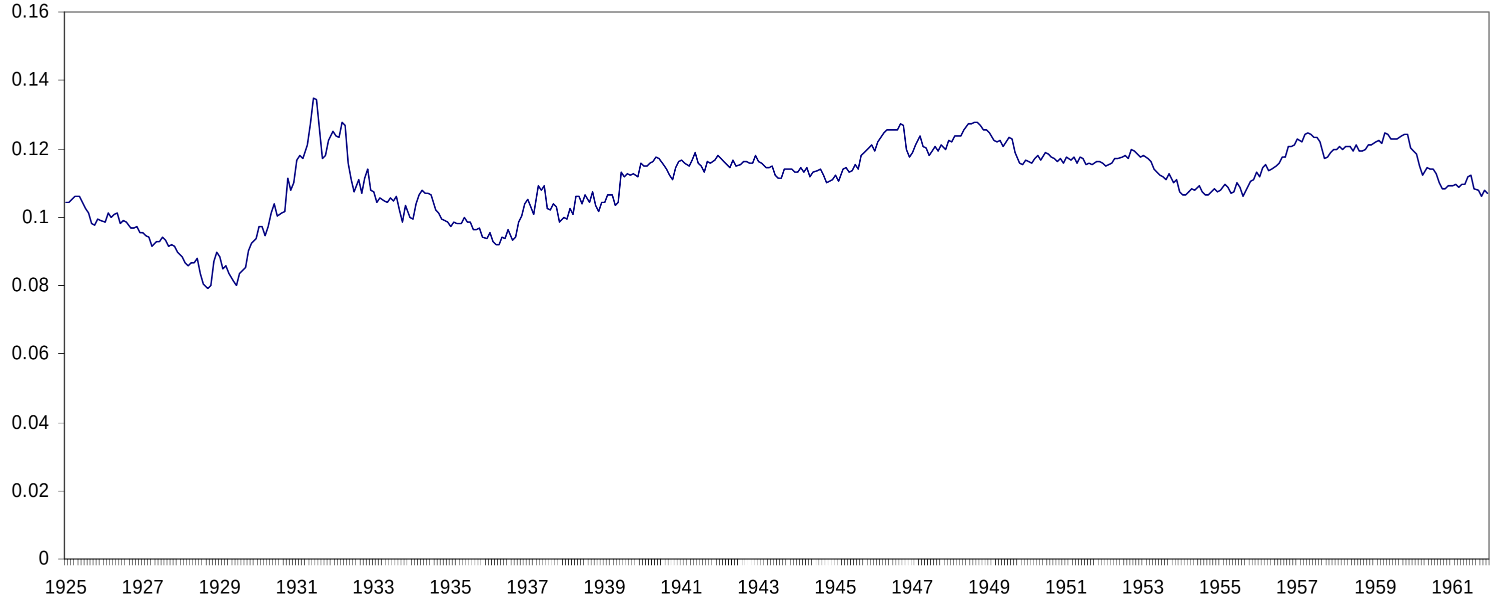


Figure 2. Accumulated wealth from $1 Monthly Contributions over a 40-year investment horizon per generation for a total of 444 generations computed two ways:

monthly contributions invested at the generation's geometric mean rate of growth and

monthly contributions invested at the generation's specific term structure of returns

(Invested 100% in S&P 500 Index)

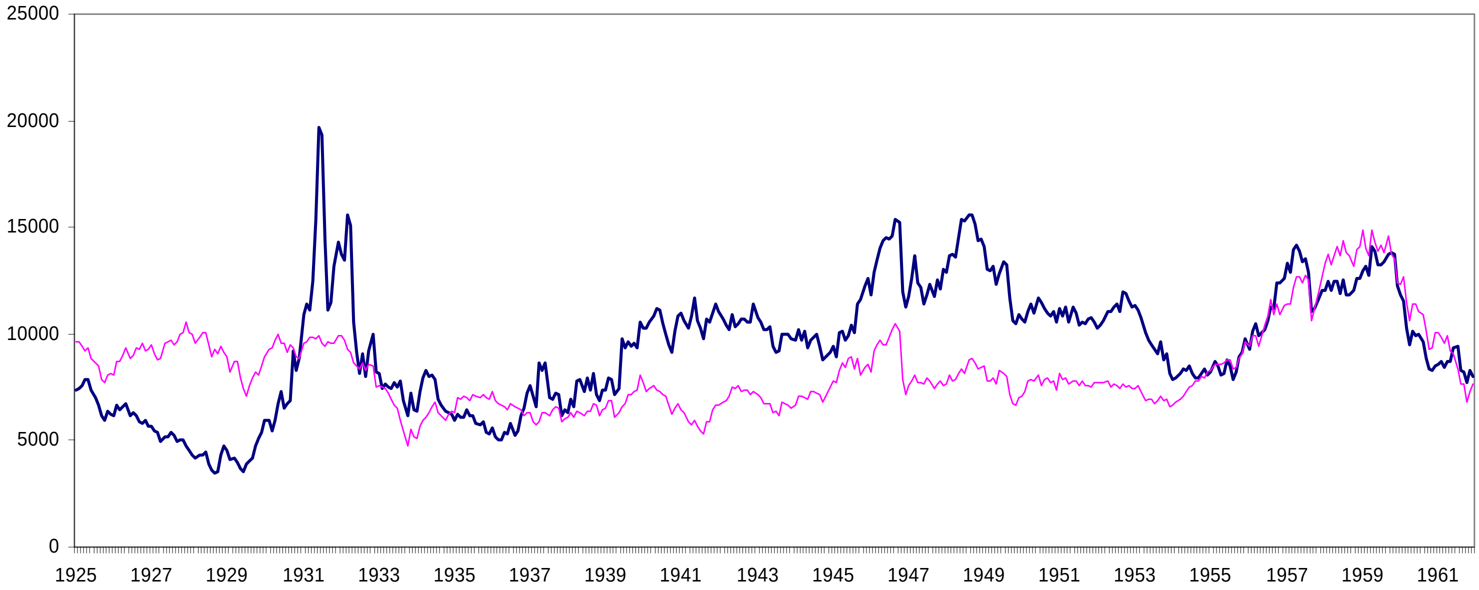


Figure 3. Accumulations from $1 Monthly Contribution (3% Inflation and 2% Productivity Adjusted) for 40 years earning S&P 500 nominal returns

First generation begins Dec 31, 1925 and ends Dec 31, 1965; last generation begins Dec 31, 1962 and ends Dec 31, 2002 (37 years x 12 months = 444 generations)

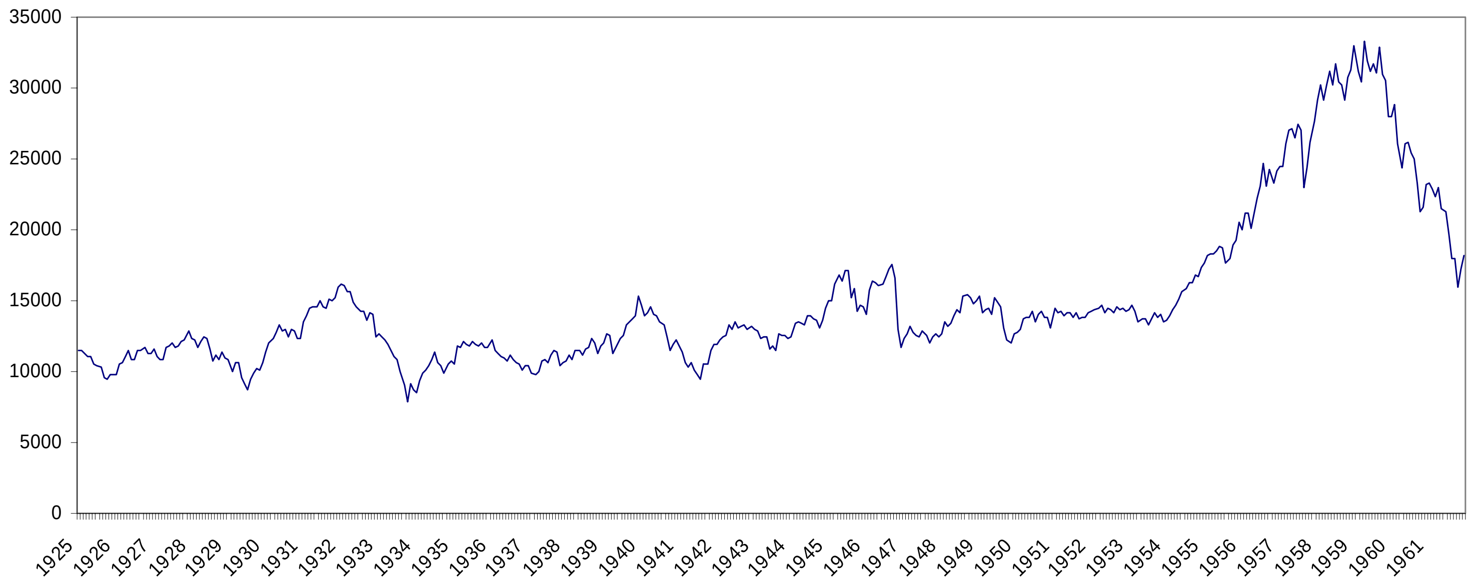


Figure 4. Accumulations from $1 Monthly Contribution (3% Inflation and 2% Productivity Adjusted) for 30 years earning S&P 500 nominal returns

First generation begins Dec 31, 1925 and ends Dec 31, 1955; last generation begins Dec 31, 1972 and ends Dec 31, 2002 (47 years x 12 months = 564 generations)

Figure 5. Distribution of Accumulations from $1 Monthly Contribution (3% Inflation and 2% Productivity Adjusted) for 40 years earning S&P 500 nominal returns

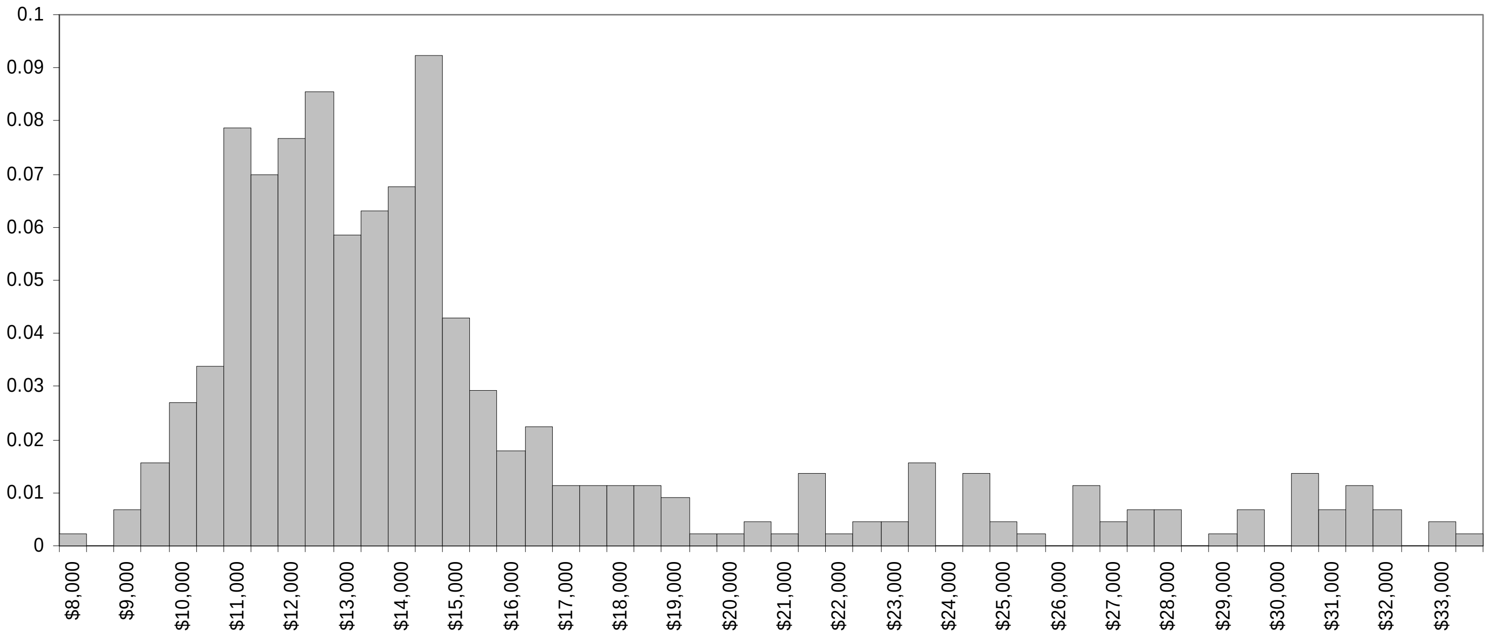
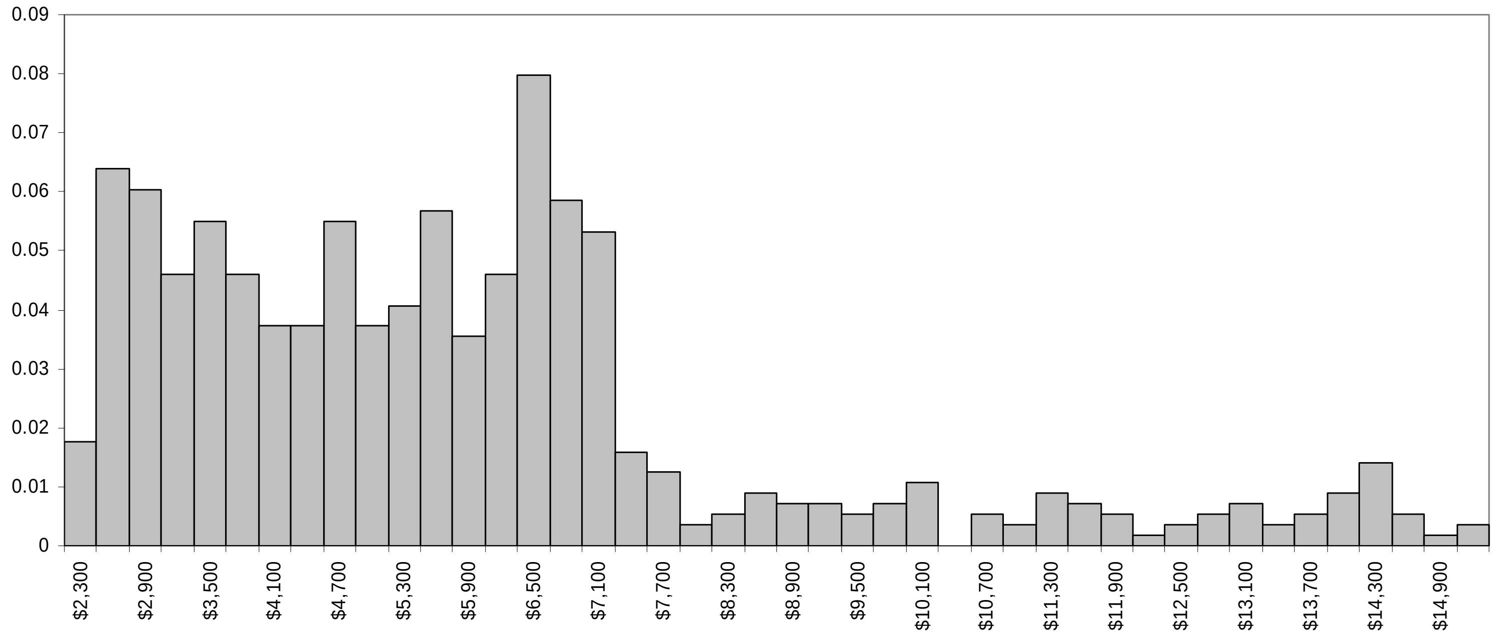


Figure 6. Distribution of Accumulations from $1 Monthly Contribution (3% Inflation and 2% Productivity Adjusted) for 30 years earning S&P 500 nominal returns



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