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Factors Affecting Arithmetical Achievement of Seventh Grade Students

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FACTORS AFFECTING ARITHMETICAL ACHIEVEMENT
OF SEVENTH GRADE STUDENTS

by
Henry Moughamian

A Dissertation Submitted to the Faculty of the Graduate School
of Loyola University in Partial Fulfillment of
the Requirements for the Degree of
Doctor of Education

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CHAPTER I

INTRODUCTION

Of the many challenges confronting educators today, a major one is that of improving the teaching of problem solving. For many years the teaching of problem solving has received much attention in the general literature, but still it persists in being the most troublesome aspect of the arithmetic program in both the elementary and the high schools. The importance of problem solving ability is evident in the voluminous amount of literature concerning some phase of this topic. There are those who would make problem solving the ultimate objective of any course in mathematics in the junior and senior high schools. As H. Van Engen states, "In its broadest sense, problem solving teaches attitudes of considerable value to a democratic society. It teaches the art of suspending judgment, carefully weighing facts, making value judgments, and arriving at conclusions on the basis of a preponderance of evidence."

Many pupils with average or above average intelligence fail to achieve as they should in problem solving. Others with lower ability often achieve to a greater degree. An attempt to identify any one cause of problem solving disability would be an oversimplification of a complex problem. Among those factors most frequently cited as contributing to problem solving disability are reading disability, mental deficiency, lack of adequate development of number concepts, the blocking of adjustments by emotional responses, the inability to sense quantitative relations, poor teaching methods and materials and over-sized classes. These factors will be discussed in greater detail in the following chapter.

Without a doubt there are many factors involved in problem solving ability. It is not within the scope of this study to deal with a great number of them. To say to what degree each factor affects pupils' achievement would be an impossibility without considering each individual case; even then, it would be a complex task.

The present study deals partially with the emotional and personal adjustment of children. To what degree do attitudes affect pupil achievement? We know that they do, but do they to a significant degree by which we could conclude that it would behoove teachers to spend more time in motivating and understanding pupils? Perhaps teachers help build negative attitudes. What role does the home play in the development of negative attitudes? Could it be that the various community agencies hold conflicting interests and
values? Grace Fernald found in one clinical study that of seventy-eight disability cases, only four had records of any disability before entering school.\(^2\) Were these acquired in school?

Teachers have many measuring instruments by which to judge pupil behavior. They have intelligence and achievement tests, personality inventories, self-rating scales, teachers' evaluations, anecdotal records and various community agencies such as psychological, correction and guidance services. However, because school services are so time consuming, and because private services such as psychological and psychiatric treatment are so costly, often pupils who need help fail to receive any at all. Parental emotional involvement makes the problem even more complex. It is hoped that this study will shed some light on means by which the classroom teacher, without clinical training, can play a more significant role in helping to provide better pupil personal and environmental adjustment, with one major purpose being to improve pupil achievement in problem solving.

Can a better understanding of the nature of problem solving ability help teachers to improve pupil achievement? Despite the fact that extensive research has been done on this subject, the relative effectiveness of various methods of teaching problem solving has not been determined. In this study the related literature

will be reviewed and the relationship between problem solving and various intellectual factors will be examined, with the hope that the results will suggest to teachers means of better diagnosing pupil difficulties in arithmetic, thus providing bases for more effective instruction.

The Purpose of the Study

It is, therefore, the purpose of this research to investigate the following problems:

1. What is the relative importance of certain abilities and aptitudes as factors in arithmetical problem solving ability?

2. What attitudinal factors significantly influence achievement in arithmetical problem solving ability?

It should be noted that the abilities, aptitudes and attitudes will be studied as objectively measured by the tests to be named in the following section. They will not be factors as defined in factor analysis.

Procedure

During the fall semester, 1961, the California Test of Personality was administered to 330 pupils in Grade 7B of District No. 2 in the Chicago Public Schools. In addition results on the California Test of Mental Maturity (Short-Form) and the Metropolitan Achievement Battery, as obtained from these same pupils in
grade 6A, were used. Of the original 355 pupils included in this study, complete information was gathered on 330.

Including twelve sub-tests, the California Test of Personality measures specific attitudes defined as types of tendencies to think, to feel and to act which reveal personal and social adjustments to life situations. (See Appendix I.) The reliability of this test is given as .88 as computed by the Kuder-Richardson formula. With respect to validity the Educational Research Bulletin of the New York City Schools carries this statement: "This procedure which is followed in the California Test of Personality is perhaps the most diagnostic of any test of this type." Taylor and Combs found a statistically significant difference between students who scored in the upper fifty percent and those who scored in the lower fifty percent on the California Test of Personality. Baker says, "There is often a theoretical but entirely invalid objection upon the part of those who have never used such tests [personality inventories] that children will not be truthful. It is generally known that children's problems are so close to their lives that they can scarcely refrain from answering what applies

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to them. This situation is similar to the quite universal tendency of most individuals to unburden themselves about their problems even to strangers if they are encouraged to talk about themselves. 5

The California Test of Mental Maturity consists of seven subtests which sample various kinds of mental processes to establish the level and rate of mental development. The seven tests contribute to scores in four factors, providing a Language I.Q., a Non-Language I.Q. and a Total I.Q. (See Appendix I.)

The Metropolitan Achievement Battery is comprised of ten subtests, measuring achievement in various school subject areas. (See Appendix I.) For the present study only three areas will be considered--arithmetical problem solving achievement, arithmetical computation achievement and reading achievement.

The product-moment correlation coefficient will be used to measure the relationships between the various factors included in the California Test of Mental Maturity and problem solving ability as measured by the Metropolitan Achievement Battery. The intercorrelations between the various factors and the correlations with arithmetical problem solving ability will be subjected to path coefficient analysis as a means of estimating the relative

contributions of the various aptitude and ability factors to individual differences in problem solving achievement.

In studying the relationships between the aptitude and attitudinal factors and arithmetical problem solving achievement, simple analysis of variance will be done for each attitudinal factor. Mental ages and attitude scores will be classified as high, average or low. The arithmetical problem solving stanine scores will be entered in the cells of $3 \times 3$ tables. It will thus be possible to ascertain how such an attitude as "self-reliance" is related to achievement for all the pupils studied and for pupils of differing levels of aptitude. For comparative purposes a similar analysis will be made with reference to the reading achievement stanine scores.
CHAPTER II

REVIEW OF THE RELATED LITERATURE

Factors Involved in Problem Solving Difficulties

Much research is available concerning the nature of problem solving disability. Those factors frequently mentioned are mental deficiency, reading difficulty, lack of development of number concepts, lack of interest and emotional maladjustment.

Those children with mental deficiency may be excusable failures in problem solving under proper learning conditions. Children of normal intelligence, however, possess the abilities that are essential in achieving success in solving problems. Fernald states: "There is no such thing as a child of normal intelligence who cannot do arithmetic."\(^1\)

Reading has often been cited as a reason why pupils fail to achieve in problem solving. A common suggestion and one supported by data is that training in reading will improve arithmetic achievement. Knight has shown that children's scores on a written-problem test may be raised as much as a full year through training.

\(^1\)Fernald, p. 213.
in reading without any training in arithmetic proper. The materials containing unrealistic problems have caused students to lose interest in problem solving. This seems to be a consensus among arithmetic experts as early as Thorndike who found that unrealistic problems made children lose confidence in themselves.

The lack of development of number concepts is another frequently mentioned reason for lack of success in problem solving. Traditional teaching methods, many using the atomistic approach of stimulus-response, have led to lack of understanding of number concepts. More emphasis is advocated upon understanding our number system and using numbers in a variety of realistic situations. The utilization of field psychology, which stresses understanding and the organization of learning through discovery of relationships and generalizations, is advocated by most mathematicians. Thorndike states:

The ordinary view of the nature of arithmetical learning is obscure or inadequate in four respects. It does not define what knowledge of the meaning of number is; it does not take account of the very large amount of teaching of language which is done and should be done as a part of the teaching of arithmetic; it does not distinguish between the ability to meet certain quantitative problems as life offers them and the ability to meet the

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problems provided by textbooks and courses of study; it leaves the ability to apply arithmetic knowledge and power as a rather mystical general faculty to be improved by some educational magic. ⁴

With the many intangibles involved in pupils' attitudes the relationship between attitudes and problem solving achievement is in doubt. The effect of interest has received much attention. For many years the development of favorable attitudes and interests has been an aim of the arithmetic program. Since interest is an emotional attitude, there is a positive relationship between interest and attitudes. The lack of interest has often been cited as an important reason for adverse pupil attitudes. Then the question arises as to the relationship between interest and ability. Interest in arithmetic and ability at arithmetic are probably correlated positively in the sense that the pupil who has more interest tends in the long run to have more ability. Wheat suggests that perhaps our efforts are partially futile because of the lack of immediate need of arithmetic on the part of the learner. ⁵

Speaking in reference to emotional difficulties in arithmetic, Fernald states: "Because the emotional problem is so serious, many investigators feel that emotional stability should be established before remedial work is attempted. It is often difficult to tell

⁴Ibid., 2.

which comes first, the failure or the emotional breakdown. Some children fail to learn because they are emotionally unstable; others become emotionally unstable because they fail to learn."⁶ With reference to failure, Fernald states: "A child who has failed repeatedly in arithmetic becomes so negatively conditioned with reference to it that he is unable to approach anything connected with number without an emotional response."⁷

In relation to arithmetic difficulties, Brueckner says: "It is one thing to discover them, but their treatment often presents a serious problem because the causes underlying them are complex and cannot easily be isolated. If certain factors within the learner himself are not taken into consideration in planning the arithmetic program, difficulties may develop. For instance, it is necessary to adjust to such items as the learner's mental level, physiological defects and handicaps, his background of experience, his attitudes and emotional reactions."⁸

Many educators and psychologists would maintain, within limits, that attitude is more important than aptitude in school work. Similarly, personality factors often seem to be even more important than abilities in determining how a student gets along

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⁶ Fernald, p. 7.

⁷ Ibid., 253.

in school and how he will get along in life. Often large differences in motivation compensate for small differences in aptitude.

Studies related to the role of personality in school achievement are inconclusive, many because of lack of proper controls. However, there are some relations between personality and achievement which are widely accepted as important. Blanchard illustrates this by cases demonstrating increased achievement in arithmetic through the development of positive emotional attitudes.\(^9\) Herriott controlled the effect of intelligence in order to study the effect of attitudes alone on scholastic success. He found that those students who possessed what he termed "evaluative and persevering attitudes" achieved greater success in school.\(^10\) Lecky studied the role attitudes play in arithmetic deficiency. He found drill in arithmetic less important than counseling or other experiences that lead to a change of attitude for those students who held a poor self-evaluative attitude.\(^11\)

The most frequently mentioned factors, outside of the learner, affecting achievement in arithmetic are the teacher and the size of the class. In the Fiftieth Yearbook of the National Society for

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\(^{11}\) P. Lecky, Self-Consistency: A Theory of Personality (New York, 1945), pp. 103-104.
Foster E. Grossnickle mentions the fact that many teachers' colleges fail to prepare prospective teachers adequately. Their poor background often becomes a source of difficulty for the pupil. Another handicap is the large size of classes. It has been shown that the range in ability in arithmetic at the seventh grade level is often between six and seven years. Brueckner states: "The most perplexing and difficult problem faced by the teacher is that of providing for the wide range of differences among the members of the class, their ability to learn mathematics, the rates at which they learn, the level to which skills have been developed, their interests, their attitudes toward the subject, and so on."

Intelligence and Problem Solving Ability

Contrary to Spearman's statement that "Many people of high general intelligence have low ability for mathematics," it is considered by most authorities that the relationship between problem solving ability and intelligence is highly positive, and that high intelligence is the most important single factor for success.


13 Brueckner, Grossnickle, and Reckzeh, p. 86.

in problem solving ability. Evidence by Wrigley also leads to the conclusion that there is a close connection between mathematical and general ability.\textsuperscript{15} This is also Thorndike's theory.\textsuperscript{16}

Engelhart employed the method of path coefficients to study the relationship between problem solving ability and four factors: (1) arithmetical problem scores, (2) intelligence scores, (3) arithmetical computation scores and (4) reading scores. He found that intelligence and computation ability are important factors in causing individual differences in problem solving.\textsuperscript{17}

A number of correlation studies have been conducted between the factors of intelligence, as measured by the various intelligence tests, and problem solving ability. Most of these studies have indicated that problem solving ability is related more closely to computational ability or number ability than any other factor. Phillips correlated the six tests of the Chicago Primary Mental Abilities with problem solving. He found the relationship between problem solving and arithmetical computation to be the

\begin{itemize}
  \item[16] Thorndike, pp. 51-69.
\end{itemize}
highest.\textsuperscript{18} This conclusion is also supported by the Thurstones in their discussion of number ability.\textsuperscript{19} In studying algebraic problem solving ability Kellar found algebraic computational facility to be the most important factor in algebraic problem solving ability.\textsuperscript{20} Thorndike states that the correlation between problem solving (part of the "g" factor of intelligence) and computation may approach .60.\textsuperscript{21} Sister Canisia studied mathematical ability and found a high correlation between the number factor and verbalization. She also found mathematical ability, the perception of relations, a part of intelligence.\textsuperscript{22}

It would seem to indicate that since studies point to a close relationship existing between number ability and problem solving, an improvement in one would improve the other. Coombs studied the nature of number ability. He found that the number factor was

\begin{itemize}
    \item \textsuperscript{19}L. L. Thurstone and T. G. Thurstone, \textit{Factorial Studies of Intelligence} (Chicago, 1941), p. 5.
    \item \textsuperscript{20}Wylma R. Kellar, "The Relative Contribution of Certain Factors to Individual Differences in Algebraic Problem Solving Ability," \textit{Journal of Experimental Education}, I (September 1929), 26-35.
    \item \textsuperscript{21}Thorndike, p. 299.
    \item \textsuperscript{22}Sister Mary Canisia Majewska, "A Study of Mathematical Ability as Related to Reasoning and Use of Symbols," Unpublished Doctoral Dissertation (Loyola University, Chicago, 1960).
\end{itemize}
most clearly identified by very simple number tests such as multiplying whole numbers.²³

In reference to the apparent relationship between intelligence and problem solving ability, and number ability and problem solving ability, Engelhart believes that instruction should be directed toward fostering whatever traits or abilities are relevant to problem solving and capable of modification.²⁴


CHAPTER III

STATISTICAL METHODS RELATIVE TO PREDICTION

Educators and psychologists use many statistical methods by which they hope, objectively and ultimately, to arrive at evaluation and prediction of human behavior. Since the origin of statistical methods relative to prediction, about the beginning of the seventeenth century, numerous methods have been devised by which we strive to predict human behavior. It is not within the scope of this research to discuss a great number of them, but only those that have a bearing on the present topic.

In dealing with one variable the fundamental statistical procedure that we use usually includes measures of central tendency, such as the mean, the median and the mode, and also measures of variability such as the standard deviation. But often science demands answers to questions whereby we need to study the relationships between variables. In trying to predict one phenomenon from another, we must try to determine how the one depends upon the other. We seek to find cause and effect, and furthermore, how a change in one is accompanied by a corresponding change in the other. Whenever it is possible, this relationship is stated in quantitative terms. This is possible when both cause and effect
are continuously variable and subject to measurement. In essence we seek a functional relationship between two variables. In seeking to find the above relationships, one of the methods used is correlation analysis.

In this chapter the writer will attempt to present a brief description of certain procedures which can be utilized in studying relationships. These include the coefficient of correlation, the tests of its significance, the t test for means and differences between means, and the analysis of variance including the F test of significance.

One of the most frequently used statistics in psychological and educational research is Pearson's product-moment correlation coefficient. It measures the degree of linear relationship between two variables which range in value from 1.00 (a perfectly positive relationship) to -1.00 (a perfectly negative relationship). A correlation of zero indicates no relationship.

Often in the physical sciences correlations may approach 1.00 because the variables involved may be experimentally controlled and precisely measured; this is practically impossible in education, and therefore the correlation coefficients that we obtain are very seldom above .90. Of the two variables that we study, one we designate as the dependent variable and the other as the independent variable, even though there may be no clear cut basis for designating them as such. In dealing with more than two variables, for example, $X_1$, the dependent variable, and $X_2$, $X_3$
... $X_1$, the independent variables, we can, through the use of partial correlation, estimate the net relationship between $X_1$ and $X_2$, or other independent variables, with the rest of the independent variables held constant. On the other hand the correlation between $X_1$ and the combined effect of two or more independent variables is measured by coefficients of multiple correlation. Walker traces excellently the history of these techniques in her book entitled Studies in the History of Statistical Methods.¹

In dealing with an independent variable, $X$, and the dependent variable, $Y$, one of our problems is to find the line of best fit that relates $Y$ to $X$. Very seldom, if ever, do we get a straight line when we plot our values of the variables graphically. Usually we have the values scattered about a straight line. The line that best fits these scattered values is called the regression line of $Y$ on $X$, and the equation for the line is called a regression equation. If we do not know which of the variables is the independent one, then we would also try to predict $X$ from $Y$, and therefore we would have two regression equations, $Y$ on $X$ and $X$ on $Y$. For an extensive discussion of linear regression and regression equations, see Statistical Methods for the Behavioral Sciences by Allen L. Edwards.²


In dealing with correlation coefficients, the sampling problem differs from the sampling problem in the case of such a statistic as the mean. The higher the correlation in the universe, the greater the skewness in the sampling distribution. For example, if the true correlation is .90, sample correlation can range far below .90, but can range above .90 only to 1.00. Hence, observed correlation coefficients are transformed to Fisher's $z'$ coefficients, since the mathematical model of these is a normal distribution before application is made of $t$ tests to evaluate their significance. We may also use the $z'$ transformation to establish the fiducial limits of the parameter, at some defined significance level. Here we establish an interval so that we can say we have a certain degree of confidence that the interval contains the population correlation. For a complete discussion of one and two tailed tests of significance as employed in significance tests for the correlation coefficient, see Experimental Design in Psychological Research by Allen L. Edwards.³

circumference of a circle relates entirely to its diameter. This is not the case of relationships between educational or psychological variables. Lack of controls and errors can mask a perfect relationship. A coefficient of correlation can be used to indicate the extent to which the variation in one variable determines the observed variation that we find in the other variable. Thus, if one variable on the basis of other evidence is assumed to be causally related to another, the coefficient of correlation is a means of estimating its importance as a causal factor. The standard error of estimate tells us how reliable our predictions are; the coefficient of correlation tells us how strong the relationship is. Often we work with two regression equations when we cannot classify our variables as dependent and independent. Therefore, we have Y regressing upon X, and X regressing upon Y. These values are called the regression coefficients and merely tell us how much a unit change in Y is accompanied by a unit change in X, and how much a unit change in X is accompanied by a unit change in Y. Using the regression equations, if the X measurements and Y measurements are given in terms of standard scores, then r (correlation coefficient) becomes the slope of the regression line. In this case it is known as a beta, or standard regression coefficient.

The methods of path coefficients, part correlation, beta coefficients and ordinary regression coefficients can be considered to constitute one family of techniques for analyzing data with each
method focusing attention on specific details: path coefficients emphasizing the variance interpretation; part correlation connecting the betas for the variability in the dependent variable unexplained by the combined influences of all the factors; beta coefficients, or ordinary regression coefficients, expressing the relative weights to be given each independent variable when the multiple regression equation is used in predicting values of the dependent variable.

The theory of path coefficients was first developed in 1921 by Sewall Wright, who applied it in his agriculture and animal studies. The equivalence of path coefficients and beta coefficients has been demonstrated by Kelly to be true for all problems. Later Engelhart established the fact that path coefficients are identical with beta coefficients by employing in his proof the semi-partial correlations given by Dunlap and Cureton. Burks and

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4 Sewall Wright, "Correlation and Causation," Journal of Agricultural Research, XX (January 1921), 557-585.

Sewall Wright, "The Theory of Path Coefficients," Genetics, VIII (May 1923), 238-255.


Heilman were the first in the field of education to apply the methods of path coefficients. In terms of standard deviations, a path coefficient is defined as the ratio of that part of the standard deviation of a variable which is due to another variable to the total standard deviation of the variable. Symbolically a path coefficient may be expressed as follows:

\[ p_{ij} = \frac{\sigma_{ij}}{\sigma_i} \]

\( p_{ij} \) = path coefficient for variable i and j.

\( \sigma_{ij} \) = standard deviation of variable, one due to the jth variable.

\( \sigma_i \) = standard deviation of variable i.

The \( \sigma_{ij} \) is a partial standard deviation—a measure of that part of the variation of one variable due to another, with other variables controlled.


8Engelhart, Psychometrika, pp. 288-290.
In terms of correlation coefficients the fundamental idea of path coefficients may be stated as follows. Let \( X_1 \) be the dependent variable, and \( X_2 \ldots X_n \) be independent variables. Then the correlation coefficient between \( X_1 \) and any of the \( X_i \) (any independent variable), or between any two such as \( X_1 \) and \( X_j \) (any two independent variables), is equal to the path coefficient connecting the two variables plus the sum of the products of the path coefficients along all paths of indirect connection, not including those through the dependent variable. Once the equations for the path coefficients are set up in terms of zero-order correlation coefficients, the set can be solved for the path coefficients considered as the unknown. Following a suggestion made by Engelhart, Monroe and Stuit have shown that the solution of the equations for the values of the path coefficients is mathematically the same problem as the solution of the normal equations for the regression coefficients.\(^9\)

It should be noted that the estimates of the relative importance of different independent variables obtained by the method of path coefficients are but estimates. When there are several measures of each of several skills or abilities considered as causal, factor analysis is probably the more appropriate technique. Engelhart states, "When the variables studied are few in number and the

interpretation will be more meaningful in terms of these variables rather than in terms of hypothetical orthogonal factors, the path coefficient technique may be preferred. Where variables may be measured with greater validity than in the fields of psychology or education—for example, in the field of economics—the technique of path coefficients may be more appropriate than the technique of factor analysis in studying certain problems. 10

When we are working with two means, if we wish to find if there is a significant difference between them, we can apply the t test. By formula and with the calculation of the standard error of the difference between two means, we can formulate the null hypothesis that there is no difference between our means. By finding the value of t and establishing confidence limits, we can ascertain whether the difference between the two means is significant, that is, not attributed to chance. If so, we can reject our null hypothesis and say that these two means do not come from the same population.

The t test is adequate if we have only a few means to compare, but often we have a number of groups involved in a study, and this method would be too laborious. The analysis of variance permits us to test differences among a group of means at the same time. The analysis of variance concerns itself with variances rather than

with standard deviations and standard errors. The rationale of
the analysis of variance is that the total sum of squares of a set
of measurements composed of several groups can be analyzed or
broken down into specific parts, each part identifiable with a
given source of variation. In the simplest case the total sum of
squares is analyzed into two parts: a sum of squares based upon
variation within the several groups, and a sum of squares based
upon the variation between the group means. Then, from these two
sums of squares independent estimates of the population variance
are computed. With these two variances, the value of $F$, tabled in
convenient form by G. W. Snedecor, can be calculated, and similarly, as with the $t$ test, our data can be evaluated to discover
whether they include significant differences or not.

The analysis of variance was largely the accomplishment of
Sir R. A. Fisher, a brilliant British statistician. Concerning
the analysis of variance, he had the following to say:

We were together learning how to use the analysis of
variance, and perhaps it is worth while stating an impres­
sion that I have formed—that the analysis of variance,
which may perhaps be called a statistical method, because
that term is a very ambiguous one—is not a mathematical
theo­ren, but rather a convenient method of arranging the
arithmetic. Just as in arithmetical text-books—if we
can recall their contents—we were given rules for
arranging how to find the greatest common measure, and
how to work out a sum in practice, and were drilled in
the arrangement and order in which we were to put the
figures down, so with the analysis of variance; its one
claim to attention lies in its convenience. It is con­
venient in two ways: (1) because it brings to the eyes
and to the mind a summary of a mass of statistical data
in which the logical content of the whole is readily
appreciated. Probably everyone who has used it has found that comparisons which they have not previously thought of may obtrude themselves, because there they are, necessary items in the analysis. (2) Apart from aiding the logical process, it is convenient in facilitating and reducing to a common form all the tests of significance which we may want to apply. I do insist that its claim to attention rests essentially on its convenience. Nearly always we can, if we choose, put our data in other forms and other language. Naturally, like other logical arrangements, it is based on mathematical theorems previously proved, and in particular the tests of significance were based on problems of distribution the solution of which was published for the most part from 1921 to 1924.\footnote{Edwards, Experimental Design, pp. 117-118.}

The present discussion of statistical techniques is by no means a very thorough one. The writer advises anyone desiring a more complete explanation to consult the bibliographical sources relevant to statistical procedure. It was the intention of the writer to merely convey to the reader a fairly simple explanation of some of the techniques that will be used in the following chapters, without going into many formulas, definition of statistical terms, and the use of too many symbols. An excellent account of most of the methods described can be found in Dixon and Massey's Introduction to Statistical Analysis.\footnote{Wilfred J. Dixon and Frank J. Massey, Jr., Introduction to Statistical Analysis (New York, 1951).}
CHAPTER IV

COLLECTION AND INTERPRETATION OF THE DATA

The data for this study were secured with the aid of personnel from four Chicago elementary schools located in Rogers Park, a community of Chicago. Complete data were obtained for 330 pupils, all in Grade 6A. Since almost all of the chronological ages fell within a range of two years, this factor was eliminated as an important consideration. Table I on page 29 shows by the use of stanine scores (one being the lowest and nine being the highest, with a mean of five) the relative position of the group in relation to national norms following the normal curve. The actual I.Q.'s and problem solving achievement scores in each stanine represent the performance of this study's 330 pupils in contrast to the theoretical number in each stanine, which would have been the case if the data of the group had conformed to a normal distribution.

The results obtained for this study were from the California Test of Mental Maturity (Short Form), the California Test of Personality and the Metropolitan Achievement Battery. These tests were administered by the adjustment teachers of each school and machine scored, with the exception of the California Test of Personality, which was scored by the writer on scoreze answer sheets.
<table>
<thead>
<tr>
<th>Stanine</th>
<th>Theoretical in Stanine</th>
<th>Actual I.Q.'s</th>
<th>Actual Problem Solving</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>f</td>
<td>%</td>
<td>f</td>
</tr>
<tr>
<td>9</td>
<td>13.2</td>
<td>4</td>
<td>68</td>
</tr>
<tr>
<td>8</td>
<td>23.1</td>
<td>7</td>
<td>55</td>
</tr>
<tr>
<td>7</td>
<td>39.6</td>
<td>12</td>
<td>72</td>
</tr>
<tr>
<td>6</td>
<td>56.1</td>
<td>17</td>
<td>63</td>
</tr>
<tr>
<td>5</td>
<td>66.0</td>
<td>20</td>
<td>49</td>
</tr>
<tr>
<td>4</td>
<td>56.1</td>
<td>17</td>
<td>16</td>
</tr>
<tr>
<td>3</td>
<td>39.6</td>
<td>12</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>23.1</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>13.2</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

Part of the handling of the data consisted in transposing the raw scores of the tests into stanine scores. The stanine scale constitutes a method of grouping scores into classes which are broad enough to permit use of a single digit to represent each class, but precise enough for many practical and simple statistical purposes.

One problem confronting the writer was that of grouping the mental age and attitudinal scores into categories of high, average and low. Even though a student was classified in the low group, his score might warrant his being in the average or high group
based on national norms. This was often the case. Since for the
present study the purpose was to ascertain the pupil's relative
position in the group studied, this factor was overlooked. Another
related problem was that in order to obtain equal or proportional
numbers of cases on each level, the splits could not be made using
arbitrary or inflexible mental age cutting scores. Also, in making
the splits, the frequencies could not be split. In grouping the
mental age scores, for example, seventy-six cases were needed for
the low ability group. Since sixty-nine cases were already in-
cluded and ten fell at the next level, even though only seven more
cases were needed, all ten were included. Conversely, if only
three or four cases were needed and ten fell at the next level,
none were added, and these ten cases constituted the lower limit
of the average group.

After the mental age and attitudinal scores had been arranged
in the cells of 3 x 3 tables, the arithmetical problem stanine
scores were entered. For comparative purposes the same was done
with the reading stanine scores. From the tabulated scores in each
of the nine cells for each attitude, using a table of random num-
bers, ten scores were selected from the outer cells and twenty from
the middle cells, totaling 140 cases. Even though each cell did
not contain the same number of cases, the cells were proportional
and thus enabled us to use a two-part analysis of variance schema.
This method was used to test the null hypotheses that (1) there
were no differences in the means of the total groups, and (2) there
was no interaction between mental age and attitude levels. The t
test was also used to compare individual means. The product moment
correlation coefficient was used to determine the relationships
between problem solving ability and those attitudes significant at
the 1% level. These results are given in Table XI on page 45.

With regard to the second purpose of this study—determining
the relationships between various intellectual factors and problem
solving ability—correlation coefficients, using the scores of all
330 subjects, were obtained between problem solving, the dependent
variable, and the four factors measured by the California Test of
Mental Maturity: spatial relations, verbal ability, numerical
reasoning and logical reasoning. Arithmetical computation consti-
tuted the fifth independent variable. The term dependent variable
is used in the sense that it is considered to be determined by
several other variables, the independent variables, even though
they may not be statistically independent, i.e., uncorrelated. The
use of the term independent variable is not intended to imply per-
fect determination of the dependent variable by a single valued
mathematical function in the sense generally attributed to it by
mathematicians.

From the intercorrelations between the variables, beta coef-
cficients, which are equivalent to path coefficients, were obtained.
Thus, it was possible to estimate the relative contribution of
each independent variable to the dependent variable.
The terms used herein will coincide with those generally accepted by most statisticians. As previously mentioned, the terms abilities, aptitudes and attitudes will not be factors as defined in factor analysis. The terms cells and groups will be used synonymously, as will mental age and ability.

The Data

Attitudes and Achievement

The following data present a quantitative picture of the relationship between arithmetical problem solving achievement and attitudes, both personal and social, and also a description of the intellectual aspect of problem solving ability. It is desired that from this statistical presentation quantitative inferences may be drawn so as to enable educators to identify more readily those pupils who will meet greater success, or be handicapped in problem solving achievement, because of their attitudes and their intellectual abilities.

It should be noted that with each attitude that will be discussed, the mental age levels and cells are significant at the 1% level, and the interactions between mental age levels and attitudinal levels are non-significant. With regard to the significant findings, since we know that achievement is substantially related to intelligence as measured by typical intelligence tests including group tests yielding mental ages, these results are not surprising.
When this type of analysis of variance is applied to an experiment where pupils of varying levels of ability are instructed by differing methods, a significant interaction indicates a relationship between methods and levels of ability. For example, one method may be more effective for superior pupils, while another method may be more effective for less able pupils. In this study a significant interaction would tend to indicate, for example, that a desirable attitude may compensate for low intelligence as measured. This does not occur. It seems evident that both capacity and attitude tend to contribute concomitantly to achievement. This is revealed by study of the analysis of variance data as reported in Tables III through X, and by the correlations between certain of the attitudes and achievement and between mental age and achievement as shown in Table XI on page 45. An illustration of the two-part analysis of variance that was calculated for each attitude in relation to problem solving achievement and reading is given in Appendix II.

Data for the twelve attitudes, total personal adjustment, total social adjustment and total adjustment are reported in Table II on page 34.

1. Feeling of Self-reliance. Observing the attitude self-reliance, we see that it is significant at the 1% level, having an F ratio of 4.90 and needing an $F = 4.78$ to be significant at the 1% level.
## TABLE II

**ANALYSIS OF VARIANCE RESULTS PERTAINING TO ATTITUDE LEVELS AND PROBLEM SOLVING**

<table>
<thead>
<tr>
<th>Attitude</th>
<th>Sums of Squares</th>
<th>Degrees of Freedom</th>
<th>Variance</th>
<th>F</th>
<th>Level of Significance^a</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-reliance</td>
<td>23.69</td>
<td>2</td>
<td>11.85</td>
<td>4.90</td>
<td>.01</td>
</tr>
<tr>
<td>Personal Worth</td>
<td>27.59</td>
<td>2</td>
<td>13.80</td>
<td>5.73</td>
<td>.01</td>
</tr>
<tr>
<td>Personal Freedom</td>
<td>7.40</td>
<td>2</td>
<td>3.70</td>
<td>1.52</td>
<td>Non-sig.</td>
</tr>
<tr>
<td>Belonging</td>
<td>19.84</td>
<td>2</td>
<td>9.92</td>
<td>3.11</td>
<td>.05</td>
</tr>
<tr>
<td>Withdrawing Tend.</td>
<td>24.22</td>
<td>2</td>
<td>12.11</td>
<td>5.11</td>
<td>.01</td>
</tr>
<tr>
<td>Nervous Symptoms</td>
<td>9.16</td>
<td>2</td>
<td>4.58</td>
<td>1.63</td>
<td>Non-sig.</td>
</tr>
<tr>
<td>Total Pers'1 Adj.</td>
<td>21.39</td>
<td>2</td>
<td>10.70</td>
<td>4.63</td>
<td>.05</td>
</tr>
<tr>
<td>Social Standards</td>
<td>1.34</td>
<td>2</td>
<td>.67</td>
<td>.31</td>
<td>Non-sig.</td>
</tr>
<tr>
<td>Social Skills</td>
<td>2.24</td>
<td>2</td>
<td>1.12</td>
<td>.43</td>
<td>Non-sig.</td>
</tr>
<tr>
<td>Anti-social Tend.</td>
<td>5.17</td>
<td>2</td>
<td>2.59</td>
<td>1.35</td>
<td>Non-sig.</td>
</tr>
<tr>
<td>Family Relations</td>
<td>22.67</td>
<td>2</td>
<td>11.34</td>
<td>4.28</td>
<td>.05</td>
</tr>
<tr>
<td>School Relations</td>
<td>27.71</td>
<td>2</td>
<td>13.86</td>
<td>5.11</td>
<td>.01</td>
</tr>
<tr>
<td>Community Rel.</td>
<td>.27</td>
<td>2</td>
<td>.14</td>
<td>.05</td>
<td>Non-sig.</td>
</tr>
<tr>
<td>Total Social Adj.</td>
<td>8.32</td>
<td>2</td>
<td>4.16</td>
<td>1.83</td>
<td>Non-sig.</td>
</tr>
<tr>
<td>Total Adj.</td>
<td>30.30</td>
<td>2</td>
<td>15.15</td>
<td>6.45</td>
<td>.01</td>
</tr>
</tbody>
</table>

^a F of 4.78 needed for 1% level and 3.07 for 5% level with 2 & 131 degrees of freedom.
Table III gives the total stanine scores and means (in parentheses) of each of the nine groups. For comparative purposes the writer has taken the liberty of dividing the scores of the average level by two, since twice as many cases constitute these groups. We can observe that the averages decrease both ways, indicating the differences in the groups. Considering only the high mental age level, we can also see that for each attitude level there are differences indicating that self-reliance does affect achievement. Our F ratio of 4.90 enables us to reject the null hypothesis as untenable that no differences exist between the groups in achievement with respect to self-reliance.

**TABLE III**

**PROBLEM SOLVING SCORES IN RELATION TO SELF-RELIANCE AND MENTAL AGE**

<table>
<thead>
<tr>
<th>Self-reliance Levels</th>
<th>Mental Age Levels</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High</td>
<td>Average</td>
<td>Low</td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>69 (6.9)</td>
<td>53 (5.3)</td>
<td>36 (3.6)</td>
<td></td>
</tr>
<tr>
<td>Avg.</td>
<td>64 (6.4)</td>
<td>50 (5.0)</td>
<td>32 (3.2)</td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>58 (5.8)</td>
<td>41 (4.1)</td>
<td>28 (2.8)</td>
<td></td>
</tr>
</tbody>
</table>

2. **Sense of Personal Worth.** Do those pupils who possess a feeling of being well-regarded by others and who have confidence in themselves tend to achieve more? Table II shows a significance
at the 1% level between those pupils possessing high, average and low sense of personal worth as measured. Again needing an F ratio of 4.73 for the 1% level, we have an F = 5.73 for the attitudinal variable. Disregarding the fact that in Table IV on page 37 those pupils with an average sense of personal worth in the high ability level scored higher than those in the high attitudinal level, we again have the scores decreasing as attitude and ability decrease. Contrasting only the high personal worth level with the low personal worth level, we can see that the pupils with a high sense of personal worth in the same mental age levels have scored more than one stanine higher in the high and average ability levels and almost one stanine (.9) higher in the low ability level than those pupils with a low sense of personal worth. Therefore, we should consider a sense of personal worth as contributing to achievement in problem solving.

3. Personal Freedom. We may observe from Table II that our F ratio of 1.52 is non-significant, indicating a lack of strong relationship between problem solving achievement and a pupil's feeling that he is permitted a reasonable share of responsibility for making his own decisions. Even though the high attitude level had means .4, .55 and .1 higher than the low attitude level, we are not able to substantiate our hypothesis that a sense of personal freedom affects significantly the problem solving achievement of seventh grade pupils.
### Table IV

**Problem Solving Scores in Relation to Personal Worth and Mental Age**

<table>
<thead>
<tr>
<th>Personal Worth Levels</th>
<th>Mental Age Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td>High</td>
<td>67 (6.7)</td>
</tr>
<tr>
<td>Avg.</td>
<td>68.5 (6.85)</td>
</tr>
<tr>
<td>Low</td>
<td>58 (5.8)</td>
</tr>
</tbody>
</table>

4. **Feeling of Belonging.** This attitude implies that one is liked by his classmates and feels secure in his relations with his family. Table II shows that this attitude is significant at the 5% level. Table V on page 38 shows that the average attitude cells for the high and average mental age levels achieved more than the high attitude cells. Once again, the high feeling of belonging levels show greater achievement than the low levels, respectively, for each mental age level. The greatest difference lies in the low ability group where those pupils with a high feeling of belonging scored 1.1 stanines higher than those pupils with a low feeling of belonging.
TABLE V

PROBLEM SOLVING SCORES IN RELATION TO BELONGING AND MENTAL AGE

<table>
<thead>
<tr>
<th>Belonging Levels</th>
<th>Mental Age Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td>High</td>
<td>65 (6.5)</td>
</tr>
<tr>
<td>Avg.</td>
<td>70 (7.0)</td>
</tr>
<tr>
<td>Low</td>
<td>63 (6.3)</td>
</tr>
</tbody>
</table>

5. **Withdrawing Tendencies.** Table II indicates at the 1% level that pupils who are withdrawn achieve less than those pupils who are not withdrawn. Consulting Table VI on page 39 we see that the means decrease in both directions. In comparing the high and low attitudinal levels, the high groups have scores more than one stanine higher than the low groups. We conclude that pupils possessing withdrawing symptoms tend to be handicapped in problem solving achievement.

6. **Nervous Symptoms.** Table II indicates that pupils who do not exhibit nervous symptoms do not significantly achieve more than those who do exhibit overt signs of being nervous. Even though there is a trend for those demonstrating nervous symptoms to achieve less, since the means decrease in most of the cells, the differences are not significant at the 1% or 5% levels. The
largest difference occurs in the high and low mental ability groups where in both cases the high attitude groups scored .7 stanine higher than the low attitude group. Therefore, even though a difference does exist, we would have to question whether pupils possessing nervous symptoms achieve less than those who do not.

**TABLE VI**

**PROBLEM SOLVING SCORES IN RELATION TO WITHDRAWING TENDENCIES AND MENTAL AGE**

<table>
<thead>
<tr>
<th>Withdrawn Tendencies Levels</th>
<th>Mental Age Levels</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High</td>
<td>Average</td>
<td>Low</td>
</tr>
<tr>
<td>High</td>
<td>70 (7.0)</td>
<td>53 (5.3)</td>
<td>36 (3.6)</td>
</tr>
<tr>
<td>Avg.</td>
<td>62.5 (6.25)</td>
<td>46.5 (4.65)</td>
<td>33 (3.3)</td>
</tr>
<tr>
<td>Low</td>
<td>58 (5.8)</td>
<td>41 (4.1)</td>
<td>28 (2.8)</td>
</tr>
</tbody>
</table>

7. **Total Personal Adjustment.** We receive an indication from Table II that one's total personal adjustment does play a significant role in problem solving achievement. The F ratio of 4.63 is significant at the 5% level (3.07 needed). Table VII on page 40 shows that the lower one's personal adjustment is, the lower the achievement. The use of the t test shows the most significant difference to exist in the low mental age group, where the high total personal adjustment group achieved 1.8 stanines higher than
the low group. This difference, $t = 2.65$, is significant at the 1% level. With 131 degrees of freedom a $t$ of 2.62 is needed for the 1% level. This significant difference may indicate that a pupil's total personal adjustment does play an important part in problem solving achievement, particularly with those pupils possessing low mental ability.

### TABLE VII

PROBLEM SOLVING SCORES IN RELATION TO TOTAL PERSONAL ADJUSTMENT AND MENTAL AGE

<table>
<thead>
<tr>
<th>Mental Age Levels</th>
<th>High</th>
<th>Average</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Personal Adjustment Levels</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High</td>
<td>67 (6.7)</td>
<td>50.5 (5.05)</td>
<td>43 (4.3)</td>
</tr>
<tr>
<td>Avg.</td>
<td>69 (6.9)</td>
<td>46 (4.6)</td>
<td>31 (3.1)</td>
</tr>
<tr>
<td>Low</td>
<td>59 (5.9)</td>
<td>43 (4.3)</td>
<td>25 (2.5)</td>
</tr>
</tbody>
</table>

8. Social Standards. We receive the impression from Table II that the strength of pupils' social standards, i.e., the subordinating of personal desires to the needs of the group and the tendency to understand what is right or wrong, does not influence problem solving achievement. The means in each ability level are almost identical except in the low ability group where the high attitude group obtained scores .7 stanine higher than the low
attitude group. The use of the t test shows this difference to be non-significant.

9. **Social Skills.** Does the tendency to act in accordance with accepted social standards and to be sensitive to the needs of others affect achievement? The results are very similar to those of social standards; they are non-significant, as shown in Table II.

10. **Anti-social Tendencies.** We may gather from Table II that no significant differences exist between the attitudinal levels with respect to pupils' feelings of hostility and achievement. The \( F \) ratio of 1.35 does not approach even the 5% level of significance. The difference between the high and low attitude groups in the high ability group is 1.3 stanines. In the low ability level the stanine difference is 1.2. From these results we could hypothesize that in these ability levels anti-social tendencies might affect achievement.

11. **Family Relations.** With an \( F \) ratio of 4.78 (1% level) and 3.07 (5% level) needed, our obtained \( F \) of 4.28 in Table II indicates that a pupil's relations at home do affect problem solving at the 5% level. Analyzing Table VIII on page 42 we see the largest difference to exist in the high ability group where the difference between the high and low attitude groups is 1.6 stanines. Even though some of the differences are small, the means show that the higher the attitude scores are, the higher the achievement scores are. These results do indicate better progress of pupils with good family relations.
12. **School Relations.** The obtained $F$ of 5.11 in Table II shows that school relations do play a part in achievement. While not too far beyond the needed ratio of 4.78 for the 1% level, our obtained ratio is significant at the 1% level. The differences between the high and low attitude groups for the high, average and low ability groups, as shown in Table IX on page 43, are 1.1, 1.15; and .8 stanines, respectively.

13. **Community Relations.** This attitude shows the lowest $F$ ratio, .05. There are very few differences among the means, except in the high ability group where those with low community relations scores did better (one stanine) than those with high community relations scores. Thus, a pupil's relationship with his community does not seem to affect achievement, either positively or negatively.

### TABLE VIII

PROBLEM SOLVING SCORES IN RELATION TO FAMILY RELATIONS AND MENTAL AGE

<table>
<thead>
<tr>
<th>Family Relations Levels</th>
<th>Mental Age Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High</td>
</tr>
<tr>
<td></td>
<td>75.5 (7.5)</td>
</tr>
<tr>
<td>High</td>
<td></td>
</tr>
<tr>
<td>Avg.</td>
<td>63 (6.3)</td>
</tr>
<tr>
<td>Low</td>
<td>59 (5.9)</td>
</tr>
</tbody>
</table>

12. **School Relations.** The obtained $F$ of 5.11 in Table II shows that school relations do play a part in achievement. While not too far beyond the needed ratio of 4.78 for the 1% level, our obtained ratio is significant at the 1% level. The differences between the high and low attitude groups for the high, average and low ability groups, as shown in Table IX on page 43, are 1.1, 1.15 and .8 stanines, respectively.

13. **Community Relations.** This attitude shows the lowest $F$ ratio, .05. There are very few differences among the means, except in the high ability group where those with low community relations scores did better (one stanine) than those with high community relations scores. Thus, a pupil's relationship with his community does not seem to affect achievement, either positively or negatively.
TABLE IX

PROBLEM SOLVING SCORES IN RELATION TO SCHOOL RELATIONS AND MENTAL AGE

<table>
<thead>
<tr>
<th>School Relations Levels</th>
<th>Mental Age Levels</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High</td>
<td>Average</td>
<td>Low</td>
</tr>
<tr>
<td>High</td>
<td>72 (7.2)</td>
<td>52 (5.2)</td>
<td>38 (3.8)</td>
</tr>
<tr>
<td>Avg.</td>
<td>68 (6.8)</td>
<td>49.5 (4.95)</td>
<td>31.5 (3.15)</td>
</tr>
<tr>
<td>Low</td>
<td>61 (6.1)</td>
<td>40.5 (4.05)</td>
<td>26 (2.6)</td>
</tr>
</tbody>
</table>

14. **Total Social Adjustment.** We may infer that with an F ratio of 1.83, total social adjustment does not significantly affect achievement in problem solving. Still, we have the means decreasing as attitudinal scores decrease. The largest of these differences is in the low ability group, where the high attitude group achieved 1.1 stanines higher than the low group. If total social adjustment does affect achievement positively, then it would seem to affect those pupils with low ability rather than those with high or average ability.

15. **Total Adjustment.** The composite of all attitudes yields an F ratio of 6.45, significant at the 1% level. The scores and means in Table X on page 44 show the highest achievement scores associated with the highest attitude scores. Comparing the high and low attitude levels, we find differences of .9, 1.1 and .9
stanines in the high, average and low ability groups, respectively. This difference of approximately one stanine in each group would seem to warrant us to say that a well adjusted pupil does significantly achieve better than a maladjusted pupil.

**TABLE X**

**PROBLEM SOLVING SCORES IN RELATION TO TOTAL ADJUSTMENT AND MENTAL AGE**

<table>
<thead>
<tr>
<th>Mental Age Levels</th>
<th>High</th>
<th>Average</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>68</td>
<td>53</td>
<td>37</td>
</tr>
<tr>
<td>(6.8)</td>
<td>(5.3)</td>
<td>(3.7)</td>
<td></td>
</tr>
<tr>
<td>Avg.</td>
<td>66</td>
<td>45</td>
<td>35</td>
</tr>
<tr>
<td>(6.6)</td>
<td>(4.5)</td>
<td>(3.5)</td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td>59</td>
<td>42</td>
<td>28</td>
</tr>
<tr>
<td>(5.9)</td>
<td>(4.2)</td>
<td>(2.8)</td>
<td></td>
</tr>
</tbody>
</table>

Table XI on page 45 gives the correlation coefficients, means, standard deviations and standard errors of the correlation coefficients of those attitudes that are significant at the 1% and 5% levels in relation to problem solving achievement and mental age. The correlation between problem solving achievement and mental age is also given. Consulting the table of values of the correlation coefficient for different levels of significance, we find that with 300 degrees of freedom (actually we have 328) a correlation of .148 is significant at the 1% level, and .113 is significant at the 5% level. Therefore, we may reject the null hypothesis that the
population correlation is zero and say that self-reliance (.2521), personal worth (.2433) and withdrawing tendencies (.1756) are significant at the 1% level with relation to problem solving ability. School relations (.1254) is significant at the 5% level. Thus, our data tend to indicate that there is a positive relationship, even though not very strong, between problem solving and these attitudes, that probably did not occur by chance. Similarly, self-reliance (.2403) and personal worth (.2045) correlate significantly at the 1% level with mental age; withdrawing tendencies (.1082), with 328 degrees of freedom, is significant at the 5% level. School relations (.0623) correlates non-significantly with mental age.

**TABLE XI**

| PROBLEM SOLVING AND MENTAL AGE CORRELATIONS WITH ATTITUDES SIGNIFICANT AT THE 1% LEVEL |
|----------------------------------------|-----------------|-----------------|-----------------|-----------------|
|                                      | Problem Solving | Mental Age      | Standard Deviation | Standard Error |
|                                      | r               | r               |                  | of Coefficient  |
|                                      |                 |                 |                  | of Correlation  |
|                                      |                 |                 |                  | Problem        | Mental        |
| Self-reliance                        | .2521           | .2403           | 5.97             | 1.48            | .052           | .052          |
| Pers'1 Worth                         | .2433           | .2045           | 6.49             | 1.83            | .052           | .053          |
| With. Tend.                          | .1756           | .1082           | 5.97             | 1.70            | .053           | .054          |
| School Rel.                          | .1254           | .0623           | 5.66             | 1.59            | .054           | .055          |

Problem Solving Correlated with Mental Age

<table>
<thead>
<tr>
<th>Mental Age</th>
<th>Problem Solving</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>r</td>
<td>of Correlation</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Problem Solving</td>
</tr>
<tr>
<td>.6267</td>
<td>---</td>
<td>.033</td>
</tr>
</tbody>
</table>

\[
\sigma_r = \frac{1 - r^2}{\sqrt{N - 1}}
\]

\(^a\) Standard error of \(r\) estimated by the general formula

When the correlations were subject to Fisher's \(z'\) transformation, even the lowest correlation was significantly different from zero.
With the use of the correlations given in Table XI, coefficients of determination were obtained showing the relative and joint contributions of attitudes (those significant at the 1% level) and mental age to problem solving achievement. An illustration of the relative and joint contributions is given in the diagram below. Direct contributions to problem solving achievement are made by attitude and mental age, and the product terms, shown by the double arrows, indicate joint contributions of attitude and mental age to problem solving achievement.

\[
\begin{align*}
\text{Mental Age} & \rightarrow \text{Attitude} \\
\downarrow & \\
\text{Problem Solving Achievement} & \leftarrow \text{Attitude}
\end{align*}
\]

Letting \( X_1 \) represent problem solving achievement, \( X_2 \) represent mental age and \( X_3 \) represent attitude, beta coefficients were calculated by the formulas below:\(^1\)

\[
\beta_{13.2} = \frac{r_{13} - r_{12}r_{23}}{1 - r_{23}^2}
\]

\[
\beta_{12.3} = r_{12} - \beta_{13.2}r_{23}
\]

\(^1\)Engelhart, Psychometrika, p. 289.
The proportions of variance of the dependent variable to be ascribed to the direct and joint influences of the independent variables were ascertained by calculating:

$\beta^2_{13.2}, \beta^2_{12.3}$ and $2\beta_{13.2}\beta_{12.3}r_{23}$

The first two terms, $\beta^2_{13.2}$ and $\beta^2_{12.3}$, represent, respectively, the direct contributions of attitude and mental age to problem solving achievement. The product term, $2\beta_{13.2}\beta_{12.3}r_{23}$, is a measure of the joint contribution of attitude and mental age to problem solving achievement. If all of the variance of $X_1$ were accounted for, even though this is not the case here, then the formula would be:

$$1 = \beta^2_{13.2} + \beta^2_{12.3} + 2\beta_{13.2}\beta_{12.3}r_{23}$$

The results of this path coefficient analysis are given in the coefficients of determination of Table XII on page 48. A percent interpretation can be given by shifting the decimal points two places to the right.

Since mental age has much the higher correlation with problem solving than each attitude, and both mental age and attitude are substantially correlated, the coefficients of determination relevant to mental age are much higher than the coefficients of determination relevant to attitudes. If one were to predict problem solving scores from mental ages and attitude measures, comparison of the regression coefficients for mental age with those for attitudes would show the former much the larger. This analysis reveals
that in the production of variation in problem solving ability, intelligence as measured is much more important than attitude, even though the analysis of variance has shown certain attitudes to be significant factors in problem solving achievement.

The method of path coefficients has certain limitations, as shown by Monroe and Stuit. The method is most effective when correlations between the independent variables are low. One of the major purposes of factor analysis is to identify uncorrelated, or nearly uncorrelated, factors.

**TABLE XII**

**COEFFICIENTS OF DETERMINATION SHOWING DIRECT AND JOINT INFLUENCES OF MENTAL AGE AND ATTITUDE ON PROBLEM SOLVING ACHIEVEMENT**

<table>
<thead>
<tr>
<th></th>
<th>Mental Age</th>
<th>Attitude</th>
<th>Mental Age and Attitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Self-reliance</td>
<td>.3610</td>
<td>.0114</td>
<td>.0291</td>
</tr>
<tr>
<td>Personal Worth</td>
<td>.3624</td>
<td>.0144</td>
<td>.0296</td>
</tr>
<tr>
<td>Withdrawing Tend.</td>
<td>.3782</td>
<td>.0119</td>
<td>.0145</td>
</tr>
<tr>
<td>School Relations</td>
<td>.3860</td>
<td>.0075</td>
<td>.0067</td>
</tr>
</tbody>
</table>

The same procedure that was used for problem solving and attitudes was also used for reading and attitudes. These results are presented in Table XIII on page 50. Those attitudes found to

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2Monroe and Stuit, p. 162.
be significantly related to reading achievement at the 1% level are personal worth, withdrawing tendencies and school relations. Self-reliance is significant at the 5% level. Total personal adjustment is significant at the 1% level, and total adjustment is significant at the 5% level. Even though family relations fails to be significant with an F of 2.96 (3.07 needed for 5% level), in the low ability group those with high attitudes scored 1.7 stanines higher than those having low scores. A similar difference exists for the attitude sense of belonging, with a stanine difference in the low ability group of 1.4. Even though nervous symptoms is non-significant with an F of .68, a comparison of the low ability group means shows a 1.3 stanines difference between the high and low attitude groups.

Our data show that self-reliance, personal worth, withdrawing tendencies and school relations are significant with both arithmetic problem solving and reading, all being significant at the 1% level, with the exception of self-reliance and reading, which is significant at the 5% level. Together with total personal adjustment, these attitudes seem to have a significant role in arithmetic problem solving and reading achievement. A sense of belonging and family relations are significant factors, at the 5% level, with problem solving, and non-significant with reading. Still, we cannot discount the hypothesis that these attitudes might affect reading achievement, since differences do occur, largely in the low ability groups.
# TABLE XIII

**ANALYSIS OF VARIANCE RESULTS PERTAINING TO ATTITUDE LEVELS AND READING**

| Attitude            | Sums of Squares | Degrees of Freedom | Variance | F   | Level of Significance
|---------------------|-----------------|--------------------|----------|-----|-----------------------
| Self-reliance       | 19.67           | 2                  | 9.84     | 4.17| .05                   |
| Personal Worth      | 39.54           | 2                  | 19.77    | 7.85| .01                   |
| Personal Freedom    | 3.04            | 2                  | 4.02     | 1.34| Non-sig.              |
| Belonging           | 12.50           | 2                  | 6.25     | 2.59| Non-sig.              |
| Withdrawing Tend.   | 25.26           | 2                  | 12.63    | 5.37| .01                   |
| Nervous Symptoms    | 3.27            | 2                  | 1.64     | .68 | Non-sig.              |
| Total Pers'1 Adj.   | 35.52           | 2                  | 17.76    | 6.65| .01                   |
| Social Standards    | 4.87            | 2                  | 2.44     | .97 | Non-sig.              |
| Social Skills       | 2.99            | 2                  | 1.50     | .60 | Non-sig.              |
| Anti-social Tend.   | 7.34            | 2                  | 3.67     | 1.35| Non-sig.              |
| Family Relations    | 12.67           | 2                  | 6.34     | 2.96| Non-sig.              |
| School Relations    | 24.50           | 2                  | 12.25    | 5.08| .01                   |
| Community Rel.      | .80             | 2                  | .40      | .15 | Non-sig.              |
| Total Social Adj.   | 12.12           | 2                  | 6.06     | 2.49| Non-sig.              |
| Total Adj.          | 20.16           | 2                  | 10.08    | 3.80| .05                   |

\*a F of 4.78 needed for 1% level and 3.07 for 5% level with 2 & 131 degrees of freedom.
The second purpose of this study was to try to get a better understanding of the intellectual factors involved in problem solving ability. As was mentioned previously, certain controls, such as administration of the tests, scoring and same grade level, were maintained to make the results as objective as possible. Arithmetical problem solving was designated as the dependent variable, and arithmetic computation, verbal ability, spatial relations, logical reasoning and numerical reasoning were designated as the independent variables. Table XIV lists these variables with each symbol, mean, standard deviation and standard error of the mean.

### TABLE XIV

**LIST OF VARIABLES**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Symbol</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Standard Error of Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arith. Problem Solving</td>
<td>X₁</td>
<td>6.5</td>
<td>1.47</td>
<td>0.081</td>
</tr>
<tr>
<td>Arith. Computation</td>
<td>X₂</td>
<td>7.6</td>
<td>1.24</td>
<td>0.068</td>
</tr>
<tr>
<td>Verbal Ability</td>
<td>X₃</td>
<td>7.3</td>
<td>1.27</td>
<td>0.070</td>
</tr>
<tr>
<td>Spatial Relations</td>
<td>X₄</td>
<td>5.1</td>
<td>1.94</td>
<td>0.107</td>
</tr>
<tr>
<td>Logical Reasoning</td>
<td>X₅</td>
<td>6.7</td>
<td>1.56</td>
<td>0.086</td>
</tr>
<tr>
<td>Numerical Reasoning</td>
<td>X₆</td>
<td>7.2</td>
<td>1.37</td>
<td>0.076</td>
</tr>
</tbody>
</table>

The correlation coefficients between the dependent and independent variables with their standard errors are listed in Table XV on page 52. We see that the independent variables that correlated
TABLE XV

ZERO ORDER COEFFICIENTS OF CORRELATION
WITH THEIR STANDARD ERRORS\(^a\)

<table>
<thead>
<tr>
<th></th>
<th>X₁</th>
<th>X₂</th>
<th>X₃</th>
<th>X₄</th>
<th>X₅</th>
<th>X₆</th>
</tr>
</thead>
<tbody>
<tr>
<td>X₁</td>
<td>1.0000</td>
<td>0.6934 ± 0.029</td>
<td>0.6971 ± 0.028</td>
<td>0.3969 ± 0.047</td>
<td>0.5423 ± 0.039</td>
<td>0.6391 ± 0.033</td>
</tr>
<tr>
<td>X₂</td>
<td>0.6934</td>
<td>1.0000</td>
<td>0.4214 ± 0.045</td>
<td>0.3539 ± 0.048</td>
<td>0.4644 ± 0.043</td>
<td>0.5303 ± 0.040</td>
</tr>
<tr>
<td>X₃</td>
<td>0.6971</td>
<td>0.4214</td>
<td>1.0000</td>
<td>0.2885 ± 0.051</td>
<td>0.4472 ± 0.044</td>
<td>0.5264 ± 0.040</td>
</tr>
<tr>
<td>X₄</td>
<td>0.3969</td>
<td>0.3539</td>
<td>0.2885</td>
<td>1.0000</td>
<td>0.3943 ± 0.047</td>
<td>0.3503 ± 0.048</td>
</tr>
<tr>
<td>X₅</td>
<td>0.5423</td>
<td>0.4644</td>
<td>0.4472</td>
<td>0.3943</td>
<td>1.0000</td>
<td>0.6271 ± 0.033</td>
</tr>
<tr>
<td>X₆</td>
<td>0.6391</td>
<td>0.5303</td>
<td>0.5264</td>
<td>0.3503</td>
<td>0.6271</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

\(^a\)Standard error of r estimated by the general formula \(\sigma_r = \frac{1 - r^2}{\sqrt{N - 1}}\).

When the correlations were subject to Fisher's z' transformation, even the lowest correlation was significantly different from zero.
Highest with problem solving are \( X_3 \), verbal ability (.70), \( X_2 \), arithmetic computation (.69) and \( X_6 \), numerical reasoning (.64). These data are compatible with those of previous studies reviewed in the chapter on the related literature.

The intercorrelations between the variables are also presented in Table XV.

In determining the relative contribution of each independent variable to problem solving ability, beta coefficients were obtained using the Doolittle Method, as shown in Table XVII in Appendix III. (The rationale of beta coefficients is also presented in Appendix III.) The results of these beta coefficients are given in the following standard score regression equation:

\[
X_1 = 0.3883X_2 + 0.4047X_3 + 0.0642X_4 + 0.0526X_5 + 0.1651X_6
\]

The general form of the multiple regression equation is:

\[
X_1 = b_{12.3456}X_2 + b_{13.2456}X_3 + b_{14.2356}X_4 + b_{15.2346}X_5 + b_{16.2345}X_6 + c
\]

The results are:

\[
X_1 = 0.462X_2 + 0.470X_3 + 0.049X_4 + 0.050X_5 + 0.177X_6 - 2.33
\]

This equation shows that for every unit increase in \( X_2 \), \( X_1 \) is increasing .462 unit; that for every unit increase in \( X_3 \), \( X_1 \) is increasing .470 unit; etc.

---

\(^3\)As in regression equations, "c" is a constant. The coefficients \( b_{12.3456} \), etc., are multiplying constants or weights for the X values; they were found by the formula:

\[
b_{12.3456} = \left( \frac{\sigma_1}{\sigma_2} \right) \beta_{12.3456}
\]
The coefficients of determination are:

\[ \beta^2_{12.3456} = .1503 \]
\[ \beta^2_{13.2456} = .1638 \]
\[ \beta^2_{14.2356} = .0041 \]
\[ \beta^2_{15.2346} = .0028 \]
\[ \beta^2_{16.2345} = .0273 \]
\[ 2\beta_{12.3456}\beta_{13.2456}\beta_{12.23} = .1324 \]
\[ 2\beta_{12.3456}\beta_{14.2356}\beta_{24} = .0176 \]
\[ 2\beta_{12.3456}\beta_{15.2346}\beta_{25} = .0190 \]
\[ 2\beta_{12.3456}\beta_{16.2345}\beta_{26} = .0680 \]
\[ 2\beta_{13.2456}\beta_{14.2356}\beta_{34} = .0150 \]
\[ 2\beta_{13.2456}\beta_{15.2346}\beta_{35} = .0190 \]
\[ 2\beta_{13.2456}\beta_{16.2345}\beta_{36} = .0703 \]
\[ 2\beta_{14.2356}\beta_{15.2346}\beta_{45} = .0027 \]
\[ 2\beta_{14.2356}\beta_{16.2345}\beta_{46} = .0074 \]
\[ 2\beta_{15.2346}\beta_{16.2345}\beta_{56} = .0109 \]

The results were checked by comparing the summation of these measures (.7111), the coefficient of multiple determination, with the square of the coefficient of multiple correlation, \( R^2_{1.23456} = (.7108) \), obtained as follows:

\[ R^2_{1.23456} = \beta_{12}r_{12} + \beta_{13}r_{13} + \beta_{14}r_{14} + \beta_{15}r_{15} + \beta_{16}r_{16} \]
\[ = .2692 + .2821 + .0255 + .0285 + .1055 \]
\[ = .7108 \]

This yields a multiple correlation of .8431, using all values correct to eight decimal places.
From these beta coefficients we see that verbal ability \((X_3)\) and arithmetic computation \((X_2)\) contribute most to problem solving ability. The relative contribution of numerical reasoning, spatial relations and logical reasoning are slight.

The coefficients of determination are listed below. The joint contributions have been broken down according to the ratios of the direct contributions and added to the direct contributions to obtain estimates of the separate effects of the independent variables on variation in the dependent variable. The contribution of unknown, or unmeasured, factors was secured by the difference from unity of the observed coefficients of determination.\(^4\)

<table>
<thead>
<tr>
<th>Independent Variables</th>
<th>Per Cent Contribution to Variance of Problem Solving</th>
</tr>
</thead>
<tbody>
<tr>
<td>Verbal Ability</td>
<td>33.39</td>
</tr>
<tr>
<td>Arithmetic Computational Ability</td>
<td>30.74</td>
</tr>
<tr>
<td>Numerical Reasoning</td>
<td>5.57</td>
</tr>
<tr>
<td>Spatial Relations</td>
<td>.84</td>
</tr>
<tr>
<td>Logical Reasoning</td>
<td>.57</td>
</tr>
<tr>
<td>Unknown or Unmeasured Factors</td>
<td>28.89</td>
</tr>
<tr>
<td></td>
<td>100.00</td>
</tr>
</tbody>
</table>

A summary and check of the beta coefficients are shown in Table XVIII in Appendix III.

\(^{4}\) - \(R^2\) is often termed a coefficient of non-determination.
Chapter V

Conclusions and Recommendations

Conclusions

Educators and psychologists have frequently expressed their views of the effects of attitudes on achievement. It is difficult or improper to be too specific in evaluating limited data. We may be guilty of over-generalizing, and on the other hand we may be handicapping ourselves by being overly skeptical in evaluating data. One of the purposes of this study was to determine the effects, if any, of certain attitudes on problem solving achievement. The hypothesis was stated that attitudes do affect problem solving achievement. Which attitudes influence achievement and to what degree we hoped to learn from the results of this study. Our second purpose was to gain a further understanding of the intellectual factors in problem solving achievement. Again, we know some of the factors involved, but we hoped to evaluate the relative contributions of some of these factors—namely, arithmetical computation, spatial relations, verbal ability, numerical reasoning and logical reasoning.

The conclusions drawn from this study, we must remember, came from the test results of seventh grade pupils from four Chicago
elementary schools. This sample population was above average in intelligence and problem solving ability as measured by the California Test of Mental Maturity (Short Form) and the Metropolitan Achievement Battery, respectively. The term, attitude, as used in this study, refers to those attitudes as defined and measured by the California Test of Personality.

Application of analysis of variance to the data showed that the attitudes self-reliance, personal worth, withdrawing tendencies and school relations were significantly related to problem solving achievement at the 1% level of confidence. These results were also verified by the correlations that were run between these attitudes and problem solving achievement. All were significant at the 1% level with the exception of school relations, which was significant at the 5% level.

The results of the minor path coefficient analyses, as given in Table XII on page 48, show that mental age contributes most significantly to problem solving achievement. The per cents are .36, .36 and .38, respectively, in dealing with self-reliance, personal worth and withdrawing tendencies. These attitudes accounted for only 1% of the variation, while the joint influences accounted for approximately 2% in each case. Thus, we would have to say that mental age is the most important factor in problem solving achievement.

The two attitudes that were significant at the 5% level were family relations and the feeling of belonging. Even though these
attitudes are identified separately, the close relationship between
them is easily understood. The other six attitudes presented small
but non-significant ratios. This is not to say that personal free-
dom, freedom from nervous symptoms, desirable social standards and
skills, freedom from anti-social tendencies and good community
relations are not desirable attitudes to possess. The data showed
differences with all attitudes, even though they were not signifi-
cant at the 5% or 1% levels.

The comparative research with reading achievement seemed to
strengthen our findings with problem solving achievement. With
reading achievement the same four attitudes, self-reliance, per-
sonal worth, withdrawing tendencies and school relations gave sig-
nificant results. The only difference was that self-reliance
showed a significance at the 5% level instead of the 1% level.
Even though the attitudes family relations and feeling of belonging
were significant at the 5% level with problem solving and non-
significant with reading, differences of over one stanine did occur
with the low ability group in reading achievement. With respect
to the total personal adjustment, total social adjustment and total
adjustment, the comparisons between problem solving achievement
and reading achievement were similar, all being significant at the
1% or 5% level with the exception of total social adjustment, which
showed non-significant differences in problem solving and reading
achievement. Thus, we may infer that personal adjustment is more
important than social adjustment in problem solving and reading achievement.

From the preceding presentation we may conclude that the pupil who is independent, reliable and relatively free from hostility and aggressiveness is more apt to be successful in problem solving than the pupil who lacks these attributes. The successful pupil is further characterized as having good school and family relations with feelings of belonging and self-respect, in contrast to having feelings of being lonely, sensitive and given to self-concern.

The path coefficient values were used in securing the coefficients of determination, the direct influences being obtained by squaring the path coefficients of the paths leading directly from each of the independent variables to the dependent variable, problem solving achievement. The coefficients of determination measuring the joint or combined influences were obtained by taking twice the product of the two paths from the given independent variables to the dependent variable times the coefficient of correlation between the two independent variables. Since we were working with five independent variables, there were ten product terms.

We may conclude from our data that verbal ability and arithmetic computation are the two most significant factors in a pupil's ability to solve arithmetic problems. The coefficients of determination between problem solving ability and these factors were .16 and .15, respectively. Thus, we may state that problem solving ability depends almost equally upon computational ability and
verbal ability. The ability to reason numerically (.03) also seems to have a positive bearing on problem solving ability. Spatial relations (.004) and logical reasoning (.002) seem to have little or no relationship.

**Recommendations**

The data of this study have indicated that certain attitudes do influence problem solving achievement, and that these results did not occur from mere chance. It was also pointed out that verbal ability and arithmetic computation are important factors in a pupil's ability to solve problems. If we, as educators and parents, wish to see greater achievement in this difficult phase of the arithmetic program, then it would behoove us to consider these points in planning and teaching the arithmetic curriculum.

It was not the purpose of this study to determine how and why negative and positive attitudes are developed. This is not to say that this problem does not provide a fertile field of investigation for educators and parents alike. Since education is a co-operative endeavor, contributed to by many social institutions of which the school and family are primary, then we must all be cognizant of the factors which influence achievement, attitudes being one. As it is the objective of all concerned with the schools to accomplish the aims of education, then it would seem that the educative process might call for greater stress on the development of favorable attitudes, especially on the elementary level.
There are certain implications apparent to the writer of this study for educators, in particular for elementary teachers. Understanding the development of positive and negative attitudes of pupils should contribute to teachers' techniques in fostering greater achievement in problem solving. This could also lead to better understanding and recognition of poor attitudes. By the establishment of an atmosphere in which pupils are free to express themselves and to participate freely, favorable attitudes, such as self-reliance and feelings of being worthy, could become a reality to those pupils who lack them, and maintained and developed in those pupils who already exhibit these qualities. By being aware of those pupils who have poor family backgrounds and feel as though they do not belong, an understanding teacher can provide experiences which can alleviate, if not eliminate, much of the harm that the lack of these attitudes presents in achievement. Teachers with knowledge, a posteriori, can without too much difficulty readily identify those pupils who manifest personal and social problems.

The preceding discussion was not meant to place the burden of pupils' personal and social adjustment only upon the classroom teacher. Many agencies must help if we are going to make great progress in this direction, not the least of which is the family. The writer is not trying to set up utopian principles which would be almost impossible to achieve, but suggestions--practical and difficult in that they take hard work and time--which we must follow to produce greater achievement. The teacher need not take the
place of a psychologist, but with the use of available test results and tests such as the California Test of Personality, or any other measuring technique, such as teacher rating scales and pupil self-inventories, the teacher can gain a better understanding of the attitudes that pupils possess. It seems that if attitudes do play a significant role in achievement, and the results of this study seem to verify this point, implications for teacher training are present. The writer suggests that additional courses in psychology and measurement could aid teachers in better understanding their pupils, especially on the elementary level during important formative years.

Education cannot be complete without the co-operation of parents. Parents must be informed to better understand their children and the educative process, of which attitudes constitute a significant aspect. Educators must work with parents to study pupils' adjustment, both personal and social. The lack of joint co-operation makes the objectives of the schools almost impossible.

Since verbal ability and arithmetic computation are significantly related to problem solving, improvement in these abilities should show an improvement in problem solving ability. Rather than continuing practice in solving problems, it would seem that teachers should spend more time working on the improvement of these two factors, especially in remedial cases. Test results showing a pupil's verbal and computational abilities are accessible early in a pupil's school life. These results can be watched carefully.
throughout elementary school, and added instruction can be given when a pupil falls below a certain level. Many cases of failure may be avoided.

With the use of personality inventories and intelligence tests educators have means by which to diagnose a pupil's chances of being successful in arithmetic problem solving. With these results they are better equipped to make the total adjustment of pupils more of a reality.

Additional questions that may be answered by further research are: Do attitudes influence achievement to a greater extent with pupils possessing lower intelligence? How, why and when do pupils acquire certain attitudes? How much do the attitudes of parents and teachers affect children's attitudes? How much does failure affect attitudes? What are some of the methods that teachers may use to improve verbal and computation abilities? To what degree is inadequate teaching responsible for poor attitudes and lack of achievement in problem solving?

The answers to these questions would certainly aid us in teaching and probably would assure pupils of greater achievement.
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III. TESTS


APPENDIX I

DESCRIPTION OF TESTS

California Test of Personality

The California Test of Personality, Kindergarten to Adult, is composed of two sections, personal adjustment and social adjustment. Each of these sections contains six components which totally measure the concept of life adjustment.

The specific tendencies to think, to act and to feel as measured by this test and considered as attitudes for the study are as follows:

A. Personal Adjustment

1. Self-reliance
2. Sense of Personal Worth
3. Sense of Personal Freedom
4. Feeling of Belonging
5. Withdrawing Tendencies
6. Nervous Symptoms

B. Social Adjustment

1. Social Standards
2. Social Skills
3. Anti-social Tendencies
4. Family Relations
5. School Relations
6. Community Relations

The definitions are those given by the authors of the test. The writer has taken the liberty of re-wording and shortening some
of the definitions. However, in general they are as found in the
manual.1

Self-reliance. An individual may be said to be self-reliant when his overt actions indicate that he can do things independently of others, depend upon himself in various situations and direct his own activities.

Sense of Personal Worth. An individual possesses a sense of being worthy when he feels he is well regarded by others, when he feels that others have faith in his future success, and when he believes that he has average or better than average ability.

Sense of Personal Freedom. An individual enjoys a sense of freedom when he is permitted to have a reasonable share in the determination of his conduct and in setting the general policies that shall govern his life.

Feeling of Belonging. An individual feels that he belongs when he enjoys the love of his family and a cordial relationship with others. Such a person, usually, will get along well with his teachers and feels proud of his school.

Withdrawing Tendencies. The individual who is said to withdraw is the one who substitutes the joys of a fantasy world for actual successes in real life. Such a person is characteristically sensitive, lonely and given to self-concern.

**Nervous Symptoms.** The individual who is classified as having nervous symptoms is the one who suffers from one or more of a variety of physical symptoms such as loss of appetite, frequent eye strain, inability to sleep, or a tendency to be chronically tired.

**Social Standards.** The individual who recognizes desirable social standards is the one who has come to understand the rights of others and who appreciates the necessity of subordinating certain desires to the needs of the group.

**Social Skills.** An individual who possesses the tendency to act in accordance with prevailing social standards and subordinates his or her egoistic tendencies in favor of interest in the problems and activities of his associates may be said to be socially skillful.

**Anti-social Tendencies.** The anti-social person is the one who endeavors to get his satisfactions in ways that are damaging and unfair to others such as bullying, disobedience and destructiveness to property.

**Family Relations.** The individual who feels he is loved and well-treated at home, and who has a sense of security and self-respect in connection with the various members of his family, exhibits desirable family relationships.

**School Relations.** The student who is satisfactorily adjusted to his school feels that he is liked by his teachers, enjoys being
with other students, and finds the school work adapted to his level of interest and maturity.

Community Relations. The individual making good adjustments in his community is one who mingles happily with his neighbors, takes pride in community improvements and is tolerant in dealing with both strangers and foreigners.

**California Short-Form Test of Mental Maturity**

The California Short-Form Test of Mental Maturity, Elementary Level (1957 Edition) consists of seven tests. These seven tests contribute to scores in four factors:

1. Spatial Relationships
2. Logical Reasoning
3. Numerical Reasoning
4. Verbal Concepts

These factors comprise a Language I.Q., a Non-Language I.Q. and a Total I.Q. The following description is basically the same as that presented in the manual.²

**Spatial Relationships Factor.** Important aspects of mental functioning involve understanding the relationships of objects in space. Involved also is the ability to recognize both differences and similarities in designs when they are presented in various positions.

Test 1, Sensing Right and Left, consists of twenty pictures of hands and feet in various positions. The items are designed to reveal the pupil's ability to discriminate between right and left, involving such mental processes as manipulation of visually presented objects and two- and three-dimensional perception.

Test 2, Manipulation of Areas, consists of fifteen items, each of which requires the pupil to identify the one among four drawings which is a different view of the first one.

Logical Reasoning Factor. The solution of almost any problem requires the making of decisions. Even the simplest situations generally involve a choice among alternatives and the drawing of conclusions from given premises. The ability to grasp relationships is tested in two patterns of logical arrangement, involving both inductive and deductive reasoning. The following tests contribute the data for the factor score for Logical Reasoning.

Test 3, Similarities, contains fifteen items, each of which consists of seven drawings. The first three are alike in some way. The pupil determines the nature of this likeness and then finds another drawing among the remaining four which is related to the first three in the same way.

Test 4, Inference, consists of fifteen items, each of which contains two premises. The pupil must select the logical conclusion, based on these premises, from the three possible responses given.
Numerical Reasoning Factor. This factor measures the ability to recognize numerical concepts and relationships, to identify the principles involved in the solution of numerical problems and to use these principles in making inferences and reaching correct conclusions.

Test 5, Number Series, consists of ten number series items which increase or decrease in various patterns. The pupil indicates his understanding of the principle governing each pattern by identifying the one number in each series which violates this principle.

Test 6, Numerical Quantity, consists of fifteen problems in quantitative reasoning. Each item contains a numerical situation, together with a question about it. The pupil must work out the problem and select the correct answer from four possible responses. The emphasis is not on testing knowledge of arithmetic fundamentals but rather on testing thinking in mathematical situations.

Verbal Concepts Factor. This factor measures the examinee's comprehension of the meanings of a carefully selected series of words. This facility is universally recognized as an important aspect of intelligence.

Test 7, Verbal Concepts, contains fifty items consisting of a key word, a synonym, and three distracter words. Two words, therefore, must be correctly comprehended as possessing a similar connotation, making a total of 100 recognition words.
Metropolitan Achievement Tests

The Metropolitan Achievement Tests comprise a co-ordinated series of measures of achievement in the important skill and content areas of the elementary and junior high school curriculum. Primary in the minds of the authors has been the intent to develop tests that would contribute most effectively to teacher understanding and analysis of pupils' achievement, and that would provide dependable data for evaluation of pupil growth. The validity of these tests is based on analysis of textbooks, courses of study, and expert formulations of the goals of instruction at the various elementary levels.

The Metropolitan Achievement Tests consist of ten tests:

1. Reading
2. Arithmetic Computation
3. Arithmetic Problem Solving and Concepts
4. Word Knowledge
5. Spelling
6. Language
7. Language Study Skills
8. Social Studies Information
9. Social Studies Study Skills
10. Science

Of these ten tests, this study makes use of only the first three. Definitions of these three tests, generally as presented in the manual, are given below.3

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Reading. This test consists of a series of reading selections, each followed by several questions designed to measure various aspects of reading comprehension, including the following:

a. Ability to select the main thought of a passage, or to judge its general significance.

b. Ability to understand the literal meaning of the selection, or to locate information explicitly set forth.

c. Ability to see the relationships among the ideas set forth in the selection and to draw correct inferences from the selection.

d. Ability to determine the meaning of a word from context, or to judge from the context which of several possible meanings of a word is the appropriate one.

Arithmetic Computation. This is a forty-eight item test which covers fundamental operations with whole numbers, decimals and fractions, through fractional parts of numbers, reading of graphs and addition and subtraction of denominate numbers.

Arithmetic Problem Solving and Concepts. This test consists of two parts. The first is a measure of understanding of concepts of the number system, arithmetic processes, vocabulary, mathematical generalizations and principles, measures and arithmetic relationships. The second measures the ability of the pupil to apply numbers in social situations and to make sound judgments with respect to quantitative problems. The emphasis is on reasoning in numerical situations; the reading load has been kept at the lowest possible level, and only very simple computational skill is required.
APPENDIX II

AN ILLUSTRATION OF
TWO-PART ANALYSIS OF VARIANCE RESULTS

TABLE XVI

PROBLEM SOLVING ACHIEVEMENT -
SELF-RELIANCE AND MENTAL AGE

<table>
<thead>
<tr>
<th></th>
<th>Sums of Squares</th>
<th>Degrees of Freedom</th>
<th>Variance</th>
<th>F</th>
<th>Level of Significance</th>
</tr>
</thead>
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<tr>
<td>Mental Age Levels</td>
<td>201.61</td>
<td>2 (M - 1)</td>
<td>100.81</td>
<td>41.66</td>
<td>.01</td>
</tr>
<tr>
<td>Attitude Levels</td>
<td>23.69</td>
<td>2 (L - 1)</td>
<td>11.85</td>
<td>4.90</td>
<td>.01</td>
</tr>
<tr>
<td>Cells</td>
<td>226.49</td>
<td>8 (ML - 1)</td>
<td>28.31</td>
<td>11.70</td>
<td>.01</td>
</tr>
<tr>
<td>Interaction</td>
<td>1.19</td>
<td>4 (M - 1) x (L - 1)</td>
<td>.30</td>
<td>.12</td>
<td>Non-sig.</td>
</tr>
<tr>
<td>Within Sub-groups</td>
<td>316.50</td>
<td>131 (N - ML)</td>
<td>2.42</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>542.99</td>
<td>(N - 1)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX III

SIX VARIABLE SCHEMA
FOR CALCULATION OF BETA COEFFICIENTS

First Order Calculations

\[ a_{13.2} = r_{13} - r_{12}r_{23} \]

\[ a_{14.2} = r_{14} - r_{12}r_{24} \]

\[ a_{15.2} = r_{15} - r_{12}r_{25} \]

\[ a_{16.2} = r_{16} - r_{12}r_{26} \]

\[ a_{43.2} = r_{34} - r_{24}r_{23} \]

\[ a_{53.2} = r_{35} - r_{25}r_{23} \]

\[ a_{54.2} = r_{45} - r_{25}r_{24} \]

\[ a_{63.2} = r_{36} - r_{26}r_{23} \]

\[ a_{64.2} = r_{46} - r_{26}r_{24} \]

\[ a_{65.2} = r_{56} - r_{26}r_{25} \]

\[ \omega_{23} = 1 - r_{23}^2 \]

\[ \omega_{24} = 1 - r_{24}^2 \]

\[ \omega_{25} = 1 - r_{25}^2 \]

\[ \omega_{26} = 1 - r_{26}^2 \]

---

First Order Calculations (cont.)

\[ \beta_{13.2} = \frac{\alpha_{13.2}}{\omega_{23}} \]
\[ \beta_{43.2} = \frac{\alpha_{43.2}}{\omega_{23}} \]
\[ \beta_{53.2} = \frac{\alpha_{53.2}}{\omega_{23}} \]
\[ \beta_{63.2} = \frac{\alpha_{63.2}}{\omega_{23}} \]

Second Order Calculations

\[ \alpha_{14.23} = \alpha_{14.2} - \beta_{13.2} \alpha_{43.2} \]
\[ \alpha_{15.23} = \alpha_{15.2} - \beta_{13.2} \alpha_{53.2} \]
\[ \alpha_{16.23} = \alpha_{16.2} - \beta_{13.2} \alpha_{63.2} \]
\[ \alpha_{54.23} = \alpha_{54.2} - \beta_{53.2} \alpha_{43.2} \]
\[ \alpha_{64.23} = \alpha_{64.2} - \beta_{63.2} \alpha_{43.2} \]
\[ \alpha_{65.23} = \alpha_{65.2} - \beta_{63.2} \alpha_{53.2} \]
\[ \omega_{24.3} = \omega_{24} - \beta_{43.2} \alpha_{43.2} \]
\[ \omega_{25.3} = \omega_{25} - \beta_{53.2} \alpha_{53.2} \]
\[ \omega_{26.3} = \omega_{26} - \beta_{63.2} \alpha_{63.2} \]

\[ \beta_{14.23} = \frac{\alpha_{14.23}}{\omega_{24.3}} \]
\[ \beta_{54.23} = \frac{\alpha_{54.23}}{\omega_{24.3}} \]
\[ \beta_{64.23} = \frac{\alpha_{64.23}}{\omega_{24.3}} \]
Third Order Calculations

\[ \alpha_{15.234} = \alpha_{15.23} - \beta_{14.23} \alpha_{54.23} \]
\[ \alpha_{16.234} = \alpha_{16.23} - \beta_{14.23} \alpha_{64.23} \]
\[ \alpha_{65.234} = \alpha_{65.23} - \beta_{64.23} \alpha_{54.23} \]
\[ \omega_{25.34} = \omega_{25.3} - \beta_{54.23} \alpha_{54.23} \]
\[ \omega_{26.34} = \omega_{26.3} - \beta_{64.23} \alpha_{64.23} \]

\[ \beta_{15.234} = \frac{\alpha_{15.234}}{\omega_{25.34}} \]
\[ \beta_{65.234} = \frac{\alpha_{65.234}}{\omega_{25.34}} \]

Final Partial Regression Coefficient

\[ \beta_{16} = \frac{\alpha_{16.234} - \beta_{15.234} \alpha_{54.23}}{\omega_{26.34} - \beta_{65.234} \alpha_{65.234}} \]
\[ \beta_{15} = \beta_{15.234} - \beta_{16} \alpha_{65.234} \]
\[ \beta_{14} = \beta_{14.23} - (\beta_{15} \alpha_{54.23} + \beta_{16} \alpha_{64.23}) \]
\[ \beta_{13} = \beta_{13.2} - (\beta_{14} \alpha_{43.2} + \beta_{15} \alpha_{53.2} + \beta_{16} \alpha_{63.2}) \]
\[ \beta_{12} = r_{12} - (\beta_{13} \alpha_{23} + \beta_{14} \alpha_{24} + \beta_{15} \alpha_{25} + \beta_{16} \alpha_{26}) \]
### TABLE XVII

**DOOITTLE METHOD FOR COMPUTING BETA COEFFICIENTS**

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<tr>
<th>Column Number</th>
<th>2</th>
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<th>4</th>
<th>5</th>
<th>6</th>
<th>1</th>
<th>Check</th>
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<tr>
<td>Variable</td>
<td>X₂</td>
<td>X₃</td>
<td>X₄</td>
<td>X₅</td>
<td>X₆</td>
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<tr>
<td>Row Instruction</td>
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<td></td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>A r₂k</td>
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<td>.4214</td>
<td>.3539</td>
<td>.4644</td>
<td>.5303</td>
<td>.6934</td>
<td>3.4634</td>
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<tr>
<td>B A ÷ (-A2)</td>
<td>-1.0000</td>
<td>- .4214</td>
<td>- .3539</td>
<td>- .4644</td>
<td>- .5303</td>
<td>- .6934</td>
<td>-3.4634</td>
</tr>
<tr>
<td>C r₃k</td>
<td>1.0000</td>
<td>.2885</td>
<td>.4472</td>
<td>.5264</td>
<td>.6971</td>
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<tr>
<td>D A x B₃</td>
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<td>- .1491</td>
<td>- .1956</td>
<td>- .2235</td>
<td>- .2922</td>
<td>-1.4595</td>
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</tr>
<tr>
<td>E C + D</td>
<td>.8224</td>
<td>.1394</td>
<td>.2516</td>
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<td>.4049</td>
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<td>F E ÷ (-E₃)</td>
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<td>- .3059</td>
<td>- .3683</td>
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<td>G r₄k</td>
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<td>.3503</td>
<td>.3969</td>
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<td>H A x B₄</td>
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<td>.1877</td>
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<td>I E x F₄</td>
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<td>-.0513</td>
<td>-.0686</td>
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<td>J G + H + I</td>
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<td>-.0182</td>
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<tr>
<td>P L + M + N + O</td>
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<td>.2636</td>
<td>.0782</td>
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<td></td>
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</tr>
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<td>-.1174</td>
<td>-.5134</td>
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<tr>
<td>U J x K₆</td>
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<td>V P x Q₆</td>
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<td>-.0309</td>
<td>-.3989</td>
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<td></td>
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<tr>
<td>W R + S + T + U + V</td>
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<td>.4883</td>
<td>.0806</td>
<td>.5690</td>
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<td>X W ÷ (-W₆)</td>
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<td>-.1653</td>
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<tr>
<td></td>
<td>$\beta_{1k}$</td>
<td>$\beta_{k6}$</td>
<td>$\beta_{1k^*k6}$</td>
<td></td>
<td></td>
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<td>$x_2$</td>
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<td>$x_6$</td>
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<td>1.0000</td>
<td>.1651</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

$\leq = .6395 = r_{16}$
The dissertation submitted by Henry Moughamian has been read and approved by five members of the Department of Education.

The final copies have been examined by the director of the dissertation and the signature which appears below verifies the fact that any necessary changes have been incorporated, and that the dissertation is now given final approval with reference to content, form, and mechanical accuracy.

The dissertation is therefore accepted in partial fulfillment of the requirements for the Degree of Doctor of Education.

11-15-1962
Date

Signature of Adviser