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The Relationship between Secondary Students' Mathematics Identities, Problem Solving, and Self-Regulation

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THE RELATIONSHIP BETWEEN SECONDARY STUDENTS' MATHEMATICS IDENTITIES, PROBLEM SOLVING, AND SELF-REGULATION

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DEDICATION

This dissertation is dedicated to the students and math teachers who I have worked with over the past ten years. I think of you often and hope you have made the most of your educational opportunities, whether in mathematics, education, or otherwise. Thank you for inspiring me!

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ABSTRACT

This mixed methods study explores secondary students' math identities. The primary purpose of this dissertation is to examine the relationships among students' math identities, their perceived problem solving practices, and their perceived self-regulated learning strategies. This study holds implications for teachers, school administrators, instructional coaches, teacher preparation professionals, policy makers, and educational researchers who influence the education of secondary math students.

This dissertation examines the following research questions: What is the relationship between secondary students' math identities, their perceived problem solving practices, and their perceived self-regulated learning strategies? What is the relationship between problem solving, self-regulation, and math identity given gender? How do secondary students articulate their math identities? Does students' articulation of the development of their math identities explain their problem solving practices and self-regulated learning strategies?

The design methods are grounded in Bandura's (1986) social cognitive theory and mixed methods methodology, which includes quantitative correlational research, qualitative interviews, and survey research. The instruments include: (1) a survey of students' math identities and perceptions of their problem solving and self-regulation practices and (2) structured qualitative interviews, of students reporting positive and negative math identities, to explain the quantitative results.

CHAPTER I

INTRODUCTION

This study examines the relationship between students' mathematics identities, problem solving, and self-regulation. In order to achieve this objective, high school students' math experiences are investigated by surveying their perceptions of their identities as doers of mathematics and their use of problem solving and self-regulation practices as well as interviewing a select group to provide depth to the survey responses. An underlying assumption is that students' learning in mathematics is a function both of the teaching they experience since instructors explicitly teach students how to learn and engage in mathematics and of students' participation in math communities of practice within their classroom learning environments.

This study investigates students' cognitive and affective domains of learning mathematics. This research does not stop at concerns about students' achievement in mathematics (i.e., achievement gap) but seeks to understand how students' math identities are developed and connected to practices they engage in to learn mathematics (i.e., problem solving and self-regulation). As a result of this study, the discussion hopes to offer insights and possible implications for instructional practice in order to support all students in developing math knowledge and positive math identities.

This chapter is outlined as follows: current state of science, technology, engineering, and math (STEM) careers, degrees, and enrollment; students' achievement in mathematics; students' interests, confidence, and dispositions towards mathematics; and mathematics identity. This chapter ends with sections on the statement of the problem; purpose of the study; research questions; significance of the study; chapter summary; and definition of terms.

Current State of STEM Careers, Degrees, and Enrollment

Careers in STEM in the United States have shown sustained growth for the last fifty years. The number of workers in these occupations grew from about 1.1 million in 1960 to approximately 5.8 million in 2011, which is an average annual rate of 3.3 percent (National Science Foundation, 2014). In 2015, the number of STEM workers was 9.0 million in the United States, and jobs in STEM fields are projected to grow by 8.9 percent from 2014 to 2024, compared to non-STEM jobs with a 6.4 percent growth rate (Noonan, 2017). Employment in STEM occupations is also outpacing non-STEM occupations at 10.5 percent, between May 2009 and May 2015, compared with 5.2 percent respectively (Fayer, Lacey, & Watson, 2017). These noteworthy growth numbers have sparked concern in filling STEM jobs; however the Bureau of Labor and Statistics reports that there are surpluses in STEM jobs. Shortages and surpluses in STEM careers depend on the specific field and geographic location (Xue & Larson, 2015).

In addition to favorable expected growth rates of STEM jobs and employment, STEM employees earned 29 percent more than non-STEM workers in 2015 (Noonan, 2017). Employees of STEM majors are also more educated that non-STEM jobs; almost three-quarters of STEM workers hold a college degree or higher, compared to just over one-third of non-STEM workers (Noonan, 2017). While STEM careers are attractive, there are benefits to simply graduating with a STEM degree, even if this does not mean a future STEM job. Regardless of whether students work in STEM occupations, graduates with a STEM degree can earn up to 12 percent more than non-STEM graduates (Noonan, 2017). Thus, there are advantages for pursing a STEM career or even a STEM degree.

In 2013-2014, of the 1.8 million bachelor's degrees awarded to American citizens, about 17 percent, or 319,000, were in STEM fields (National Center for Education Statistics, 2017). In 2013, about 28 percent of students pursuing a bachelor's degree entered a STEM field (i.e., chose a STEM major) at some point within six years of entering postsecondary education in 2003-04 (Chen, 2013). Other data, specific to science and engineering, show that for the 35-year period from 1972 to 2007, about 30 percent of all first-time freshmen at 4-year institutions began college enrolled in science or engineering. The proportion increased to 40 percent in 2011 and then declined to 39 percent in 2012 (Chen, 2013). Thus, there is a gap between students who originally enroll and those that graduate with a STEM degree. Instead of persisting in STEM majors, students change to a non-STEM major or do not complete their degree. The National Center for Education Statistics found that 48 percent of students who entered STEM undergraduate majors between 2003 and 2009 had left by spring 2009. About half of these students switched majors while the others left college without a degree (Chen, 2013). Thus, we need to look at what leads to attrition in STEM majors; one common area to start is American students' achievement in mathematics.

Students' Achievement in Mathematics

Looking at national and international standardized tests, students from the United States have room for improvement but are performing in the middle of the countries tested. This section reports student achievement data from the National Assessment of Educational Progress (NAEP), the Program for International Student Assessment (PISA), and Trends in International Mathematics and Science Study (TIMSS).

In 2015, approximately 13,200 students took the National Assessment of Educational Progress (NAEP) mathematics assessment. This assessment measures students' knowledge and skills and their ability to solve application problems. The NAEP is given every two years to 4th and 8th grade students, and 12th grade students are assessed periodically. The achievement levels are basic, proficient, and advanced. These levels are cumulative; for example, a student performing at the proficient level also demonstrates the competencies at the basic level. At each grade level–grades 4, 8 and 12–separate definitions and cut scores are provided.

In 2015, the percentage of 12th grade students at or above proficient achievement level was 25 percent, which is a slight decrease from 2013 with 26 percent and also lower than 33 percent at or above proficient achievement level in 8th grade and 40 percent in 4th grade. The percentage of students at the below basic achievement level was 38 percent. For higher-performing students at the 75th and 90th percentiles, no significant difference was seen when comparing 2015 to 2013. However, the NAEP estimates that only 37 percent of students are academically prepared for college math, with a score of 163 or above. Even worse, the gap between the high-performing and low-performing students is growing. In 2015, the mathematics scores for 12th grade students performing at the 10th, 25th, and 50th percentiles were lower in comparison to 2013.

Internationally, the United States does not fair much better. On the Program for International Student Assessment (PISA) from 2015 that tests the mathematics literacy of 15-year-olds, the United States' average was 470 compared to the average score of 490 for the international Organization for Economic Co-operation and Development (OECD). This ranks the United States 31st in mathematics out of 35 OECD countries. In 2015, just nine percent of 15-year-olds in the United States achieved at Level 5 or 6 to be considered top performers. Although American students performed above the OECD average of eight percent, over 15 percent of 15-year-old students in Japan, Singapore, and Taipei were top performers (OECD, 2015).

The 2015 Trends in International Mathematics and Science Study (TIMSS) revealed slightly higher averages for elementary school students in math and science. The TIMSS assessment is administered every four years to 4th and 8th grade students. The United States mathematics scores of 4th grade students averaged 539, and for 8th grade students, the average score was 518. These are higher than the TIMSS scale center point score of 500. Compared to other countries, the United States ranks 15th out of 54 countries testing 4th grade students and 12th out of 43 countries testing 8th grade students. In 1995 and again in 2015, TIMSS assessed 12th grade students in advanced mathematics and physics. The United States average was 485 with 500 as the TIMSS scale center point.

Although American students are in the middle of international countries in mathematics and the United States produces some high performing students, all high school students should be competent and confident in mathematics and 21st century skills (e.g., critical thinking, problem solving, digital literacy skills, flexibility and adaptability) so that they can be successful in college and career. Therefore, we must look beyond students' academic performance and to the affective domain.

Students' Confidence, Interests and Dispositions towards Mathematics, and Recognition

Besides achievement, researchers have studied other influences on students' persistence in STEM, including perceived abilities, beliefs and interests, and recognition by oneself and others. A study by Boaler and Staples (2008) found that students who engaged in more questioning and justification within their high school classes were more persistent than other students. Their findings indicated persistence in classroom activities, such as problems and tasks, and students' confidence and positive feelings towards math may have positively affected their achievement on tests and future plans to pursue advanced mathematics courses. Li, Swaminathan, and Tang (2009) looked at characteristics that are predictive of persistence; they found that mathematics preparation, self-ratings of mathematical ability, and enjoyment all contributed.

Looking at 2,266 undergraduate students at 129 two- and four-year colleges and universities who were enrolled in Calculus I, Ellis, Fosdick, and Rasmussen (2016) found that a lack of confidence in mathematical ability–not mathematical ability itself–deters female students from pursuing STEM. Although male and female students lost confidence at similar rates throughout the course, females started with lower levels of confidence (Ellis et al., 2016), and thus, females' lower confidence at the end of the course did not bode well for their persistence in more advanced STEM courses. Another study uses the Education Longitudinal Study (ELS) 2002 data to examine perceived mathematical ability under challenge in secondary schools and found perceived ability was highly predictive of choosing PEMC (physics, engineering, math, and computer science) and health sciences majors and varied by gender. For example, a 12th grade female's positively perceived mathematics ability under challenge increased her probability of selecting PEMC majors over biology (Nix, Perez-Felkner, & Thomas, 2015). Thus, one reason students, especially females, might choose to pursue or not pursue mathematics is confidence in mathematical ability. Women often report lower self-confidence in mathematics compared to their male counterparts (Piatek-Jimenez, 2015).

Besides perceived ability, students' interests, motivations, and beliefs also affect persistence in STEM. Boaler and Greeno (2000) argue that students' different levels of participation and persistence in mathematics were related to students' perception of mathematics. Students were more interested in persisting in math when it was not portrayed as an established set of rules but open for debate, creativity, and discussion. Thus, students wanted to be participants in mathematics instead of passive observers. Boaler's (2015) more recent work finds that students with growth mindsets, who believe their intelligence is not fixed but can grow and change, are more persistent. Besides positive beliefs and attitudes, undergraduate mathematics majors found math to be enjoyable; they were interested in using problem solving and critical thinking to solve complicated problems as well as understanding how math might be used in their lives (Piatek-Jimenez, 2015).

Lastly, recognition–how an individual or others see oneself–has been tied to persistence in mathematics. Cribbs, Cass, Hazari, and Sonnert (2016) used data from the Factors Influencing College Success in Mathematics (FICSM) project of 10,437 college calculus students and found that recognition in mathematics significantly predicts the choice of an engineering career, controlling for SAT/ACT math scores and student backgrounds.

Mathematics Identity

Perceived ability to perform and understand mathematics, beliefs and interests, and recognition are all factors of an individual's mathematics identity, and thus, there is a connection between identity and persistence in mathematics. Gee (2000) describes identity as "being recognized as a certain 'kind of person', in a given context" (p. 1). For example, we have all heard someone say, "I am a math person" or "I am just not a math person." Identity can be viewed by nature, institution, discourse, and affinity. Identity by nature is an individual's born state whereas identity by institution is authorized by a position. Discourse identity is acquired through dialogue with rational individuals, and affinity identity comes from shared experiences with a group (Gee, 2000). Although these four ways may seem discrete, they should be viewed as interrelated and complex. Some studies have found students' math identities had a positive correlation with their persistence in mathematics and other STEM fields (Cass, Hazari, Cribbs, Sadler, & Sonnert, 2011). Therefore, this study focuses on mathematics identity.

Statement of Problem

Jobs in the United States and abroad are increasingly requiring more than basic skills and knowledge but instead 21st century skills. Students "need to be able to find, evaluate, synthesize, frame, and use knowledge in new contexts, and to be able to solve non-routine problems and produce research findings and solutions" (Conley & Darling-Hammond, 2013, p.1). To complete non-routine tasks, they also need to be able to "demonstrate well-developed thinking skills, problem-solving abilities, design strategies, and communication capabilities" in the workplace (Conley & Darling-Hammond, 2013, p. 1). According to the Bureau of Labor Statistics, communication skills and critical and creative thinking are essential in STEM (science, technology, engineering, math) fields (Vilorio, 2014). Students must be able to problem solve and work through failure, gather data and research solutions, and comprehend interconnected systems. In addition, students need to effectively communicate their ideas and information to others, verbally and in writing. Thus, students must have the necessary skills and positive math identities so that they can face non-routine tasks in future careers.

Math teaching and classroom experiences shape students' ability, understanding, interests, confidence, and dispositions; in other words, their math identities. Persisting in mathematics depends on the extent students identify with mathematics content and the practices promoted within their classroom learning environments. Yet how are students' math identities developed? Teachers need to understand how instructional practices and interactions with others in the classroom influence students' math identities. Two ways students learn math and engage with others is through problem solving and self-regulated learning practices, and therefore, these practices can offer a starting point for understanding students' development of their math identities.

The Study

Purpose of the Study

The purpose of this mixed methods study is to understand the relationship between students' math identities and their perceived problem solving and self-regulation practices as well as students' articulation of their mathematics identities, either positively or negatively. This study's participants are secondary mathematics students at an urban high school on the West Coast. I argue that while mathematics identity has been studied extensively in relationship to classroom communities and teacher instruction, teachers' math identities, multiple identities, and career choices, there are few studies that look at the relationship to specific ways to learn and engage in mathematics, i.e., student practices of problem solving and self-regulation.

This study uses social cognitive theory as its theoretical framework. Social cognitive theory considers an individual's self-beliefs in addition to behaviors and environmental factors. Reciprocity and self-efficacy are two key concepts of Bandura's social cognitive theory (Zimmerman, 2001). Bandura's (1989) model of triadic reciprocity shows "behavior, cognition and other personal factors, and environmental influences all operate as interacting determinants that influence each other bi-directionally" (p. 2). An individual's self-efficacy, or the beliefs in one's abilities to perform, affects an individual's actions. Individuals with high self-efficacy take action and continue to improve their understanding, but individuals with low self-efficacy do not take productive steps to further their learning. Individuals with low self-efficacy do not think highly of their capabilities to successfully perform the task and, thus, are not proactive. Building on Bandura's social cognitive theory, other social cognitive theorists argue that there are more factors, including motivation and self-regulation that influence an individual's learning.

With social cognitive theory as the foundation, this study utilizes student surveys about math identity, problem solving, and self-regulation followed by interviews to further understand students' math identities and how they are developed through engaging in classroom practices. These data are then connected and used to answer the research questions.

Research Questions

This mixed methods research study includes quantitative, qualitative, and mixed methods research questions. The research questions for this study are:

1. What is the relationship between secondary students' math identities, their perceived problem solving practices, and their perceived self-regulated learning strategies? What is the relationship between problem solving, self-regulation, and math identity given gender?
2. How do secondary students articulate their math identities?
3. Does students' articulation of the development of their math identities explain their problem solving practices and self-regulated learning strategies?

Methods

The quantitative purpose of the study is to analyze the relationship between secondary students' math identities, their problem solving practices, and their self-regulated learning strategies using a social cognitive theory for learning. Data are analyzed with Spearman correlations of aggregated and disaggregated data. To study math identity, questions focus on the factors that make up math identity–perceived mathematical ability, interest and dispositions towards mathematics, and recognition in math. Following the analysis of the quantitative data, qualitative data are collected in the form of structured interviews about students' math identities. Once the quantitative and qualitative data are collected and analyzed separately, the qualitative results are used to add depth to the quantitative data. Thus, the qualitative data are used to triangulate and validate the quantitative findings, benefiting from the strengths of both quantitative and qualitative research.

Significance of the Study

This dissertation study holds significance for theoretical research and the instructional practice of secondary mathematics teachers. A more detailed discussion of the significance and implications based on the study's findings are included in Chapter V. Based on the literature review, the study is expected to (1) add to research on math identity by comparing the experiences of students with positive and negative math identities, (2) partially fill a need for mixed methods studies about math identity, and (3) inform secondary math teachers' instructional practice to develop the math identities of all students and provide information on how math identity, problem solving, and self-regulation influence one another.

Studies about math identity focus on classroom interactions between students and the content as well as the teacher and students, students' experiences with peers during discussions and collaborative activities, and comparisons between students' success and the math identities of teachers or parents. The literature also relates students' math identities with students' future plans to use mathematics in their careers or lives. This study collects data on all students' math identities through the form of a survey and then compares the experiences of students with positive and negative math identities from interview data.

The majority of math identity studies are qualitative, utilizing interviews, observations, and document analysis. One reason for this is that identity is dynamic, changing over time and by situation. Thus, many researchers use narrative inquiry to understand students' identity trajectories. Of the limited quantitative studies on math identity, many surveys determine math identity by a simple phrase, "I am a math person" or "I am not a math person." Further questions ask about influential factors of an individual's math identity but do not allow for a more in-depth explanation of the responses. This mixed methods study about math identity benefits from the strengths of both qualitative and quantitative research and partially fills the need for mixed methods studies about math identity.

Third, there is limited information for teachers about how to develop students' math identities. Studies focus on understanding students' math identities, sometimes in relation to race/ethnicity or normative identities in the classroom, as well as connections between identity and career choices. Yet many studies do not provide recommendations for secondary math teachers' instruction to develop the math identities of all students. Using the findings from Chapter IV, the last chapter offers suggestions for further studies and implications for instructional practice based on the results.

Chapter Summary

The following chapters include the review of literature (Chapter II), a description of the research methods for this study (Chapter III), quantitative and qualitative results from this mixed methods study (Chapter IV), and a discussion of the findings related the reviewed literature, possible further studies, and implications for practice (Chapter V).

Definition of Terms

Throughout this study, there are several terms and definitions that will be used and are defined below.

Mathematics identity is an individual's beliefs, attitudes, emotions, and dispositions towards mathematics. Students develop math identities individually and within mathematics classroom communities by engaging in shared interactions and social processes.

Problem solving is a process through which individuals move from an unknown to a known solution. Some problems have clear pathways from the problem to a specific solution, while novel problems have unclear paths to the answer. Problem solving requires setting goals when approaching a problem, creating a plan using heuristics and knowledge, executing the plan, and monitoring thinking.

Self-regulated learning is a self-driven process through which individuals take ownership of their learning and actions, evaluate goals and strategies, analyze tasks and create strategic plans, goal set, self-monitor while implementing strategies, and engage in sense-making and seeking help.

CHAPTER II

REVIEW OF LITERATURE

The current literature on students' mathematics identities explores connections to classroom communities and teacher instruction (Boaler, 2000, 2002a, 2002b; Boaler, Wiliam, & Zevenbergen, 2000; Grootenboer, 2013; Grootenboer & Zevenbergen, 2009; Schoenfeld, 2014; Solomon, 2009), teachers' math identities (Grootenboer & Zevenbergen, 2008), multiple identities (Cobb & Hodge, 2010; Martin, 2000; Solomon, 2009), and career choices (Cass et al., 2011; Cribbs, 2012). To promote students' persistence in mathematics, researchers encourage practitioners to promote positive relationships with mathematics and create classroom communities that support students' learning and belonging within classrooms. For example, effective instruction might include access to rigorous math problems, support for productive struggle, and opportunities to engage in practices of a mathematician. But the field of study has yet to examine the relationship between students' math identities and the practices and ways of learning that students are engaging in from this type of instruction, i.e., problem solving practices and self-regulated learning strategies. Also, most studies utilize qualitative measures only, such as narrative inquiry (Sfard & Prusak, 2005), and there exist few quantitative measurements for math identity.

This chapter is divided into two sections: the literature review and the theoretical framework. The literature review is based on the following research questions: What is the relationship between secondary students' math identities, their perceived problem solving practices, and their perceived self-regulated learning strategies? What is the relationship between problem solving, self-regulation, and math identity given gender? How do secondary students articulate their math identities? Does students' articulation of the development of their math identities explain their problem solving practices and self-regulated learning strategies? To answer these questions, I identified and organized literature concerning the following sections: (1) mathematics identity and identity development, (2) implications for instructional practice, (3) problem solving, and (4) self-regulated learning.

I begin by providing an overview of the literature that explains these areas, themes and trends in current literature, and implications for instructional practice. The second section offers a description of my theoretical framework of social cognitive theory (Bandura, 1986).

Mathematics Identity and Identity Development

In the last two decades, mathematics identity has been widely researched in literature and discussed in practice. Yet there is no agreed upon definition of mathematics identity (Darragh, 2016). Much on the literature on identity in mathematics draws its foundation from identity theory, taking on psychological/developmental, sociocultural, or poststructural perspectives (Grootenboer, Lowrie, & Smith, 2006). From the psychological/developmental perspective, Erikson (1968) describes students' learning and thinking with developmental stages. In the early stages, an individual does not comprehend his or her own identity in relation to a social or cultural group.

Throughout life, an individual becomes more aware and also more committed to a community. To comprehend and explain identity, some researchers attempt to compartmentalize and categorize aspects of identity while other researchers create models of the individual and variables that influence the individual's self-concept (Marsh, Graven & Debus, 1991). From either approach, formation of an individual's identity is self-determined since the individual adjusts or grows to align with specifics events, situations, or contexts (Grootenboer et al., 2006).

From the sociocultural perspective, Wenger (1998, 2010) and E. Wenger-Trayner and B. Wenger-Trayner (2015) articulate identity development within a community of practice, or a "group of people who share a concern or a passion for something they do and learn how to do it better as they interact regularly" (p. 1). Identity is constructed within a community, whether at home, at school, or within a network. However, the relationship between the individual and group is reciprocal because "the trajectory of an individual in a community of practice is influenced by their identification with that community, and an individual's trajectory influences their participation within that community of practice" (Wenger, 1998, p. 1050). As individuals engage in these social learning systems, they utilize Wenger's (2010) modes of identification or belonging, which include engagement, imagination, and alignment. Engagement involves participating in the activities of the group, imagination involves interpreting an individual's role in the larger world, and alignment is connecting individual's goals with broader group, organization, or system goals/laws. An individual's identity trajectory emerges from these modes.

Scholars of the poststructural perspective often draw from Foucault (1984), who believes identity is formed not by the individual nor social phenomenon. Instead, identity formation is dynamic and “a continuing process of becoming” (Grootenboer et al., 2006). Educational structures, such as curriculum, school policies, and classroom routines, position students and influence the roles the students play in classrooms. Unlike the other perspectives on identity, poststructuralists acknowledge the ways individuals become subjective through power and discourse (Goos, 2005).

After considering the math identity literature and the other constructs of this study–problem solving and self-regulation–I chose the psychological/developmental perspective because the purpose of the study is to explore the relationship between students’ mathematics identities and how they are developed during certain situations and contexts in the classroom setting. Students’ cognitive and affective processes as well as interactions with others within learning environments shape their beliefs about themselves, mathematics, peers, and learning. An in-depth discussion of this study’s theoretical framework is included at the end of this chapter.

Descriptions of identity in mathematics include an individual’s beliefs about mathematics, dispositions towards mathematics, abilities to learn and do mathematics, and sense of belonging within the field of mathematics. Students develop positive math identities when they believe they can do math and believe that they belong (Boaler, 2015). Yet what does it mean to be a “doer” of mathematics? Mathematics is more than domain knowledge and procedural skills. When doing math, students engage in conjecturing, explaining ideas, and constructing mathematical arguments (Schoenfeld, 2014). Students move between what they know and do not know to make sense and work through a problem (Boaler, 2003). This is challenging and at times extremely frustrating for students. However, classrooms that support students in these practices, engage students in the work of mathematicians. According to Burton (1999), to know mathematics, research mathematicians engage in collaboration, have emotional responses to mathematics, use intuition and insight, try different approaches, and desire connections between mathematics and other disciplines. Thus, doing mathematics involves engagement, participation, and persistence in the practices of mathematicians.

Beliefs about Relationship between Math and Self

One aspect of math identity is an individual's self-concept in relation to doing mathematics. Before Martin (2000) coined "mathematics identity", Schoenfeld (2014) articulated this idea as the "belief systems regarding mathematics and one's sense of self as a thinker in general and a doer of mathematics" (p. 4). Thus, identity is not only an individual's beliefs about their abilities and practices in mathematics but also how the individual views mathematics content and learning. For example, a student may be good at math, i.e., achieving high grades or success on tests, which show the student's abilities, but the student may not view mathematics knowledge and practices as an important component influencing the future. Without this strong belief system, Schoenfeld (1988) says students do not take ownership of their learning but become "passive consumers of others' mathematics" (as cited in Solomon, 2009, p. 118).

Therefore, Schoenfeld has continued to advocate for students' agency, authority, and identity in mathematics classrooms by promoting teaching practices that support students in developing their math identities. His Teaching for Robust Understanding of Mathematics (TRU Math) framework provides guiding questions for teachers: Do students have the opportunity to engage productively in mathematics, and feel that they can do so (Agency)? Do they have the opportunity to make the content their own (Authority)? Do they have they have opportunities to see themselves as people who can do mathematics, and to develop positive mathematical identities (Identity)? The framework also provides questions to think about instruction through the students' eyes: What opportunities do I have to explain my ideas? In what ways are they built on? How am I recognized as being capable and able to contribute? (Schoenfeld & the Teaching for Robust Understanding Project, 2016) However, the framework is still in alpha form, and using this framework to engage students in articulating their identity in mathematics has not been studied yet.

Grootenboer and Zevenbergen (2008) incorporate beliefs within their definition of math identity as "students' knowledge, abilities, skills, beliefs, dispositions, attitudes and emotions, that relates to mathematics and mathematics learning" (p. 244). Some may state that identity formation or classroom pedagogy is the same for any subject, just different content. However, within their first model of math identity, Grootenboer and Zevenbergen (2008) argue that the discipline of mathematics is crucial within an identity framework and classroom instruction. Secondary classrooms have lost the nature of mathematics, resulting in school mathematics being much different than the math that research mathematicians undertake (Burton, 1999). Yet to develop students' math identities, mathematics must be central to pedagogy. In agreement, Boaler (2003) found that the teachers' mathematical epistemology that drove their pedagogy significantly influenced their students' mathematics identities. Building on this work, Grootenboer and Zevenbergen (2009) created a "theory of identity and agency in coming to learn mathematics" (p. 341) and offered a model in Figure 1.

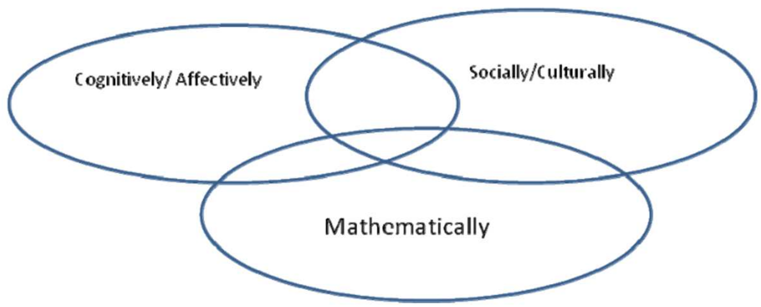

Figure 1. Aspects of working as a mathematician

Similar to Schoenfeld, Grootenboer's (2013) recent work centers on teacher practice. Using qualitative data from effective mathematics teachers, he examined classroom practice that engages students' mathematics identities. Grootenboer (2013) described moral and ethical issues of developing students' mathematics identities; students were not just learning math but also engaging in collaborative group work with peers. Thus, the teachers tried to balance growth and comfort as students learned math and developed their identities in a social context. The findings also revealed the importance of context when developing math identities; students were not learning alone but working with pairs, groups, and outside of the classroom.

Martin also emphasizes the individual and mathematics; his (2000) description of mathematics identity includes four specific areas: beliefs about an individual's "(a) ability to do mathematics, (b) the significance of mathematical knowledge, (c) the opportunities and barriers to enter mathematics fields, and (d) the motivation and persistence needed to obtain mathematics knowledge" (p. 19). In other words, math identity is beliefs about doing math, why it is important, and resistance and perseverance in math. Martin's (2006) work looked at three ethnographic and participant observation studies of African American students and their parents, as they conceptualized mathematics learning as racialized experiences. Through a narrative approach, this study explored mathematics socialization, specifically looking at people's experiences that influence their participation in mathematics, and how these experiences were interpreted and internalized to shape an individual's mathematics identity, or their self-concept and self-understanding about math. To support African American students' success in math, Martin (2006) advocates for "leveraging knowledge about (a) the mathematical experiences of African American parents, (b) their perceptions of school-based mathematics, (c) how parents situate school-based mathematics in their lives and their children's lives relative to their socioeconomic and educational goals, and (d) their resulting advocacy practices" (p. 224). Parents can be key partners in this reform effort in math education, especially considering that math identity is influenced by others' views of an individual's relationship with mathematics.

Expanding on Martin's earlier work about taking action on an individual's opportunities, Varelas, Martin, and Kane (2012) studied elementary students' learning as a process of content learning (CL) and identity construction (IC) for meaning making through narratives. Content learning is developing disciplinary concepts, processes, tools, language discourse, and norms within practices whereas identity construction is defined as seeing oneself in relation to communities. They found that how students view their own learning, how they understand their knowledge as well as knowledge gaps, and how they position themselves as learners in relation to others is pivotal to building a positive mathematics identity.

Researchers (Aguirre, Mayfield-Ingram & Martin, 2013; Grootenboer et al., 2006; Piatek-Jimenez, 2015) acknowledge that these perceptions are twofold: how students see themselves and how others, including teachers, parents, and peers, see them as doers of mathematics. Therefore, it is important that students are encouraged to engage in mathematics from a young age.

Belief that One Belongs

Math identity can also be formed when students engage in social interactions and learn mathematics as members of a community of practice (Wenger, 1998, 2010; Wenger-Trayner, E., & Wenger-Trayner, B., 2015). As in any classroom, some students identify with the content and others do not feel that they belong. Belonging to a group may result in feelings of security, commitment, value, self-esteem, or other positive attitudes (Boaler et al., 2000). Not separate from an individual's beliefs previously described, belonging is part of one's sense of self and self-concept.

Boaler has written extensively about math identity from the student perspective and how it is formed within classroom communities. To understand why some students would want to continue studying math after their senior year and others would not, Boaler et al. (2000) interviewed 120 secondary students, aged from 14 to 18, in England and the United States. One group was 48 Advanced Placement (AP) calculus students in six Northern California public schools and the other group was 72 students from six schools in the United Kingdom. The first group was interviewed about their confidence in mathematics whereas the second group was interviewed about issues related to their math learning experiences. Findings included insights about math identity, factors that influence math identity, and indication that students do not struggle from a failure of ability; rather, they struggle with belonging to community of practice because learning is a social practice.

Boaler (2002a) explored how students increase their competency, shifting from solely developing math knowledge to looking at students' dispositions towards mathematics and practices to engage in mathematics. Thus, she challenges what it means to know and do mathematics, incorporating practices, norms of the classroom, and learning practices. Considering how students engage in these practices influenced Boaler's (2002a) definition of mathematics identity–"the knowledge they possess, as well as the ways in which students hold knowledge, the ways in which they use knowledge and the accompanying mathematical beliefs and work practices that interact with their knowing" (p. 16-17). Reflecting on three different studies, including the calculus one above, Boaler (2002b) attempts to make sense of how mathematical practice influences knowledge and identity and create a model. In discussion-oriented classrooms, students formed relationships with mathematics that did not conflict with their other identities. Although these students were scoring similar levels on assessments as students in traditional classrooms, they developed active relationships with the mathematics. In these discussion-oriented classrooms, where they were invited to participate and contribute thoughts, their own and disciplinary agencies were supported. Boaler (2002b) calls the connection between identity and knowledge a "disciplinary relationship" (p. 10).

Boaler also examined the impact of ability tracking on math identity. Boaler and Staples' (2008) mixed methods study, often known as the Railside study, looked at three different schools with various degrees of tracking and traditional versus inquiry-based instruction. They studied student achievement and attitudes and documented teacher and student practices. Findings indicated school tracking has a negative effect on identity development in the lower tracks. Railside heterogeneous classrooms were multicultural and multilingual. These inquiry-based classrooms also supported and valued students using different methods and approaches, sharing ideas with others, and making mistakes and offering incorrect ideas. Thus, students felt they belonged to their classroom community of math learners and were more likely to succeed in their careers and jobs. Thus, Boaler discusses students' mathematical knowledge, beliefs, and practices; much of her work has revealed effective teaching practices that provide students with a sense of belonging to the math community within the classroom.

Solomon (2007) also emphasizes the community aspect. During interviews, undergraduate math students expressed feelings of not belonging or marginalization. Although students did well in math, they did not feel that they could make constructive connections or contributions in mathematics. These findings were gendered and have implications for math instruction in higher education. Solomon's (2009) work focuses on elementary school to undergraduate students and the stories they tell about their relationship to mathematics. This book included multiple past studies to analyze relationships between language, learning, and mathematical knowledge and between identity, equity, and processes of exclusion/inclusion. Solomon (2009) describes mathematics identity as beliefs about an individual's self as a mathematics learner, perceptions of being seen by others as a mathematics learner, beliefs about the nature of mathematics, engagement in mathematics, and perceptions of oneself as a potential participant in mathematics.

Given the previous descriptions, we know math identity is based on more than just having knowledge and skills. Math identity also refers to an individual's beliefs, attitudes, emotions, and dispositions towards mathematics. While self-perceptions are part of students' math identities, they also include social identity, which students develop through shared interactions and social processes. As students develop their math identities, they make sense of their relationship with mathematics, understand their own learning practices, and feel a sense of belonging. With this understanding of math identity, this study analyzes how students articulate their own math identities and offers a way to measure students' math identities.

Measuring Math Identity

As seen in the previous studies cited, most math identity studies are qualitative, using case studies that utilize narrative inquiry, counternarratives, interviews, focus groups, and observations. Those who take a quantitative approach use surveys; however, asking about a students' math identities often manifests itself with a simple statement of "I am a math person" or "Others see me as a math person" (Alexander, 2015; Cass et al., 2011; Cribbs, 2012; Heller, 2015). "Math person" is vague language so it is unknown how the respondent is perceiving "math person," i.e., scores well on math tests, displays characteristics and practices of mathematicians, or can simply do quick mental math. Thus, there is a need to use a mixed methods approach to better understand what students mean by the term “math person”.

Implications for Instructional Practice

Researchers provide evidence of a strong relationship between learning mathematics and developing a mathematics identity. Thus, given what is known about math identity, researchers and educators need to understand how to develop students’ math identities through mathematical learning. Aguirre et al. (2013) recommend five equity-based teaching practices that align with the National Council of Teachers of Mathematics’ (NCTM, 2014) *Principles to Action* teaching practices. First, it is recommended that teachers “go deep with mathematics,” meaning problems must promote reasoning. Schoenfeld (2013) attributes powerful instruction to developing powerful thinkers and problem solvers, and therefore, advises teachers to provide good problems and instruction that engages students in problem solving strategies and making sense of the mathematics. Second, good instruction leverages multiple mathematical competencies. Students enter the classrooms with various skill and knowledge, so teachers are challenged with supporting students in linking their informal knowledge and skills to the formal rules, notations, and procedures to make strong conceptual connections (Bruer, 1993).

Third, teachers “affirm students’ mathematics identity and help them develop a sense of agency by promoting persistence and reasoning during problem solving and encouraging students to see themselves as confident problem solvers and as active participants in mathematics" (Berry, 2016 as cited in Larson, 2017, p. 8). Martin (2006) found that perceptions of parents and teachers about students in mathematics influenced students' academic competence and performance. Math is a complex subject and learning is messy, so students need support to work through productive struggle. Support and motivation are key as students engage as doers of mathematics, and this relies on the teacher knowing the students as learners and also having a clear understanding of their math knowledge. Both students and teachers can use evidence of students' thinking to affirm students' knowledge of certain math concepts and encourage flexible thinking about math or going for the answer. Diane Briars (2016), former NCTM president, states that teachers send implicit and explicit messages about mathematics identity every day. Educators might consider which students work together, who shares their work in partners or whole class, and which students are asked higher- and lower-level questions.

Fourth, teachers can challenge spaces of marginality by using students' experiences and knowledge within classroom math discussions. All students come with knowledge about mathematics that reflects their background and experience; these can be used as assets in the classroom. Lastly, instruction can incorporate multiple resources of knowledge from math to language to culture to family. In math, multiple representations support students in making sense of a problem, working through a barrier in problem solving, or allowing for visualization. Only by understanding students' representations of math can teachers truly make sense of the evidence of students' thinking.

Looking at teachers' practices that develop students' math identities, Grootenboer's (2013) found the need for a delicate balance. When learning mathematics, teachers experience a tension between "protecting students' (often fragile) mathematics identities and facilitating unease and discomfort so growth can occur" (p. 330). By involving students in the cognitive labor of a rigorous math task or lesson, students do not just memorize the skill or algorithm but improve their math identities (Grootenboer & Zevenbergen, 2009). Thus, by working through the discomfort of a challenging math problem, students engage in meaningful discourse of ideas, questions, and solutions, which in turn develop broader and well-rounded math identities. Teachers who engage students in this work typically have well developed mathematics identities, enjoy working on math topics themselves, and facilitate caring teacher-student relationships within their classrooms (Grootenboer & Zevenbergen, 2008).

Although many studies connect classroom communities and instruction to positive math identity, the instructional recommendations, including those from Aguirre et al. (2013) above, focus on grades K-8. This study will focus on 9-12 math education. Also, the relationship between students' math identities and the practices that students are learning from this instruction, i.e., their problem solving and self-regulation strategies, has yet to be studied. In today's workplace, students are required to demonstrate well-developed thinking skills, problem-solving abilities, design strategies, and communication capabilities (Conley & Darling-Hammond, 2013). Thus, students must be able to work through new situations, monitor their knowledge, and evaluate their work and solutions. Problem solving and self-regulation are critical skills for students' futures.

Mathematical Problem Solving

Problem solving is a cognitive process that is considered an essential skill in school but also in everyday life situations (Jonassen, 2003). Problem solving involves refining, combining, and modifying knowledge to obtain successful solutions and reach a goal despite the solution pathway being unknown at the outset (Bransford & Stein, 1993; NCTM, 2014; Newall & Simon, 1972). Thus, the process is complex and often difficult.

Why is Math Problem Solving Important?

It is not surprising that American students do not excel at problem solving. According to OECD (2010 as cited in OECD, 2013, p. 6),

> Problem solving competency [is]…an individual's capacity to engage in cognitive processing to understand and resolve problem situations where a method of solution is not immediately obvious. It includes the willingness to engage with such situations in order to achieve one's potential as a constructive and reflective citizen. (p. 6)

In 2012, 15-year-old American students who took the PISA test, which examines students' application of knowledge to real-world problems, averaged 508 in problem-solving skills, which is slightly above the 500-point average of the 28 participating OECD countries but below high-performing countries like Japan, China, and Finland. Specifically, a PISA report states that students in the United States did not perform well on "higher cognitive demands, such as taking real-world situations, translating them into mathematical terms, and interpreting mathematical aspects in real-world problems" (OECD, 2013, p. 1). Yet in the past ten years, with an increase in STEM careers and occupations, jobs require high problem solving skills for non-routine tasks (OECD, 2013). Thus, it is critical that students learn problem solving and critical thinking to prepare for future careers in the changing economy.

Over the last 100 years, the value of teaching problem solving has been debated. Researchers and practitioners teeter between advocating for curriculum based on basic skills and procedural understanding to more conceptual understanding and practice-oriented curriculum, including mathematical thinking and problem solving. In 1989, NCTM published *Curriculum and Evaluation for School Math*, which emphasized the process of doing mathematics and gave value to problem solving in the classroom. In 2010, the Common Core State Standards articulated the practices of mathematically proficient students with the eight Standards for Mathematical Practice:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Within the classroom, teachers may explicitly teach these practices, and students also engage in them while completing tasks. Many of these practices are especially useful when students are solving problems they have not previously encountered. The practices are not a linear checklist but describe how students might engage in doing mathematics.

Since NCTM emphasized problem solving with its 1989 math standards, pedagogy, and assessment recommendations, researchers found that mathematics curricula could teach students to problem solve successfully (Senk & Thompson, 2003). Curricula emphasizing problem solving is correlated to students' math success on rigorous problems, interpretation of mathematical representations, and conceptual understanding. Still if practitioners are to teach problem solving to K-12 students, they must understand what problem solving is. However, Chamberlin (2008) states mathematics faculty and researchers have not agreed on one definition, and there is little hope that an agreed upon definition of math problem solving will ever exist. Examining types of problems and how to solve them is one way to understand problem solving.

Types of Problems

Not all math questions are created equal–some are problems while others are simply exercises. George Pólya (1945, 1957), considered by many to be the father of problem solving, calls exercises "routine problems," meaning a task that "can be solved either by substituting special data into a formerly solved general problem, or by following step by step, without any trace of originality, some well-worn conspicuous example" (p. 171). To solve exercises, learners can follow the teacher's step-by-step process, a common algorithm, or textbook examples. For example, a worksheet filled with problems such as find the remainder of 276 divided by 21 or solve the equation $2x + 5 = -11$ would be considered exercises.

Exercises or routine problems could also be considered well-defined or well-structured problems because the problems are constrained and have a right answer. Well-defined problems are constrained by a specific topic in a textbook, clear goals, defined solution pathways, and expected solutions (Schacter, Gilbert, & Wegner, 2009). All factors of the problem are described in detail, which allows for more initial planning and certainty. These problems have right answers, meaning the goal or solution is recognizable and can be found by the application of an appropriate algorithm. Textbook problem sets are typically well-defined problems. Thus, a certain degree of novelty is needed for mathematical problem solving (Pólya, 1945, 1957) because an individual is trying to find a solution that is unknown, or trying to achieve something without knowing a straightforward way (Schoenfeld, 2013).

An individual confronts a "problem" when a defined path for solving problems is not known (to the solver). When individuals have to convince someone else or work to make sense of the problem and solution pathway, these are problems. Solving problems is messy, non-linear, and at times, a lengthy process. Only in the end does the final solution pathway become concise and elegant. Schoenfeld (1992) calls these types of problems non-routine (novel) versus routine (exercise). Non-routine problems require learners to apply content knowledge and practices, i.e., mathematical thinking and problem solving.

Problems or non-routine problems could also be considered ill-defined or ill-structured or even messy problems. Kyung, Jiyoung, Jiyeon, and Eunkyung (2011) characterize these problems with authenticity, complexity, and openness (as cited in

Byun, Kwon, & Lee, 2014). Authenticity means the task reflects real life situations and are not constrained to the classroom or textbook. In the real world, data are conflicting or inconclusive, people disagree about appropriate assumptions or theories, and values are in conflict, so when students work on ill-defined problems in class, they experience problems that they might encounter in the future.

Complexity means there is uncertainty in the concepts or procedures to complete the task and inconsistent relationships between these concepts or procedures. Because these problems address complex issues, they cannot easily be described in a concise, complete manner. These problems often have unclear definitions, uncertain goals, and no limiting conditions. Without detailed constraints, students can benefit from understanding the "problem context that may involve social, economic, cultural, etc. issues and are open to solvers' interpretations and negotiation" (Goel, 1992 as cited in Toy, 2007, p. 26).

Openness offers students a chance to make their own assumptions, interpretations, and conclusions, provided they give proper justification. Because ill-defined problems do not end with one clear answer but may have a range of acceptable solutions, students may debate the strengths and weaknesses of these solution options. Simply telling procedural steps is not enough to convince another student of a solution, students must consider others' views, create claims for their proposed solution pathway, and justify their arguments.

Solving ill-defined problems is not an easy undertaking and may require judgment, planning, multiple strategies, and use of previously learned skills or knowledge of concepts (Huba & Freed, 2000). Sinnott (1989) recommends the construction of a problem space, choice and creation of a solution, monitor/memory/non-cognitive factors, and use of think-aloud protocol (as cited in Byun et al., 2014). Ge and Land (2003) offered a similar approach: problem representation, developing solutions, developing justification, and monitoring and evaluation, and Jonassen (1997) proposed a model for solving ill-defined problems:

> a) Presentation of problem space and contextual constraint, b) verification of alternative views, standpoints and perspectives, c) creation of possible problem solving methods, d) evaluation of feasibility of alternative solution methods through construction of disputes and expression of personal belief, e) monitoring of problem space and choice of a solution, f) execution of a chosen solution and monitoring, and g) the process of applying the chosen solution. (as cited in Byun, et al., 2014, p. 293)

Solving a Problem

Although many textbooks show problem solving as a linear step-by-step way to get a solution, Pólya intended his problem solving phases to be stages not steps. In his 1945 book *How to Solve It*, Pólya provides the foundation for problem solving research with four phases:

1. Understanding the problem: What is the unknown? What are the data? What is the condition?
2. Devising a Plan: Do you know a related problem? Look at the unknown! Here is a problem related to yours and solved before. Could you use it?

3. Carrying Out the Plan: Carry out. Check each step.
4. Looking Back: Examine. Check the result.

These phases support students in working from an unknown, where the path to the answer was unclear, through a process to get to a solution. Building on Pólya's principles, Schoenfeld (1985, 1992) developed a framework to explore problem solving and, more broadly, mathematical thinking. He captured these characteristics in four categories of problem solving activity: knowledge of the content, heuristic strategies, monitoring and self-regulation, and student beliefs. Yet Schoenfeld (2010) found this framework lacking–it was not a theory of problem solving that explained how and why student made choices and decisions.

Shifting to a theory of "goal-oriented decision making in complex, knowledge-intensive, highly social domains" (Schoenfeld, 2013, p. 15), his book *How We Think* provides a basic theory of "in-the-moment decision making" (p. 17). This shift to decision making aligns with Schoenfeld's view that mathematics is about sense making. Students need to engage with the content, work through misunderstandings and new ideas, and come to their own conclusions and questions. To be a good problem solver, an individual must be willing to dig into new problems, be a flexible thinker, and be willing to persevere in the face of difficulty (Schoenfeld, 2013). The typical psychological traits that may benefit a successful problem solver are as follows: correctly identify problem goals, be persistent, adopt efficient strategies in search, and be able to trace back to a certain previous point in the solution process. Thus, Schoenfeld's (2010) current theory of problem solving includes:

1. The goals the individual is trying to achieve;
2. The individual’s knowledge (including resources and heuristics);
3. The individual’s beliefs and orientations (about him- or herself, about mathematics, about problem solving); and
4. The individual’s decision-making mechanism (including metacognition aspects, i.e., monitoring and self-regulation)

Goals. Problem solving is a cognitive process “that searches a solution for a given problem or finds a path to reach a given goal” (Wang & Chiew, 2010, p. 82). Thus, when there is a problem, there is a goal, which cannot immediately be attained (Newall & Simon, 1972; Wilson, Fernandez, & Hadaway, 1993). To work towards the goal, or the desired state of a solution to a problem, the problem itself typically provides givens and operations, which are possible moves or actions to work towards solving the problem. These actions exist within the problem space, where all the possible goals and paths potentially related to the problem known by a problem solver exist (Wang & Chiew, 2010). To support thinking through a problem, many problem solvers set subgoals that break down the problem into smaller goals.

Goals are aligned to types of thinking. Directed thinking is goal-oriented and rational whereas undirected thinking is unclear, does not move towards to goal but wanders and drifts. Although undirected thinking may be helpful for creative endeavors, it can be unproductive for goal-oriented problems.

Knowledge and heuristics. Mathematics knowledge is critical to success in problem solving. Content knowledge includes definitions, formulas, notations, tools, and key concepts but also drawing on prior content knowledge connected to the problem. Cognitive scientists have developed explicit models of expert knowledge and skills in a number of mathematical domains (Bruer, 1993). Experts tend to have more knowledge, better knowledge, and more inter-connected knowledge structures (Kellogg, 2016). Boaler (2000) found that students who were not experts at the content could still engage in the practices of mathematics, and Resnick (1988) found that students lacking essential content knowledge struggle with problem solving. Yet to transfer knowledge and skills to a new problem being a flexible thinker and able to use mathematical practices is important.

Individuals also use problem solving strategies. These heuristics are informal strategies or approaches that work under some circumstances, unlike algorithms which are a set of rules guaranteed to produce the correct answer (Kellogg, 2016). Pólya (1945, 1957) recommended a variety of strategies: guess and check, look for a pattern, draw a picture, solve a simpler problem, use a model, and work backwards. Although the strategies may seem easy, i.e., draw a picture; an individual needs to know if that is an appropriate strategy and how to use it to move towards the solution. Pólya's strategies are not step-by-step processes. However, many practitioners (Schoenfeld included) found these to be too broad. Schoenfeld used Pólya's original list as categories to create more specific strategies that were useful for students. Looking at problem solving research, Kantowski (1977) noticed it was focused on the product and not the process of working through unknown paths towards a solution. Using a pretest-posttest, she found problem solving skills, specifically heuristics, were related to measurable student outcomes, as shown by student success solving problems.

Although students need knowledge and problem solving strategies, providing students with different resources, tools, and strategies for every problem is not helpful. Lester (1988) found that this support is piecemeal and not productive for students learning to problem solve and think mathematically. Thus, learners need math content knowledge and problem solving tools, strategies, and resources, but it is advisable to teach and use these in thoughtful and systematic ways.

Beliefs and orientations. Another category of Schoenfeld's (2010) theory of problem solving is beliefs and orientations. Thus, this echoes the previous discussion of mathematics identity as beliefs one can do math and one belongs. These beliefs may be about the individual's personal strengths and capabilities in math or based on past successful and failed experiences within the classroom.

Researchers (Garofalo & Lester, 1985; Hoffman & Spatariu, 2008) found that problem solving success was a combination of ability, estimation of task success, and beliefs about subject and test. Hoffman and Spatariu (2008) found self-efficacy, or the belief that one has the ability, increases problem solving efficiency, and reflective hints facilitate problem solving success. A study of college students by Shen, Miele, and Vasilyeva (2016) found problem solving ability is related to mindset and previous experiences of success and failure.

Decision making. Throughout problem solving, individuals engage in metacognition, or thinking about their thinking. While problem solving, they think about and reflect on learning and understanding, asking: Do I have all the information? How are these two components connecting? Within the problem space, an individual's monitoring consists of assessing, controlling, and directing one's progress in understanding and solving the problem. Good problem solvers have well-developed metacognitive skills–see the gaps in their thinking, understand and verbalize their thinking processes, and make corrections to their thinking (Brown, Bransford, Ferrara, & Campione, 1983). Teachers play a key role in supporting and developing students' metacognition by helping students select learning strategies and asking monitoring questions about students' learning approaches. With this support, students are able explore new connections between concepts and transfer their knowledge to new contexts or tasks (Bransford, Brown, & Cocking, 2000). Schoenfeld (1992) recommends the following monitoring questions when students are problem solving: "What (exactly) are you doing? (Can you describe it precisely?) Why are you doing it? (How does it fit into the solution?) How does it help you? (What will you do with the outcome when you obtain it?)" (p. 397). These questions can be asked by the teacher or peers to support individuals in thinking through a problem and adjusting their thinking.

Measuring Problem Solving

The benefits of teaching students to problem solve are apparent; problem solving supports students' mathematical learning of both concepts and procedures and accurately reflects what it means to do mathematics (Wilson et al., 1993). Yet problem solving itself is hard to measure besides achievement on non-routine problems.

Often in studies, “good” problems are created and students are observed as they complete these chosen problems. Observers may review student work and ask students to talk or “think” aloud. Schoenfeld (2013) acknowledges that most of his research on problem solving focuses on individual students doing provided problems with clear goals (“solve this problem”), but this has its limitations. This limits problem solving to a point in time and ignores the learning and development process of problem solving, which occurs over time inside and outside of class. He acknowledges that his past work has been at the micro-level but sees the need for macro-level studies. Therefore, he questions how issues of learning and development might be incorporated into a theory of decision-making. Another challenge in problem solving research is collaboration. As students learn and make sense of mathematics through problem solving, they interact with peers and the teacher, providing ideas, gathering feedback, and critiquing others’ reasoning. Through this collaborative process, students refine and reorganize the structure of their mathematical knowledge and problem solving skills.

Another way to measure problem solving is analyzing students’ views of “themselves as capable of using their growing mathematical knowledge to make sense of new problem situations in the world around them” (NCTM, 1989, p. ix).

Connection to Math Identity

Although researchers recommend developing students’ math identities and know problem solving is a key skill for mathematics understanding as well as 21st century careers, few studies look at the relationship between the two. Greeno (1997) examined middle school learning environments using communities of practice and problem solving sessions. The findings indicated significant gains in students' problem solving (as cited in Quinn, 2005). Another study analyzed the math identities of six Black male students through videotaped problem solving sessions (Grant, Crompton, & Ford, 2015). In this study, math identity was defined as "participation through interactions and positioning of self and others" (p. 87). Over four years of the study, the students' self confidence and engagement in mathematics increased while their reliance on others decreased. Houston (2017) studied the influence of a metacognitive strategy instruction on elementary students' problem solving achievement and mathematical agency by rating students' agency with a rubric during problem solving activities and scoring students' problem solving skills. Findings indicated that this type of instruction positively influenced students' math agency.

However, other studies focus on teachers' math identities and incorporate problem solving as instruction or professional development. Frank (2013) observed problem solving in classrooms while focusing on middle school teachers' math identities while Johns (2009) investigated the relationship between teacher identity and problem solving instruction occurring in math class communities. In Gujarati's (2010) study, problem solving was part of teachers' professional development. Thus, there is a need for more studies that relate math identity and students' math problem solving.

Self-regulated Learning

Self-regulated learning, or self-regulation, is the "self-directive process through which learners transform their mental abilities into task-related academic skills"

(Zimmerman, 2001, p. 65). By engaging in self-regulated learning, students seek to manage affective, cognitive, motivational, and behavioral strategies to attain a goal (Cleary & Zimmerman, 2004).

Why is Self-regulated Learning Important?

According to the U.S. Department of Education (2011), secondary students need to be college and career ready. In high schools today, students are focused on meeting the academic achievement requirements to attend college. Yet grades or tests often symbolize the end of learning instead of a road map to increase mastery or maintain a high level of content knowledge and understanding. The Career Readiness Partner Council (CRPC, 2012) states:

> Career readiness has no defined endpoint. To be career ready in our ever changing global economy requires adaptability and a commitment to lifelong learning, along with mastery of key knowledge, skills, and dispositions that vary from one career to another and change over time as a person progresses along a developmental continuum. (p. 8)

In order to do this, students must engage in continuous improvement and learning; learning over time helps build understanding (Dunlosky, Rawson, Marsh, Nathan, & Willingham, 2013).

Therefore, focusing on the process of learning, assessing, and improving is another option instead of teachers and students stressing only achievement. According to Nicol and Macfarlane-Dick (2006), teachers are reluctant to give students more control within the learning process. Yet one cause of academic failure is the lack of self-regulation and motivation (Cleary, 2006; Cubukcu, 2009; Schunk, Pintrich, & Meece, 2008). According to Borkowski and Thorpe (1994), underachievers, who lack self-regulation, are "more impulsive, have lower academic goals, are less accurate in assessing their abilities, are more self critical and less efficacious about their performance and tend to give up more easily than achievers" (p. 54, as cited in Cubukcu, 2009).

There are many theoretical perspectives and models of self-regulation, but all emphasize several critical elements: learners are proactive and exert control on their learning, behaviors, and environments; learners actively develop their skills, strategies, and metacognition; and learners are motivated to participant in the learning process (Schunk, 2005; Zimmerman, 1989). Two core components of self-regulated learning models are self-regulated learning strategies and motivational beliefs. As active participants in the learning process, learners utilize self-regulated learning strategies, e.g., make choices about how to learn, seek additional instruction or challenges as needed, and structure and organize their environment to support their learning. For learners to attain their selected goals, they must be motivated. One motivational belief is self-efficacy, or the "the perceived ability to implement actions necessary to attain designated performance levels" (Bandura, 1977 as cited in Zimmerman & Schunk, 2001, p. 10). Another belief is perceived responsibility, or when learners feel they have the ability to choose outcome expectations and successfully use a particular strategy.

Metacognition

Metacognition means thinking about one's thinking. Flavell (1979) describes three kinds of metacognitive knowledge: awareness of one's knowledge and other's knowledge, awareness of thinking, and awareness of thinking strategies. Pintrich (2002) recommends that students use metacognitive strategies for learning and thinking but also know about them and their benefits. In other words, students do not just use the strategies because their teachers instructed them, but they consciously use the strategies (Zohar & David, 2009). Also, teaching and using metacognition is to be embedded within content so that it is not generic (Bransford et al., 2000). Zohar and David (2009) agree and argue that metacognition is most effective when it reflects the specific discipline, context, class, or concept.

There are many benefits to metacognition. By having awareness of knowledge, thinking, and thinking strategies, students are not only learning the content but also thinking about the content in different contexts and thinking about themselves as learners within these contexts. Weimer (2012) recommends teachers or students ask: "What are you learning?" and "How are you learning?" (p. 1) With this depth of thinking, students understand their strengths and weaknesses and are able to "actively monitor their learning strategies and resources and assess their readiness for particular tasks and performances" (Bransford et al., 2000, p. 67). Thus, when students are aware of their knowledge, thinking, and thinking strategies (metacognition), they can regulate their learning. "Metacognitive regulation involves the ability to think strategically and to problem-solve, plan, set goals, organize ideas, and evaluate what is known and not known. It also involves the ability to teach to others and make the thinking process visible" (Darling-Hammond, Austin, Cheung, & Martin, 2003, p. 161), and these same strategies are echoed in the self-regulation learning strategies as students self-direct their own learning.

Self-regulated Learning Model and Strategies

Although there are many models of self-regulation, e.g., Boekarts, Borkowski, Pintrich, Winne, and Zimmerman (Puustinen & Pulkkinen, 2001), the focus of this study is Zimmerman's self-regulated learning model. His (1989) first model of self-regulation was built from Bandura's (1986) triadic analysis of self-regulated functioning, which included personal, behavioral, and environmental determinants (as cited in Usher, 2009). Self-regulated learning, or self-regulation, is the "process by which learners personally activate and sustain cognitions, affects, and behaviors that are systematically oriented toward the attainment of learning goals" (Schunk & Zimmerman, 2008, p. 1). In 2000, Zimmerman expanded the model with a cyclical feedback loop to show complex, dynamic interactions between motivational, strategic, and metacognitive processes (Lubin, 2015). In 2003, Zimmerman and Campillo updated the model to be a three-phase cycle to incorporate the phases of forethought, performance, and reflection.

In the forethought phase, the learner determines a goal within a set time period. Setting a goal is critical because later the learner self-evaluates his or her learning and performance from this standard. During this phase, the learner also creates a strategic plan to identify specific strategies, behaviors, or thoughts that are used during performance. While goal setting and planning, the learner considers self-motivation beliefs, including self-efficacy, outcome expectations, intrinsic interest/value, and goal orientation. The learner asks, "Can I do it?" and "Why is this important?"

In the performance phase, the learner is engaging in self-generated actions and self-observation. While performing, the learner is self-monitoring as he or she is metacognitively aware of the quality of his or her competency and skill levels. Throughout this phase, the learner asks, “Do I think I have performed a flawless process thus far or have I made any mistakes?”

In the self-reflection phase, the learner self-judges his or her learning and self-reacts to the performance. The learner self-evaluates based on the goal for performance and notes perceived causes of success or failure. The learner also reflects on his or her satisfaction with the performance (Cleary, Callan, & Zimmerman, 2012). In 2009, the self-regulation model was again refined to showcase how these processes interact (Panadero & Alonso-Tapia, 2014). While these models do not indicate the importance of context, Schunk (2005) stated that self-regulated learning is situationally specific in a social environment.

“Self-regulated learning strategies can be conceptualized as purposeful actions and processes directed at acquiring skill or information” (Zimmerman, 1989 as cited in Cleary, 2006, p. 309). Strategies have been described by Zimmerman and Martinez-Pons (1988, 1990) and streamlined from 15 to ten general categories of self-regulation strategies by Cleary (2006). Strategies include task analysis and strategic planning, goal setting, self-monitoring as implement strategies, sense making and seeking help as needed, ownership of learning and actions, and evaluation of goals and strategies.

Motivational Beliefs

Zimmerman (2002) and Pintrich (2004) emphasized how motivation interacts with cognitive, behavioral, and environmental factors, while other models centered on cognitive processing (Winne, 1996) or emotions (Boekaerts & Nievimirta, 2000 as cited in Panadero & Alonso-Tapia, 2014). Motivational beliefs include self-efficacy and perceived responsibility, which have been found to predict motivation and academic success (Lubin, 2015). These correlate with self-regulated learning strategies and achievement levels, meaning all predict motivation and academic success.

Self-efficacy. According Luszczynska and Schwarzer (2005), Bandura predicts "expectations of self-efficacy are self-regulatory cognitions that determine whether instrumental actions will be initiated, how much effort will be expended, and how long it will be sustained in the face of obstacles and failures" (p. 128). Thus, self-efficacy beliefs about personal abilities to learn and perform behaviors to outcome expectations may come from mastery experience, social modeling, social persuasion, or psychological or physiological responses (Bandura, 1997; Zimmerman & Schunk, 2001). Mastery experiences have been found to have the greatest influence on self-efficacy (Briggs, 2014).

Students' self-efficacy beliefs have been linked to positive math achievement. Pajares and Graham (1999) found that sixth grade, middle school students' self-efficacy was the sole motivation variable that predicted students' performance, when also looking at anxiety, self-concept, and self-regulation. Also, positive self-efficacy along with goal setting has been linked to quality of decision-making, goal setting, and academic achievement (Maddux, 1995 as cited in Luszczynska & Schwarzer, 2005, p. 128).

Perceived responsibility. Perceived responsibility is the extent to which learners should control their lives and learning. When learners feel they can take action, they have a sense of control over the environment or personal agency. However, if learners do not feel confident in their ability to meet the expectation, they may feel hopeless or depressed. Thus, self-efficacy and perceived responsibility are closely tied. Learners who take ownership of their own development move from passive learners in static learning environments, waiting to respond to teacher prompts, to actively learners and thinkers within a process. By engaging in metacognition, learners monitor, direct, and regulate actions toward goals (Paris & Paris, 2001). When learners feel they have the ability to choose outcome expectations and successfully use a particular strategy, they feel motivated. This in turn results in students having a sense of responsibility, and "when students feel a sense of ownership, they want to engage in academic tasks and persist in learning" (McCombs, 2012, p. 1). This is in opposition to meeting an external standard (grade) and getting extrinsic rewards (Tzohar-Rozen & Kramarski, 2014). While the teacher plays a key role in instruction and assessment, "classroom environments and experiences should show each student that he or she can gain control over their own learning outcomes if they adopt self-regulatory strategies" (Borkowski, Chan, & Muthukrishna, 2000, p. 34).

Perceived responsibility is highly correlated with grade-point-average (GPA) and predicted 22 percent more variance in grade point average than homework (Zimmerman & Kitsantas, 2005). According to Rattan, Good, and Dweck (2011), research has shown that students' perceptions of their ability affect their motivation: students who believe their ability is fixed draw conclusions about their ability from setbacks and give up quickly when challenged, as compared to students who believe their abilities can grow and change.

Measuring Self-regulated Learning

Self-regulation can be measured through event or aptitude measures (Winne & Perry, 2000). Event measures are moments in time focusing the micro-level, while aptitudes measures are self-report questionnaires asking about retrospective, macro-level behaviors. One tool is Zimmerman and Martinez-Pon’s (1986) observation tool for certain self-regulation strategies. Teachers or observers look for student use of these strategies, the frequency with which students use various self-regulation strategies in a specific academic subject, and how students respond. Other event tools include think-alouds, diaries/logs, and interviews before, during, and after events. Although capturing self-regulation in the moment may seem ideal, self-report measures have higher reliability than interviews (Pintrich, 2000) and can capture the unobservable.

Aptitude measures include self-report surveys to ask students what motivates them and why they are using a specific strategy; these are most common since they capture both individual knowledge and strategies. Measure options include Pintrich’s *Motivated Strategies for Learning Questionnaire (MSLQ)* (Pintrich, 2004; Pintrich & De Groot, 1990; Wolters, Pintrich, & Karabenick, 2003), the *Learning and Study Strategies Inventory (LASSI)* (Weinstein, Palmer, & Acee, 2016), the *Junior Metacognitive Awareness Inventory (Jr. MAI)* and the *Metacognitive Awareness Inventory (MAI)* (Sperling, Howard, Miller, & Murphy, 2002) for elementary and secondary students, *Rating Students Self-regulated Learning (RSSRL)* (Zimmerman & Martinez-Pons, 1988) for rating individual students, and *Self-Regulation Strategies Inventory - Student Version (SRSI-SR)* (Cleary, 2006). Each of these surveys captures beliefs, attitudes, and perceived behaviors and asks students to retrospectively rate self-regulation behaviors. In recent years, Cleary's (2006) SRSI-SR self-report measure of self-regulation strategies has been studied, validated, and used with teacher and parent comparison data. This tool analyzes self processes, i.e., goal setting, learning strategies, and self-recording, with an internal consistency of alpha = .92.

Connection to Math Identity

Few studies make connections between math identity and self-regulation. Briggs (2014) used social cognitive theory to determine if a relationship exists between mathematics self-efficacy and mathematics identity to mathematics achievement. His quantitative data were from the High School Longitudinal Study of 2009 (HSLS:09), specifically focusing on Black males. Findings indicated a positive relationship between mathematics self-efficacy and mathematics identity to mathematics achievement. Peterson (2016) investigated an Algebra II program's effects on promoting motivation and achievement by facilitating math identity exploration. Surveys were used to measure participants' beliefs, goals, self-perceptions, and perceived action possibilities. The study found the intervention to effect some students' math identity exploration but not all. A study by Rashid (2014) focused on parental involvement but also made connections between students' self-regulation and persistence. Literature on math identity was minimal, with only Martin's work included. Thus, there is a need for studies to relate students' math identities and their self-regulated learning.

Theoretical Framework: Social Cognitive Theory

There are many theoretical perspectives from which to view identity and identity development, including the psychological/developmental, sociocultural, and poststructural perspectives. After considering these three perspectives described earlier in this chapter, I chose the psychological/developmental perspective. From this viewpoint, the individual is the focus of identity, related to self, self-concept, and self-efficacy in specific contexts; an individual's beliefs, interactions within a culture and with others in a community, and a learning environment all influence identity formation. Thus, social cognitive theory describes how an individual's cognitive and affective processes, social interactions within an environment, and behaviors influence thinking and learning.

Origins of Social Cognitive Theory

Social cognitive theory originated from the work of psychologist Neal Miller and sociologist John Dollard in the 1940s. They proposed a theory of social learning and imitation that revealed four aspects of learning: drive, cue, response and reward (Rolnick, n.d.). They also showed that fear can be a learned response and operate as a reinforcing agent. Psychologist Albert Bandura, probably the most famous developer of this theory today, studied the topic of fear as well.

In the 1960s, Bandura began to study the acquisition of behaviors, which he called social learning theory and later became social cognitive theory. Bandura's initial study was the Bobo doll experiment, in which children watched adults behave aggressively when playing with a Bobo doll and then the children displayed this aggressive behavior. The children learned by observing and reinforcement. In 1977, Bandura added learners' thoughts, beliefs, and emotions to his theory, which set him apart from previous behavioral research that only studied observable, external behavior. However, this reflects a paradigm shift in the 1970s from a focus on behaviors to a focus on cognition (Luszczynska & Schwarzer, 2005). Thus, identity is individual but shaped by observations of others' behaviors as well as inner cognition, emotions, and beliefs of control and ability.

Key Concepts of Social Cognitive Theory

Bandura's social cognitive theory includes four key concepts: enactive and vicarious learning, modeling, reciprocal determinism, and self-efficacy. Because he thought trial and error was ineffective, he proposed vicarious learning, or learning through social observation and imitating. However, watching others and mimicking alone was insufficient, but when combined with learning by doing, or enactive learning through personal experiences, learners extend their understanding to create new meaning. Therefore, a learner's identity is formed within a social context (i.e., observations), but the individual retains the executive function of learning by his or her own actions. Although others' modeling may be influential in learning, the individual is ultimately responsible for processing others' modeling and his or her own sense making. This may cause discomfort between a learner's core identity and the observed or taught normative identity (Cobb & Hodge, 2010). Yet it is important to remember that learning and identity development are dynamic. Social cognitive theory utilizes reciprocal determinism, which means different factors that influence learning are reciprocal–cognitive and affective factors influence behaviors and the behaviors influences these personal factors. Environmental factors also influence personal factors and behaviors and vice versa. Lastly, learning is influenced by an individual's self-efficacy (Bandura, 1977).

Model of Triadic Reciprocity: Personal Factors, Behavior, and Environmental Influences

Tying these components into a framework for social cognitive theory, Bandura (1986, 1997, 2001) designed a model of triadic reciprocity that includes behavior, personal factors including cognitive and affective factors, and environmental influences in Figure 2.

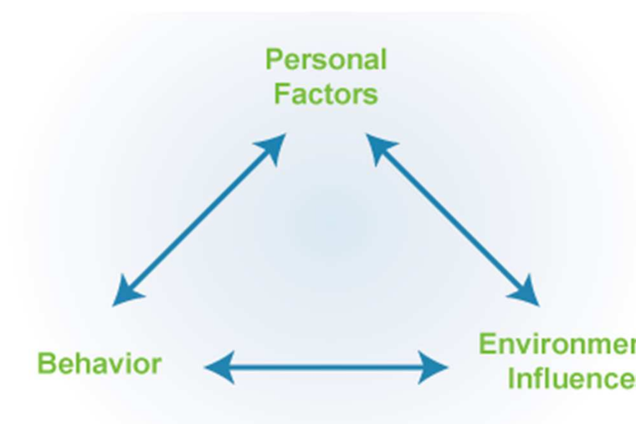

Figure 2. Model of triadic reciprocity

To show reciprocal determinism, the model's three components interact bidirectionally within a specific context or situation. Bandura (1986) believed that "a theory that denies that thoughts can regulate actions does not lend itself readily to the explanation of complex human behavior" (p. 15), and he acknowledged that learners are both products of and interacting agents with the environment (Luszczynska & Schwarzer, 2005). Learners are learning and acting from their own thinking and emotions but also from observing others within a certain context. Thus, their identity is defined and developed by these three factors.

Connecting Math Identity, Problem Solving, and Self-regulation to Social Cognitive Theory

Social cognitive theory provides a frame for explaining how learners regulate their behavior over time through cognitive and affective processes and interactions with the environment. Elements of the three main constructs of this study–math identity, problem solving, and self-regulation–are connected to the three components of social cognitive theory–personal factors, behaviors, and environmental influences–as in Figure 3.

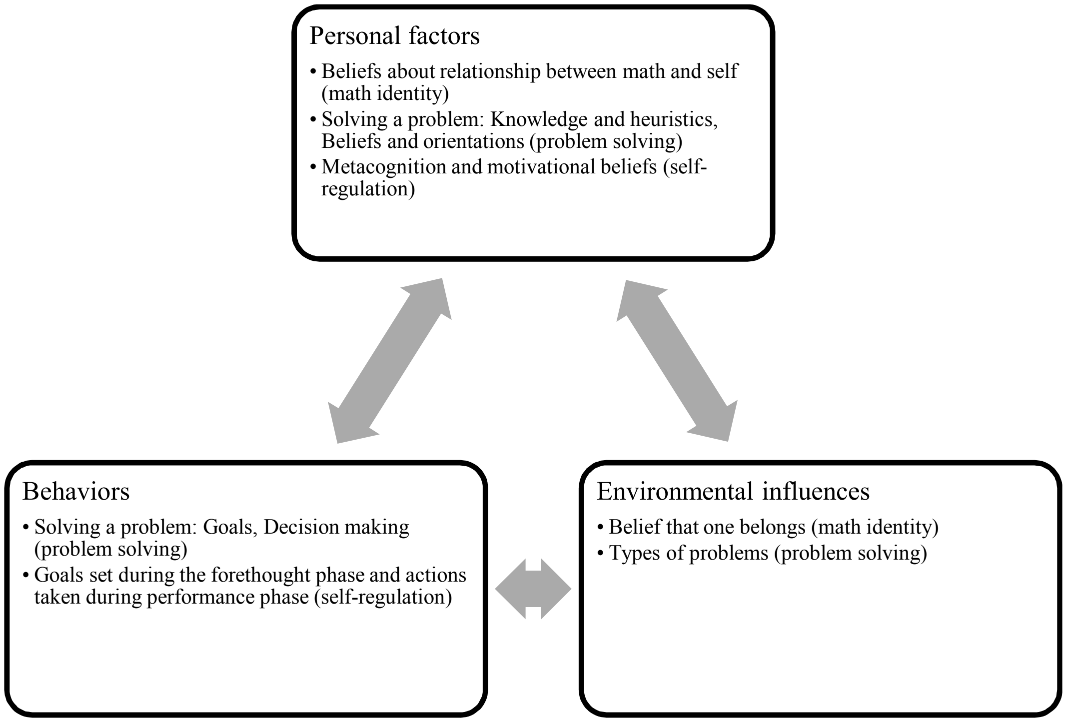

Figure 3. Connections between constructs and theoretical framework

One component is personal factors; individuals come to a situation with learned experiences and, thus, also come with their own math identities. An element of math identity–beliefs about the relationship between mathematics and oneself–is formed over time from experiences with mathematics as well as interactions with teachers, peers, and family and friends. Similarly, how individuals solve problems also relies on their learned experiences; problem solvers may use past mathematical knowledge and heuristics, or strategies for working through novel problems. They also begin problems with certain positive or negative beliefs and orientations about problem solving and mathematics. Individuals come to new situations with a certain way of thinking about their own thinking, or metacognition, which influences how they build from successes and work through challenges of learning. Along with metacognition, individuals' motivational beliefs vary from high to low self-efficacy and their perceived responsibility lies with themselves or others.

As previously described, reciprocal determinism means that the three components of person, environment, and behavior interact in a dynamic and reciprocal fashion. Therefore, the personal factors are interacting with behaviors, or responses individuals receive and respond to after performing a behavior. Thus, this second component–behaviors–closely aligns with goals set during the forethought phase and actions taken during the performance phase of the self-regulation process. Behaviors may also reflect the problem solving process that incorporates goals and decision-making throughout to monitor thinking and work through various approaches and solution pathways. Not all of these behaviors will be successful, but through multiple chances, an individual can modify behaviors and experience success of correct performance.

The third component is environmental influences, or the aspects of a setting or specific context that influence an individual's ability to successfully perform a behavior. The classroom environment and those with whom an individual interacts influence math identity by impacting the belief that one belongs with a community of mathematicians. When individuals are problem solving, the types of problems attempted and how these problems are set up within a classroom can also influence their ability to successfully complete behaviors. Individuals benefit from appropriate support and materials provided within the classroom, which improve self-efficacy and may maintain the behavior.

Even though identity is ultimately the individual's, personal factors, behaviors, and the environment influence identity and whether an individual engages in behaviors and finds success.

Math Agency

Social cognitive theory takes an agentic perspective towards an individual's abilities, developments, and changes (Bandura, 1986, 2001, 2006). Therefore, Bandura (2001) states:

> Through agentic action, people devise ways of adapting flexibly to remarkably diverse geographic, climatic and social environments; they figure out ways to circumvent physical and environmental constraints, redesign and construct environments to their liking...By these inventive means, people improve their odds in the fitness survival game. (p. 22)

Individuals are not bystanders or products of society but active participants, influencers, and decision makers within their lives. Characteristics of these individuals include self-organizing, pro-active, self-regulating, and self-reflecting. Bandura (2006) describes four key agentic properties:

- Intentionality: People form intentions that include action plans and strategies for realizing them.
- Forethought: People set themselves goals and anticipate likely outcomes of prospective actions to guide and motivate their efforts.
- Self-reactiveness: Agency thus involves…the ability to construct appropriate courses of action and to motivate and regulate their execution.

- Self-reflectiveness: Through functional self-awareness, they reflect on their personal efficacy, the soundness of their thoughts and actions, and the meaning of their pursuits, and they make corrective adjustments if necessary. (pp. 164-165)

Agency in mathematics has been extensively researched through Pickering's (1995) studies of mathematicians. He describes interplay of human agency and the agency of the discipline. Similar to Bandura's work, mathematicians display human agency by being pro-active as they create new knowledge, self-regulate their actions, and work to achieve their goals. However, this is not outside of the context of mathematics; the agency of the discipline of mathematics, or the normative processes and standards of mathematics (e.g., mathematical proof) is guiding their work. To describe this back and forth, Pickering (1995) coined the phrase "dance of agency" (p. 116). Thus, mathematics is more than just taking in knowledge of what is known or solved but involves practicing mathematics in such a way that knowledge is created, changed, or advanced. When secondary students do mathematics, they engage in mathematical practices, such as conjecturing, explaining ideas, and constructing mathematical arguments (Schoenfeld, 2014). They may collaborate with others, respond emotionally to mathematics, use their instincts, attempt multiple solution pathways, and make connections within mathematics and across disciplines (Burton, 1999). Doing mathematics boosts students' interest (Boaler & Greeno, 2000; Martin, 2000 as cited in Sengupta-Irving & Enyedy, 2015), achievement, and persistence in mathematics (Boaler & Staples, 2008).

In the classroom, students often passively learn mathematics by sitting quieting, watching the teacher do problems, and listening for steps and directions. Only afterwards do they try a problem on their own (Sengupta-Irving & Enyedy, 2015). When students engage in doing mathematics on rigorous tasks, they create a high sense of agency. Specifically, agency is built in the classroom when students make choices, are given opportunities for self-exploration and self-direction, seek their own resources, and feel a sense of authority.

Students make choices about their learning and the content. When teachers encourage students to make their own learning choices–i.e., choosing content, a process for making sense of the content, or how to show what they have learned–this leads to greater student engagement and interest in taking further mathematics classes (Boaler & Greeno, 2000 as cited in Sengupta-Irving & Enyedy, 2015) and has been shown to have a positive effect on math learning (Boaler, 2015; Boaler & Staples, 2008). Students can also make choices within the discipline; Fiori and Selling (2015) call these aesthetically guided choices when an individual "act[s] with agency in ways that are authentic to the discipline itself (doing mathematics)" (p. 232). These choices are influenced by the agency of the discipline, or the normative practices of mathematicians who emphasize "elegance, precision, lucidity, coherence, unity" (Bass, 2011, p. 4 as cited in Fiori & Spelling, 2015, p. 232). Although a student may choose to solve a problem in his or her own way, the student is guided by the discipline norms. Yet relying solely on norms is not advised because a critical part of the dance of agency is knowing when to draw on mathematical ideas (Boaler, 2003). Therefore, it is beneficial for students to work in collaboration. When students share their own solution pathways to a problem, they defend their perspectives with justification as others determine the validity of the responses. Thus, students act with agency.

Agency is also built in the classroom when students are given opportunities for self-exploration of who they are and their individual capabilities as well as self-direction (Côté & Schwartz, 2002). Students not only investigate new math concepts but also what they can and cannot do, how learning works for them, and why they succeed or experience challenges. By understanding themselves, they are able to take action towards their potential, and instead of using a pre-determined, structured plan, students navigate options to work towards success–e.g., how to participate in mathematics or how to learn new, challenging content.

As students self-direct their own learning, they may need to acquire supports to strengthen or sustain their mathematical understanding. Thus, some turn to peers, teachers, tutors, textbooks, or online resources. McGee and Pearman II (2015) found students demonstrated significant agency in gaining material resources, and in their study, seven of the thirteen students expressed preferences for working with peers and within collaborative settings as opposed to with teachers.

Lastly, authority is connected to agency because students are the ones making choices, directing their actions, and seeking support (Engle, 2011). When tasked with a math or learning problem, students define it, plan for a solution, adjust their pathways as needed, and ultimately solve the problem. Even though the process occurs within the norms of the discipline of mathematics, students are the main decision makers and have control over the solution, their knowledge, and their future.

Since learning is occurring within the classroom, it is important to consider the environment in which students are doing mathematics, building their agency in math, and collaborating with others. Engle and Conant (2002) provide four principles for creating this type of learning environment:

> (1) Problematizing, where students are encouraged to take on intellectual problems; (2) authority, where students are given authority to address those problems; (3) accountability, where students are held accountable to others and to disciplinary norms; and (4) resources, which refers to students having sufficient materials for inquiry. (pp. 400–401)

Connections between these principles and how to build students' agency in mathematics are apparent. Authority and resources are included above, and as students explore and direct their learning, they problematize to confront challenges to their thinking. Collaboration with peers and the norms of mathematics hold students accountable for their mathematical reasoning and the accuracy of a solution pathway (Greeno, 2011 as cited in Fiori & Selling, 2015). Therefore, a learning environment that supports students' agency in mathematics cannot be restrictive but should give freedom of movement and tools (Fiori & Selling, 2015). When working on problems, students may choose to stand or move to converse with peers; novel problems may necessitate the use of pencils and paper, whiteboards and markers, rulers, calculators, other technology, visuals, objects, etc. as students pursue meaningful mathematical work.

It is pertinent that teachers reflect on their instruction and classroom environments. Only when teachers are willing to engage in their own "dance of agency" can they adequately support their students in doing the same (Grootenboer & Zevenbergen, 2009). This requires teachers to understand and reflect on their human agency and also the agency of the discipline of mathematics.

CHAPTER III

RESEARCH METHODS

The previous chapter described the current literature on mathematics identity, problem solving, and self-regulated learning and gave an overview of the social cognitive theoretical framework that is used in this study. In this chapter, I provide my rationale for using mixed methods as a methodology to understand secondary students' math identities in relationship with their problem solving and self-regulated learning practices.

To begin, I provide a detailed description of the research design and methodology, including the reasons for using a mixed methods design, the research questions, and key constructs of the quantitative portion. Then I explain the school setting where this research is conducted, the research sample, and the participants. Following this, I detail my data collection and analysis methods for the quantitative and qualitative phases of the study. This chapter ends with a description of the reliability and validity in the quantitative strand, the trustworthiness in the qualitative strand, and the limitations of my research design.

Research Design and Methodology

This study addresses secondary students' math identities and the relationship to problem solving and self-regulated learning. As a sequential explanatory mixed methods design with quantitative correlational research, qualitative interviews, and survey research, it involved collecting quantitative data from surveys first and then explaining the quantitative results with in-depth qualitative analysis. In the first phase of the study, ordinal survey data were collected from secondary high school math students to examine whether math identity relates to problem solving and self-regulated learning. As nonexperimental, this study looked at the relationship between variables but did not include the manipulation of an independent variable or random assignment of participants to specific conditions or interventions (Shadish, Cook, & Campbell, 2001). The second phase was qualitative and conducted as a follow-up to help explain the quantitative results. In this explanatory follow-up, through structured interviews, I planned to explore math identity with students at the school site: six students who indicated positive math identities, as evidenced on the quantitative survey, and six students who indicated negative math identities.

In support of mixed methods, Creswell (2015) states, “The use of quantitative or qualitative research alone is insufficient for gaining an understanding of the problem” (p. 15). While quantitative research provides close-ended data and allows generalization from a small sample to a large population (Creswell, 2009), qualitative research offers open-ended responses, portrays stories and meanings, and facilitates an understanding of the perspectives and experiences of individuals. Even though quantitative instruments–such as surveys or observation tools–provide meaningful data, they lack information about the setting and context, which qualitative instruments offer. Since qualitative research provides participants’ views, perspectives, and experiences yet lacks generalizability, mixed methods design builds on the strengths of both types of research. By using a mixed methods design, I gathered quantitative and qualitative data about the research questions, connected and interpreted the two data sets, and used the strengths of the collective data set to understand and address the research questions (Creswell, 2015). Thus, mixed methods research provides a more complete approach to data collection and analysis than either quantitative or qualitative methods alone.

In this two-phase design, quantitative data were collected through surveys, and the results were analyzed to determine quantitative results that would benefit from more explanation. Then the qualitative data were collected through structured interviews, and the qualitative findings were analyzed and interpreted to explain the quantitative results. For Phase 1, the surveys asked about demographics, math identity self-perception and perspectives of others, problem solving practices, and self-regulated learning strategies. Surveys provide a "numeric description of trends, attitudes, or opinions of a population by studying a sample of that population" (Creswell, 2009, p. 145). These surveys were initially used to gather data from a larger group of participants. Then after analyzing the quantitative survey data, I planned to select specific members of the group, based on their math identities, for interviews to explain their survey answers.

For Phase 2, data from structured interviews add depth, support triangulation of results, strengthen conclusions, and provide trustworthiness to the findings. The purpose is to use the qualitative findings to triangulate the quantitative data from the surveys in order to describe and interpret the results from the quantitative phase (Creswell, 2015). With structured interviews, the list of questions includes direct and open-ended questions to gather data relevant to my topic. However, I may adjust based on the participants' responses and may explore certain ideas or survey questions in more depth (Merriam & Tisdell, 2015).

To study secondary students' math identities and the connection to their perceptions of their own problem solving and self-regulation skills, I chose the explanatory sequential design in Figure 4. An explanatory sequential design is beneficial because I was collecting and analyzing quantitative data and then qualitative data to explain the quantitative results in more depth.

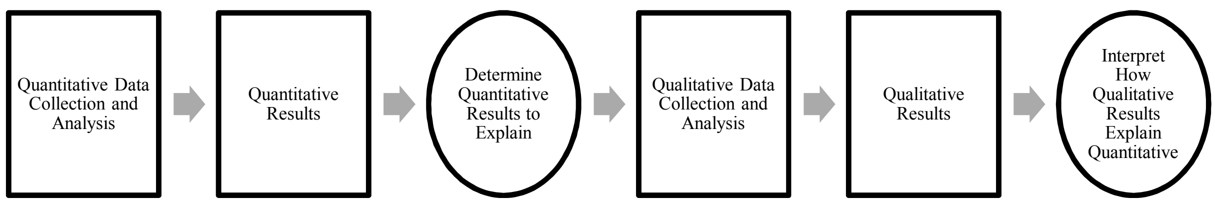

Figure 4. Explanatory sequential design

Research Questions

This mixed methods research study includes quantitative, qualitative, and mixed methods research questions. The research questions are:

1. What is the relationship between secondary students' math identities, their perceived problem solving practices, and their perceived self-regulated learning strategies? What is the relationship between problem solving, self-regulation, and math identity given gender?
   a. Hypothesis 1: Students who report higher use of problem solving practices have positive math identity.
   b. Hypothesis 2: Students who report higher use of self-regulated learning strategies have positive math identity.

c. Hypothesis 3: Students who report higher use of problem solving practices report higher use of self-regulated learning strategies.

2. How do secondary students articulate their math identities?
3. Does students' articulation of the development of their math identities explain their problem solving practices and self-regulated learning strategies?

Definition of Constructs

The main constructs studied are math identity, problem solving practices, and self-regulated learning strategies. However, as seen in Chapter II, the definitions of these constructs vary in educational research. Therefore, this section describes the definitions used for the quantitative strand of this study as well as other key constructs.

Age is the self-reported age of participants and is nominally coded 1 = 15 years old, 2 = 16 years old, 3 = 17 years old, and 4 = 18 years old.

Gender is the self-reported sex of participants and is nominally coded 1 = Male, 2 = Female, 3 = Transgender, 4 = Non-binary, and 5 = Other.

Grade is the self-reported grade in school of participants and is nominally coded 1 = Freshman, 2 = Sophomore, 3 = Junior, and 4 = Senior.

Math class is the self-reported current math course of participants and is nominally coded 1 = Algebra II, 2 = Precalculus, and 3 = Precalculus Honors.

Race/ethnicity is self-reported by participants and is nominally coded 1 = Black or African American, 2 = American Indian or Alaska Native, 3 = Asian, 4 = Filipino, 5 = Hispanic or Latino, 6 = Native Hawaiian or Other Pacific Islander, 7 = White, 8 = Two or More Races, and 9 = Other.

Mathematics identity is a latent construct associated with a student's perception of himself or herself as a math person. Indicators of this construct include numeric records of a student's perceived characteristics of a "math person," belief that he or she can do math, belief that he or she belongs within a community of math people, and belief that "I am a math person." Responses are recorded using a Likert scale, 1 = exactly me to 5 = not me.

Problem solving is a latent construct associated with a student's perceived use of problem solving practices over time. Indicators of this construct include, but are not limited to, use of mathematical tools, understanding what is known and unknown, trying multiple strategies, and thinking through possible ways of solving the problem. Responses are recorded using a Likert scale, 1 = almost always to 5 = almost never.

Self-regulated learning is a latent construct associated with a student's perceived use of self-regulated learning strategies over time, or from a macro level. Indicators of this construct include, but are not limited to, setting a learning goal for what to study, making choices and a plan to meet that goal, engaging in actions and monitoring while working towards the goal, and evaluating progress and reflecting on errors and successes. Responses are recorded using a Likert scale, 1 = almost always to 5 = almost never.

Research Setting

This study examined high school math classrooms in a mid-sized, urban, ethnically diverse K-12 school district on the West Coast. In 2015-2016, the high school served a student body of 2,082 students; one middle school and five elementary schools feed into the high school. The high school's demographics have not changed much since

2010; in 2015-2016, the demographics were: Black or African American (21.4%), American Indian or Alaska Native (0.5%), Asian (10.4%), Filipino (2.2%), Hispanic or Latino (39.2%), Native Hawaiian or Other Pacific Islander (0.4%), White (23.6%), Two or More Races (2.3%), and Other (0.0%) (California Department of Education, 2017a). The graduation rate in 2015-2016 was 97.4 percent with 57 percent of graduates meeting state required courses (Visiting Committee Members, 2016). The number of English Language Learners (ELL) has steadily declined since 2010 as a result of an increase in language fluency reclassification. On the state English language development test, ELLs mostly score in the intermediate, early advanced, and advanced ranges (84% average). The high school student body is 52 percent female and 48 percent male. Roughly four out of every 10 high school students receive free or reduced lunch. To qualify for free lunch, children's family income must be under $15,171 in 2015 (California Department of Education, 2017b), and 29.3 percent of students at the high school receive free lunch. To qualify for reduced lunch, children's family income must be below $21,590 annual income in 2015 (California Department of Education, 2017b), and 6.9 percent of students at the high school receive reduced lunch.

Looking at the students' achievement, grade 11 students take the state math test. Forty percent of students met or exceeded the state standards on the 2014-2015 math test, compared to the state as a whole in which 34 percent of students met or exceeded the state standards. In 2015-2016, 35 percent of the high school students met or exceeded the state standards. Comparing male and female students, female high school students in the district slightly outperform males on the state math test; there is about a five percent achievement gap (California Department of Education, 2017a).

At the high school, the math course sequence is Algebra I, Geometry, Algebra II, Precalculus or Finite Math, and AP Calculus AB/BC or AP Statistics. Honors courses are offered at the geometry level and above. From the student body, 22 percent are enrolled in advanced math courses, meaning Algebra II or above, which is a higher percentage than the average for the state (13%). For this study, participants are students, who consent (if over 18 years old) or assent and whose parents consent (if under 18 years old), from four math classrooms taught by two high school math teachers. One teacher teaches a section of Algebra II, and the other teacher teaches one section of Precalculus and two sections of Precalculus Honors. These four math classrooms were chosen for the two teachers' implementation of instruction that explicitly teaches problem solving and self-regulation skills.

Research Sample

Math Instruction

Two secondary math teachers at the high school site have spent extensive time developing their instructional practices to support students' problem solving and self-regulated learning. These teachers teach the four mathematics classrooms in this study. The Precalculus and Precalculus Honors teacher has been teaching high school math for 13 years. The Algebra II teacher has seven years experience teaching high school math.

The two teachers plan together, collect and analyze student data over time, attend conferences, and continue to make adjustments to their lesson plans and assessments.

Their work has focused on targeted planning and implementation specifically using the Math Learning by Design (MLD) Instructional Moves. These moves are meant to engage students in communicating their mathematical thinking and problem solving through rigorous mathematics by also supporting students in becoming self-regulated learners. The MLD Instructional Moves follow an engineering design approach to support students in Figure 5.

Figure 5. Engineering design process

The instructional cycle flows through four moves that align with the engineering design process, and improve is embedded throughout the flow. Teachers use this structure to design, plan, and implement their lessons. When planning instruction, it is recommended that they think about interactions between the content, students, and teacher within the instructional core as well as what success looks like for mathematical thinking, problem solving, or self-regulation. Typically moves one to four take two fifty-minute periods with assessment and improvement components incorporated throughout.

For Move 1: ASK/Hook, teachers design and implement a hook to evoke emotion and promote student reasoning, curiosity, and questioning skills. Students interpret, problem pose, and communicate about a culturally relevant prompt. In real time, teachers observe and informally assess students' structure of knowledge to inform the sense making process in the investigation task (Move 2). Data may include student diagrams with precise mathematical language, student academic/non-academic questions, or student categorized questions and adjustments.

For Move 2: IMAGINE/Investigation before Explanation (IBE), teachers design and implement a task to teach students to creatively problem solve and self-regulate. Students engage in the problem solving framework to interpret, communicate, formulate a plan, and self-monitor their progress. In real time, teachers assess students' structure of knowledge to inform closing the gap between what they know and need to know in notes. Data may include student evidence of an approach that successfully leads to a plan (problem solving), student monitoring questions, student evidence of time spent in each part of the problem solving strategy, and student questions that have them move back in the problem solving strategy. Students act on feedback from teachers, peers, and self.

For Move 3: PLAN/Notes, teachers design and implement notes for students to learn mathematical thinking, which may include direct instruction or modeling based on student data. Students interpret multiple representations, personalize their notes to adjust their structure of knowledge and deepen their reasoning for problem solving. In real time, teachers check for understanding around student solution pathways to inform directed next steps. Data may include student evidence of highlighted notes, student evidence of thinking through the problem solving framework, student evidence of engaging in their own solution pathway, student evidence of reasoning questions and answers in notes, and student evidence of summaries. Students act on feedback from teachers, peers, and self.

For Move 4: CREATE/Active Practice, teachers design and implement practice for students to own their learning within a directed goal and leveled choices. Students compare and analyze their structure of knowledge, communicate, and make choices in challenge levels to move their learning. They also clear up misconceptions and/or extend their reasoning to adjust the way they structure their knowledge before they reflect on effective strategies and effectively setting and adjusting goals. In real time, teachers use students' misconceptions and questions to inform direct whole class summarizing. Students act on feedback from teachers, peers, and self.

Throughout Moves 1-4: IMPROVE, teachers design and implement an assessment system that supports students in reflecting on the gap between what they know and need to know to create a goal that can be acted upon. Assessments include summative assessments of concept categories that are made up of clusters of the Common Core State Standards (CCSS) and formative assessments on learning targets (LT) that are road markers used to support students in goal setting and monitoring growth towards mastery of concept categories. Students set mathematical goals, compare their work to success criteria, and reflect on effective strategies and personal actions to learn how to monitor their progress in attaining their goals. This helps them create an action plan by effectively self-evaluating the methods selected, and adapting future methods based on what was learned. In real time, teachers support students' self-regulation as they use evidence of goal setting to inform actions that measurably move student learning forward in both the short and long term. Evidence may include students' goals on post-its, index cards or pictures, or students' goals with feedback from teachers, peers, and self.

Sampling Procedures

With permission of these two high school teachers, I invited all students enrolled in their math courses–Algebra II, Precalculus, or Precalculus Honors–at the research site to participate in the study. I followed the informed consent procedures (Appendix A and Appendix B), approved by the Institutional Review Board (IRB), at the research site. To recruit participants, I visited their classrooms and invited the students to participate in the study and gave a description of the study, including the quantitative and qualitative strands. As I explained the study, students read the description and asked any questions. I planned to survey and interview youths, ages 14 to 18. If students are 18 years or older, they can give consent; however, those under 18 years old are minors and a protected class that cannot consent. Their parents must give consent for them and the students give assent. With the study description, I also asked the students to take home an informed consent form to be signed by their parent or guardian. Students under 18 years old give assent prior to taking the survey and participating in the interview, regardless of if their parents previously consented. The informed consent form explains that participation in the study is completely voluntary and has no perceived risks beyond normal classroom activity at the school. Participants may benefit from the results of the study, since the study has implications for secondary math teachers' instruction to support students' development of their math identities, problem solving skills, and self-regulated learning strategies. Parents are given the option of consenting their child for the survey only, the survey and interview, or not consenting. Additional written invitation letters with informed consent documentation were provided to the two teachers for any students absent on the day of my initial invitation.

Expectations of the two participating classroom teachers included: (1) providing time for my brief invitation visits during class time, (2) collecting signed informed consent forms, (3) allowing 20 minutes during class for students to complete the survey, and (4) working collaboratively with me to coordinate interviews with a few select students after the quantitative data analysis.

For the qualitative interviews, I planned to use a stratified purposeful sampling procedure to capture variations between students with positive and negative math identities. This type of sampling allows me to identify similarities and differences in the phenomenon of interest, math identity (Palinkas et al., 2015). Patton (2002) explains, "The purpose of a stratified purposeful sample is to capture major variations rather than to identify a common core, although the latter may also emerge in the analysis" (p. 240). Within two weeks after the invitation visit, students were asked to complete a survey about their perceived math identity, problem solving practices, and self-regulated learning strategies. After the analysis of the quantitative survey data, my plan was to sort participants who report a positive or negative math identity into two groups, and then six students from within each of these groups would be purposely sampled to participate in the qualitative interviews. The goal of these interviews is to gain a better understanding of the factors that influence students' math identities by asking follow-up questions about the previous survey and quantitative results.

Sample Size

There were four participating math classrooms total of Algebra II, Precalculus, and Precalculus Honors, and the school has a student to teacher ratio of 24 to 1. Since there were not specific criteria for participating in the study and students cannot take more than one of these math classes at the same time, there was a potential sample of approximately 113 unique students. From this group, I expected 50 percent or greater to consent to participate and respond to the survey; this percentage is based on the acceptable response rates for email, classroom paper, and face-to-face surveys (Division of Instructional Innovation and Assessment, The University of Texas at Austin, 2007).

Data Collection and Analysis

Since the mixed methods design I used includes multiple data collection and analysis phases, I provide an overview of the design timeline in Figure 6 and then detail each phase. The timeline below includes administering informed consent procedures, collecting survey data (background and math identity, problem solving, self-regulation), and recruiting and interviewing participants. To minimize interruptions to the research setting but still collect valid data, the surveys and interviews were carefully spaced out during the end of the fall semester. The surveys were completed during class while the interviews were conducted outside of class time, either during a lunch or before/after school.

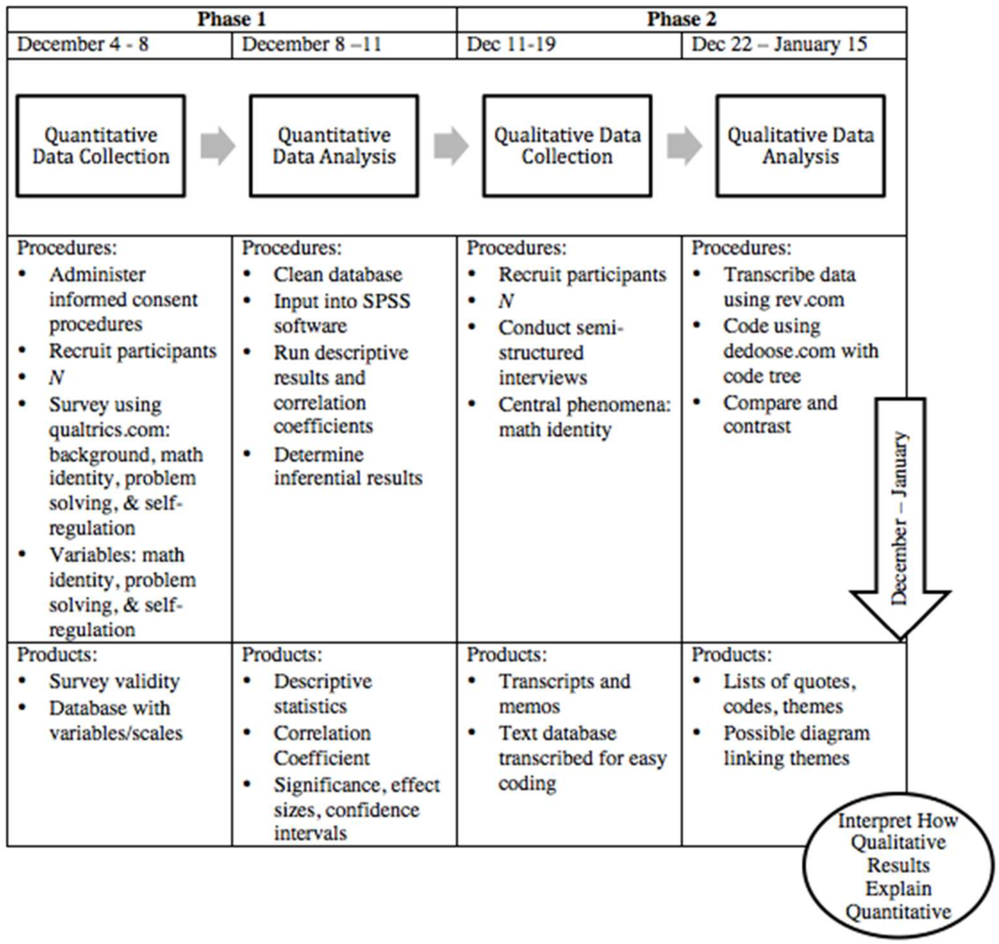

Figure 6. Model of mixed methods design

Phase 1 Data Collection: Quantitative

Within two weeks after the invitation visit, students completed the survey about their background, math identity, perceived problem solving practices, and perceived self-regulated learning strategies (Appendix C). The survey takes approximately 20 minutes, and I planned for the participants to receive an email invitation to fill out the survey online through Qualtrics (www.qualtrics.com). Participants did not have Internet access, so they completed the survey on paper. The background section includes questions about students' age, gender, grade level, math class, and race/ethnicity. These variables have been previously described in the constructs section. The math identity section includes questions about what students think “math persons” are like and if these characteristics are like them, if they feel they can do math and if they feel they belong within a community of math people (Boaler, 2015), and if they see themselves as a “math person.” These questions reflect the literature on positive math identity–having a belief that I can do math and belong within the community–and other quantitative surveys that use “I am a math person.”

The problem solving questions are adapted from a survey by the Math Leadership Corps (MLC), which was used to assess students’ self-perceptions of problem solving. The problem solving questions reflect practices recommended by Pólya (1945, 1957) and Schoenfeld (1985, 1992, 2010). The original questions were used by MLC in 2016-2017 to survey 686 K-12 students; Cronbach’s alpha was .79. The self-regulation questions are developed from Zimmerman and Campillo’s (2003) model of self-regulation. Three math education experts reviewed and provided feedback on the survey questions and the length of the survey. The survey was revised to better reflect the practices recommended by Pólya, Schoenfeld, Zimmerman, and Campillo, incorporate student-friendly language, and work within the given survey time. Cronbach’s alphas are reported for the final survey questions in Chapter IV. At the end of the survey, I asked if the participant would be interested in an interview and for preferred times.

Phase 1 Data Analysis: Quantitative

I compiled my quantitative database in an Excel document. Then I cleaned the database by updating row labels and looking for missing or duplicate data. The Excel file was then uploaded to SPSS, a quantitative software data analysis program, and the data and column labels were checked for accuracy prior to running analyses.

To begin analysis in SPSS, I checked response statistics, including the *N* and return rate. I carried out a descriptive analysis to look at the means of each variable and the standard deviation of each variable to note if the means and the error were similar within the group. These descriptive statistics are represented in a table in Chapter IV, since these comparisons are useful to begin analyzing the variables and their relationships.

To analyze the relationship between students' math identities, problem solving, and self-regulation, I planned to use either Pearson's or Spearman's correlation coefficient. To use Pearson's, the following assumptions must hold: interval or ratio level, linearly related, and bivariate normally distributed. I began by creating a scatterplot of the data and looking for a positive or negative correlation between two variables. I also considered if there is evidence of non-linearity. If the data are non-linear, I would use Spearman's correlation coefficient, but if I was uncertain, I also would check the normality assumption by creating a boxplot. A boxplot for normal distribution shows the median near the center of the box and the whiskers are of approximate equal length. If the median is near either end of the box or the whiskers are of very different lengths, this indicates possible skewness. If the data are not normally distributed but instead skewed, I would use Spearman's correlation coefficient.

Spearman's correlation coefficient (r_s) measures the strength and direction of association between two ranked variables. Spearman's correlation determines the strength and direction of the monotonic relationship between two variables whereas Pearson's correlation determines the strength and direction of the linear relationship between two variables. A monotonic relationship is a relationship that does one of the following: (1) as the value of one variable increases, so does the value of the other variable; or (2) as the value of one variable increases, the other variable value decreases. In other words, a monotonic function is one that either never increases or never decreases as its independent variable increases. Thus, a relationship may be monotonic but not linear. To use Spearman's correlation, the following assumptions must hold: interval or ratio level or ordinal and monotonically related. Because there is no requirement for normality, Spearman's coefficient is a nonparametric statistic.

In SPSS, I ranked the data by ranking the scores for each variable separately. Scores with highest values are labeled "1" and data are ranked until the lowest score. If some scores are the same, labels are the average of the ranks. Then I ran the Spearman's correlation analysis and analyzed the output. Since a correlation is an effect size, I described the correlation's strength using the following guide for the absolute value of the Spearman correlation: .00-.19 very weak, .20-.39 weak, .40-.59 moderate, .60-.79 strong, and .80-1.00 very strong. The Spearman's correlation analysis includes a significance test to determine whether there is any or no evidence that linear correlation is present. With a p-value less than .05, there is less than a five percent chance that there is no monotonic correlation. Using Spearman's correlation analysis provides the strength and significance of the relationship between math identity, problem solving, and self-regulation. During the interviews, I asked questions about any results that stood out, i.e., had very weak or very strong correlations or were surprising compared to the literature review.

To determine who might be interviewed, the math identity questions only were the focus. Looking at participants' individual means, I planned to choose six students who showed positive math identities, as evidenced by higher mean scores, and six students who showed negative math identities to participate in interviews in order to add depth to the quantitative data. As described in Chapter IV, only 10 students participated in the interviews.

Phase 2 Data Collection: Qualitative

Prior to the study, I designed interview questions to learn about the students' math identities and ask follow-up questions about the survey questions (Appendix D). Interview questions were created using the student survey and qualitative questions asked by other researchers (Boaler, 2000, 2003). Questions progress from their general understanding of mathematics and feelings of belonging to their math identities and factors that influence their identities. After the quantitative data were analyzed, I made any necessary modifications to the interview questions, since the goal is to explain the survey responses in more depth through the interviews.

Since only 10 students were interested in participating in an interview and also had signed informed consents, I used their preferred interview days/times and my availability to schedule structured interviews in person or via Skype in a private, quiet location at the school that was supervised by a teacher. Within two weeks after the quantitative survey, I planned to facilitate the 45-minute to one-hour interviews outside of class time, either during a lunch or before/after school, audio record the discussions, and take notes within my research journal, writing down key phrases/points. In order to keep the research questions in mind, I used a graphic organizer for my interview notes (Appendix E). This allowed me to gather corresponding data for each question and ensure that I was not missing a key area before concluding my interview. If any part of the template was blank, I could ask further questions during an interview to complete the organizer. Following the interviews, I wrote down my first reactions as a memo within my research journal and reflected on my research questions and collected data.

Phase 2 Data Analysis: Qualitative

I used a professional transcription service, Rev (www.rev.com), to transcribe the audio recordings. Once the data were transcribed, I inputted the data into Dedoose (www.dedoose.com). For the first coding round, I highlighted key words/phrases that stood out and used open coding by jotting down information next to quotations that might be useful in answering my research questions to see what categories or themes emerged. After I examined the entire transcript, I reviewed my notes and began to group some of the codes together, engaging in a process of axial or analytical coding. Once I had a general idea of the categories and initial names for each category, I set up my code tree and families. Codes may include my theoretical framework, parts of the research questions, and noteworthy quotations to incorporate into the results or discussion sections. Then I sorted all the interview quotations using these codes. As a self-check, I reviewed my categories with the criteria: “be responsive to the purpose of the research, be exhaustive, be mutually exclusive, and be conceptually congruent” (Merriam &

Tisdell, 2015, p. 212).

To analyze my codes, I exported the codes, looked for generative themes from within each code, and used my theoretical framework to guide data analysis. I referred to some of the following questions: What themes arise within each code? Are there any outliers? Are the themes what I expected? How does my data address my research questions? What other data sources and types will I use for triangulation? What should I do next with this knowledge? (Merriam & Tisdell, 2015). Although this is a long process of coding and developing categories, I expected it to help make sense of the data and thus the students' math identities and factors that influence their beliefs in doing math and belonging.

Reliability and Validity in the Quantitative Strand

An instrument should be both valid and reliable; therefore, I report the reliability and validity of the quantitative instrument for this study. The reliability or consistency of the quantitative strand over time and over similar samples (Cohen, Manion, & Morrison, 2007) is important because this means the instrument consistently measures what it is intended to measure. Since the quantitative instrument consists of three different sections–math identity, problem solving, and self-regulation–each section was analyzed and reported separately. I examined internal consistency reliability, or the consistency of results in measuring a construct or idea, often measured with Cronbach's Alpha. The description of Phase 1 Data Collection: Quantitative included the Cronbach's Alpha for the problem solving, since a version of this survey had been used previously. I also analyzed the data from my participants and report Cronbach's Alpha for each survey section in Chapter IV.

The extent to which an instrument measures what it is designed to measure is called validity. There are various threats to validity, which are "specific reasons why we can be partly or completely wrong when we make an inference about covariance, about causations, about constructs or about weather the causal relationship holds over variations in persons, settings, treatments and outcomes" (Shadish et al., 2001, pg. 39). Thus, I looked at statistical conclusion, internal, construct, and external validity.

Statistical conclusion validity is the correlation (covariation) between treatment and outcome, and internal validity is the validity of inferences about whether the relations between two variables are causal (Shadish et al., 2001). I should be able to account for how students' problem solving and self-regulation in a particular community relate to the students' mathematics identities. However, using a nonexperimental design, I did not control or manipulate the independent variable or participants, and thus, I was not looking for causation. Instead, I used the data from all three variables–math identity, problem solving, and self-regulation–to observe and interpret correlations to form my conclusions. Internal validity is low.

Construct validity is the degree to which inferences are warranted from the observed persons, settings, and operations sampled within a study to the constructs that these samples represent (Shadish et al., 2001). There is a dual problem–understanding the constructs and assessing them–because there are many ways to define constructs and there is not always a clear relationship between the study's methods and the constructs being measured. Hence, there are many threats to construct validity (Shadish et al., 2001). One is inadequate explication of construct, meaning the construct definition is too general, specific, or wrong. In response, I defined my study's constructs previously in this chapter. This was especially important for latent or abstract constructs, which are unobservable in nature, so there is a shared understanding of the terms (Cohen et al., 2007). Operational definitions were based on the terms' common descriptions in the literature.

Another threat is mono-operation bias, or only one way of measuring the construct. To address this threat, I used multiple survey questions to measure the more complex constructs, such as math identity, problem solving, and self-regulation. Given that these constructs are complex and have many layers, they are confounding constructs. To address this, I defined the specific components to be studied in the construct definitions. For example, self-regulation can include self-efficacy and motivation, but I chose to focus on students' use of self-regulated learning strategies. I also ensured content validity by asking committee members and experts in the field of mathematics education, including university professors and high school teachers, to critique of the content of the instruments, including surveys and the interviews protocol. Prior to beginning data collection, experts' suggestions were considered when finalizing the survey and interview questions.

External validity is about whether the relationship holds over variations in persons, settings, treatments, and measurements (Shadish et al., 2001). In other words, external validity is connected to generalizability, since the goal is to generalize findings to a population besides the sample participants at the given research site. Since this study's setting is four secondary math classrooms of approximately 113 students total in an urban district, the target of generalization might be narrow to broad (class size to school possibly) or at a similar level (other high school students). Threats to external validity may be with units, outcomes, or settings. This study's setting is high school, so the results may not generalize to middle or elementary school. Also, the findings might differ with other surveys about math identity, problem solving, and self-regulation. Since this study is conducted in an urban setting, suburban or rural locations might yield different outcomes.

Trustworthiness in the Qualitative Strand

For this study, I am concerned with producing consistent and dependable knowledge from the qualitative strand in an ethical manner so that the study's findings are trusted (Merriam & Tisdell, 2015). To ensure reliability, I used the investigator's position since "the trustworthiness of a qualitative study depends on the credibility of the researcher" (Merriam & Tisdell, 2015, p. 265) and also an audit trail. Thus, I practiced reflexivity by acknowledging my experiences working with students, teachers, and instructional coaches in collaborative or training roles, biases, and assumptions about the topic. During my data collection and analysis phases, I continuously reflected by writing memos after interviews in my research journal, when viewing the initial data, and throughout the analysis process as I made sense of the data and formed interpretations of the results. This information was later used to provide a thick description of the participants and classroom contexts, so that connections can be made to similar cases or phenomenon. While the findings of a qualitative study such as this study may not be widely transferrable, the use of thick descriptions can help enhance the external validity of the findings (Anfara, Brown, & Mangione, 2002). These results also are strengthened by the connection to the quantitative data of this mixed methods study.

In addition, I made public the instrument development process to improve dependability (Merriam & Tisdell, 2015). The creation of and prior use of the problem solving survey questions described above provide clarity around the quantitative tool. The previous discussion of the interview protocol includes how the initial draft was developed, how it was updated based on the quantitative analysis and results, and the comprehensiveness of its content aligned to the research questions. Also, throughout the process of data analysis, a detailed account of the methods, data collection protocols, and data analysis procedures was kept. These "running notes" provide an audit trail for the data collection and analysis procedures and allow for a peer audit of the procedures (Merriam & Tisdell, 2015).

As a comprehensive check, I considered Patton's (2015) "Ethical Issues Checklist" (as cited in Merriam & Tisdell, 2015). Table 1 captures the use of the checklist in this study.

Table 1

Patton's (2015) "Ethnical Issues Checklist"

	In this study
1. Explain the purpose of the research	I explained this to the participants in the written informed consent form as well as verbally prior to the study.
2. Promises and reciprocity	I made recommendations to the site's math teachers about better ways to support students in learning and succeeding in mathematics.
3. Risk assessment	There were no perceived risks beyond normal classroom activity at the school.
4. Confidentiality	Pseudonyms were used for the participants, and at the conclusion of the study, all data will be destroyed within five years of the study.
5. Informed consent	IRB guidelines and procedures were followed. Parents or guardians received a description of the study and were asked to sign a consent form prior to their child beginning the study.
6. Data access and ownership	Only my dissertation chair and I have access to the data.
7. Interviewer mental health	I used reflexivity and talked with my dissertation chair about any issues.
8. Advice	I asked my dissertation chair as well as a committee member who is on the IRB team.
9. Data collection boundaries	Participants were not pressed for data. During both the survey and interview, participants might end the data collection process at any time. Participation was voluntary.
10. Ethical vs. legal	Since there were no perceived risks posed to the participants, I did not have a professional or disciplinary code of ethics as a guide. I followed the procedures approved by the IRB.

Research Design Limitations

As noted in the introduction of Chapter III, there are many advantages to using a mixed methods research design. However, there are some limitations to this design. First, this is a nonexperimental design study, meaning there is no control or manipulation, which are necessary to claim causation (Frankfort-Nachmias & Nachmias, 2000). Thus, the findings only claim correlations between variables and explain relationships among math identity, problem solving, and self-regulation. Second, although the survey covered three areas of students' academic behaviors and perceptions–math identity, problem solving, and self-regulation–student behaviors and perceptions are not limited to these. Research on developing students' math identities is underdeveloped, and, therefore, the literature does not provide much insight into variables to investigate. Problem solving and self-regulated learning were chosen because the literature contains few or no connections to math identity. Third, because the data were collected at one point in time, the study is not longitudinal. Instead, the study gives a snapshot of students' academic behaviors and perceptions at one point (Creswell, 2009). Over the semester or during the following semester, relationships among the variables may change. To see a change over time, it may be advisable to give the survey at different points throughout the semester or year.

CHAPTER IV

RESULTS AND FINDINGS

This chapter provides an analysis of the data described in Chapter III, a detailed report of the results and findings, and how these relate to the research questions from Chapter I. The chapter begins with a brief description of the purpose and research questions of this study. Then the chapter is organized in the following way: (1) internal consistency reliability of the quantitative survey, (2) descriptive results, (3) quantitative results related to Research Question 1, parts 1-3, (4) qualitative findings related to Research Question 2, and (5) qualitative findings related to Research Question 3.

Overview of the Study

The purpose of this study is to explore the relationships among students' math identities, their perceived problem solving practices, and their perceived self-regulated learning strategies. The goal of the study is to gather information from students within a secondary school regarding strategies and practices that they use to engage with mathematics. This study uses mixed methods methodology, which includes quantitative correlational research, qualitative interviews, and survey research. The study examines the following research questions:

1. What is the relationship between secondary students' math identities, their perceived problem solving practices, and their perceived self-regulated

learning strategies? What is the relationship between problem solving, self-regulation, and math identity given gender?

2. How do secondary students articulate their math identities?
3. Does students' articulation of the development of their math identities explain their problem solving practices and self-regulated learning strategies?

Internal Consistency Reliability

Before analyzing participants' quantitative survey results, I examined internal consistency reliability, or the consistency of results in measuring a construct or idea, by determining Cronbach's Alpha. Since there were specific questions for each of the three constructs–math identity, problem solving, and self-regulation–Cronbach's Alpha is reported separately for each construct. For the eight questions about math identity, Cronbach's Alpha was .92, meaning 92% of the variance in that score would be true score variance, or internally consistent reliable variance. Cronbach's Alpha of .70 or above is the most cited, with .70 to .80 considered to be acceptable. For the 12 questions about problem solving, Cronbach's Alpha was .73, which is in the acceptable range. For the 16 questions about self-regulated learning, Cronbach's Alpha was .89, which is above the acceptable range.

Descriptive Results

Participants

Secondary math students took the Math Identity, Problem Solving, & Self-regulated Learning Survey. Although I set out to gather data from 113 students, the final participant sample for the study included 28 secondary math students. Students were recruited during the first week of December 2017. Although many students asked questions about the study and expressed a desire to participate, all students were under 18 years old and needed informed consent from a parent or guardian. Their classroom teachers and I reminded students to turn in their consents to participate, but the response rate for the informed consents was low. This may have been due to finals within the next three weeks; however, the survey was completed during class time and did not distract from outside activities or extra academic support.

All quantitative analyses were conducted using IBM SPSS Statistics (Version 24). Two different participants each left a question blank, but each of the variables had under four percent of values missing. The percentages of missing values for each of the variables used in this study can be seen in Table 2.

Table 2

Percentages of Missing Values

	% Missing
PS4. I know when to ask myself if I have solved a similar problem.	3.6
SR9. I assess my own understanding and progress toward the mathematics learning goals.	3.6

Because of the small sample size (N=28) and small percentages for missing values, all participants were included in the data and not deleted if they had missing data values.

The participants' demographics vary by math class, age, grade level, gender, and race/ethnicity. Of the 28 participants, six (21.4%) are in Algebra II, six (21.4%) are in Precalculus, and 16 (57.1%) are in Precalculus Honors as seen in Table 3. More than half of the participants were in one of the most rigorous math classes the school offers

(Precalculus Honors). Yet these data reflect the recruitment process; students from one Algebra II, one Precalculus, and two Precalculus Honors classrooms were recruited for the study.

Table 3

Math Class of Study Participants

	(N = 28)	
	N	%
Algebra II	6	21.4
Precalculus	6	21.4
Precalculus Honors	16	57.1

In Table 4, participants' ages are shown to range from 14 years old to 17 years old. One participant (3.6%) is 14 years old, two (7.1%) are 15 years old, 16 (57.1%) are 16 years old, and nine (32.1%) are 17 years old.

Table 4

Age of Study Participants

	(N = 28)	
	N	%
14	1	3.6
15	2	7.1
16	16	57.1
17	9	32.1
18	0	0.0

Participants are in high school grade levels: freshman, sophomore, junior, and senior. One participant (3.6%) is a freshman, two (7.1%) are sophomores, 20 (71.4%) are juniors, and five (17.9%) are seniors as displayed in Table 5.

Table 5

Grade Level of Study Participants

	(N = 28)	
	N	%
Freshman	1	3.6
Sophomore	2	7.1
Junior	20	71.4
Senior	5	17.9

Of the 28 participants, nine identify as males (32.1%), 18 identify as females (64.3%), one identifies as non-binary (3.6%), and no one identifies as transgender or other (0.0%), as seen in Table 6.

Table 6

Gender of Study Participants and Students at the Research Site

	Study Participants (N = 28)		Research Site
	N	%	%
Male	9	32.1	48.0
Female	18	64.3	52.0
Transgender	0	0.0	
Non-binary	1	3.6	
Other	0	0.0	

Shown in Table 7, within the sample, four participants (14.3%) identify as Black or African American, four (14.3%) as Asian, five (17.9%) as Hispanic or Latino, seven (25.0%) as White, seven (25.0%) as Two or More Races, and one (3.6%) as Other.

Table 7

Race/Ethnicity of Study Participants and Students at the Research Site

	Study Participants (N = 28)		Research Site
	N	%	%
Black or African American	4	14.3	21.4
American Indian or Alaskan Native	0	0.0	0.5
Asian	4	14.3	10.4
Filipino	0	0.0	2.2
Hispanic or Latino	5	17.9	39.2
Native Hawaiian or Other Pacific Islander	0	0.0	0.4
White	7	25.0	23.6
Two or More Races	7	25.0	2.3
Other	1	3.6	0.0

Math Identity, Problem Solving, and Self-regulation

The main variables or constructs of the study are secondary students' math identities, their perceived problem solving practices, and their perceived self-regulated learning strategies. These variables are scored using the following scales:

- Math Identity scoring: 1 = exactly me, 2, 3, 4, 5 = not me
- Problem Solving scoring: 1 = almost always, 2 = very often, 3 = somewhat often, 4 = not very often, 5 = almost never
- Self-regulation scoring: 1 = almost always, 2 = very often, 3 = somewhat often, 4 = not very often, 5 = almost never

This was done so that analysis could be more meaningfully interpreted, since previous quantitative surveys on math identity have used only dichotomous variables. To compare the descriptive statistics of these variables, I created composite scores of each variable:

MathIdentityCompositeScore contains 12 survey questions, ProblemSolvingCompositeScore contains eight survey questions, and SelfRegulationComposite contains 16 survey questions. For each composite score, I first ran descriptive statistics to analyze the data. For each variable, the mean, median, standard deviation, and range can be seen in Table 8.

Table 8

Descriptive Statistics of Math Identity, Problem Solving, and Self-regulation

Variable	N	Mean	Median	SD	Min	Max
MathIdentity CompositeScore	28	2.792	2.542	.899	1.25	4.58
ProblemSolving CompositeScore	28	2.211	2.188	.686	1.13	4.14
SelfRegulation CompositeScore	28	2.571	2.469	.526	1.75	3.56

The mean for math identity is 2.792, which is between 2 and 3 and slightly closer to the "exactly me" side of the scale. However, the standard deviation is almost a point (.899), which the largest standard deviation of the three constructs, so participants' scores varied more than within the problem solving and self-regulation questions. The composite score means range from 1.25, or almost "exactly me," to 4.58, or almost "not me." The mean for problem solving is 2.211, which is between "very often" and "somewhat often" on the scale. The median is almost the same as the mean, meaning the data likely have a symmetrical distribution. The standard deviation is .686, and within the range from 1.13, or close to "almost always" to 4.14, or around "not very often," some variation in scores is evident. Self-regulation's mean is 2.571, which is between "very often" and "somewhat often." The standard deviation is .526, so the data have little variation. This is evident from the range between 1.75 and 3.56; there were no averages in the 4-range or “not very often.”

Research Questions

1. What is the relationship between secondary students’ math identities, their perceived problem solving practices, and their perceived self-regulated learning strategies? What is the relationship between problem solving, self-regulation, and math identity given gender?
    a. Hypothesis 1: Students who report higher use of problem solving practices have positive math identity.
    b. Hypothesis 2: Students who report higher use of self-regulated learning strategies have positive math identity.
    c. Hypothesis 3: Students who report higher use of problem solving practices report higher use of self-regulated learning strategies.
2. How do secondary students articulate their math identities?
3. Does students’ articulation of the development of their math identities explain their problem solving practices and self-regulated learning strategies?

Results of Analysis of Data

Research Question 1

Part 1. What is the relationship between secondary students’ math identities, their perceived problem solving practices, and their perceived self-regulated learning strategies?

- Hypothesis 1:

- H_{o1} (Null hypothesis): Secondary students' math identities are independent from their perceived problem solving practices.
- H_{a1} (Alternative hypothesis): There is an association between secondary students' math identities and their perceived problem solving practices.

- Hypothesis 2:
  - H_{o2}: Secondary students' math identities are independent from their perceived self-regulated learning strategies.
  - H_{a2}: There is an association between secondary students' math identities and their perceived self-regulated learning strategies.
- Hypothesis 3:
  - H_{o3}: Secondary students' perceived problem solving practices are independent from their perceived self-regulated learning strategies.
  - H_{a3}: There is an association between secondary students' perceived problem solving practices and their perceived self-regulated learning strategies.

An alpha level of .05 is used to compare for statistical significance. If $p < .05$, then the null hypothesis is rejected in favor of the alternative hypothesis. If $p > .05$, then the null hypothesis is accepted.

Pearson vs. Spearman. To use Pearson's correlation coefficient, the following assumptions must hold: interval or ratio level, linearly related, and bivariate normally distributed. Looking at scatterplots of pairs of the three variables–math identity, problem solving, and self-regulation–the relationships did not appear linear but were monotonic.

Pearson requires normal distribution but Spearman's does not, so I also checked the normality assumption by creating boxplots of each variable. Normal distribution shows the median near the center of the box and the whiskers of approximate equal length. Although most of the medians were near the center of the box, some boxplots have whiskers of different lengths. Examining histograms of the each variable confirms that the data are not normally distributed but skewed. Thus, I used Spearman's correlation coefficient.

Spearman's correlations between variables. Spearman's correlation coefficients were utilized to examine significant relationships among the study's variables in Table 9.

Table 9

Spearman's Correlation Coefficients for Math Identity, Problem Solving, and Self-regulation

	MathIdentity CompositeScore	ProblemSolving CompositeScore	SelfRegulation CompositeScore
MathIdentity CompositeScore			
Correlation Coefficient	1.000	.256	.070
Sig. (2-tailed)		.189	.722
ProblemSolving CompositeScore			
Correlation Coefficient	.256	1.000	.722
Sig. (2-tailed)	.189		.000***
SelfRegulation CompositeScore			
Correlation Coefficient	.070	.722	1.000
Sig. (2-tailed)	.722	.000***	

Note: * $p < .05$. ** $p < .01$. *** $p < .001$

For Hypothesis 1, Spearman's rho (r_s) is .256 and the significance is .189. It is not statistically significant ($p > .05$) and, thus, the null hypothesis is accepted. Secondary students' math identities are independent from their perceived problem solving practices. For Hypothesis 2, Spearman's rho (r_s) is .070 and the significance is .722. It is not statistically significant ($p > .05$) and, thus, the null hypothesis is accepted. Secondary students' math identities are independent from their perceived self-regulated learning strategies. For Hypothesis 3, a significant positive relationship was found between problem solving and self-regulation ($r_s = .722$, $p = .000$). The null hypothesis is rejected, indicating that there is an association between secondary students' perceived problem solving practices and their perceived self-regulated learning strategies. This indicates a strong positive relationship between the ranks that participants perceived their problem solving practices and self-regulated learning strategies, meaning the higher one ranked perceived problem solving practices, the higher one ranked perceived self-regulated learning strategies, and vice versa.

Spearman's correlations between survey questions. Since only one pair of the three main variables is correlated, correlations between survey questions from different constructs may reveal statistically significant results if there are any associations. I analyzed the survey questions using Spearman's coefficient; any in the ranges .40-.59 are moderate, .60-.79 are strong, and .80-1.0 are very strong.

Looking at math identity (MI) and problem solving (PS) survey questions in Table 10, "MI10. My parents/relatives/friends see me as a math person" and "PS5. I think of several ways to try to solve this problem and select a plan that might work" display a positive, moderate correlation ($r_s = .530$, $p = .004$). These questions do not immediately show a connection but may with the qualitative findings.

Table 10

Math Identity and Problem Solving Survey Questions

	Spearman's Correlation	Significance
MI10. My parents/relatives/friends see me as a math person. PS5. I think of several ways to try to solve this problem and select a plan that might work.	.530	.004**

Note: * p < .05. ** p < .01. *** p < .001

Comparing math identity and self-regulation (SR) survey questions in Table 11, the two statements "MI3. I belong within a community of math people" and "SR16. I provide feedback to my peers so they can revise their actions" yield a positive, moderate correlation (r_s = .566, p = .002), indicating students are working with others to make sense of mathematics. This finding can be likely attributed to the fact that students are not learning math alone but with others in a classroom. The questions "MI2. I can do math" and "SR8. I set a mathematics learning goal of what I want to accomplish before studying" are negatively, moderately correlated (r_s = -.584, p = .001). Based on the literature review, the opposite was expected: a positive correlation between students' ability to do math and set goals. Because goal setting is a key element of problem solving and self-regulation, it seemed logical that it would be correlated with doing mathematics. However, participants may not be in the practice of setting goals. Therefore, someone good at math would not set goals, and thus, show a negative correlation.

Table 11

Math Identity and Self-regulation Survey Questions

	Spearman's Correlation	Significance
MI3. I belong within a community of math people. SR16. I provide feedback to my peers so they can revise their actions.	.566	.002**
MI2. I can do math. SR8. I set a mathematics learning goal of what I want to accomplish before studying.	-.584	.001**

Note: * p < .05. ** p < .01. *** p < .001

As previously discussed, Spearman's correlation was statistically significant for the problem solving and self-regulation composite scores. In Table 12, many survey questions show statistically significant correlations, and some are strong correlations (.60-.79). The questions "PS1. I think about what formulas, tools, or strategies I have learned that can help me solve the problem" and "SR10. I check if my thinking is on the right track for a specific concept" are positively, strongly correlated (r_s = .658, p = .000), showing that knowing and reflecting on formulas, tools, or strategies may be part of working through a problem. Looking at "PS8. I ask myself if there might be an error in my thinking" and "SR14. I seek to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing how other strategies are derived," there is a positive, strong correlation (r_s = .651, p = .000), indicating errors may come up through conversations with peers about questions, strategies, and other ways of thinking. "PS7. I follow the plan to solve the math problem until complete" and "SR10. I check if my thinking is on the right track for a specific concept" are positively, strongly correlated (r_s = .638, p = .000), revealing that executing a plan and thinking through a concept may be similar activities. Also, "PS1. I think about what formulas, tools, or strategies I have learned that can help me solve the problem" and "SR3. I choose and prioritize which concepts I need to study" show a positive, strong correlation (r_s = .630, p = .000). This finding indicates formulas, tools, or strategies may be integral to the concepts students need to know and study. Another positive, strong correlation is "PS8. I ask myself if there might be an error in my thinking" and "SR2. I reflect on the effectiveness of my study methods after an assessment" (r_s = .620, p = .000), revealing a relationship between understanding errors and reflecting on study habits.

Other correlations between problem solving and self-regulation survey questions are moderate. "PS4. I know when to ask myself if I have solved a similar problem" and "SR1. I determine the causes of my mistakes and misconceptions to avoid them in the future" display a positive, moderate correlation (r_s = .595, p = .001), demonstrating mistakes and misconceptions may have been made in previous, similar problems. Another positive, moderate correlation "PS8. I ask myself if there might be an error in my thinking" and "SR1. I determine the causes of my mistakes and misconceptions to avoid them in the future" (r_s = .562, p = .002) means that finding an error in a problem may be similar to seeing mistakes on assessments or reflecting on ineffective study habits. The questions "PS1. I think about what formulas, tools, or strategies I have learned that can help me solve the problem" and "SR1. I determine the causes of my mistakes and misconceptions to avoid them in the future" are also positively, moderately correlated (r_s = .558, p = .002). This result reveals mistakes and misconceptions may be from not knowing formulas, tools, or strategies. "PS4. I know when to ask myself if I have solved a similar problem" and "SR5. I choose and prioritize personally effective study methods" give a positive, moderate correlation (r_s = .544, p = .003), indicating a relationship between reflecting on the solution and determining a study method. Lastly, "PS8. I ask myself if there might be an error in my thinking" and "SR5. I choose and prioritize personally effective study methods" show a positive, moderate correlation (r_s = .511, p = .005), revealing those who find errors in their thinking also make decisions about their study methods.

Table 12

Problem Solving and Self-regulation Survey Questions

	Spearman's Correlation	Significance
PS1. I think about what formulas, tools, or strategies I have learned that can help me solve the problem. SR10. I check if my thinking is on the right track for a specific concept.	.658	.000***
PS8. I ask myself if there might be an error in my thinking. SR14. I seek to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing how other strategies are derived.	.651	.000***
PS7. I follow the plan to solve the math problem until complete. SR10. I check if my thinking is on the right track for a specific concept.	.638	.000***
PS1. I think about what formulas, tools, or strategies I have learned that can help me solve the problem. SR3. I choose and prioritize which concepts I need to study.	.630	.000***
PS8. I ask myself if there might be an error in my thinking. SR2. I reflect on the effectiveness of my study methods after an assessment.	.620	.000***
PS4. I know when to ask myself if I have solved a similar problem.	.595	.001**

SR1. I determine the causes of my mistakes and misconceptions to avoid them in the future.		
PS8. I ask myself if there might be an error in my thinking. SR1. I determine the causes of my mistakes and misconceptions to avoid them in the future.	.562	.002**
PS1. I think about what formulas, tools, or strategies I have learned that can help me solve the problem. SR1. I determine the causes of my mistakes and misconceptions to avoid them in the future.	.558	.002**
PS4. I know when to ask myself if I have solved a similar problem. SR5. I choose and prioritize personally effective study methods.	.544	.003**
PS8. I ask myself if there might be an error in my thinking. SR5. I choose and prioritize personally effective study methods.	.511	.005**

Note: * $p < .05$. ** $p < .01$. *** $p < .001$

Part 2. The second part of Research Question 1 focuses on gender: What is the relationship between problem solving, self-regulation, and math identity given gender?

- Hypothesis 4:
    - H_{o4}: There is no difference between male and female secondary students' math identities.
    - H_{a4}: There is a difference between male and female secondary students' math identities.
- Hypothesis 5:
    - H_{o5}: There is no difference between male and female secondary students' perceived problem solving practice.
    - H_{a5}: There is a difference between male and female secondary students' perceived problem solving practice.

- Hypothesis 6:
    - H_{o6}: There is no difference between male and female secondary students' perceived self-regulated learning strategies.
    - H_{a6}: There is a difference between male and female secondary students' perceived self-regulated learning strategies.

An alpha level of .05 is used to compare for statistical significance. If $p < .05$, then the null hypothesis is rejected in favor of the alternative hypothesis. If $p > .05$, then the null hypothesis is accepted.

Descriptive statistics. Since only one participant identified as non-binary and no one identified as transgender or other, these data were removed for analysis specific to gender. Thus, the N value for males is nine, and the N value for females is 18. For each composite score, given gender, I analyzed the mean, median, standard deviation, and range. Descriptive statistics were run first to analyze the data; these can be seen in Table 13.

Table 13

Given Gender, Descriptive Statistics of Math Identity, Problem Solving, and Self-regulation

	N	Mean	Median	SD	Min	Max
Male						
MathIdentityCompositeScore	9	2.630	2.500	.696	1.83	3.58
ProblemSolvingCompositeScore	9	2.544	2.375	.747	1.75	4.14
SelfRegulationCompositeScore	9	2.986	3.125	.602	2.00	3.56
Female						
MathIdentityCompositeScore	18	2.773	2.667	.923	1.25	4.50
ProblemSolvingCompositeScore	18	2.007	1.938	.596	1.13	3.13
SelfRegulationCompositeScore	18	2.336	2.250	.323	1.75	2.88

For math identity, males' mean is 2.630, and females' mean is 2.773; both are between 2 and 3 and slightly closer to the "exactly me" side of the scale. This indicates that, on average, males have more positive math identities. The standard deviation for males is .696 and the standard deviation for females is .923, which means that females' data vary slightly more. The range for males is 1.83 to 3.58; the females have a larger range of 1.25 to 4.50. The difference between these ranges demonstrates more variation in the data; at least one female has an average of 4.50, close to "not me" overall.

Looking at problem solving, males' mean is 2.544, and females' mean is 2.007, indicating that males and females use problem solving practices "very often" to "somewhat often." On average, females use problem solving practices more often than males. Male and female data vary slightly and also similarly; the standard deviation for males is .747 and the standard deviation for females is .596. The range for males is 1.75 to 4.14, and the range for females is 1.13 to 3.13. Unlike the math identity ranges, females have a smaller range than males; for females, the lowest average use of problem solving skills is "somewhat often" to not "very often" (between 3 and 4).

For self-regulation, males' mean is 2.986, and females' mean is 2.336, indicating that, like problem solving, females' average use of self-regulated learning strategies was higher or occurred more often than males'. Data vary more for males than females, with standard deviations of .602 and .323, respectively. The range for males is 2.00 to 3.56, and the range for females is 1.75 to 2.88. Similar to problem solving, females had a smaller range than males, and the lowest average was the use of problem solving skills "very often" to "somewhat often" (between 2 and 3). Also, females' median score for self-regulation was .875 higher than males.

Mann-Whitney U Test. The Mann-Whitney U test is employed due to the ordinal, non-normal distributed data. This nonparametric, inference test compares outcomes between two independent groups to test if two samples are likely to derive from the same population. This test, found in Table 14, helps to identify any differences between gender groups and can be used with unequal group samples, but it cannot be used to analyze relationships.

Table 14

Mann-Whitney U Test

	Mean Rank	Z	Asymp. Sig. (2-tailed)
MathIdentityCompositeScore			
Male	12.94	-.490	.624
Female	14.53		
ProblemSolvingCompositeScore			
Male	17.56	-1.655	.098
Female	12.22		
SelfRegulationCompositeScore			
Male	19.44	-2.524	.012*
Female	11.28		

Note: * $p < .05$. ** $p < .01$. *** $p < .001$

The results of the Mann-Whitney U test indicate a significant difference in the SelfRegulationCompositeScore, $z = -2.524$, $p = .012$, between males, which had a mean rank of 19.44, and females, which had a mean rank of 11.28. The null hypothesis is rejected in favor of the alternative hypothesis. These results indicate there is a statistically significant difference between male and female secondary students' perceived self-regulated learning strategies. Females have the higher perceived use of self-regulated learning strategies, overall, since the mean rank of females was 11.28 and males' mean rank was 19.44 with the scale 1 = almost always to 5 = almost never. When males and females were examined for math identity, the results were not significant, z = -.490, p = .624. Since p > .05, the null hypothesis is accepted. When males and females were examined for perceived use of problem solving practices, the results were not significant, z = -1.655, p = .098, and the null hypothesis is accepted. Since multiple hypothesis tests were run using the Mann-Whitney U test, there is a possibility of an increase in Type I errors. However, controlling for a family-wise error rate, or a t-test divided by three, would still result in a statistically significant result.

Upon finding a statistically significant difference between male and female secondary students' perceived self-regulated learning strategies, I looked for gender differences within the qualitative data about self-regulated learning strategies. I will describe my qualitative analysis below, after the results for Research Question 1, Part 3.

Looking at the surveyed participants' reported study methods, five of nine males stated that they only study during class or do not study at all due to other priorities or lack of incentives in their math classes. One female stated she studies only in class, and one female does not study. Eleven of 18 females talked about completing practice problems compared with three of nine males. While the majority of females cited practicing problems, they also talked about using a variety of study methods, and some use more than one method. Other study methods include creating model cards, learning charts, concept maps, if-then diagrams, or summaries; memorizing formulas; asking for help from the teacher or peers; reviewing notes or online materials; checking answers for errors; and tutoring. One female participant explained that these methods have been explicitly taught in recent classes:

> As a senior, many of my math teachers throughout the years have advised me to just practice worksheets that have similar problems on them. Then when it comes to the test, there is a problem that we have not covered. But with Algebra II and Pre-calc[ulus], I have learned how to use model cards, learning charts, and even how to prioritize my math problems (content). This has really helped me to stay organized.

Surveyed participants were also asked how they create their study plans and learn math. Seven of 18 females and three of nine males reported prioritizing key math concepts. One female student described this process:

> I usually look through all of the learning targets in order to understand what information I'm lacking. After, I categorize each concept category and decide which problems I need to revise and which ones I need to relearn, I usually just teach myself the basics.

However, only five of 18 females and one of nine males explained that they actually create a study plan. One of 18 females and three of nine males stated they do not use plans or do not study at all.

Findings were similar to the interview responses. Two of six females and two of four males talked about doing practice problems, two of six females and two of four males seek help, six of six females review their notes while only one of four males does, and four of six females and two of four males prioritize their concepts. When reflecting on the effectiveness of their study plans, three of six female students find their current plans to be effective, and the others explained changes to their current study plans to better learn the math content. Three of four males have not studied throughout the semester, and two feel pressure to study intensely now for their upcoming final.

Although females and males prioritize concepts to study and practice problems, females report using a greater variety of study methods, including model cards, summaries, and error analysis. Males may seek help from online videos, but overall, use a limited number of study methods, lack study plans, or do not study outside of class. This result is consistent with the Mann-Whitney U test finding described above.

Part 3. The last part of Research Question 1 is hypotheses about which variables are higher:

- Hypothesis 7: Students who report higher use of problem solving practices have positive math identity.
- Hypothesis 8: Students who report higher use of self-regulated learning strategies have positive math identity.
- Hypothesis 9: Students who report higher use of problem solving practices report higher use of self-regulated learning strategies.

Percentages. For these hypotheses, “positive math identity” and “higher” use of problem solving and self-regulated learning strategies are defined using cut off scores; then percentages are found. “Positive math identity” is defined as scores 1, 2, and 3 on the MathIdentityCompositeScore scale: 1 = exactly me, 2, 3, 4, 5 = not me. “Higher” is defined as scores 1 and 2 on the ProblemSolvingCompositeScore scale: 1 = almost always, 2 = very often, 3 = somewhat often, 4 = not very often, 5 = almost never, or scores 1 and 2 on the SelfRegulationCompositeScore scale: 1 = almost always, 2 = very often, 3 = somewhat often, 4 = not very often, 5 = almost never.

For Hypothesis 7, nine of 28 students (32.1%) who report higher use of problem solving practices have positive math identities. For Hypothesis 8, one of 28 students (3.6%) who report higher use of self-regulated learning strategies has positive math identity. For Hypothesis 9, three of 28 students (10.7%) who report higher use of problem solving practices report higher use of self-regulated learning strategies. Thus, there is not strong evidence to support any of the three hypotheses. Students who reported higher use of problem solving practices or self-regulation strategies do not have positive math identities, and students who report higher use of problem solving practices did not report higher use of self-regulated learning strategies.

Qualitative Analysis

To investigate research questions 2 and 3, I included open-ended questions on the survey and interviewed some of the survey participants. I used a structured interview format to interview each participant. The goal of these interviews was for participants to describe their experiences in mathematics during this school year and previous years. During the interviews, I probed for their understanding of their mathematics identities and how they learned by problem solving and self-regulating their learning. As stated earlier, 28 secondary students ranging from 14 to 17 years old participated in the survey. I planned to interview six students reporting positive math identities and six students reporting negative math identities, but only 10 students participated in the interviews. Students varied in their math identities, as evidenced by the quantitative data as well as their responses to the interview question, "Do you consider yourself a math person?" Table 15 provides descriptive data about the students who participated in the interviews.

Table 15

Study Participants, in order of math identity composite scores (most positive to least positive)

Student Pseudonym	Math Class	Age	Gender	Grade Level	Survey*	Interview**
Monica	Precalculus Honors	14	Female	Freshman	1.25	Yes
Brady	Precalculus Honors	16	Male	Junior	1.92	Yes
Jeffry	Algebra II	16	Male	Junior	2.50	Yes
Ingrid	Precalculus	17	Female	Senior	3.33 Stated "no" in open-ended	Yes in class, No in everyday
Janice	Precalculus	16	Female	Junior	3.33	Yes/No, more no
Noel	Precalculus Honors	17	Male	Junior	3.33	No
Sonja	Precalculus	17	Female	Senior	3.58	Yes
Kenny	Precalculus Honors	16	Male	Junior	3.58	No
Kasey	Precalculus Honors	16	Female	Junior	3.83	No
Liz	Algebra II	16	Female	Junior	3.92	No

Note: * Math Identity Composite Score: 1 = most positive to 5 = least positive. ** Do you consider yourself a math person?

To analyze the qualitative data, I transcribed the interview audio using Rev (www.rev.com) and read through my memos within my research journal, the open-ended survey responses, and interview transcripts to highlight key words/phrases. After this open coding, I reviewed my notes, research questions, and theoretical framework to set up an initial code tree in Dedoose (www.dedoose.com) (Appendix F). Then I sorted all quotations by these codes and exported the codes and their excerpts to look for themes within each code. I found that some codes had an abundance of excerpts and, thus, multiple themes within one code. However, other codes had fewer excerpts, and, upon further review of the surveys and interviews, these categories could be collapsed into one or excerpts were outliers. Throughout the process, I reflected on what I expected and what I was finding. For example, I expected grades to be a common topic, since students are in advanced math classes; yet when explaining math ability during the interviews, I was surprised by how students talked about achievement, successes, and challenges.

While coding, developing themes, and reviewing the data multiple times, I looked for themes within the open-ended survey responses and interview transcripts separately and also across data sources. Specifically, I looked to see if responses were similar in both sets of data and also if questions that I had from the brief survey responses were answered or clarified in the interviews. Since the survey and interview questions were not set up to ask directly about my theoretical framework of social cognitive theory, I also reviewed my themes with the theory's three components in mind. For example, I focused on excerpts about belonging (math identity) and types of problems (problem solving) to get a better sense of the environmental influences and effects on their students' agency in mathematics, and I also compared these to what students said helps and does not help their learning. Through this lengthy process of coding and developing themes, I gained a better sense of how students' math learning experiences influenced their math identities.

Research Question 2

How do secondary students articulate their math identities?

Participants described math identity in general as beliefs about their abilities in and interests towards mathematics. However, when asked about their own math identities, some interviewed participants focused solely on their ability while others articulated skills of mathematicians. A few participants talked about enjoyment found only by succeeding on problems or tests that confirmed their abilities, while others expressed joy in struggling through challenges and engaging in the content. They explained their learning by emphasizing the roles that classroom instruction and teachers have played throughout their math education.

Describing a person with positive math identity. One survey question asked participants to explicitly describe a person with positive math identity, so findings below reflect all participants' voices. Following this, I include views of those interviewed, seeing if there are any connections to how they articulated their own math identities.

Ability and interest. "They have good math skills and like solving math problems," a description of math identity by one of the survey participants, was echoed by the majority of the participants. In fact, competence or performance in mathematics was described by 20 of the 28 students who took the survey. Many attributed being good at math to natural talent, or done "easily" without hard work while a few explained the mathematical skills and effort involved in learning mathematics: "They probably try to take the hardest math classes possible and view themselves as mathematically minded. They are good with numbers and relationships between graphs, charts, etc." Another participant explained, "A math person is very much more analytical and like[s] structure and discipline."

Participants described people who like math by their enjoyment of the content or their interest in solving challenging problems. Also, math might be their favorite subject. Although liking math was included in 18 out of 28 responses, many used "or" in their descriptions, i.e., "They are good at math <u>or</u> like math." Some explicitly questioned attaching enjoyment with mathematical skill. As one participant wrote:

> I'm almost certain people who enjoy math are a rarity. We all struggle–even if it is in the subject we are strong in–and that's another thing, if a math person enjoys math does that mean they necessarily must be strong in math? I know people who enjoy math, but aren't very good at it. Does this mean they aren't a math person?

Participants did not agree if ability should come with hard work or ease and if enjoyment should stem from only successes or some struggle.

Overall, those interviewed reflected the views of the surveyed participants: strong ability and enjoyment. However, responses from the three participants with the least positive math identity composite scores stood out. Kenny and Kasey focused on ability only while all but one interviewee mentioned both skill and enjoyment in their survey responses. It was also surprising that Liz, who had the least positive math identity composite score, only included enjoyment and interest in her response. During her interview, she explained that her family members enjoy mathematics but not her and made no mention of her family's math abilities.

Describing one's own math identity. Since the survey question asked about describing math identity in general, interviews shed more light on individuals' views of their own identities. Thus, the data below are directly from interviews.

Ability. Because the majority of participants' descriptions were rooted in competence and performance, it was not surprising that articulations of interviewees' own math identities were also grounded in this idea. Many attributed others' math identities to natural ability instead to time and effort taken to learn math. Monica, Brady, and Jeffry, with the most positive math identity composite scores, discussed being good at math; liking it and being good at it; and math coming easy, respectively. Those with less positive math identity composite scores explained that their own assessments of their performance changed by concept, problem, exam, or class. For example, Kasey felt her natural math ability had been strong in the past and described her current experience differently:

> I think I lack confidence because ever since I was a little kid, I've always been naturally good at math and then when I started algebra it was different because that was the first time I had to really work to understand something in math. I have a lot of friends who still don't have to work at understanding things in math, so I think that makes me feel, I guess, a little bit insecure because I have to work really hard at something that might come easily to somebody else.

Noel, Janice, Kenny, and Ingrid also articulated that if they were performing well in math this year or solved a recent problem correctly, they had a positive view of their math identities. However, a recent failure caused them to question their math identities.

> Interviewer: Do you see yourself as a math person?
>
> Janice: Not really. I mean, it's funny because this answer will vary based on if I'm doing good in a topic or not.
>
> Ingrid: I do see myself as a math person when I get…what the person is asking me to solve and I feel like a math person.

Interest. As expected from on the general descriptions of positive math identity, enjoyment was also closely tied to ability for many participants. Noel, with a neutral math identity, succinctly explained how interest is tied to success in mathematics: "When I can do it, yes [I like problem solving]. When I can't, no. Normally, it's I can't." Others with less positive math identity composite scores agreed:

> I like solving problems when I get them right because then it's really satisfying. But if I'm solving a problem and it's on a test and I'm feeling very overwhelmed and confused, then I get very stressed out. (Kasey's interview)

> When you know how to solve something it feels great and you're like, I could do a thousand of these. If you don't know how to do it and you know you don't know how to do, even though you're working through it, you're still stuck. That's when it becomes not fun at all. You're just like, I don't want to solve anymore of these. (Sonja's interview)

However, not everyone was interested in being correct or solving quick, easy problems. Some participants expressed joy in the content itself and satisfaction from struggling through challenging problems that might take them more time and effort to think through, as evidenced by Brady's comment:

> I just find math interesting. I like math. So, learning new concepts is always fun for me. I do like solving problems…I get a very good sense of satisfaction if I finish up a problem…Definitely not as much [satisfaction if solving an easy problem]. If a problem takes me a long time and I eventually solve it,…that's way better.

Sonja also explained the benefits of struggling through problems:

> You just have to be able to enjoy it and respect what you're learning and be able to be like, yeah, this is something that may be a challenge but I know if I work at it, in the future it won't be as difficult for me.

This was surprising because she had a lower positive math identity composite score but self-reported "I am a math person" after talking through her math experiences in the interview. It may seem obvious that two participants who expressed positive math identities–Brady and Sonja–would be positive about working through difficulties in mathematics. However, none of the interviewees mentioned skipping problems because they grew disinterested or did not know what to do; everyone said they would try a challenging problem, using all problem solving methods they knew, before moving on to other problems. A few said they would work on a problem for an extended period of time, during a good part of a class period or over a few days, while others talked about taking a break from a problem in order to return with fresh eyes and ideas.

Interest in getting problems correct or struggling through challenges were both within the classroom; no one articulated using mathematics outside of the classroom or relevance of the content to their everyday lives or future careers. Two participants were considering Science, Technology, Engineering, or Math (STEM) careers. Yet both stated that they would not pursue mathematics in college, and they did not expand on connections between math and these STEM areas.

Development of math identity. Students' math identities are dynamic and continue developing with new teachers and classes over time. Participants' descriptions of their experiences in math classrooms centered on two areas: the classroom instruction and structures as well as their teachers. In terms of classroom instruction and structures, common themes were the lesson structure, freedom and choice within practice, and peer collaboration. Participants also explained their relationships with their teacher and their teacher's understanding and engagement during class.

Classroom instruction and structures. All students articulated a similar structure to their teacher's lessons, as Liz described:

> [The teacher] starts our mornings off with a superhero video for some encouragement. We analyze the video, and the videos are usually connected to our lesson for that day. This is on days where we're learning a new concept. And then we begin our notes, she gives a problem that we've never seen before and we break it down to our best ability on our own…Once we start to learn how the

> problems are supposed to be solved, our notes get longer and longer and longer…We spen[d] [class] doing many problems to understand [the notes]. [A few days later] she showed us the problem again. We were all able to get a lot further on the problem because we had discussed what we needed to know…to solve them.

For some students, this was a new way of learning mathematics; Monica explained the difference from past years: "Last year was a lot of take notes, take notes. Now it's more discussion around why does this sort of thing work and that sort of thing, which I find interesting." For others, this teaching method was familiar. Ingrid, Sonja, and Janice had both teachers over the past two years for Algebra II and now Precaclulus or Precalculus Honors. They felt that they benefited from similar expectations and teaching styles in these classrooms. Ingrid explained the experience:

> [Both teachers] work together so those two classes have been very similar, which is really good. I really like that because I'm able to understand what they're doing and I [can] connect previous lessons that I've had over the past two years.

The structure of the lessons and classrooms allowed for freedom and choice during "self-guided" practice. Participants were given worksheets with various types of practice problems, from simple equations to applications, and they decided which ones to start with and how to go about completing the practice. While participants may work together during practice, this was not a requirement, and they did not feel the need to rely on their partner at all times. Therefore, students collaborated to varying degrees and at various points during practice. Some students appreciated being given the chance to think through problems on their own prior to asking for support from their peers or given steps or hints by their teacher. Ingrid found that the benefit of starting on her own before collaborating was choosing to work with a group struggling with the same ideas as she. Monica agreed:

> I think what's working is being able to individually and with peers to figure it out yourself a little bit. Then if you can't get it, then it's helpful to have a teacher there who you can ask, so figuring it out yourself, but then if you can't, having help.

Jeffry and Brady mainly preferred to work alone, but they expressed some benefits to working with others. Of note, the students who preferred to work alone, at least at first, articulated more positive math identities. On the other hand, Kasey preferred to start solving problems together by suggesting strategies to one another and spotting errors in each other's work and thinking. Similarly, Kenny believed that he benefited from learning with his friends in class, stating, "It's a lot easier to learn math when you're with your friends then it is to be forced to sit still and only learn people around who you might not know in the first place." Overall, students appreciated choosing when and how to work with peers, even though some took time to adjust to this freedom during class practice.

Peer collaboration was repeatedly cited as a support when making sense of challenging or new problems. When working together, participants either brought a question or their own work to their partner for feedback. When Ingrid got stuck on a problem and needed help, she described her next steps:

> I'll usually bring the work and the equation that I've been working on. I'll show them exactly where I got stuck. I'll explain my thinking behind that. My peer will usually look at it and read it and then…sometimes they'll show me how to do it and other times they'll ask me questions to see how much I actually know. Then from there we end up solving the problem together if I need more help. If not, I go back to my seat and try to solve it by myself.

Monica described a similar process, saying, "If we can't solve problems, we're supposed to ask questions about the problems and think about it a lot. That's the deal." Thus, most students do not approach one another for support empty handed; rather, they come with questions and a start on the problem.

Others found value in seeing different viewpoints and strategies. None of the interview participants emphasized talking about the correct answer, although they mentioned determining if an answer was reasonable and looking for errors in thinking. Kasey explained that working together helped her "strategize when solving questions so…that way you get different perspectives and maybe find a new way to solve a problem that you wouldn't have thought of before." Janice agreed, and she was trying to be more open to others' suggestions while also cognizant of responding to her peers: "I am sort of giving them the direct answer [but] I know that isn't as helpful… I'll try and like give a diagram or tell them my thinking to see if they can come up with it on their own." Liz's partners patiently explained problems step by step but also made her explain the process and reasoning for the steps, so she was confident in her ability and able to extend her knowledge.

With liberty to choose problems and a practice partner, more responsibility was on the students; the teacher was not micromanaging their groups or conversations. Although students know the expectation for learning the material and practicing problems, some were frustrated with this freedom. In their interviews, a few started by saying the teacher needed to provide the focus to keep everyone working, but they ended their statements by acknowledging their roles and responsibility in the situation. For example, Janice saw opportunities for the teacher and classmates to make changes:

> [Y]ou got to have a balance between…letting the student go and know what they need to do and…bring[ing] the student back in and…tell[ing them] this is what you need to do. Please get on task….If the teacher doesn't see that they're not on task, most of the time they're just going to keep going unless a friend [says] help yourself…I want you to do better.

Noel also appreciated being able to collaborate with peers but said that has led to a lack of focus at times, and he and his peers have questioned what they are supposed to be doing and learning. Yet he admitted that this issue was partially his fault, since at any time he can ask other classmates or the teacher for clarification.

Teachers. Participants also talked about how their relationships with the teachers and their teacher's style have had an impact on their math experiences. Brady described really liking his past math teachers but that his experience this year was different since his relationship with his current teacher was more academically focused:

> [T]his [relationship] is more purely just math…In previous years, I think I've been one of the best students in the class. [My past teachers] talk to me about stuff

> that isn't math or whatever, and this [class] I'm definitely challenged a lot more, so I'm a lot more focused on my work instead of just trying to pass the time.

Liz appreciated her current teacher because, unlike last year, she felt that she has more help and an extra push that she really needs. Thus, her outlook on math has changed so much that she no longer tells people that she hates math, and instead says, "I'm getting better, and I think it's because I'm preparing myself to have to be more advanced in math, to do harder stuff. My teacher's not gonna hold my hand. So [my teacher's] preparing me for that."

Other participants described that their teacher understood they were not going to "get it right" all the time and were okay with students struggling productively. As Sonja put it:

> If you have a teacher that has that mindset that everybody is going to pick this up the first time, you're not going to get far. But if you have teacher who is like, I'm going in knowing that a lot of students are going to have questions and I have to be prepared for that, then that's when you see a lot of improvement. I think that goes hand in hand how I would or my peers would be able to have that mindset to learn. You get that same energy. If your teacher doesn't know what's happening you will take that on and not know what['s] happening.

She felt that her teacher knew students would experience challenges and frustrations, but her teacher also has prepared supports and other methods for students to show improvement. Sonja embodied this same approach to her own learning, knowing that she will make mistakes yet still grow along the way. Kasey has seen growth in how she asks and answers questions because her current teacher has encouraged her to ask herself questions first. She reflected that she is “definitely challenged a lot more, but [she’s] also…asking more questions than [she] normally would,” which has helped in thinking through problems and errors.

Thus, classroom instruction and teachers have had an impact on students’ math identities. However, unique impacts were not noted by varying degrees of positive math identity. The lesson structure, independence within classroom practice, and collaborative nature encouraged students to work together and take ownership of their learning. The students’ relationships with their teacher engaged students in thinking about changing their views and improving their learning of mathematics.

Research Question 3

Does students’ articulation of the development of their math identities explain their problem solving practices and self-regulated learning strategies?

Participants articulated common problem solving practices, including relating concepts to a challenging or novel problem and utilizing resources. Self-regulated learning strategies often cited were practicing problems, asking others for help, reviewing notes and online resources, and selecting key concepts to study. Yet participants articulated these separately from their math identities because, regardless of their perceived ability and interest in mathematics, the majority of students knew and used problem solving and self-regulated learning practices.

Problem solving practices. In the survey, participants explained their problem solving practices. Their methods varied, but common themes from the data included: thinking about connections between the problem and prior knowledge; using a method their teachers called "given, want, know"; and seeking help from other sources.

Almost half of the survey participants talked about using what they know and relating that to the problem. They made connections to concepts learned earlier in the year or prior math classes, rules and formulas, and ideas or approaches from different types of problems. One participant explained the process:

> I try to think of concepts and formulas that connect with the problem and I can use to find its solution. Once I have linked it to a concept/function, I recall how we solved a similar problem in class and usually pick a few steps from there. I continue to find solution with the help of the information I had jot[ted] down, the connections I had made and any visual that I can draw for the problem.

Relationships were shown symbolically and visually, as students described thinking about the mathematics in the problem as it connected to their own mathematical knowledge. Some participants also explained using what they were given, what they wanted to find or solve, and what they knew. Although this method is very similar to the previous method, the relationship between what is given in the problem and what the problem solver knows was emphasized more since students saw these as a trio: "what I know, what information I am given, and what I want to find." A few mentioned that they learned this approach in their previous math classes. Lastly, students sought help from notes, the teacher or peers, or online. No one said they would turn immediately to support, but instead all talked about trying the problem alone first. However, some mentioned getting stuck and not being able to find an error in their thinking, and at that point, they would turn to other resources.

Connections and relationships. All interview participants articulated the importance of making connections between known concepts and the problem. Yet only about half of the students explained the "given, want, know" strategy when describing their process for completing challenging math problems. Monica described this process as filling in a gap of knowledge between what she wanted to find and what she knew:

> I look at the problem, and it's like what information do I have, is really what I look at. Then what am I trying to find? Then I want to look at what's in between those two and how I could get there, ideally, and think about things I know how to do, if there [are] any words that might associate with a concept that I know and can use.

Kasey's process to think through a problem was similar, but she used her guiding questions that she outlined from previous information and problems: "What's my given information? Is there any pattern that I recognize from previous problems that I've done that are similar to this?" When stuck on a problem, she used these questions to think through the process and see if she made an error. Brady and Janice explained their uses of connections with specific example math problems. Brady remembered the concepts of past problems to see if the current problem was a "more abstract version of a problem we've done before." For example, he described that solving an exponential equation with the number e was challenging, but then he that "it looked like a problem we'd done before but there was just x normally, [be]cause you solve for x, and then there's *e* and that was more complicated." Because he knew how to solve for x, he applied this method to a more complex problem. Janice articulated recent success with factoring–a topic that she struggled with in the past–because she found it useful for verifying and solving trigonometric equations. She commented, "As you get older in school, you realize that…everything you've learned in the past just builds upon what you're already learning…I'm seeing stuff come back from eighth grade that I'm like oh, okay, this still exists. Cool." All students with either a greater positive math identity or a lower positive math identity articulated making connections between the problem and known concepts.

Support resources. Half of the interviewed participants talked about using supports, including peers, the teacher, and online resources. When working on challenging math problems, participants appreciated hearing their peers' strategies, finding errors in their thinking, and discussing their process and reasoning together.

Connection between math identity and problem solving. From research question 2, participants described math identity by ability and interest in mathematics. Those who were good at and enjoyed mathematics were viewed with positive math identities. Yet analyzing interviewees' math identity data with problem solving, whether or not they were good at and interested in mathematics did not determine their use of problem solving practices or perseverance in solving problems. All interviewees attempted to make connections between the problem and their own mathematical knowledge, half used the "given, want, know" strategy, and half sought support from peers or the teacher. Students who used "given, want, know" articulated various degrees of math identity, while all students who sought resources expressed neutral positive math identities. In other words, interviewed participants with more positive or less positive math identities did not seek support from others.

Self-regulated learning strategies. When asked about their study methods, surveyed participants mainly discussed doing practice problems; seeking help from peers, the teacher, or online resources and videos; reviewing notes for examples, concepts, and formulas; and prioritizing concepts for review.

As expected, practice problems were the most common study method. However, students found practice problems from various places; some referred to old worksheets or redid examples in notes, while others searched online for problems and video solutions. Since students found value in working together, with the teacher, and with online resources to solve problems, it was not surprising that students also cited these methods for their general studying before an assessment. The four classroom cultures appeared to be collaborative, encouraging students to ask questions of themselves, peers, and the teacher as students made sense of the mathematics. Many students reviewed their notes, seeing them as a beneficial resource, possibly for connections between concepts or detailed solution pathways. Students cited use of notes both for problem solving and studying. Also, participants articulated prioritizing concepts or problems for review. Students used past tests to understand where they made mistakes and where they could improve; this error analysis helped them focus on specific concepts and organize their studying.

Prioritizing concepts. Interviewed participants elaborated on similar themes: practicing problems, seeking support, reviewing notes, and prioritizing concepts, although the last theme was identified most frequently. Participants explained in more detail how they prioritize content when studying. Brady described starting with the "hardest" concepts first, and Noel and Liz began with concepts they did not understand so they could ask for help during class. Ingrid and Janice both mapped out their priority concepts and planned specifics days or nights for review and practice.

Making decisions. Self-regulated learning is defined as using and managing affective, cognitive, motivational, and behavioral strategies to attain a goal. Even though the teacher provided direction for studying, participants felt part of the decisions about what and how to study. Many understood that the content standards came from the school, district, or state, and some also mentioned that their teachers worked together to decide what to teach. Yet they felt their voices were heard in the classroom. Liz explained, "I get to put in the amount of effort that I want to and she gives us the start. I have to remember everything that I know to keep going forward." Brady described this experience as, "I make decisions on how to study, and then the mutual understanding of what I'm going to study because it gets harder and I've gotten worse test grades on CC2, [concept category 2], than other stuff. But yeah, it's pretty much up to me." Kasey described the teacher and her classmates deciding together if they needed more notes or it was time for practice.

Taking ownership. While Monica, Ingrid, and Liz felt their current study methods and action plans were effective, others reflected on pitfalls and proposed changes for finals' studying based on how they were performing in class. Citing experiences with other teachers and in past math classes, Noel and Kenny explained that they performed well in math in the past but have not found the same success this year. In the past, they did not need to study or try because math came easily or the topics were simpler. However, this year they had a hard time adapting to their teacher's style and expectations; specifically, they struggled with the freedom during practice and not receiving frequent grades but instead formative assessments and feedback. Noel explained:

> This year's definitely harder. Generally, in math, in the past, I've been able to understand the concepts without having to do much studying. I think that's just because it's been simpler in the past. I've generally been able to get all A's. This year, I am struggling a little bit more. I can definitely feel it. I think that this semester I did slack off a little bit too much in the beginning. Even though I have been working harder in the end, you know, putting whole effort into it, recently, I still feel like I could've done better at the beginning. I think that I'm going to correct that next semester.

Although they would like to see changes in instruction, they also acknowledged that they need to improve their own actions by paying attention more, focusing during practice instead of leisurely working, and not falling behind on the content. They planned to make changes in the spring semester.

Kasey and Brady both reflected that their action plans were ineffective this semester, based on the grades they received on assessments. Kasey asked her teacher about adding something new to her study methods, and her teacher worked with her to create if-then diagrams to map out her thinking and guiding questions. She found recent success with this method and planned to continue. In previous classes, Brady only did practice problems, but that method did not work this year. His teacher provided recommendations based on common errors, as he explained:

> When we go over tests she'll have a little chart up on the board on the types of the mistakes that you would make, and that will have recommended plans on what to do depending on the types of mistakes that you make. If it's like a procedural mistake then…just keep practicing the problems and get it more consistent. But if it's a misconception, like if you don't understand a concept or you have a misconception with the problem then…you would review your notes and get [a] better understand[ing] and…annotate a problem…You can just write down what each step is doing.

Along with using the suggestions in his teacher's chart, Brady also hoped to work more with his peers. Although he typically preferred to work alone on practice, he knew that some of his peers have knowledge and ideas that could help him. Therefore, instead of searching for support online, his current method of support, he would like to work with his peers on challenging problems.

Arguably, Sonja has experienced the most change this semester. Before this school year, she was against math, thinking it was too hard and was not for her. Now she is a willing to learn. When describing what worked and did not work for her when learning mathematics, she explained a shift in mindset:

> I definitely stopped blowing off [math] ... I feel like in previous years I was always like, math is just challenging. When you have that mindset where you think math is just hard and it's not for you, you start to doubt yourself and lower your self esteem and you become more anxious when you take tests. I feel like this time around I was more confident. Even though it was a synthetic confidence where I had to pretend that I was confident for it to actually work… Doing that, I tell myself, "This is easy." I tell myself, "It's easier than I'm thinking it is." I just have to stop overthinking and actually work and not just walk away from it or flip the page or just start copying. I have to sit there and work through the problem. If I don't finish it, then I better go home and finish it.

Although Sonja praised her teacher's patience when answering questions and energy when teaching, she ultimately took ownership of her learning by deciding to put in the time and effort to work through problems and stay positive when faced with difficulties.

Connection between math identity and self-regulated learning. As previously stated, participants described those with positive math identities having good abilities in and enjoyment of mathematics. Yet analyzing interviewees' math identity data with self-regulation, there was not a clear connection between the two. Students who were actively managing study strategies and making changes to attain their goals did not have the most positive math identities, and students who were not changing their action plans did not necessarily report less positive math identities.

CHAPTER V

DISCUSSION

Overview of the Study

The focus of this study was to examine the relationships among secondary students' math identities, their perceived problem solving practices, and their perceived self-regulated learning strategies. This dissertation examined the following research questions:

1. What is the relationship between secondary students' math identities, their perceived problem solving practices, and their perceived self-regulated learning strategies? What is the relationship between problem solving, self-regulation, and math identity given gender?
2. How do secondary students articulate their math identities?
3. Does students' articulation of the development of their math identities explain their problem solving practices and self-regulated learning strategies?

In Chapter I, I provided my rationale for this study and introduced the construct of mathematics identity. Existing research on mathematics identity analyzed its relationship to classroom communities and teacher instruction, teachers' math identities, multiple identities, and career choices; however, there are limited studies about how math identities are developed through instruction and interactions with others. Two specific ways that students learn and engage in mathematics are problem solving and self-regulation. The purpose of this mixed methods study was to understand the relationship between students' math identities and their perceived use of problem solving and self-regulation practices as well as students' articulation of their mathematics identities, either positively or negatively.

In Chapter II, I reviewed existing literature relevant to this study and described the theoretical background. I identified and organized literature about mathematics identity and its development, implications for instructional practice, problem solving, and self-regulated learning. Then I presented a description of my theoretical framework of social cognitive theory (Bandura, 1986) and its relationship to agency in mathematics.

In Chapter III, I detailed my research methodology to include reasons for using a mixed methods design. This study was a sequential explanatory mixed methods design with quantitative correlational research, qualitative interviews, and survey research. My data collection plan was to gather quantitative data from surveys first and then explain the results with in-depth qualitative analysis from interview data. I provided a description of my data collection and analysis methods for the quantitative and qualitative phases of the study prior to discussing the reliability, validity, trustworthiness, and limitations of my research design.

Conclusions

In Chapter IV, I addressed my research questions in order, starting with the quantitative focus: What is the relationship between secondary students' math identities, their perceived problem solving practices, and their perceived self-regulated learning strategies? What is the relationship between problem solving, self-regulation, and math identity given gender? I analyzed the quantitative data using descriptive statistics, Spearman's correlation coefficient, the Mann-Whitney U test, and percentages. From my analysis, I found two key results. First, secondary students' math identities were independent from their perceived problem solving as well as their perceived self-regulated learning strategies. However, there was an association between secondary students' perceived problem solving practices and their perceived self-regulated learning strategies. This meant that the higher an individual ranked perceived problem solving practices, the higher that individual ranked perceived self-regulated learning strategies. Second, my analysis indicated a statistically significant difference between gender groups' perceived use of self-regulated learning strategies. This meant that females have the higher perceived use of self-regulated learning strategies, overall, since the mean rank of males was 19.44 and females' mean rank was 11.28 with the scale 1 = almost always to 5 = almost never. Analyzing the qualitative data for gender differences in self-regulated learning strategies, females use a greater variety of study methods, and males use a limited number of study methods, lack study plans, or do not study outside of class. Thus, this result is consistent with the Mann-Whitney U test finding described above.

Then I addressed my second research question using the qualitative data: How do secondary students articulate their math identities? I described the students' articulation of their mathematics identities based on survey and interview responses in which they discussed their past and present mathematics experiences. Two key findings emerged from this analysis. First, the majority of participants described an individual's math identity by ability and interest, and interviewed participants used these components to analyze their own math identities. Participants explained ability from natural skills and assessment performance and interest for being correct, learning new content, or productively struggling. Second, classroom instruction and teachers had an impact on the development of students' math identities, but students with more or less positive math identities did not report different influences.

Finally, I addressed the third research question using both quantitative and qualitative data: Does students' articulation of the development of their math identities explain their problem solving practices and self-regulated learning strategies? Looking at the quantitative results and qualitative findings, I described the relationship between math identity and problem solving practices and the relationship between math identity and self-regulation learning strategies. First, I found that whether or not students felt they were good at math or enjoyed it was not correlated to their use of problem solving practices or perseverance in solving problems. All interviewees attempted to make connections between the problem and their own mathematical knowledge, and no one skipped challenging problems completely. Second, there was not a clear connection between students' math identities and their perceived use of self-regulated learning strategies. For example, students, who were actively monitoring study strategies and performance, did not have the most or least positive math identities. These two findings are consistent with the quantitative correlational analysis described in the first research question. Thus, the triangulation of the data support the conclusion that secondary students' math identities were independent from their perceived problem solving as well as their perceived self-regulated learning strategies.

Discussion of Findings Related to the Extant Literature

As detailed in the literature review, math identity encompasses two main areas: (a) beliefs about the relationship between math and self and (b) belief that one belongs. Some of the findings of this study were supported by the existing literature while others conflicted with previous insights.

Math Identity: Beliefs about the Relationship between Math and Self

I analyzed this first component of math identity–beliefs about the relationship between math and self–by referring back to the math identity definitions by Schoenfeld (2014) and Martin (2000). The existing literature defines math identity as one's "belief systems regarding mathematics and one's sense of self as a thinker in general and a doer of mathematics" (Schoenfeld, 2014, p. 4). This description includes not only an individual's beliefs about abilities and practices in mathematics but also how the individual views mathematics content and learning. Most participants articulated that those with positive math identities enjoyed math or were good at math, mainly citing natural ability or performance on a recent problem, assessment, or class. Interviewed participants used ability and interest to judge their own math identities. Although participants explained the mathematical practices they engaged in and how they learned math, they did not view these as influencing their mathematics identities. Nor did many participants see struggling through challenging problems as a component of doing math or interest.

Yet participants engaged in mathematical practices as evidenced by their survey responses. The mean for perceived use of problem solving practices was 2.211 with 1 =

almost always to 5 = almost never. Their use was elaborated on in the interviews. When problem solving, all students reported making connections between the problem and concepts, rules, and ideas they knew from previous problems and instruction. This aligns with Boaler's (2003) finding that students move between what they know and do not know to make sense and work through a problem. When discouraged or without further solution pathways to attempt, students sought assistance from peers, their teacher, or online resources. Thus, they definitely had both positive and negative emotional responses to mathematics and were willing to engage in collaboration, which are common practices of research mathematicians (Burton, 1999). Although making connections and seeking help were the two most common responses, students also reported using heuristics, such as guess and check, look for a pattern, draw a picture, or solve a simpler problem as Pólya (1945, 1957) recommends. Therefore, evidence indicated that students considered their prior mathematical knowledge and used a variety of heuristics during problem solving.

Participating students possessed at least neutral if not positive dispositions towards math content and learning. The quantitative analysis showed that students were interested in learning more about math and enjoyed learning math (means 2.57 and 2.50 respectively, with 1=exactly me to 5=not me). Not everyone had positive experiences, and hard problems were still frustrating, but some students changed their feelings about math. Although Liz hated math in the past, she no longer felt this way and saw this class as preparing her for advanced mathematics. Brady, having had great success in the past, embraced and enjoyed struggling through math problems, and Sonja believed she would improve her mathematical knowledge with support from her teacher and peers.

Within the classroom, students had opportunities to learn math by engaging productively in mathematics (agency)–i.e., working on new problems before teacher explanations, using their resources to learn a new topic, and learning from mistakes. This aligns with Schoenfeld and the Teaching for Robust Understanding Project's (2016) recommendations for promoting agency, authority, and identity in mathematics classrooms. Students also had the opportunity to make the content their own (authority). Liz "created" her notes as she learned more about the mathematics involved in the novel problem at the beginning of class. Kasey put past problems and questions together into her if-then diagrams, and Ingrid carefully prioritized her concept categories for studying. Lastly, Schoenfeld recommends giving students opportunities to see themselves as people who can do mathematics to develop their positive mathematics identities (identity). As seen in the data, this is not happening, since students are not making the connection between problem solving and collaborating with their peers to seeing themselves as people who can do mathematics. One reason may be that teachers have not found a balance between growth and comfort as students productively struggle through learning math and developing their identities in a social context (Grootenboer, 2013). Another reason may be secondary students' identities are influenced by the pressure of grades and doing well in preparation for college. Thus, there is room of improvement of instructional practices, and these concerns are addressed as implications for those involved in math education later in this chapter.

To expand on Schoenfeld's definition of math identity, Martin (2000) considers beliefs about "the motivation and persistence needed to obtain mathematics knowledge" and "the significance of mathematical knowledge" (p. 19) as key elements to mathematics identity. There was evidence of students' motivation and persistence needed to obtain mathematics knowledge. When talking about problem solving and self-regulated learning, students explained what motivated them. They were driven to complete a problem because they wanted to find the answer or get adequate practice on a certain type of problem. When studying, their self-motivation came from these desires: to get the best grade, feel they knew the math content, or show improvement. Some were motivated by working with others, but a few had limited or no motivation to study. Also, the majority of participants persisted in the face of challenging math problems by working through errors and misconceptions and staying on problems for large portions of a class period or returned to problems hours or days later.

None of the participants mentioned being motivated by using mathematics in their future, and very few articulated the importance of math within their daily lives. In other words, math content was useful during math class, but students did not articulate that it was as meaningful outside the classroom. However, research shows there is a reciprocal relationship between students finding use of mathematics and valuing its role in their future careers to displaying a more positive math disposition (Martin, 2000). Without seeing the significance of the mathematical knowledge, Schoenfeld (1992) warns that students do not take ownership of their learning. In this study, there was no evidence of students' finding significance of mathematical knowledge in their future. Because students' beliefs about math and their self-regulation skills might benefit, this concern will be addressed as an implication below.

Math Identity: Belief that One Belongs

As students develop their math identities, they not only make sense of their relationship with mathematics and understand their own learning practices but also feel part of a group engaging in and learning mathematics. I analyzed the students' experiences in a community of practice (Wenger, 1998, 2010; Wenger-Trayner, E., & Wenger-Trayner, B., 2015), described the classroom instruction, and considered if students feel a sense of contribution.

The existing research focuses on a struggle with belonging instead of failure of ability (Boaler et al., 2000; Solomon, 2007). Within the quantitative data, the mean for "I belong within a community of math people" (mean = 3.71, with 1=exactly me to 5=not me) showed that on average they did not feel part of this group. However, it is possible that students did not fully understand the statement because they extensively described their work with peers and the teacher. Based on their articulation of collaboration during the interviews, I would argue that students felt a sense of belonging within the classroom. They had tools at their disposal to learn on their own, seek help, and advance their knowledge and cited examples of using these tools regularly. Yet failure of ability was still present in their discussions about math identity. Participants separated themselves from their peers during interviews by ability alone, but this often reflected performance on assessments and not use of mathematical practices. When describing problem solving and self-regulation, they highlighted peer collaboration and their contributions to the class and one another.

To understand how a sense of belonging is created within a classroom, I examined at how students are learning math. In the two classrooms, students were given daily opportunities for investigation, conversation with others, and questioning. This instruction is supported by previous research that finds discussion- and inquiry-based classrooms have positive influences on students' dispositions towards mathematics and their engagement with the content (Boaler, 2002a; 2002b). In the four classrooms, students were not learning alone but working with pairs, groups, and the teacher to make sense of the mathematics, and we know that context is greatly important when developing math identities (Grootenboer, 2013). Therefore, even though a strong sense of belonging within a math learning community did not come out in the quantitative data, peer collaboration was a large part of students' learning experiences and successes in math class.

Besides feeling that one belongs with others doing mathematics, Solomon (2007) explores students' experiences in making constructive connections or contributions in mathematics. In this study, students articulated that they could solve problems multiple ways if they were able to explain their reasoning. They were also encouraged to make sense of the content in their own way by putting content and questions in their own words. Students also felt that their voices were heard in the classroom in terms of what and how to learn mathematics–e.g., deciding if the class should move on to practice or continue with another example. Participants were very supportive of their peers, offering new ideas, answering questions, or modeling on the board. Still, no students had plans for contributing to mathematics in a broader way or for a longer term. It is possible that they did not feel they belonged in more advanced math or they did not see the significance of mathematics in their futures. This study did not go in-depth about students' future careers, so it might be a topic for further study.

We know that students develop positive math identities when they believe they can do math and believe that they belong (Boaler, 2015). Participants had mixed views about their own math abilities, but all articulated ways in which they used mathematical practices when problem solving or self-regulating their math learning. Peer collaboration was an important part of their math experiences. To address some of these findings, implications are discussed below.

Discussion of Findings Related to Social Cognitive Theory

As described in Chapter II, elements of the three main constructs of this study–math identity, problem solving, and self-regulation–are connected to the three components of social cognitive theory: personal factors, behaviors, and environmental influences. Considering these connections deepened my analysis and understanding of the quantitative and qualitative data.

The first social cognitive theory component is personal factors. Although many participants did not see themselves as having positive math identities, they showed positive dispositions towards math, used mathematical practices, and articulated what supported them in learning mathematics. Of the interviewed participants, 9 out of 10 stated that they believed they could solve challenging math problems, and they articulated connecting prior knowledge, using heuristics and flexible thinking, or showing determination in continuing a challenge. Individuals with high self-efficacy take action and continue to improve their understanding (Bandura, 1989), and self-efficacy has also been found to increase problem solving efficiency (Hoffman & Spatariu, 2008). In discussing their study methods and action plans for the final, students reflected on what was working or not working for them in learning mathematics. When students understand their strengths and weaknesses, they are able to "actively monitor their learning strategies and resources and assess their readiness for particular tasks and performances" (Bransford et al., 2000, p. 67). Three students found their plans to be effective, while the others proposed changes. Thus, most believed that they could perform skills or understand content and took action to further their learning, thereby displaying high self-efficacy.

The second component of social cognitive theory is behaviors, or the responses an individual receives after they perform a behavior. I analyzed students' goals and decision making during problem solving as well as their goals and actions in the self-regulation process, but evidence was not as clear for this component. When talking about solving a challenging or new problem, participants explained the need to understand what the problem was asking and then try various approaches prior to seeking help. Yet only three mentioned setting subgoals for problem solving and few detailed how they made a plan when seeking a solution. Therefore, it was unclear how well the students could define a problem space of possible goals and paths potentially related to the problem. Some students engaged in decision-making and metacognition as they problem solved, which is beneficial for seeing the gaps in their thinking, understanding and verbalizing their thinking processes, and making corrections (Brown et al., 1983). Students who engaged in metacognition asked themselves questions, looked for errors in their thinking, and worked through different methods. Yet extensive evidence of goal setting and decision-making during problem solving was lacking, and this might merit examination in future studies.

As part of the forethought phase of the self-regulation process, an individual sets goals and later self-evaluates learning and performance from this standard (Zimmerman & Campillo, 2003). Because participants articulated content priorities as opposed to specific goals to accomplish, they may not see value in goal setting, or this may not be how teachers articulate goals in their classrooms. There was also no evidence that participants were creating strategic plans to identify specific strategies, behaviors, or thoughts in preparation for the performance phase (Zimmerman & Campillo, 2003). This was confirmed in their description of the performance phase; the main methods used were practicing problems, seeking support, reviewing notes, and prioritizing concepts. While these actions seem logical, they are generic and disconnected responses to perform a specific behavior more successfully. In other words, participants noted specific concepts to take action on, but the actions did not depend on past errors, the concept itself, or the best method for students to learn or build their understanding of the content. Instead of using "self-regulated learning strategies…as purposeful actions and processes directed at acquiring skill or information (Zimmerman, 1989 as cited in Cleary, 2006, p. 309), many students explained that they "usually" or "always" took that action.

The third component of social cognitive theory is environmental influences, or the context that influences an individual's ability to complete a behavior. In all four classrooms, students were not only completing routine or well-defined problems but also engaging in non-routine or ill-defined problems. Ill-defined problems are characterized by their openness, meaning students have a chance to make their own assumptions, interpretations, and conclusions with proper justification (Kyung et al., 2011 as cited in Byun et al., 2014). Environmental conditions, such as support and materials, can promote an individual's learning, improvement, and continued success. Participants explained that they could come up with their own methods for solving, check errors and ideas with peers, and make choices about how and when to seek support within the classroom. They had opportunities to work alone, with pairs, in groups, and ask the teacher. This collaborative classroom culture allowed students to feel challenged but also comfortably engage in different types of problems and ask for support, which Grootenboer (2013) recommends. Thus, I would argue that students felt a sense of belonging to their classroom community, since they reported working with peers who had similar needs, could answer their questions, or provided new strategies prior to asking the teacher for help. The classroom environments provided both support and freedom to work on well- and ill-defined problems.

Math Agency

Within social cognitive theory, agency is an awareness of performing and controlling one's own actions. Within mathematics classrooms, agency is developed by student choice, self-exploration and self-direction, the acquisition of resources, and authority. Using this description, I reflected on if and how students were becoming agentic and not simply reactive or responsive to the surrounding world.

First, participants talked about making choices about the content and their learning in a variety of ways. The lesson structure allowed for students to make sense of a novel problem before the teacher provided definitions, visuals, and processes in the notes. Students tried any methods to begin the problem, discussed their thinking with others, and then were introduced to more formal mathematical knowledge. Thus, the learning environment adhered to the problematizing and accountability principles (Engle & Conant, 2002); the teacher encouraged students to think independently on challenging problems by justifying their reasoning to peers or comparing to disciplinary norms presented in teacher's notes. During "self-guided" practice, students chose what concepts to start with, which types of problems to practice, whether to practice alone or with a partner, and when to seek support from peers or the teacher. Participants' descriptions reflected Fiori and Selling's (2015) recommendation for a learning environment that allows students to move around the room and provide necessary tools. However, not all students benefited from this freedom and requested that the teacher provide more structured groups and assignments.

Second, agency in mathematics is strengthened when students self-explore and self-direct (Côté & Schwartz, 2002). Participants' preferences for learning new material varied; some benefited most from visuals, others from the textbook, and still others from questioning and if-then diagrams. Students articulated that they came to understand their learning methods through experiences in different math classes, from error analysis activities, and when action plans failed and they needed to rethink their study strategies. Thus, the data revealed that students were able to articulate their own abilities and preferences. As previously noted, the learning environment gave students the freedom to choose their own solution pathways and study methods. Yet the data indicated that few students are mapping out concepts to review and the majority are making connections between the problem and prior content to solve novel problems and doing practice problems to study. In other words, most appear to be using the same strategies, not considering how they best learn or what recent successes or challenges they have experienced. However, some students reflected that they were unsatisfied with their current progress in class and had asked the teacher for guidance in adjusting their action plans.

As students engage in rigorous mathematics or work to understand concepts, they may need to use resources or collaborate with others. Participants demonstrated agency since they were able to self-reflect and regulate when they needed support. No one expressed the need to turn immediately to help; instead, all described attempting problems independently first. As students got stuck and did not find support within their own notes or textbook resources, most of them turned to peers, which was the same student preference in McGee and Pearman II's (2015) study. When asked about these interactions, some participants asked questions about what a concept was or how to do a step, but others explained that these interactions were not as beneficial for their own understanding. Instead, they preferred to explain their current thinking and ask specific questions about why or to understand their peers' perspectives or strategies. Thus, the resources that students were seeking and using were not answer-driven but collaboration to continue their mathematical investigations.

Lastly, authority is connected to agency since students decide their own actions, direction, and support. When tasked with a challenging problem, few planned to complete the problem step by step as they were taught. Instead, participants talked about coming up with their own solutions by using what they knew, connections they saw, and different approaches. When asked about their study methods, students acknowledged that the state, district, or teachers determine standards and curriculum, but the majority of the interviewed students said they decided what to practice or how to study. Thus, they felt ownership for deciding solution pathways and study practices to improve their learning.

Implications

This study holds implications for teachers, school administrators, instructional coaches, teacher preparation professionals, policy makers, and educational researchers who influence the education of secondary math students.

Implications for Teachers

To support students' math learning and development of their math identities, teachers are encouraged to understand students' math identities, create collaborative classroom environments that engage students in doing mathematics, and give students the responsibility to take action.

Students' math identities are deeply rooted in emotions, as evidenced by the energy and passion interviewed participants displayed when describing their experiences in and beliefs towards mathematics. A first step may be listening to and understanding students' math journeys to make sense of students' positive and negative experiences. This is important because identity is dynamic, changing over time and by situation, and even though students may not identify with being mathematicians or "math people," they are capable of doing mathematics. Aguirre et al. (2013) recommend that teachers affirm students' math identities because perceptions of parents and teachers influenced students' academic competence and performance in math (Martin, 2006). For students with very negative math identities, teachers may give individual attention to understand factors that affect these identities and provide multiple opportunities to learn and experience success in math. Participating students suggested that they would benefit from teachers who show empathy towards their needs and struggles as well as teachers who care about their success in mathematics, overall wellbeing, and future.

Traditional math instruction is often void of the discipline of mathematics because it does not teach students how mathematicians do mathematics (Grootenboer, 2013). Researchers warn of a similar situation when teaching and using metacognition; it must be embedded within content so that it is not generic (Bransford et al., 2000). Therefore, researchers (Aguirre et al., 2013; Schoenfeld, 2013) recommend that meaningful math learning instruction engage students in practicing mathematics and making sense of the content to become powerful thinkers and problem solvers. An environment conducive to doing mathematics facilitates collaboration, values students' voices, and embraces mistakes. Students have a variety of skills and knowledge, so teachers are encouraged to create a collaborative culture in which individuals are challenged to think flexibly and supported to transform informal knowledge and skills to strong conceptual understanding (Bruer, 1993). To do this, teachers may use formative assessment data to make instructional decisions, offer practice choices for content or peer interactions, or emphasize growth and perseverance instead of academic grades. Teachers' support and guidance can also facilitate students' understanding of their own strengths and weaknesses.

Once students can differentiate their strengths from their weaknesses, teachers can give them responsibility for taking actions of their own learning, making changes to their study methods, and asking for specific supports aligned to their needs. However, some may need explicit instruction on how to take initiative after an absence or when falling behind during a class period. By explicitly teaching students to take responsibility and monitoring their use of these strategies, students become the decision makers, determining how, what, and when to learn.

Implications for School Administrators, Instructional Coaches, and Teacher Preparation Professionals

The instructional changes described above are not quick fixes; they require effective professional development focused on mathematics procedural and conceptual understanding, implementation of instructional routines, and adjustments to ensure student learning. Instead of stand-alone professional development, researchers recommend job-embedded professional development that is ongoing, within the school day, and tightly connected to the daily work of teachers (Borman, Feger, & Kawakami, 2006; Killion, Harrison, Bryan, & Clifton, 2012). It has been found that a teacher's greatest struggle is not in learning a new instructional practice but in implementing it; this challenge is often referred to as the "implementation dip" (Fullan, 2001). One form of professional development that meets these criteria is coaching, which supports teachers with content and data-analysis to plan instruction for their students as well as reflect on their instruction to determine next steps for improving teaching practices and increasing student learning. Thus, coaching "fosters meaningful, personalized, professional growth opportunities for staff; increases the influence of exemplary teaching; and magnifies the collective propensity of schools to be able to provide responsive, high-quality learning experiences to ensure that every student succeeds" (Robbins, 2015, p. 8).

When teachers learn a new idea, their exposure is active and collaborative because they are engaged through varied approaches as they make sense of a new practice within their school context. Exposure specific to teachers' academic discipline for middle school and high school teachers allows them to make direct connections to their daily work with students. When teachers attempt to implement a change in classroom practice, coaches provide meaningful, timely formative feedback (Kanold, 2016) and opportunities to learn from other colleagues' modeling. Finally, effective professional development is connected to school initiatives and encourages strong relationships between colleagues within a culture of trust.

Similar professional learning components can be built within teacher preparation programs and taught by teacher preparation professionals instead of waiting until teacher candidates finish their degrees to then retrain them to develop students' math identities. Programs taught by teacher preparation professionals can focus on the same topics as in-service professional development: mathematics procedural and conceptual understanding, implementation of instructional routines, and adjustments to ensure student learning. Teacher candidates will also benefit from a coach's timely formative feedback during fieldwork.

Implications for Policy Development

It is a complicated process to change policies that affect classroom instruction or professional development facilitated by administrators and coaches. The process of change includes the following stages of innovation: initiation, implementation, and continuation (Fullan, 2001). During the initiation stage, Fullan (2001) recommends reviewing the "existence and quality of innovations, access to innovations, advocacy from central administration, teacher advocacy, and external change agents" (p. 200). Therefore, it is important to have advocacy from all stakeholders and understand instructional and professional development options. Policies that emphasize achievement will continue to bolster students' fears of failing in an already ability-focused environment whereas policies that support stakeholders in creating environments for students to do mathematics, learn from mistakes, and grow their math knowledge can develop students' positive math identities.

To plan for innovation, those involved might consider relevance, or the practicality and need for change; readiness, or the capacity and need for change; and the availability of resources. For example, stakeholders might consider current evidence of students growing their math knowledge, developing positive math identities, and becoming problem solvers and self-regulated learners–and how instruction is influencing this learning. This creates a need by showing there is a gap between what students are learning and teachers' current instruction. Besides the need, policymakers might also consider the support teachers require to implement new instructional practices and the capacity, time, and money for providing professional development (i.e., coaching).

For the implementation stage, it is important to reflect on the following factors: characteristics of change to each stakeholder involved in the policy; local characteristics and context; and external factors, such as local and federal government and other agencies (Fullan, 2001). It may also be necessary to identify which kind of problem is occurring: technical, which will need targeted re-training; political, which will require more power/people on board or to minimize distractions; or cultural, which will demand more positive energy around the idea and/or alignment to values/ideologies. A policy to support teachers in developing students' positive math identities will address a technical problem since professional development will be critical (Yow, 2010) as well as a cultural problem since teachers and administrators will need to shift their thinking about instruction. The problem may also be political, and thus, communication among parents, teachers, principals, and district leaders will be essential from the birth of the policy.

For the continuation stage, sustaining change will rely on the organization's ability to adapt internally to external changes. Robertson and Choi (2010) describe organizations that do this by (1) adopting a stakeholder approach; (2) moving toward a team-based design; (3) empowering employees; and (4) facilitating continuous improvement and organizational learning. At a school site, a core group of teachers may be active participants in adapting instruction to develop students' positive math identities.

Ideally, the policy will motivate all site teachers to continually improve their instruction and support students' positive math identities. A next step may be to look beyond sites and consider system-wide policies to benefit both younger and older students on their mathematics journeys.

Implications for Math Education Research

Mathematics identity is currently a popular topic in math education research. Boaler continues to research students' relationships with mathematics and create resources to support students in growing their math knowledge, Schoenfeld and the Teaching for Robust Understanding Project's (2016) TRU Math framework is in alpha form for classroom use, and NCTM and other national math organizations provide recommendations around equity and access that incorporate math identity. This mixed methods study moved beyond researching students' math identities and achievement (i.e., achievement gap) to understand how students' math identities are developed and connected to practices they engage in to learn mathematics (e.g., problem solving and self-regulation). This study aimed to add to research on math identity by comparing the experiences of students with positive and negative math identities and partially fill a need for mixed methods studies about math identity. Findings revealed that even though students articulated more positive or negative math identities in the quantitative results, a variety of students were engaging in mathematical practices in the classroom. Their views on math identity were based mostly on ability and interest instead of how they were learning mathematics.

Knowing that identity is dynamic, the mixed methods design compared survey and interview results to fully comprehend students' math identities, benefitting from the strengths of both qualitative and quantitative research. Although this study was nonexperimental and did not plan to conclude any causes and effects, it did provide insights about how students viewed math experiences as well as their relationships with their teachers and engagement in classroom instruction. Students have a wealth of information to share about creating meaningful math experiences that engage, inspire and challenge them, and we can learn a lot from listening. I encourage more mixed methods studies about math identity and studies that incorporate students' powerful voices.

Limitations and Recommendations for Future Research

In Chapter III, I provided the research design limitations, including that the nonexperimental design study meant it was impossible to claim causation, the survey covered only three of many areas of students' academic behaviors and perceptions, and data were collected at one point in time. During the data collection and analysis, other limitations surfaced.

First, there is a need for a larger sample size in future studies. After extensive recruitment efforts, this study drew on the math experiences of only 28 students for the survey and 10 students for the interviews. These students were from a mid-sized, urban, ethnically diverse K-12 school district on the West Coast, so even though context may be common for other communities, the results cannot be widely generalized. Second, the data collection took place over the three-week long duration of the study, and thus, this study provided a snapshot of students' math experiences. Although I collected all the data following the research design, a second iteration of data collection at the end of the year would have added depth to the data analysis. I may have seen shifts in mathematics identities and student learning that were not apparent after only one semester with a teacher. Third, this study has not been replicated and therefore serves as a pilot study. Replication might occur during the next school year with the same teachers and allow me to compare findings and better understand students' math learning experiences.

Besides adjusting the research design to address these three limitations above, I recommend that the interview protocol be expanded to determine if students' perceived problem solving practices and their perceived self-regulated learning strategies are associated. As noted in the conclusions, the quantitative and qualitative data triangulated to support the conclusion that secondary students' math identities were independent from their perceived strategies. However, the qualitative interview responses did not provide enough data about the association between perceived problem solving practices and their perceived self-regulated learning strategies. A possible interview question might be: When you are studying for mathematics, how do you use the problem solving strategies you described? Provide specific examples.

Another recommendation for future research is to add data from observations, documents of students' problem solving and self-regulation, or interviews of the teachers to the current research design. This may help authenticate some of my preliminary interpretations of students' mathematics identities; get a better sense of the classroom environments, interactions, and engagement; and visualize how students are making informed decisions and changes to their behavior. This would assist me in getting a more complete picture of social cognitive theory components within the classrooms.

Third, I recommend tracking students' experiences for two or more years with two teachers who use similar instructional practices for teaching problem solving and self-regulation and compare these experiences to students who learn from teachers with very different teaching styles. In this study, a few participants articulated challenges with learning new expectations and processes each year whereas others who learned in similar classrooms expressed being able to make more connections and changes in their views towards mathematics.

The final recommendation, which was actually mentioned by a few participants, is to expand the study to more grade levels and compare students' math experiences in elementary, middle, and high schools. Some participating students were able to pinpoint when in their years of school that math came together or became a struggle, and it may be interesting to analyze this throughout the K-12 district system. This study may also involve students as researchers, since participants expressed curiosity in understanding past math experiences, interest in providing valuable insights to support teachers, and hope that future students might benefit from their successes and challenges on their math journeys. Therefore, teachers and students might work together as a community of researchers to study this common problem in math education.

Concluding Remarks

Through this study, I found that the majority of secondary math students viewed math identity as ability and interest. Yet students' math experiences influenced their beliefs about themselves and mathematics, their engagement in mathematical practices, and their feeling of belonging in a community of mathematicians in a variety ways. In looking specifically at their problem solving and self-regulated learning practices, I was able to understand both how they learned mathematics and the impact these experiences had on their abilities and interest.

The goal of this small mixed methods study was not to make generalizable conclusions regarding the relationship between mathematics identities, problem solving practices, and self-regulation strategies of secondary students, but to analyze themes of students' beliefs about, engagement in, and learning of mathematics and interpret findings based students' past and present mathematics experiences. Besides making connections and improvements to my own math instructional knowledge and practice, I hope these results will give insights to this study's readers who are interested in furthering their own teaching to promote students' math identities or in studying students' math identities. By providing details about the research setting and context, readers may make their own meaning according to how relevant the study is to their situations.

This mixed methods study benefited from advantages of quantitative and qualitative research designs to obtain a fuller picture of students' math experiences. Yet it is important to remember that learning and identity development are dynamic, and this study looked at data and results from one moment in time. As students learn more mathematics, continue in their math education with new courses and teachers, and experience mathematics outside the classroom, their math identities will evolve from those described and analyzed in this study. Participating students explained that their mathematics identities changed after successes and failures on tasks, in classes, and over a school year. Even within the three weeks of this study, I saw some differences in students' math identities as initially reported on the survey and as described later during interview conversations. Acknowledging students' shifting math identities does not mean that we cannot take action on the study's results. This knowledge can be used for further research as well as improvements in classroom instruction to build students' math identities and improve their math learning experiences. My hope is that more mixed methods studies are done in the future to enhance our understanding of students' development of their math identities and that the results of this study shed light on how math identity is related to pedagogical practices, such as teaching and engaging students in problem solving and self-regulation, and can ultimately improve the effects of these practices on students' math learning, growth, and success.

APPENDIX A

INVITATION: SCRIPT TO PARTICIPATE IN RESEARCH

Project Title: The Relationship Between Secondary Students' Mathematics Identities, Problem Solving, and Self-regulation
Researcher: Katie Laskasky
Faculty Sponsor: Dr. James Breunlin

Introduction:
I am a doctoral student at Loyola University Chicago and, with my faculty sponsor Dr. James Breunlin, am leading a study on students' mathematics identities. As a high school mathematics student, you are being asked to participate in a research study about developing students' math identities in relationship to their perceived problem solving practices and self-regulated learning strategies. Participating in this study includes taking an online survey only or taking a survey and participating in an interview.

Procedures:
By participating in this study, you will complete the online survey during class time within the next two weeks. The survey takes approximately 20 minutes. Then based on the results of this survey, you may be asked to participate in a structured interview to explain your survey responses in more detail. You will meet with me for a 45-minute to one-hour interview in person or via Skype. The interview takes place within two weeks after the survey, outside of class time, either during a lunch or before/after school, and in a private, quiet location at the school. Participants will be audio recorded for the interview.

Confidentiality:
To ensure your confidentiality, no personal identifiable information will be used as part of the data analysis or dissemination efforts.

Risks and Benefits:
I anticipate no perceived risks beyond normal classroom activity at your school. Although there are no immediate benefits to you from participation, the study results may provide recommendations to your math teachers about better ways to support you and your peers in learning and succeeding in mathematics.

Voluntary Participation:
Participation in this research is completely voluntary. You may participate, decline, or withdraw from participation without any effect on your status within the classroom or school. You may withdraw from this study **at any time**. To withdraw, please inform your teacher or me.

Do you have any questions?

At this time, I would like you to read over the consent form and ask any questions you may have regarding your participation. If you are 18 years or older, you can give consent to participate in the study. If you are under 18 years old, your parent or guardian must give consent for you and you may give assent.

Consent:
If you (if 18 years or older) or your parent or guardian agrees to your participation in this study, please have your parent or guardian sign and date the provided consent form and return it to your teacher. If you do not wish to participate or your parent or guardian does not wish you to participate in this study, please return the consent form unsigned.

APPENDIX B

INFORMED CONSENT FORM FOR PARENT OR GUARDIAN

Project Title: The Relationship Between Secondary Students' Mathematics Identities, Problem Solving, and Self-regulation
Researcher: Katie Laskasky
Faculty Sponsor: Dr. James Breunlin

Introduction:
Your child, as a high school mathematics student, is being asked to participate in a research study about developing students' math identities in relationship to their perceived problem solving practices and self-regulation strategies. Your child is being asked to participate because as a student, he or she can provide valuable information about experiences in learning mathematics and the formation of identity within a mathematics classroom. Please read this consent form carefully and feel free to ask any questions you may have before you decide whether your child may participate in this study.

Purpose:
The purpose of this study is to explore the relationships among students' math identities, their problem solving practices, and their self-regulated learning strategies. The goal of the study is to gather information from students within your child's school regarding strategies and practices that students use to engage in mathematics. The contributions your child shares are important for this study to generate an accurate understanding of students' math identities.

Procedures:
As a high school mathematics student, your child is being asked to participate in a survey only or a survey and an interview. If you agree for your child to participate in this study, your child will be asked to complete a survey to gather information about students' math identities, problem solving practices, and self-regulated learning strategies. Your child will complete the online survey during class time and data will be anonymous. The survey takes approximately 20 minutes. Reporting of any data will be in aggregate form. Then based on the results of this survey, your child may be asked to participate in a structured interview to explain her or his survey responses in more detail. Your child will meet with me for a 45-minute to one-hour interview in person or via Skype. The interview takes place within two weeks after the survey, outside of class time, either during a lunch or before/after school, and in a private, quiet location at the school. Participants will be audio recorded for the interview. Pseudonyms will be used to report interview data. Should you choose not to sign a consent form, your child's survey data will be eliminated from the study and your child will not be asked to participate in an interview.

Prior to taking the survey and participating in the interview, your child will be asked to give her or his own assent to begin data collection.

Voluntary Participation:
Participation in this research is completely voluntary. Your child may participate, decline, or withdraw from participation without any effect on her or his status within the classroom or school. Your child may withdraw from this study **at any time**. To withdraw, please inform your child's teacher, Katie Laskasky, or Dr. James Breunlin.

Confidentiality:
In this study, every effort will be made not to reveal personally identifiable information in publications based upon this research. To accomplish this, no records will be created or retained that could link your child to personally identifiable descriptions, paraphrases, or quotations. Your

child's actions or things he or she says may be presented without specific reference to your child, reference only by pseudonym, or combined anonymously with the actions and words of other participants. All data related to this study will be destroyed within three years of its completion. Until that time, the data will be stored either in password-protected computer files on secure computers or in locked file drawers. **Only the researchers who have signed an informed consent will have access to this material.**

Risks and Benefits:
Your child's participation in this project should not involve risks beyond those experienced in her or his everyday classroom. Although there are no immediate benefits to your child from participation, the study results may provide recommendations to better support all students in learning and succeeding in mathematics. By identifying factors that influence students' math identities, this study will provide implications for secondary mathematics instruction.

Compensation:
Your child will receive no direct compensation for participation in this research project.

Contacts and Questions:
The Loyola University Chicago Institutional Review Board for the Protection of Human Subjects has approved this study. If you have questions about this research project, please contact Katie Laskasky (klaskasky@luc.edu) or her faculty sponsor Dr. James Breunlin (rbreunl@luc.edu or (312) 915-7747). If you have questions about your child's rights as a research participant, you may contact the Loyola Office of Research Services at (773) 508-2689.

Statement of Consent:
My signature indicates that I have read the consent form for this research project, including information about the risks and benefits of my child's voluntary participation, and all of my questions have been answered to my satisfaction. I voluntarily agree that my child may participate in this study by signing the consent form.

- ☐ I consent for my child to participate in the survey only for this research study.
- ☐ I consent for my child to participate in the survey and interview for this research study.
- ☐ I consent for the interview to be audio recorded.

____ ID Number: ____________
Child's / Participant's Full Name (Printed)

____
Participant's Parent or Guardian Signature Date

____
Researcher Signature Date

____
Faculty Sponsor Signature Date

APPENDIX C

STUDENT SURVEY

Student ID number: ______________________

Period: _____

Math Identity, Problem Solving, & Self-regulated Learning Survey

Dear High School Math Student:

You are invited participate in a research study. The study explores how high school students' math identities are developed and how students problem solve and take responsibility for their own learning. You are being asked to participate because as a math student, you can provide valuable information about your experiences in learning mathematics.

To participate in the study, you may participate in a survey only or a survey and an interview. This survey is taken during class time today. The survey takes approximately 20 minutes. Then based on the results of this survey, you may be asked to participate in a structured interview to explain your survey responses in more detail. You will meet with Ms. Laskasky for a 20-25 minute interview in person or via Skype. The interview takes place within two weeks after the survey, outside of class time, either during a lunch or before/after school, and in a private, quiet location at the school. Participants may be audio recorded for the interview.

Participation is completely voluntary. You may participate, decline, or withdraw without any consequences. You may withdraw from this study at any time during the survey or interview. To withdraw, please inform your teacher or me.

Every effort will be made not to publicly share personally identifiable information, such as your name.

There are no perceived risks beyond normal classroom activity at your school. Although there are no immediate benefits to you from participation, the study results may provide recommendations to your math teachers about better ways to support you and your peers in learning and succeeding in mathematics.

If you have any questions, please ask your teacher before starting the survey.

Statement of Assent: Starting this survey is providing your assent and consent.

Survey Directions

Please answer the survey questions honestly. The survey should take approximately 20 minutes to complete. If you have any questions or want to withdraw while taking the survey, please ask your teacher for help.

Background Information

1. What math class are you currently in?
   - Algebra II
   - Precalculus
   - Precalculus Honors
2. What is your age?
   - 15
   - 16
   - 17
   - 18
3. What grade are you in school?
   - Freshman
   - Sophomore
   - Junior
   - Senior
4. How do you identify?
   - Male
   - Female
   - Transgender
   - Non-binary
   - Other
5. With which group do you identify?
   - Black or African American
   - American Indian or Alaska Native
   - Asian
   - Filipino
   - Hispanic or Latino
   - Native Hawaiian or Other Pacific Islander
   - White
   - Two or More Races
   - Other

Math identity

6. If someone says "I have a positive math identity" or "I am a math person", what do they mean?

7. How well do the following describe the way you think of yourself?

	1 = Exactly me	2	3	4	5 = Not me
1. I am a math person.	O	O	O	O	O
2. I can do math.	O	O	O	O	O
3. I belong within a community of math people.	O	O	O	O	O
4. I am interested in learning more about math.	O	O	O	O	O
5. I enjoy learning math.	O	O	O	O	O
6. I am confident that I can understand math in class.	O	O	O	O	O
7. I am confident that I can understand math outside of class.	O	O	O	O	O
8. I understand concepts I have studied in math.	O	O	O	O	O
9. I can overcome setbacks in math.	O	O	O	O	O
10. My parents/relatives/friends see me as a math person.	O	O	O	O	O
11. My classmates see me as a math person.	O	O	O	O	O
12. My math teacher sees me as a math person.	O	O	O	O	O

Problem Solving

8. Describe how you solve a challenging math problem. How do you think through the solution path?

9. How often do you engage in the following practices when you solve math problems?

	1 = Almost always	2 = Very often	3 = Somewhat often	4 = Not very often	5 = Almost never
1. I think about what formulas, tools, or strategies I have learned that can help me solve the problem.	O	O	O	O	O
2. I try several approaches in finding a solution, and only seek hints if stuck.	O	O	O	O	O
3. I ask myself how the information in the problem is related.	O	O	O	O	O
4. I know when to ask myself if I have solved a similar problem.	O	O	O	O	O
5. I think of several ways to try to solve this problem and select a plan that might work.	O	O	O	O	O
6. I apply a variety of approaches over time, and study previous solution attempts to try a new approach.	O	O	O	O	O
7. I follow the plan to solve the math problem until complete.	O	O	O	O	O
8. I ask myself if there might be an error in my thinking.	O	O	O	O	O

Self-regulated learning

10. What study methods do you use to learn math?

11. How often do you engage in the following practices when you are learning math?

	1 = Almost always	2 = Very often	3 = Somewhat often	4 = Not very often	5 = Almost never
1. I determine the causes of my mistakes and misconceptions to avoid them in the future.	O	O	O	O	O
2. I reflect on the effectiveness of my study methods after an assessment.	O	O	O	O	O
3. I choose and prioritize which concepts I need to study.	O	O	O	O	O
4. I do not study concepts that I have trouble learning.	O	O	O	O	O
5. I choose and prioritize personally effective study methods.	O	O	O	O	O
6. I wait to the last minute to start studying for upcoming math assessments.	O	O	O	O	O
7. I try to see how my notes from math class relate to things I already know.	O	O	O	O	O
8. I set a mathematics learning goal of what I want to accomplish before studying.	O	O	O	O	O
9. I assess my own understanding and progress toward the mathematics learning goals.	O	O	O	O	O
10. I check if my thinking is on the right track for a specific concept.	O	O	O	O	O
11. I teach myself by asking self-questions and adding/adjusting my initial thinking.	O	O	O	O	O

12. I quiz myself to see how much I am learning for a mathematics learning goal.	O	O	O	O	O
13. I avoid asking questions in class about things I don't understand.	O	O	O	O	O
14. I seek to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing how other strategies are derived.	O	O	O	O	O
15. I ask my peers questions about things that confuse me.	O	O	O	O	O
16. I provide feedback to my peers so they can revise their actions.	O	O	O	O	O

12. How do you create a plan to study and learn math? Describe how you take ownership of your learning.

Interview

13. Would you be interested in participating in a 20-25 minute interview?

- O Yes
- O No

14. If you answered **YES** to #13, when are you available for an interview?

	Best time	Second best time	Third best time	Does not work for me
Before school	o	o	o	o
During lunch	o	o	o	o
After school	o	o	o	o

Survey Complete

Thank you again for taking this survey! Good luck with the rest of your semester!

APPENDIX D

STRUCTURED INTERVIEW PROTOCOL

Prior to starting the interview, researcher asks for participant's assent and consent:

You are participating in a research study. The study explores how high school students' math identities are developed and how students problem solve and take responsibility for their own learning. You are being asked to participate because as a math student, you can provide valuable information about your experiences in learning mathematics.

To participate in the study, you took a survey and are now being asked to participate in a structured interview to explain your survey responses in more detail. You will meet with me for a 20-25 minute interview today in a private, quiet location at your school. Interviews will be audio recorded.

Participation is completely voluntary. You may participate, decline, or withdraw without any consequences. You may withdraw from this study **at any time** during the interview. To withdraw, please inform me now.

Every effort will be made not to publicly share personally identifiable information, such as your name.

There are no perceived risks beyond normal classroom activity at your school. Although there are no immediate benefits to you from participation, the study results may provide recommendations to your math teachers about better ways to support you and your peers in learning and succeeding in mathematics.

If you have any questions, please ask me before we start the interview. Are you ready to begin the interview? If yes, begin introduction:

Introduction:
"Thank you for agreeing to participate in this interview. The goal of this interview is to talk to you about your experiences in mathematics. The interview is expected to take between 45 and 60 minutes. You do not have to answer any questions that make you feel uncomfortable.

Is it okay if I record our discussion? *[If yes, turn on microphone and repeat the question so it is recorded]*

Statement of Assent:
Do you provide your assent and consent to participate in this interview? Please say yes or no.

When I transcribe this interview, meaning type up the audio recording with your responses, I will replace your name with a pseudonym. Both the audio file and the transcription will be saved on a password-protected hard drive, only accessible to my faculty sponsor and me.

Do you have any questions before we begin?"

1. What is working/not working for you in learning math this year?
2. Describe typical day in math class.
    a. What did you like and dislike about the math lessons, cite particularly good and bad examples. (Boaler, 2000)
    b. How do you interact with your peers and the teacher?
    c. Compare your current experiences in math with experiences in previous years.
3. When faced with a difficult math problem, what has helped you work through the problem (make sense of math and persevere)

Self-regulation:

4. You are preparing for the final. What is your action plan? Is there anything different about this plan compared to previous actions?
    a. How well are your study methods working? What changes should you make, if any?
    b. Who makes the decisions when you learn?

Problem Solving:

1. When you encounter new mathematical problems that you have not seen before, what is your approach? How do you do to solve the problem? (Boaler, 2003)
    a. Has this changed over the semester?
    b. Do you like solving problems? [math identity]
    c. Do you believe you can solve challenging problems? [math identity]

Math Identity:

2. On the survey, you considered yourself (a math person/not a math person).
    a. Can you explain your response?

"That's all I have for now. Do you have any questions for me?

Is it all right if I follow up with you if I have any questions about what we talked about today? Thank you for taking the time to talk with me, and good luck in your class."

APPENDIX E

GRAPHIC ORGANIZER

Research Questions	RQ1: What is the relationship between secondary students' math identities, their perceived problem solving practices, and their perceived self-regulated learning strategies? What is the relationship between problem solving, self-regulation, and math identity given gender?	RQ2: How do secondary students articulate their math identities?	RQ3: Does students' articulation of the development of their math identities explain their problem solving practices and self-regulated learning strategies?
Tallies and notes			

APPENDIX F

CODE TREE

Initial Codes	Description	Social Cognitive Theory Components
Math identity		
Development of math identity	How and when	
Described own math identity	Participant's individual identity	
Relationship between math & self	Dispositions towards math	Personal factors
One belongs	Community of learners	Environmental influences
Interest / enjoyment	Positive or negative	
Competence & performance	Natural ability, grades, success/fail	
Recognition	Parents, Friends, Peers, Teachers	
Implications for instruction		
Curriculum	Lessons, standards, grading	
Teacher	Style and relationship with	
Learning		
Typical lesson	New topic and lesson days	
Helps learning	Positive	All
Doesn't help learning	Negative	All
Problem solving		
Types of problems	Well vs. ill-defined	Environmental influences
Solving problems		
Goals	What, how, when	Behaviors
Knowledge & heuristics	Strategies used	Personal factors
Beliefs and orientations	Positive or negative	
Decision making	How and when	Behaviors
Self-regulation		
Metacognition	Reflection of own thinking	Personal factors
Strategies	Self-regulation phases	Behaviors
Motivational beliefs	Who and how	Personal factors
Great quotations		

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VITA

Katie A. Laskasky is the daughter of Cheryl and John Laskasky. She was born on September 15, 1982, and she grew up in the Chicago suburbs with her brother, Michael. She currently resides in Chicago, Illinois with her husband, John; they are expecting a baby this spring. Katie attended K-12 public schools and graduated from the University of Notre Dame in 2005 with a Bachelor of Arts in Management-Consulting and Mathematics. In 2007, Katie earned a Master of Arts degree in Secondary Education–Mathematics from Loyola Marymount University.

Katie has worked in the field of education for the past 13 years. She began her career as a math educator at a Catholic high school in Los Angeles, California before teaching in charter schools in Los Angeles and Chicago. Katie has worked for Loyola Marymount University's Center for Math and Science Teaching and Math Leadership Corps for the past ten years as Clinical Assistant Faculty and a Senior Research Associate and Systems Leadership Specialist. She developed and taught graduate courses in mathematics instruction and leadership, and she has presented at numerous regional, national, and international conferences. Her work in education focuses on building students' math identities, teacher leadership, adult professional learning, and complex problem solving to support systems change.

As a consultant, Katie provides professional development in mathematics and, through Loyola's Center for Science and Math Education, evaluates education programs.

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