A Study of Two Methods of Teaching Problem-Solving in Arithmetic

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A STUDY OF TWO METHODS OF TEACHING PROBLEM-SOLVING IN ARITHMETIC

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CHAPTER I

THE PROBLEM

The purpose of this study is twofold; first, to show the progress made in scientific studies in the field of problem-solving in arithmetic by a summary of the more important investigations; secondly, to carry out a controlled experiment which will attempt to measure the relative merits of two methods of teaching in their effects on problem-solving ability.

The aim of the experimental study is to determine whether pupils become more efficient in solving verbal problems if the mechanics of arithmetic are applied to practical problems from the beginning of the learning period, or if better results are secured when practical application is postponed until mastery of the mechanics is attained.

The experiment has been delimited to the study of one phase of sixth-grade arithmetic in order to control more perfectly the factors that might exert a disturbing influence. Case II of percentage, what percent one number is of another, was chosen because it was an entirely new problem for the pupils and because it appeared to be sufficiently difficult and clear-cut to provide suitable material for an experiment of this type.

The experiment was conducted in the sixth grade of a public school in Chicago, Illinois. The school was situated in a typically American
section of the city, and the pupils have the advantage of very good home environment. There were no widely scattered types in the school.

The first step in the study was to test the pupils of the grade and divide them into two groups with the average of each as nearly equal as possible in chronological age, mental age, ability in arithmetic fundamentals, and problem-solving ability. Standardized tests designed for these specific purposes were used in measuring all abilities except that of problem-solving. The problem-solving test consisted of ten problems selected on the basis of frequency of use in several modern arithmetic textbooks. An effort was made to equate the groups in reasoning ability, but that was not possible. However, the scores of the initial and final tests in reasoning were compared at the end of the experiment. Tables I and II show how the groups compared at the beginning of the experimental period.

The second step of the study was the actual teaching procedure. The study was a group experiment set up with the aim of using scientific procedure as much as possible and yet of keeping the teaching on a practicable basis. It was not an elaborate study, for it was limited as to time and number of cases. It aimed to measure the merits of the two methods of teaching procedure. The method used for the experimental group was based upon the theory that pupils will learn better to apply the mechanics of an arithmetic process to practical problems if such application is made from the beginning of the learning period. The method used for the control group, Method II, was based upon the assumption that pupils will best learn to apply the mechanics of a new process if such application is
delayed until proficiency is attained in the mechanics. The latter method
is typical of the procedure used throughout the school wherein the
experiment was conducted. In fact, it is the same as that employed by
many teachers of arithmetic. The technique appears to have been based
upon the organization of the most widely used arithmetic texts, which
advocate a great deal of practice and drill in the mechanics of each new
process followed by the solution of a list of "applications" or "exercises."
The actual classroom procedure for each method will be explained fully
in Chapter III.

Throughout the experimental period an effort was made to keep all
factors identical for both groups except the differences in methods which
constituted the experiment. Each group devoted the same amount of time
to the work; the same teacher taught both groups no out-of-school time
was devoted to practice; and physical factors were controlled as far as
is possible in a public school. For the time being, the regular course
of study in arithmetic was set aside for the experiment.

The third and last step in the study was to measure the amount of
gain made by each group and, on the basis of the results obtained, to
determine the relative merits of the two methods employed. The actual
findings will be discussed in Chapter IV.
CHAPTER II

PROBLEM SOLVING IN ARITHMETIC

During the past fifteen years there have been numerous investigations relating to problem-solving in arithmetic. These investigations have arisen from the fact that teachers have become more and more aware of the unusual difficulties which pupils encounter in this phase of arithmetic work. They have become aware also that pupils achieve less satisfactory results in this connection than in abstract work or example-solving. Test results have shown that pupils who gain a comparatively fair rate of speed and accuracy in the latter do not always succeed equally well in problem-solving. In other words, success in the fundamentals is no absolute guarantee of satisfactory results in arithmetical situations involving problem-solving ability.

It has been suggested that problem-solving presents new and different types of habitual reactions which require scientific methods of drill and instruction. This probably would account for the fact that in the testing of arithmetical abilities it is usual to discriminate between "mechanical arithmetic" or example-solving and "arithmetical reasoning" as found in problem-solving. In this study example-solving will be defined as the manipulation of figures or the use of an indicated arithmetical procedure while problem-solving will be understood to mean a process of reasoning which may involve mechanical manipulation but whose primary purpose is to
develop modes of thought.

The relative importance of example-solving and problem-solving is revealed through an examination of the opinions of numerous authorities as expressed in textbooks on arithmetic, textbooks on methods of teaching, and articles in periodicals. An examination of such sources reveals a remarkable agreement as to the importance of the two phases of the subject. Practically all include both phases in their aims for teaching the subject. The aim as stated by Overman (70) is typical of that given by most authorities:

The first social aim of instruction in arithmetic is to give the pupils a mechanical, automatic mastery of the fundamental facts and processes (70:11).

The second social aim of instruction in arithmetic is to develop in the pupils the ability to grasp, interpret, and master the simple arithmetical situations that are of common occurrence in life (70:12).

The third social aim of instruction in arithmetic is to so teach that the pupils develop their inborn power to think, form a habit of thinking things out for themselves and of verifying their conclusions, and form a just estimate of the usefulness of thinking (70:13).

D. H. A. Greene (29:13) sums up the situation in the less elaborate, yet fully explanatory statement that

Teachers of arithmetic are coming to consider that their two most important teaching tasks are that of increasing the skill with which their pupils use the fundamentals, and that of increasing their knowledge of when to use them.

Many educators would give primary importance to the mechanical phases, if one were to judge by the space devoted to them in the textbooks on method
Yet, it appears that the fundamentals, as such, are subordinate to the real problem in arithmetic. The division which has been set up between the so-called fundamentals and problem-solving does not seem justifiable. Usually when the term "fundamentals" is mentioned it refers to the abstract processes of addition, subtraction, multiplication, and division. These may be necessary processes, but there is a sense in which they are not "fundamentals." Never in life situations do we add or multiply two numbers except for a specific purpose. Never do we figure anything without having a concrete reference. What is called abstract work or mere figuring takes place only in the classroom. In life the real fundamentals in arithmetic are the problems. With this in mind it seems reasonable to suppose that the abstract work should be taught only when problems and applications are used to give it meaning. As Buckingham (12:358) points out in answer to the question of when to begin problem work:

> It should begin at the beginning and proceed 'pari passu' with the abstract work as long as the subject is taught. There has been a disposition to look upon abstract work as fundamental and problem work as derived. This is not true. Abstract forms and processes are by their very nature generalizations from concrete experience. Problems are fundamental; abstract processes are used in their solution. If, therefore, it is the business of the school to teach the fundamentals first and foremost, it is its business to teach problems first and foremost.

With each new abstract process should be taught the meaning of the procedure. How can this be done if not through a problem approach? After meaning has been given to the process and its usefulness established, the abstract process may be isolated for drill if necessary. Even this drill should be motivated by problem drill at intervals to insure memory of the
conditions under which the mechanics may be applied.

In the light of this discussion of aims it is interesting to see how our present-day aims originated. In the Middle Ages arithmetic teaching was dominated by the scientific scholastic attitude. As long as it was regarded as the science of numbers, the only requisite of a good problem was that it contain the desired number relations. As a result the puzzle type of problem predominated. In an effort to defend the teaching of the many obsolete and useless topics which crept into the course, educators called to their aid the doctrine of formal discipline. This doctrine of mental training satisfied the people for a time, but gradually the protests of the business man and the practical man in all lines of work became so strong that it was overthrown in favor of a new doctrine. This has been called "the doctrine of social usage." Educators of today have come to realize that it is the business of the schools not to teach the science of arithmetic as such to the pupils, but rather, through the teaching of arithmetic, to prepare the pupils for life. This attempt to prepare the pupil for adult life should not rely completely upon the setting up of habits or reactions to meet certain known conditions. No one knows what situations this changing world will present to the adults of twenty years hence. Therefore, the pupils' training must be generalized. He must be trained to think for himself, to have the correct attitude toward unfamiliar situations which will lead him to seek for known facts in these situations and from them to arrive at conclusions. His study of mathematics should develop in him a sense of orderliness and exactness which will help him to solve life-problems unlike those presented in the class-
room. The change in aim from formal discipline to utilitarian and social values made scientific investigation a necessity.

The mechanical phases of arithmetic were the first to receive attention. A number of studies were made to determine the relative difficulty of the number combinations and their frequency in textbooks. Other investigations endeavored to determine the effect of a period of systematic drill on achievement in arithmetical computation. The results obtained by investigations by Thorndike (86), Brown (7), Burton (13), Kerr (41), and Phillips (72) support the widespread belief that ability to add, subtract, multiply, and divide may be greatly increased by drill.

Another large group of educators have produced evidence in support of the use of learning exercises scientifically constructed as against exercises formulated by the teacher. Knight (45), Newcomb (65), Evans and Knoche (25), Mead and Johnson (54), and Kulp (47) are representative investigators in the field. One result of these and other studies has been the development of drill material to supplement the teaching of arithmetic. These materials have brought about greater efficiency in teaching the mechanical phases of arithmetic.

However, this study does not concern itself with the difficulties in the fundamental operations. It seeks to review the literature in the field of the interpretation or understanding of concrete problems. There is little evidence of any activity in the field of problem-solving before the year 1924. Not until that time do we find any comprehensive scientific study of this phase of arithmetic teaching and learning. Even since that time the number of such studies is comparatively small when it is compared
with the total number of arithmetical investigations. Stretch (85:13) reports that of 584 investigations classed as "quantitative or critical in character" which were reviewed at the close of the year 1929, only 42, or slightly more than seven per cent, dealt mainly with problem-solving.

The first of the studies in this phase concerned themselves largely with the elimination of obsolete problems and the attempt to make verbal problems more practical. Others gave careful consideration to the question of how pupils solve problems. Another group studied the factors which influence pupil performance in problem-solving. The latest experiments were conducted for the purpose of testing different methods in teaching the subject. The remaining sections of this chapter will present an examination of the outstanding studies which have been made in each of these phases.

How Pupils Solve Problems in Arithmetic

In most subjects teachers assign fairly definite tasks to pupils. They expect them to work at details only after some kind of a plan is understood. But, when it comes to the solution of problems in arithmetic, they suddenly change their tactics. They assign problems, but few of them provide any plan or general method of attack. When the results of such a procedure showed failure in a large number of cases, efforts were made to discover, if possible, just what procedure pupils used in solving problems and to utilize this knowledge in revising methods of instruction.

Some writers state dogmatically that problem-solving is purely a guessing process with pupils, especially the younger ones, and that system-
atic attacks are impossible. Thorndike (108:438) appears to support this view when he states:

Mathematical reasoning is successful guessing. To the psychologist it appears that the procedure of all pupils is largely guessing or making an hypothesis until you guess right........

In treating of this subject Knight (44:355) writes:

It is obvious to many that our present conventions relative to problem-solving seriously overestimate the ability of children to indulge in the types of thinking involved.

Bradford (6) conducted an experiment wherein he attempted to discover whether or not pupils employed reasoning in solving problems. He administered tests to several hundred pupils, the problems of which tests were impossible of solution. The extent to which attempts were made to solve such problems was taken to indicate that "arithmetical work is not done in any critical frame."

These conclusions have been substantiated by a more comprehensive investigation by Monroe (57). He made a study of pupils' responses in solving problems for the purpose of discovering the extent to which pupils use reason or apply habitual methods in solving problems. He selected problems from seventh-grade textbooks and constructed four tests arranged as follows:

Test A. The problems were stated in simple terminology, the data relevant, and the setting concrete.

Test B. The problems were stated in technical terminology, the data relevant, and the setting concrete.
Test C. The problems were stated in simple terminology, the data relevant, and the setting abstract.

Test D. The problems were stated in technical terminology, the data irrelevant, and the setting abstract.

He tested 9,256 pupils, representing forty-one cities in Illinois. Most of the pupils were selected from seventh-grade classes and were divided into four experimental groups by means of random sampling. The data which he secured from this investigation led Monroe to conclude that a large percentage of seventh-grade pupils do not reason in attempting to solve arithmetic problems. The responses they make seem to be determined purely by habit. Many appear to perform almost random calculations upon the numbers given. When they do solve a problem correctly, the response seems to be determined by habit. If the problem is stated in the terminology with which they are familiar and if there are no irrelevant data, their response is likely to be correct. On the other hand, if the problem is expressed in unfamiliar terminology or if it is a new type of problem, relatively few pupils appear to attempt to reason. They either do not try to solve it at all or else give an incorrect solution.

A recent study of the difficulties in problem-solving was made by Lenore John (38). The technique used consisted in observation of individual pupils. The following problems were investigated:

1. What are the errors made by pupils in the intermediate grades in solving two-step problems?

2. How do pupils in Grades IV, V, and VI differ in the types of errors which they make?
3. Do pupils from two schools show significant differences in the types of errors which they make? (38:202)

The subjects used were sixty pupils in the University Elementary School of the University of Chicago and in a nearby public school. Half of the number chosen were those who did exceptionally good work in arithmetic; and the other half, those who did very poor work in the same subject.

A detailed report was kept of the observation of the work of each pupil as he solved orally a list of fifteen practical problems. An analysis of these records yielded information regarding the subject's method of reasoning which could not be secured in any other way. The records of the method used by each pupil were studied, and all errors or peculiar methods were tabulated. The errors were divided into four groups: errors in reasoning, errors in fundamentals, errors in reading, and miscellaneous errors.

Results showed that the errors made by the greatest number of pupils were errors in reasoning. From a total of 699 errors made, 383, more than 50 per cent, fell under this division. It is most interesting to note the types of errors which were made most frequently under the classification of "reasoning" errors. Seventy-two errors were due to the use of a wrong process; fifty-four, to disregard of a significant fact which was stated in the problem; fifty-two, to the combination of numbers not directly related; forty-six, to disregard of a fact to be supplied; forty-two, to hesitation in choice of a method of solution; twenty-nine, to use of a
longer method than necessary; and twenty-two, to confusion in method.

Although Miss John draws no such conclusion from her data, they appear to support the views of Monroe, Thorndike, and Bradford that pupils do not employ reasoning to any great extent in solving problems. The question of whether or not this conclusion is correct is by no means settled. The authorities cited appear to agree that reasoning is not used in obtaining answers to verbal problems. This may be due to the fact that it has only recently been recognized that training in arithmetical judgment, analysis, and organization are as necessary as mere mastery of number facts. When all educators and teachers become aware of the importance of this problem, they may become as successful in training the reasoning power of pupils as they are in training the habits or skills needed in the mechanical phases of arithmetic. Pyle (74:328) expresses his view of the type of training needed when he says:

To be a good reasoner in arithmetic a child should have had abundant experience in dealing with arithmetical situations, should have his experience well organized with reference to use, should be trained in analyzing real situations and in reading printed problems, should be trained to be cautious and to find some way to check his conclusions or results, and should have the fundamental operations so well habituated that no thought need be given to them.

Thorndike (88:193), in speaking of reasoning, says:

Reasoning is not a radically different sort of force operating against habit, but the organization and cooperation of many habits ...... Reasoning is not a negation of ordinary bonds, but the action of many of them, especially bonds with subtle elements of the situation. An outside power does not enter to select and criticize; the
pupil's own repertory of bonds relevant to the problem is what selects and rejects.

In the light of these views, does it not seem possible that pupils' inability to use reasoning in solving problems is due to the fact that they have not received the correct training? The teacher's task should be to foresee the child's needs in the early days of problem-solving and to help him to couple his experiences in such ways that the right ideas will come when needed. In order to teach intelligently she must understand the process by which a pupil is doing his work and the difficulties which he encounters.

Again, the real difficulty may not lie with the pupil nor with the type of training he receives, but in the nature of the written problem used as a basis for reasoning work. The usual problem is meaningless and formal and often tends to confuse the pupil instead of aiding him in reasoning.

Factors Which Influence Problem-Solving

Up to the present time no satisfactory results have been attained in the teaching of problem-solving. As has been pointed out in the preceding pages, part of the difficulty lies in the ignorance on the part of teachers of the real difficulties encountered in the subject. They do not know how to go about the task of improving the reasoning power of their pupils. Either they fail to see the real aim of arithmetic teaching and devote all the class time to abstract work and drill, or they proceed to attack the
subject of problem-solving without any systematic gradation of their material. Just as work in the mechanical phases of arithmetic is graded, so should problems be classified and graded. Teachers could use problems more intelligently if they know how difficult they are. This can only be realized by an analysis of the factors which make problem-solving difficult for pupils.

Certain studies have been made pointing out some of the factors which influence problem-solving. One factor which has been shown to affect results greatly in this phase of arithmetic is mental ability. In a discussion of this factor Reed (75:120) shows that speed and accuracy increase with the amount of intelligence. He says:

Intelligence appears to increase the amount of work done per unit of time rather than the rate of improvement, although there are cases when it also increases the latter. It plays a greater part in problems which require reasoning than in computation problems; so great a part, indeed, that in the former it is a matter of great importance to adjust the difficulty of the problems to the mental level of the pupils.

A study by Morton (60:297), in an attempt to determine to what extent problem-solving ability is related to other factors, shows the correlation between problem-solving ability and verbal intelligence to be .78; and between problem-solving ability and non-verbal intelligence .52. The conclusion arrived at was that stupid children cannot solve difficult, complicated problems.

Stevenson (82:96) recognizes the influences of intelligence when he states:
Practically all dull pupils can be taught to solve correctly examples involving fundamentals. Solving verbal problems is a different matter. To read the problem, to find out what is wanted, and to choose the correct process or processes, are abilities which many of the duller pupils do not possess.

He reports a scientific study to support his contention that lack of mentality ranks among the important causes of failure in problem-solving.

In one of the very early studies an outstanding conclusion reached by Bonser (4) was that native intelligence is a determining factor and a fairly important one in arithmetic reasoning. Thorndike (88) attaches much importance to this factor. He concludes his book, *Psychology of Arithmetic*, with this statement:

Finally, it may be noted that ability in arithmetic, though occasionally found in men otherwise very stupid, is usually associated with superior intelligence in dealing with ideas and symbols of all sorts, and is one of the very best early indications thereof.

Osburn and Drennon (68) conducted an experiment to determine the amount of transfer, if any, which takes place from arithmetic problems which are specifically taught to those which are not given particular mention in instruction. The results of the experiment will be explained in detail later, but it is interesting to note that among other significant findings, the data showed that the more intelligent pupils did markedly better work in problem-solving.

These scientific investigations into the factor of intelligence merely reflect the findings and opinions of most writers in the field of mathematics, that mental ability is an influential factor in arithmetical
reasoning. Nearly all studies conducted in the phase of problem-solving give either direct or indirect evidence of the importance of this factor. Aside from that mentioned, there seems to be additional evidence that problem-solving ability requires intelligence because of the frequency with which accepted tests of intelligence include reasoning problems among their questions.

A second factor which has been said to affect pupil performances in solving problems is sex. Bonser's (4) study of the reasoning ability of school children reveals some striking sex differences in the ability to solve problems. It was found that boys are considerably superior to girls. This conclusion has been supported by the scores made on the section of the Army Alpha Test which deals with arithmetic problems. Investigation has shown also that even in classes in which the girls surpass the boys in the fundamental operations of arithmetic, the latter make higher scores on the Buckingham problem test (7:349).

Buswell (15:466) states that such sex differences do exist, and, while they are not large, they are in favor of the boys.

Not all authorities agree with the opinions cited above. Read (75:120) finds that sex differences in arithmetic are rather small. Boys usually make better scores than girls in solving problems, but the differences are not large enough to justify segregation for purposes of instruction. This question is still open for investigation. Sex differences may or may not be an influential factor in solving reasoning problems.
A third factor which influences performances in problem-solving is the degree of skill in the fundamentals. Morton's Table of Correlation (60:297) shows a correlation of .70 between problem-solving ability and arithmetic skill. In another study Morton (59:188) found that 15 percent of the errors in solving problems was due directly to errors in computation. In Miss John's experiment (38), previously referred to, the report of pupils' errors in solving arithmetic problems showed that the group with the second highest frequency was that of errors in fundamentals. Miss John found that a total of 160 pupils, or 24 percent, made errors in this classification. Others would place the figure higher, but this is sufficiently high to show the significance of the factor.

Winch (105:557) obtained results in an experiment to determine the amount of transfer between numerical accuracy and reasoning ability, which led him to make the following statement:

It would seem that some sort of connection may exist between improvement in numerical computation and mathematical reasoning. The results are too irregular to warrant the conclusion of any definite 'transfer,' but the improvement in reasoning may be due either to release of mental energy resulting from improved facility in computation, or to the association established between the two kinds of functions usually at work together . . . . .

Washburne and Osborne (97:303) are of the opinion that lack of facility in the accurate use of the mechanics of arithmetic is a very common source of error in solving problems. Stevenson (82:96) agrees that it is not only a very common source of error, but it also leads pupils to attack problems by peculiar or round-about methods.
Lutes (52) conducted an experiment in the sixth-grade classes of twelve schools in Des Moines, Iowa, to determine the relative value of three different methods of teaching problem-solving. He used the following methods:

1. Improvement of computation by drill in the four fundamental processes.
2. Selection of correct operation from many suggested operations.
3. Selection of correct solution from several given solutions.

The pupils were separated into four equal groups. Groups A, B, and C were each taught by one of the above methods; while the fourth group was taught by the regular classroom method. A preliminary test, Stanford Achievement Test, was used before beginning work. After twelve weeks the test was repeated. Results showed that the group using Method 1, the computation-drill method, made the greatest gain; and the group taught by regular classroom procedure attained second highest results. Lutes concluded that drill in computation does increase ability to solve problems. Studies by Wilson (101) and Osburn (66) support the findings stated above.

Another investigator who holds to the opinion that training in fundamentals affects reasoning ability is Haertter (31:166). His findings led him to conclude:

The individual reasons best who has at his command a very large number of facts and skills and who has used them extensively in a variety of situations......He can improve his reasoning ability, and therefore his ability to solve problems, provided he is equipped with the necessary skills and facts with which to reason, and has had training in organization of these facts and skills.
On the other hand, Courtis (20) and Stone (83) report studies which do not agree with the above conclusions. These studies which were made between 1908 and 1911 proved rather conclusively (1) that arithmetic is not a general ability but a number of special abilities; (2) that there is no correlation between accuracy in the fundamentals and in reasoning ability; and (3) that there is very little correlation between accuracy in the combinations as such and in problems involving the same combinations.

A more recent study has been made by Greene (29), in which he experimented with a group of twenty sixth-grade pupils for a five-week period. His data showed that the increase in skill in the fundamentals from drill on the Courtis Practice Pads did not appreciably affect the reasoning scores of the pupils on the Stone Reasoning Test and the Monroe Reasoning Test. In this experiment the small number of cases limited the conclusions, but Greene felt that the results raise a strong suspicion that other more complicated abilities underlie success in problem-solving.

Despite these opinions to the contrary, two of which were made in the pioneer days of arithmetic investigation, most educators today support the theory that skill in the fundamentals is an important factor in solving reasoning problems, and, if inaccuracy in computation is eliminated, a large increase in scores in problem-solving will result.

In considering the factors which affect pupil performance in the analysis of verbal problems, one very significant factor was found to be the type of problem given or the variations in which the problem was expressed. An exhaustive investigation was conducted by Hydle and Clapp (36), who made a study of eight characteristics of arithmetical problems in an effort to
determine whether or not such characteristics were the cause of difficulty in solving problems. The definite elements which were tested as possible causes of difficulty were:

1. Objective setting
2. Size of numbers
3. Unfamiliar objects
4. Arrangement in series
5. Non-essential elements
6. Visualization versus Experience
7. Project versus problem-form of statement
8. Use of symbolic terms

The results in regard to each were:

1. Out of twenty-five pairs of percentages, twenty-four support the thesis that "the objective setting of a problem is an element of difficulty in its interpretation" (36:28).

2. "Results tend to show that the size of numerical terms is a real element of difficulty in the interpretation of concrete problems" (36:34).

3. Results based upon the testing of 6,412 bear out the supposition that the use of names of unfamiliar objects in a problem increases the difficulty of the problem (36:41).

4. Results from scores on over five thousand papers show that problems presented with others of the same type are easier than when presented with other problems of a different type. The specific difference in errors for problems presented in the two ways ranged from 10.0 to 16.4 (36:48).

5. The difficulty of a problem is materially increased when a non-essential element is included in its statement. However, this difficulty tends to decrease for pupils in the higher grades (36:...
6. No consistent difference was found between the difficulty of the problem with respect to visualization and its difficulty with respect to experience.

7. Of 39 pairs of problems used, results of 32 tend to show that a problem stated in project form is more difficult than one stated in ordinary textbook form.

8. The use of symbolic terms such as "x" or "y" increased the difficulty in problems.

This rather lengthy statement of the findings of Hydle and Clapp points to the fact that variations in the statements of problems are statistically significant causes of difficulty in problem-solving.

Bowman (5) attacked the same problem in a different manner. He tested 564 pupils for the purpose of finding the effect of preference in problem-solving. His results show that pupils of high mental ability perform equally well on any form of problem, whether stated in unfamiliar terminology, or based upon adult or childlike activities, or of the puzzle type, or of the purely computation type. Pupils of lower I.Q. showed a relatively higher degree of performance on the problems of the purely computation type.

Wheat (99:2) conducted a very elaborate experiment to determine the effect of type of problem on pupil performance. His purpose was to determine which type of problem, the conventional or the imaginative, possesses the greater value as an aid in generalizing knowledge of the fundamentals and also to determine the degree of helpfulness of the two types of prob-
lems for pupils of slow, average, and superior attainments.

The results of the study showed that the differences between the totals of responses to the two types of problems were negligible (99:61). This is in agreement with Bowman's results. Wheat's data showed, however, that much less time was needed in solving the conventional type of problem than the imaginative.

Many other investigators, among whom are Washburne and Morphett (96), Klapper (42:271), Morton (60:300), Overman (70:240), and Reed (75:155), support the principle that pupils do best work when working with familiar problems or those stated in familiar terminology. The results of studies as to the effect upon pupil response of the concrete versus the imaginative type of problem do not lead to any such definite conclusion. It remains for further studies to throw light on this phase of problem-solving.

A fifth factor which affects problem-solving is reading ability. As early as 1912, Thorndike (87:293) recognized the relationship between the ability of pupils to comprehend in reading and their ability to solve arithmetic problems. The correlation between the two abilities was found by Morton (60:297) to be .61. It is usually observed that skillful reading precedes a pupil's effort to think out the steps in the solution of a problem. Later, accurate computation may be called to his assistance; but, unless he can read comprehendingly, his efforts will be of little avail. Lessenger (50) believes that reading ability affects not only work in solving problems but in handling the fundamentals as well. He states:
Arithmetic computation, although farther divorced from reading than from the solution of verbal problems, does involve certain specific skills in the field of reading.

That teachers and educators recognize the relation existing between these two abilities is evidenced by the many investigations made of the effectiveness of various reading exercises in improving problem-solving ability. These studies will be discussed in another section of this chapter.

In concluding the examination of the factors found to influence pupil performance in solving reasoning problems, it may be said that intelligence, mechanical ability in arithmetic, the type of problem used, and reading ability have a decided influence. No doubt there are other factors involved which do not lend themselves to observation as readily as do those found. However, with these factors in mind, the textbook-writer and the teacher can go a long way in eliminating causes of difficulty and in gaining more satisfactory results in problem-solving.

Technique for Improving Problem-Solving Ability

The investigations which group themselves around the topic of methods of improving pupils' ability to solve problems in arithmetic are numerous. In this study an attempt will be made to consider the findings of the most significant ones.

For years no attempt was made to give definite instruction in problem-
solving. Teachers felt that if sufficient drill were given in the fundamentals the pupils would recognize and use the correct combinations when met in problems. That such transfer really does occur is still a theory unsupported by scientific evidence. Educators have studied the fundamental operations with respect to the amount of transfer, but no reliable data have been obtained in the case of the amount of transfer, if any, between skill in the fundamentals and in problem-solving. Hamilton (32:139) writes:

A good deal of this problem-solving attitude can come from the right method of handling and thinking about the fundamental operations themselves.

Most authorities agree that skill in the mechanical phases affects the scores on problem-solving tests, but they do not feel that the scores on problem-solving tests, but they do not feel that training in the former will eventually lead pupils to reason correctly.

In studying techniques for improving problem-solving ability nearly all educators appear to agree on the effectiveness of the use of correct type of situations as problems in securing good results. Stone (83:542) writes:

One of the biggest problems that confronts us today in selecting the problems of arithmetic is finding those that shall deal with the pupil's own affairs, with what he is trying to do. .......... One of the most serious errors of the past has been that we have tried to force application relating to adult arithmetic into the first six grades rather than draw upon the activities of childhood.
When real life situations are used in problems, it is possible to secure imagery and better comprehension of the problem and, therefore, better results in solving it. Freeman (28:232) expresses his opinion thus:

Understanding a problem usually involves a clear grasp of concrete objects and relationships. This very frequently means the ability to form an image of certain concrete objects.

Hydle and Clapp (36:11) contend that, if visual imagery is clear and distinct, reflective thinking works at maximum efficiency. But if visual imagery is vague, reflective thinking is impeded. They base their contention on data secured in the experiment previously referred to in this study. Out of twenty-five pairs of percentage problems twenty-four support the thesis that the objective setting of a problem is an element of difficulty in its interpretation. (36:28)

Wilson (102:336) states:

An experience basis is a significant factor, possibly the determining factor, in successful written work. Written problems should be developed in the form of significant units, based upon community contacts. The isolated textbook problem should be abolished, since it leads the child to figure in hundreds or even thousands of unfamiliar situations.

In discussing this topic of the nature of problems, Brueckner (10) advances the idea that the word "problem" is a misnomer when applied to some of the verbal exercises in arithmetic which are given the pupil to solve. They are often merely statements of certain facts and a question based on these facts. No "felt difficulty" exists in many of these verbal statements; all that is required is that the pupils manipulate figures.
Brueckner suggests that the teacher look about her and discover the many real-life situations which present themselves within the school -- even within the classroom itself.

An amusing account of the lengths to which a class of pupils went when they faced with a real life situation in which they were interested is recounted by Piper (73). She tells of an attempt on her part to enliven the arithmetic lesson by suggesting to the pupils the problem of renting and furnishing an apartment for a given sum of money. Not only did the pupils estimate the cost of furnishings, and so forth, but they canvassed the neighborhood and even made an appointment with the superintendent of an exclusive apartment building for the busy supervisor to look at a very expensive apartment, with a view to renting it.

This is but a single example; yet it illustrates the type of problem which would vitalize the curriculum. If more problems of this type were included in the daily arithmetic lesson, the pupils would be better prepared to meet the economic needs of daily life. Problems which present real-life situations do not only augment the interest but are solved with greater ease and a fuller satisfaction. The writer feels that, while much the greater portion of them should be of the type mentioned, drill on many types of conventional problems is a great aid in the teaching of problem-solving. The use of conventional problems should be so minimized that it does not constitute the greater part of the arithmetic lesson, but it may be justified for drill purposes.

Proceeding upon the supposition that actual instruction in solving problems is necessary, many educators set up experiments to determine, if
possible, just what methods give the most desirable results. Before considering these scientific efforts, it is interesting to see what methods are advocated in the textbooks on methods of teaching arithmetic. An examination of twelve texts revealed that ten of them advocate the teaching of a definite plan of attack. Such a plan of analysis may be briefly outlined as follows:

1. Getting a clear understanding of the problem.
2. Planning the solution.
3. Executing the plan.
4. Checking the results (65:256).

Most of the textbooks advocated a certain amount of drill in problem analysis as one way of securing satisfactory results. This theory has been the subject of scientific study by many educators. An examination of some of the outstanding studies reveals valuable suggestions for the teacher of arithmetic.

A rather lengthy experiment was conducted by Ligda (51), who felt that the four-step or five-step plan of solving problems was not satisfactory. In his experiment he used a systematic method of analysis which had been used for algebra. The problems were taken from the Stone Reasoning Test No. 2. The pupils were instructed to:

1. Read the problem.
2. State a verbal equation.
3. Identify the quantities with the terms used in step 2.
4. Substitute the quantities and do the calculating required.
5. Interpret the result, check, and prove.
It may be noted that in steps two and three of this procedure the pupils were taught to recognize and state concisely the essential thoughts and to reject the irrelevant features of the problem. Instead of using the conventional procedure of analyzing what is given, what is required, etc., they state an equation in words and later substitute the numbers to be used. An example of this equation method is as follows:

**Problem:** A man had $240 and spent $143 for rent. How much money had he left?

**Procedure:**
1. Had less spent equals what is left.
2. Had - Spent equals left.
3. $240 - $143 equals left.

Ligda's use of this method is based upon the assumption that in the early stages of problem-solving "forms" is more important than speed. Ability to find and state concisely essential thoughts and to follow a definite procedure is one that increases with every problem solved. This emphasis on a plan of work is the first step toward mastery. At first it may appear to be cumbersome and slow, but that is true of any new method when compared with one already well known. Furthermore, experience has shown that slowness in learning a process for the first time is not a criterion when judging the merits of a method. Work on the formulation of brief, quantitative statements will eventually lead to increased ability to solve reasoning problems. Ligda claims great gains from the use of his method of procedure.

Adams (1:57) upholds the use of special training in the detailed method of analysis. An experiment conducted by him showed definite gains from such a method, especially in third-grade, where definite instruction in
problem-solving begins.

McNair (53:210) defends the use of a definite plan of work when he says:

Training in thinking results from deciding on the operations to be performed as well as on the order of their performance.

One study shows disagreement with the theory of the use of the conventional-formula plan. A very fine experiment was conducted by Hanna (34), who compared the merits and demerits of three methods of problem-solving. The methods tested were:

1. The dependencies (graphic method, which instructed the pupil to follow a particular thought pattern in each solution.

2. The conventional-formula method, which directed the pupil to follow the four-step plan.

3. The individual method, which permitted the pupil to use any desired method of analysis.

The experiment was conducted with 1,000 children of the fourth and seventh grades from public schools in the city of New York. The pupils of each grade were divided into three experimental groups. After the initial tests, for the purpose of equating groups, each group was instructed in one of the above methods and then given mimeographed sets of practice problems to be used for a six-week practice period.

The results showed that use of conventional-formula method gave the least mean gain (7.7). The statistical results were sufficient to demonstrate a significant difference in favor of the dependencies and the individual methods; the mean gain for both was 11.7. There was practically
little difference in favor of either of these last two methods. When data for each grade were considered separately, Hanna found that there was some promise in the dependencies method when it was used with children who were learning for the first time to solve two-step problems.

In an attempt to determine the extent to which analysis of a problem aids the pupil in solving it, Claude Mitchell (55) carried out the following experiment. He gave to 117 pupils of the seventh and eighth grades Form I of the Arithmetic Reasoning Test of the Public School Achievement Tests by Jacob Orleans. The tests were scored and the difficulty of the various problems determined on the basis of the number of incorrect solutions. The five most difficult problems were then analyzed and constructed into a new test. This newly constructed test included analytical questions on each problem.

The results showed that the pupils in Grade VII raised their average score from 0.45 of a problem on the pretest to 1.26 problems on the test containing analytical questions, a gain of 180 percent. Grade VIII raised the average score from 1.24 to 2.45, a gain of 98 percent.

Although the number of cases in this experiment was small, the large percentage of gain led Mitchell (55:466) to draw this conclusion:

1. Detailed analytical questions on problems seem to aid the pupils in the solution of their problems.

2. Since for many teachers the textbook is the sole guide in the teaching of arithmetic, the results given indicate that more arithmetical analyses of problems would be a valuable addition to textbooks in arithmetic.
Newcomb (65) substantiates the above statement in his study of teaching pupils how to solve problems. Among other factors he discovered that pupils who were supplied with sheets of general information for solving verbal problems achieved better results than pupils who were not so supplied. The pupils of the experimental group, which used sheets of directions, improved 22.3 percent in speed and 5.5 per cent in accuracy; whereas the pupils of the control group improved only 5.1 percent in speed and 2.9 percent in accuracy.

A recent discussion by Haertter (31) registers agreement with the findings above cited that some plan of analysis is necessary for good work in problem-solving. He does not agree, however, that the use of one or two "patterns" or types will help pupils to solve all problems. He believes that there are only a very few distinct types under which all of the problems met in daily life fall. Each type makes use of a body of specific facts and abilities, a knowledge of which is essential to the solution of problems of that type. His method as stated in his article is:

It seems, therefore, a more desirable method of treatment would be to study problems by types, presenting carefully to our pupils the fundamental facts and relationships common to such types. This should be followed by a careful analysis of the problem, after which numerous exercises should be solved to fix the distinctive features of that type of problem. When all the various types have been thus considered, a miscellaneous list should be presented (31:167).

Haertter does not include in his article scientific evidence to support this method, but he gives very definite directions illustrating how it could be carried out.
Buckingham (12:401) is of the opinion that an important step in teaching the solution of problems lies in helping the pupil to see that certain problems involve the same principle, that they may be classified as of the same type. He feels that the means of arriving at the type solution may vary, but that the pupil must be brought to realize that the method applied to one problem may be used in solving many other similar problems. The fundamental idea behind this is transfer, and, in order to facilitate transfer through an appreciation of likenesses and differences, the classification of problems is imperative. Buckingham feels that a great fault, perhaps the greatest, of our program of problem-solving is that each problem is treated as a separate item.

While the above-mentioned writers agree on the use of "type" procedure, one must not be led to feel that the agreement is unanimous. Some writers emphatically disapprove of such a procedure. Lazerte (48:266) concludes from a study of pupils' errors:

> It seems useless to give pupils stereotyped forms of solutions. They do not profit by being drilled in type procedures. They need practice in independent problem-solving. After the independent practice has been obtained, economical forms of solutions should prove advantageous.

An extensive study conducted by Washburne and Osborne shows further disagreement with those who advocate the use of formal analysis and "type" procedure as a method of solving reasoning problems. This investigation had for its purpose a study of three methods of training children to solve problems. A total of 763 pupils from Grades VI and VII and representing 18 different schools were used during the experiment. The pupils were
divided into parallel groups, the average score of each being as nearly equal as possible in mental age, chronological age, and problem-solving ability. The three methods used were:

Method 1. To train the child in the solving of problems by giving him large numbers of problems to solve without any special technique. He was to generalize for himself.

Method 2. To train the child to attack each problem according to a definite plan of analysis.

Method 3. To train the child to see the analogy between difficult written problems and corresponding easy oral problems. This method was called the "analogy" method.

After a six-week period of work with the three groups Washburne found that the pupils who were taught no special technique, but who simply solved many problems, surpassed those who had spent time learning a method of solution. In all cases the pupils made remarkable gains. This seems to indicate clearly that concentrated attention, even for a few weeks, on solving problems by any method brings a rich reward. To train all pupils through a formal method of analysis is less effective, according to Washburne's results, than simply to give them many problems and to help each pupil with any special difficulty that he may encounter. This direct training seems to produce better results than does the indirect training involved in teaching a special technique of analysis.

In a later investigation Washburne (92) studied the effects of teach-
ing the mechanics of a new process and practical applications of the process together. He found that pupils do equally good work whether new processes in arithmetic are associated with practical problems from the beginning of the learning period or whether such application is delayed until the mechanics have been mastered.

Rolker (77) reports good results from the use of specific individual help on difficulties in problem-solving. After a diagnosis was made, various forms of class lessons were used to insure aid for each type of difficulty. For instance, one day the pupils were grouped according to their needs, and specific help was given each group; another day the entire class met to work problems and correct errors in class; on a third day individual pupils worked on assignment sheets according to their needs; and on a fourth day, an oral lesson on problem-solving to determine how the pupils actually worked problems. Miss Rolker's results showed definite gains for all groups.

The majority of the studies of this method of teaching problem-solving contribute evidence in support of the use of systematic training in a definite procedure for attacking problems. They show that pupil performance is greatly aided by the use of definite formal technique, even if it be of the conventional formula plan only. It is, undoubtedly, a logical procedure and may train in reasoning ability. At least, in the beginning it gives the pupil some method of approach to an unknown subject. But there is a danger involved in its use which may be avoided by the skillful teacher. Since the purpose of the analysis is to expose the pupil's thinking, it follows that it should be in the pupil's own words
and should not be formalized. It is true that several lessons in formal procedure are useful and necessary to make clear the "form" of the procedure; but such lessons should be discontinued when it is evident that the "form" is understood by the pupil. Nothing is so deadening to reasoning as insistence on the part of the teacher that a set formal procedure be memorized and used almost word for word by each pupil in analyzing a problem. Analysis is of value only when used to help pupils to solve problems. Much valuable time is lost in drill in formal analysis when the thing that is needed most is practice in solving problems. It is only by solving many problems that proficiency may be acquired. It would seem desirable, therefore, to train pupils in systematic analysis, taking care to use a variety of methods rather than a few stereotyped ones and to meet individual needs as they arise.

John (37:101) very adequately sums up the situation when she says:

The problem of teaching a child to reason in arithmetical situations is, therefore, the problem of giving him an understanding of the processes in terms of the situation in which they are applicable, rather than of developing a special technique of problem-solving involving detailed analysis and formal procedure.

In discussing the factors which influence problem-solving mention was made of the relation of reading ability to ability to solve reasoning problems. Many studies have dealt with the effectiveness of the use of various types of reading exercises in improving problem-solving.

Lessenger (50:291) describes an investigation made in the public schools of Radcliffe, Iowa. In this study emphasis was placed upon instruction in reading alone. A considerable proportion of actual errors in prelimin-
ary tests given was found to be the direct result of misreading the directions for the examples. The results of special remedial work in reading showed that, while 40 percent of the pupils were totally free from reading errors due to faulty reading, the remaining 60 percent were practically cured in nine months by skillful training without special training on the specific reading skills needed in arithmetic.

Other studies give results of definite gains from drill in reading. Miss Wilson (101) gives an account of an experiment by which she found that significant gains were made when thirty-four sixth-grade pupils were drilled in reading such problems as those in the Stone's Reasoning Test. She used the following types of exercises:

1. Estimating the answers and judging absurdities.
2. Asking pupils to restate the sentence using other words than the specific terms underlined. (The underlined words were terms peculiar to arithmetic which might have added to the difficulty of the reading.)
3. A third exercise asked the pupils to read the problem and to indicate the process necessary to its solution.

The use of such exercises as these would be very practical in an arithmetic lesson and might show fine results in improving problem-solving.

Greene (29) conducted an experiment with a small number of sixth-grade pupils to determine the effects of drill in the reading of arithmetical problems. He used three experimental groups and one control group. The experimental groups were drilled ten minutes a day for eight days in
reading problems, selecting and recognizing the process involved in their solution. His results caused him to conclude that it is better to spend even a limited amount of class time in drilling on the selection of the processes necessary to solve a verbal arithmetic problem than to follow normal classroom procedure.

A somewhat similar experiment is reported by Stevenson (82), who used the following types of reading exercises which yielded excellent results, especially with dull pupils:

1. Systematic training in finding the facts pertaining to the problem, in deciding upon the processes to be used, and in finding the approximate answer,

2. Solving problems without numbers,

3. Vocabulary exercises on difficult words,

4. Reading and solving a large variety of problems arising out of immediate life needs.

Robertson (76) reports the results of an investigation which compared the ability of pupils to solve a series of problems read aloud by the teacher with their ability to solve problems of equal difficulty but read by themselves. Forty problems from the Oral Problem Scale were selected for use in the experiment; the odd-numbered problems were used for the test read by the teacher; the even-numbered ones, for that read by the pupils themselves. A third test, the Otis Arithmetic Reasoning Test, Form B, was administered also. The grades tested were the fourth and fifth.

Results showed that the pupils made consistently higher mean scores on the test which they read themselves than they made on the teacher-read
test. The differences in scores between the two tests ranged from 1.88 problems in Grade IV to 4.22 problems in Grade V. These differences were probably due to the fact that, in the one test, the child could re-read the problem if he did not understand it at the first reading, while in the other test there was but one reading by the teacher and no opportunity to get the problem if its meaning was not grasped at the first reading.

Still another experiment, which did not use reading exercises but which measured reading ability, was conducted by Stretch (85). She attempted to increase, by special training, the problem-solving ability of a group of pupils and to determine, by scientific procedure, the extent to which problem-solving ability is related to comprehension in reading. Her results are summarized in the statement:

When students increase in problem-solving ability, they also increase in reading comprehension, though the increase in reading comprehension is not equivalent to the increase in problem-solving ability ....... This gives evidence that special training produces the most significant results in the field of its direct application. (85:43).

In general it may be stated that a marked growth in problem-solving ability has resulted from the use of reading exercises of the type considered above, especially those used for purposes of analysis.

A small number of studies have been reported which deal with the use of practice exercises as a means of teaching and improving ability to solve verbal problems. One such study was made by Rosse (78), who measured the amount of increase in reasoning ability from the use of such material as is found in the Lennes Test and Practice Sheets in Arithmetic.
For the purpose of his experiment he used an experimental group, which employed the Lennes Pads according to directions, and a control group, which made use of the problem work provided by a textbook. The results showed that the control group made very little gain, while the experimental group made a mean gain of 12 percent. When the amount of gain for the lower and upper halves of each group was measured, it was found that the lower half of the control group had gained slightly but the upper half had lost. The experimental group showed gains of 7.5 percent and 14.9 percent for the upper and lower levels respectively. On the basis of these results Rosse (78:213) concluded:

The use of carefully prepared test and practice sheets similar to the ones used may be expected:
1. to increase the reasoning ability of the class;
2. to increase the reasoning efficiency of the class in relation to both chronological age and mental age; and
3. to allow for individual difference in ability.

Kulp (47) attempted to determine which of two types of drill material was more effective in developing skill in computation. Type "A" material consisted of a practice pad of purely abstract computation; type "B", a practice pad, with each sheet providing abstract computation on one side and arithmetic reasoning on the other. After a specific drill period 113 pupils in Grade IV were tested for gain in computation and in reasoning ability. Type "B" material was found to be superior not only in training reasoning ability but in improving ability in computation as well.

Another experimental study in the use of practice material was
reported by C.W. Stone (84:589), the purpose of which was to determine the value of the *Stone Diagnostic and Practice Tests* as a means of improving ability in arithmetic. A survey test was given to equivalent groups of pupils to afford a measure of each pupil's reasoning ability prior to the experiment proper. Diagnostic tests were then given for the purpose of locating more specifically each pupil's difficulties in reasoning. Then the practice tests were used for a limited period of time to provide needed practice on the specific difficulties located by the survey and diagnostic tests.

Stone measured the effects of the diagnostic and practice tests by comparing scores on survey tests before and after experimentation. He measured also the permanency of gains made by giving a survey test one year after the experiment. Transfer was measured in scores from problems on which no practice work had been given. As a result of his study, the writer concluded that the use of the tests named produces greater gains in reasoning ability in arithmetic than does regular classroom work. He found also that the gain in reasoning ability secured by these tests transfers to reasoning demanded by other problems of different content, though of similar nature, and that such transfer is greater in amount than that secured by ordinary classroom procedure. The entire study shows great promise for practice tests and materials if correctly used.

In the field of problem-solving certain studies center around another problem, that of transfer of training. Although the doctrine has been somewhat discredited, there are certain facts about it which cannot be disregarded. It is not only possible but probable that certain habits
and methods of work carry over from one type of problem to those of similar nature. In fact, much reliance is placed upon this assumption in teaching arithmetic. Everyone knows that pupils will have to use applications of arithmetic never encountered in the classroom. Again, in school comparatively simple types of problems and small numbers are used; while, in adult situations, the pupils must meet complex situations often involving large numbers. Their reactions to these situations will depend to a great extent upon transfer.

Despite the possibilities and the need for research in it there have been relatively few scientific studies made in respect to the transfer of training in arithmetic. Most of these deal with the mechanics or the fundamental operations. In this phase of arithmetic teaching definite evidence of transfer has been shown. Beito and Brueckner (3) found that the bonds formed in learning the direct form of addition combinations carry over almost completely to the reverse form. Overman (69), in considering the factors which affect transfer, reports that a large and useful amount of transfer can be obtained under proper methods of teaching. While it is not the purpose of this study to consider experiments made with the mechanics of arithmetic, mention is made of these two studies because they are two of the finest examples of the measurement of transfer in arithmetic.

As has been stated previously, there is no scientific evidence to show that there is any transfer from the learning of the fundamentals to reasoning ability. Skill in the former affects the scores on problem-solving tests, but it has yet to be proven whether training in the funda-
mentals will eventually lead pupils to reason correctly. There is, however, a strong suspicion that in the field of problem-solving itself training in one type of problem may lead to ability to solve other types.

A study of this question was made by Osburn and Drennan (68). The purpose of the study was to discover the amount of transfer which takes place from arithmetic problems which are specifically taught to those which are not given particular mention in instruction. The experiment was conducted in the following manner: A set of problem "cues" in addition, subtraction, multiplication, and division was so arranged as to include the problems which were most representative of the work done in third grade. A "cue" was defined as that part of a problem which was expressed in language; for example,

How much will .......... apples cost at .......... cents each?
One hen has .......... chickens. Another has .......... How many chickens have both hens?

A series of such cues was used in teaching third grade classes for a period of six weeks. At the end of the drill period two examinations were administered; the first consisting of entirely new "cues", with no new difficulties, the second containing new "cues" and added vocabulary difficulties. None of the "cues" in either examination had been taught previously to any of the pupils.

The scores on Test I of this study showed that 70 percent of the pupils received passing grades. When only errors of method were counted, the percentage who received passing marks was more than 80. On Test II the pupils made even better scores. This is surprising when it is noted
that this test contained vocabulary difficulties. Approximately 90 percent attained passing marks. Osburn attributes this to the possibility that the pupils sensed the similarity of Test II to Test I, which had been given the day previous. These results led Osburn (68:127) to conclude that there was a marked amount of transfer. The pupils made substantial marks on problems that had not been taught to them. This transfer was so marked, even in the case of pupils of low intelligence, that he advises teachers to teach a few of the most important types thoroughly and depend upon transfer for the remainder.

Stone (84) reports some evidence of transfer in an experiment to improve reasoning ability in arithmetic. When pupils' ability to solve problems in which they had had no specific teaching was measured, the gains were so apparent that Stone felt that increased ability obtained through the use of specially designed practice exercises transferred to the solution of problems dissimilar to those in the practice exercises.

The results of Overman's (69) study of the factors affecting transfer of training in arithmetic indicated that the effects of instruction and practice on certain types of examples are not confined to those types, but spread in considerable amounts to other related types of examples; and that aid in generalizing an arithmetic process is an effective method of increasing the amount of such transfers. The study showed evidence of an increase in the amount of transfer with an increase in mental age. Although the experiment was conducted only in the fundamentals, it suggests general conclusions which may lead to similar investigations in the field of problem-solving.
Claude Mitchell (56) does not agree with the above conclusions. He devised a test to determine whether problems with numbers or problems without numbers are more readily understood by pupils. On these tests the pupils made higher scores on the lists of specific problems, those with numbers. The differences between the number of problems solved per pupil were marked, ranging from 1.9 problem per pupil in the test of general problems to 3.5 problems per pupil in the specific problem test. There was a correlation of .52 between responses to verbal problems with numbers and verbal problems without numbers. Mitchell (56:596) interpreted his results to mean that the fact that pupils can solve a specific problem does not indicate that they have formed a generalization which they can apply to all other similar problems.

Lazerte (48:264) states that pupils may become expert in solving individual problems without appreciating the generalizations that fit the particular case. Ability to deal with particular situations is not accompanied by ability to recognize and solve general cases. This view coincides with Thorndike's ideas on the subject of transfer. Thorndike tends to emphasize the great variety of bonds involved in arithmetic and calls attention to the necessity of giving each bond separate emphasis. He says that the mind works not only by association but also by dissociation. It is by separating a situation into its elements as well as by putting things together that mental concepts are formed. As he says:

The degree of efficiency shown by persons in any intellectual function is a result chiefly of specific training in it or the elements of it and only slightly of the transfer to it of the effects of training other functions (86:483).
Judd holds to Thorndike's idea of dissociation, but adds that after
the dissociation takes place, the child must apply the dissociated element
to many new situations in order to generalize it. It is only after this
generalized element is used without conscious effort that we can say the
child is able to apply that element to new situations. He disagrees with
the views of Mitchell and Lazerte in regard to generalization. He states:

Fortunately, the mind of man is so
organized that it generalizes. Even if
the curriculum-makers resolve to train
nothing but particular abilities, pupils
will generalize and will continue to do
what the race has done throughout its
history, that is, abstract from particular
situations those aspects which are most
universal. Some children will acquire the
general idea of mathematical exactness no
matter how far curriculum-makers go in
running counter to human history.

The studies cited do not justify the formulation of any general
conclusions with regard to this subject of transfer in problem-solving.
It is one to which will have to be added the results of many more scientif-
ic studies and experiments. Perhaps the dearth of studies up to the
present time in this subject is due to the difficulty of detecting actual
transfer and measuring its spread. A suggestion advanced by Hedrick (35)
may be "the true key to a great part of the theory of transfer." He
believes that there are a great many processes in mathematics which, quite
aside from any facts with which they are commonly associated or from any
definite technical skill, are very real and very important. Examples of
such processes are precise statements and precise reading of statements,
generalization from particular instances to a general idea, and distinction
between necessary and sufficient conditions. These processes, while common to other subjects, are best emphasized and illustrated in mathematics. Such processes and ways of thinking seem to be so much more necessary in human life than single facts and given skills that Hedrick thinks transfer of training seems to be more possible with respect to them than to facts and skills. Training in the use of a process may enable one to apply that process to a different set of activities, while it is problematic whether or not a particular fact may be transferred.

Whether or not the theory of transfer suggested by Hedrick is the correct one, it offers possibilities for future study.

A summary of studies in the field of problem-solving would not be complete without mention of a very extensive piece of work by the Committee of Seven. This committee is composed of members of the Northern Illinois Conference on Supervision who have conducted a series of investigations extending over a period of five years to date and involving the cooperation of 148 cities and many thousands of children. The Committee of Seven proposes to make a thorough study of arithmetic. It prefaced its work by an extensive survey of current practices, dealing with the question of placement of the various topics in arithmetic in 125 school systems in the Middle West. The survey brought out a rather surprising diversity of practice with regard to the grades in which some topics are taught, and an equally surprising uniformity with regard to the grades in which other topics are taught. The most striking difference among school systems was that found in the grade in which arithmetic is first introduced. In some places addition facts were presented in first grade; in others the begin-
ning of teaching was delayed until second grade; and some school systems made no provision for it until third grade. These findings led the Committee to launch an investigation to find out if there was any definite and important gain in children's arithmetical knowledge as a result of beginning formal arithmetic instruction as low as the first grade, or if children would learn more quickly and economically if arithmetic were postponed until third grade.

This investigation necessitated the testing of the arithmetical ability of about five thousand pupils in Grade VI. One third of this number had begun arithmetic in first grade; one third in second grade; the remaining third had not begun introduction to formal arithmetic until third grade. In almost every case the pupils who began arithmetic in first grade made better scores in sixth-grade arithmetic than did those who began in second grade. Likewise, those who began in Grade II made better scores than those whose arithmetic had been postponed until Grade III.

Washburne reports the results of another investigation of the Committee of Seven (92). The work of the Committee for the past year or two has been directed toward finding if there is such a thing as "mental readiness" for learning certain topics in arithmetic. In other words, at what stage in a pupil's development may each arithmetic topic be taught most effectively? From the study many interesting facts were noted. For instance, it was found that long division, which is usually taught so laboriously and with such apparently poor results in fourth grade, would be much more effectively taught in Grade VI or even VII. Short division was found to be misplaced about one and one-half years. The Committee
There is a point in a child's mental growth before which it is not effective to teach a given process in arithmetic and after which that process can be taught reasonably effectively.

It prescribes reorganization of the arithmetic curriculum in respect to pupil readiness for each topic as a cure for the high percentage of pupil failure in arithmetic.

A series of experiments dealing with causes of poor work in problem-solving and with the effectiveness of certain methods of teaching problem-solving has also been made by this same Committee. When the work is completed, the Committee's findings may lead to some needed changes in arithmetic curriculum and methods of teaching.
Summary

In all the experimental studies outlined there has been an attempt to analyze the specific difficulties encountered by pupils in solving reasoning problems and to evaluate various methods of teaching the subject. There seems to be agreement that the most common source of error in solving verbal problems is the inability of pupils to use reasoning power in analyzing the problems and arriving at conclusions. Other very important causes of difficulty are poor mental equipment, lack of skill in the fundamental processes, the use of problems involving unfamiliar settings, lack of skill in reading comprehension, and the absence of actual instruction in problem-solving.

With respect to relative evaluation of comparable methods of teaching pupils to solve problems no definite conclusions can be made because the results of so many of the studies are not dependable. However, most of them offer suggestions which may be used to advantage in teaching problem-solving. Many of them urge specific teaching of problems according to types and the use of formal analysis at least in the early stages of the teaching. A few advocate simply the solving of many problems for attaining proficiency. One point made clear by all is that a conscientious attack on problem-solving, regardless of method, will produce good results in improving pupil performance. It is possible that much of the work in this field of problem-solving lies in future research rather than in the results of past or present attempts.
CHAPTER III

THE EXPERIMENT

Purpose

This experiment proposed to teach two groups of sixth-grade pupils in Case Two of Percentage by two different methods, to measure and analyze the results obtained, and from this analysis to evaluate the relative effectiveness of the two techniques in developing the ability of the pupils to solve verbal problems.

Case Two of Percentage, or the problem of finding what percent one number is of another, was chosen because it presents one of the most difficult problems that the sixth-grade pupil has to solve. Experience has shown that, after many weeks of abstract drill in the mechanics of this phase, pupils became somewhat adept at working examples; but, when concrete problems involving Case Two were presented, the pupils seemed unable to recognize the process involved. No such difficulty attended any other phase of percentage. It was with the hope of obtaining results which would simplify the teaching of the process that this somewhat limited experiment was undertaken.
Equating of Groups

For the purpose of the experiment seventy pupils of the sixth-grade were placed in two groups of thirty-five each, so arranged that the average of one group matched that of the other in (a) mental age, (b) general ability in the fundamentals of arithmetic, (c) problem-solving ability, and (d) general ability in school work as judged by teacher rating. An effort was made to equate the groups in arithmetical reasoning ability, but, as this was not possible, it was decided to measure the amount of gain or loss in this respect for each group at the close of the experiment.

The tests used for the preliminary and final measurements were the following:

a. National Intelligence Test, Scale A, Form I.
b. New Stone Reasoning Test in Arithmetic, Forms A and B.
c. Woody-McCall Mixed Fundamentals, Forms I and II.
d. Two tests in problem-solving consisting of type problems selected from arithmetic texts.

Table I presents the average scores for the two groups in Mental Age, Chronological Age, and I.Q.
TABLE I

Average Mental Age, Chronological Age, and I.Q. for Two Groups of Sixth Grade Pupils

<table>
<thead>
<tr>
<th>Group</th>
<th>Mental Age in Years and Months</th>
<th>Chronological Age in Years and Months</th>
<th>I.Q.</th>
</tr>
</thead>
<tbody>
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<td>11 - 10</td>
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<tr>
<td>II. Experimental</td>
<td>13 - 5</td>
<td>11 - 5</td>
<td>117.5</td>
</tr>
</tbody>
</table>

The average mental age for the control group was 13 years 7 months, and that for the experimental group, 13 years 5 months. The average chronological age for the control group was 11 years 5 months. The average I.Q. for the control group was 114.7; that for the experimental group 117.5.

Table II shows how the two groups compared in reasoning ability, in ability to perform the fundamental operations, and in problem-solving ability at the beginning of the experimental period.
**TABLE II**

Average Scores for Two Groups of Pupils on Initial Tests

<table>
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<tr>
<th>Group</th>
<th>Reasoning</th>
<th>Fundamental Operations</th>
<th>Problem-Solving</th>
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<td>Experimental</td>
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<td>Norms for Grade VI</td>
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</table>

An examination of Table II discloses the facts that the average scores for the control group were as follows:

(a) Reasoning ability, 8.57 problems or 41 percent of the total number of problems;
(b) Fundamental operations, 28.4 problems of 81 percent;
(c) Problem-solving ability, 7.54 problems or 75 percent.

For the experimental group the average scores were the following:

(a) Reasoning ability, 9.63 problems or 46 percent of the total number of problems;
(b) Fundamental operations, 27.7 problems or 79 percent;
(c) Problem-solving ability, 7.74 problems or 77 percent.
When the scores in the fundamental test and the test in reasoning were compared with standard scores for the grade, both groups were found to be above the norms set for the sixth-grade. In reasoning ability the pupils rated as high as the norms set for low seventh grade, i.e., the average score was equal to that of the norms set for pupils who had nine months more of school training. In fundamentals both groups rated as high as the standard score for eighth-grade pupils. However, both groups made poor scores on the problem-solving test, which included only processes with which the pupils were familiar.

The exceptionally high scores in fundamental operations proved what had been suspected in the school for some time - that too much time had been given to drill on the mechanics of arithmetic. This last fact accounts, in a measure, for the low scores on the problem-solving tests. The pupils were more or less reluctant to attack concrete problems dealing with the very same material as the abstract work because they had had so little training in this phase of arithmetic work. The standardized reasoning test, on the other hand, began with problems of such a simple nature that they had solved most of the problems before they allowed their aversion to worded problems to manifest itself.

The individual scores for all pupils on the initial and final tests are recorded in Tables III and IV, which may be seen on pages 56 and 57.
TABLE III

Chronological Age, Mental Age, Intelligence Quotient, and Scores for the Thirty-five Pupils in the Control Group

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TABLE IV

Chronological Age, Mental Age, Intelligence Quotient

and Scores for the Thirty-five Pupils in

the Experimental Group

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<th>Pupil</th>
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<td>-1</td>
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<td>2</td>
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<td>13-6</td>
<td>113</td>
<td>27</td>
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<td>-3</td>
<td>9</td>
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</tr>
</tbody>
</table>
Method

The unit of work chosen for the experiment was the process which is commonly known as Case Two of Percentage, viz., what percent one number is of another.

During the course of the experiment precautions were taken to isolate the experimental factor. Both groups were taught by the same teacher for a forty-minute period daily. The control group met during the first period while the experimental group met during the second period. This procedure was reversed during the latter half of the experiment. Both groups used the same materials of instruction. Both took the same tests: verbal for the experimental group, abstract for the control. All factors were kept as nearly alike as possible except the differences in method which constituted the experiment.

The experimental group was taught by a method which consisted of introducing the new process in its simplest form and each new step thereafter through practical problems. This method was based upon the theory that pupils become more proficient in solving verbal problems if each new fundamental operation is applied to practical situations from the very beginning of the learning period. For three weeks this group was taught the new process in percentage through the use of verbal problems. The procedure of instruction which was followed may be summarized thus:
1. On the first day the teacher read through with the class several problems involving the new process. These problems were worked out on the board and carefully explained to the pupils. An effort was made to call the pupils' attention to the situations involved and to help them to see the importance of learning how to solve the problems. After this presentation, individual members of the class were asked to work and explain similar problems. Formal expression was dispensed with in the solution of the problems, but an effort was made to show the use of the equation whenever possible. This was done in order to overcome the aversion of the class to concrete problems, for it was discovered that this was their main difficulty. To them concrete problems meant a most formal and meaningless series of expressions which, by the time the pupil had finished writing them in the required form, had caused him to lose all interest in the problem. Errors were corrected by the teacher and difficult steps explained. After all questions had been settled, the remainder of the period was devoted to practice on a set of problems of the same degree of difficulty as those explained. No abstract drill was given the class, but individual pupils were given help and drill when it was apparent that they could not proceed without them.

2. On the second day a short review of the first day's work was given. Pupils explained the process to the class. Many new problems were worked out. No attempt was made to proceed to problems of a higher degree of difficulty until all pupils were fairly proficient in working those of the simpler type.
3. As each step of difficulty was mastered, problems involving
greater difficulty were presented and explained in the manner outlined
above.

4. Reviews and tests were given from time to time, verbal problems
being used in every case.

An entirely different method was used for the control group. It was
based on the theory that, during a period involving the learning of a new
process, the pupils can devote their time entirely to drill in the ab­
stract process with little or no problem-solving and yet gain a certain
proficiency in applying the process to concrete problems. The new process
was presented to the pupils by means of a short explanation which showed
how the process could be used and which was intended to arouse interest
in the new work. Without any preliminary problem-solving, the teacher
explained how the abstract process should be performed. Each step in the
work was thoroughly analyzed, and the pupils proceeded in the same manner
as did those of the experimental group save that no concrete applications
were given during the three-week instruction period. The greater part of
each day's time was devoted to individual drill in abstract work.

At the end of the three-week instruction period, both groups devoted
two additional weeks to practice on a set of mixed problems involving all
the fundamental processes with many applications to the specific success
just completed.

Both groups were then tested in (a) the fundamental operations,
(b) problem-solving, and (c) arithmetic reasoning. The tests used were
second forms of those administered at the beginning of the experiment.
Results

The individual scores in both initial and final tests may be seen in Tables III and IV. Table V shows the average scores made by the two groups on initial and final tests. Table VI shows the same scores in percentage form.

TABLE V

Average Scores for Two Groups of Sixth-Grade Pupils on Initial and Final Tests

<table>
<thead>
<tr>
<th>Group</th>
<th>Initial Tests</th>
<th>Final Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Reasoning</td>
<td>Fundamentals</td>
</tr>
<tr>
<td>Control</td>
<td>8.57</td>
<td>28.4</td>
</tr>
<tr>
<td>Experimental</td>
<td>9.63</td>
<td>27.7</td>
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</tbody>
</table>
TABLE VI

Average Percentage Scores for Two Groups of Sixth-Grade Pupils on Initial and Final Tests

<table>
<thead>
<tr>
<th>Groups</th>
<th>Initial Tests</th>
<th>Final Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Reasoning</td>
<td>Fundamentals</td>
</tr>
<tr>
<td>Control</td>
<td>41</td>
<td>81</td>
</tr>
<tr>
<td>Experimental</td>
<td>46</td>
<td>79</td>
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</table>

An analysis of the data in Tables V and VI reveals the fact that neither of the two methods proved to be significantly better than the other. The comparison is as follows:

1. The gains made by both groups in reasoning ability were insignificant. The control group showed a slight loss of 2.3 percent while the experimental group showed practically no gain.

2. In abstract work, or the mechanics of arithmetic, the control group remained at the same level; the experimental group showed a slight gain of 1.9 percent.

3. When tested for ability to solve problems, the control group showed a gain of 1.66 problems or 17 percent gain.
The experimental group gained 16 percent. The gains in both cases were noticeable, but the difference does not show superiority for either method.

A comparison of the mean gains of the scores for both groups may be made from Table VII. The technique employed in measuring the reliability of the differences of the means was that recommended by McCall in his book, How to Experiment in Education.* Briefly stated, it is as follows: The total initial score in each arithmetic process was set opposite the total final score for each individual and an algebraic subtraction made. The differences or gains for each group of pupils thus found were then treated for the mean gain, the standard deviation of the gains, the probable error, and the probable error of the differences of the mean gains.

TABLE VII

Reliability of the Differences of the Mean Gains for Two Groups of Pupils

<table>
<thead>
<tr>
<th>Ability Group</th>
<th>Total No. of Pupils</th>
<th>Total Problems Solved</th>
<th>Total Mean Gain</th>
<th>Total Dev. of Gains</th>
<th>Total Mean Gain*</th>
<th>Total Dev. of Difference of Mean Gains*</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(M₂ - M₁)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(P.E. M₂ - M₁)</td>
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<tr>
<td>Control</td>
<td>35</td>
<td>4</td>
<td>2.2</td>
<td>0.11</td>
<td>0.25</td>
<td>0.55</td>
</tr>
<tr>
<td>Experimental</td>
<td>35</td>
<td>23</td>
<td>3.3</td>
<td>0.66</td>
<td>0.38</td>
<td>0.51</td>
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<td>Control</td>
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<td>-16</td>
<td>2.0</td>
<td>-0.46</td>
<td>0.23</td>
<td>0.57</td>
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<tr>
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<td>4</td>
<td>2.2</td>
<td>0.11</td>
<td>0.37</td>
<td>0.44</td>
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</tr>
<tr>
<td>Control</td>
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<td>1.9</td>
<td>1.77</td>
<td>0.22</td>
<td>-0.07</td>
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<tr>
<td>Experimental</td>
<td>35</td>
<td>58</td>
<td>1.9</td>
<td>1.7</td>
<td>0.22</td>
<td>0.65</td>
</tr>
</tbody>
</table>
An examination of Table VII discloses the following facts:

1. The mean gain for the control group in the fundamental operations was 0.11 problems; that for the experimental group, 0.66 problems. The difference between the mean gains was .55 in favor of the method used with the experimental group.

2. In reasoning ability the control group showed a slight loss, -0.46, while the experimental showed a gain of 0.11. The difference was 0.57 in favor of the method employed with the latter group.

3. There was no difference between the mean gains of the two groups in the problem solving test. However, the importance of the gains made in problem-solving lies not in the difference between the two groups, but in the relatively high scores made by them at the end of the experimental period. The gains for both in this respect were very much higher than the gains made in the other abilities measured. The data reveal a gain of 17 percent for the control group and of 16 percent for the experimental group. The gains might have been found to be greater had the tests in problem-solving been longer.

4. The differences between the gains made by both groups may be regarded as statistically insignificant in every case because of the fact that a statistical constant of any sort is not
significant unless it is at least four times its probable error.

As a final step in the problem, the scores for the lower and upper half of the groups were compared in an effort to determine which method of instruction proved more effective with pupils of high and low I.Q. The pupils were arranged according to I.Q. and the gains for each section computed. Table VIII shows the mean gains for each group.

Table VIII

Mean Gains for Lower and Upper Half of Two Groups of Sixth-Grade Pupils

<table>
<thead>
<tr>
<th>Group</th>
<th>Ability</th>
<th>Reasoning</th>
<th>Fundamentals</th>
<th>Problem-Solving</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Control</td>
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<tr>
<td>A. Upper Half</td>
<td>Mean Gain.............</td>
<td>0.39</td>
<td>-0.11</td>
<td>1.2</td>
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<tr>
<td>B. Lower Half</td>
<td>Mean Gain.............</td>
<td>-0.55</td>
<td>0.35</td>
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<tr>
<td>II. Experimental</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A. Upper Half</td>
<td>Mean Gain.............</td>
<td>0.29</td>
<td>-0.35</td>
<td>0.88</td>
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<tr>
<td>B. Lower Half</td>
<td>Mean Gain.............</td>
<td>0.55</td>
<td>1.61</td>
<td>1.92</td>
</tr>
</tbody>
</table>

* (continued from page 63).
Holzinger, K. Statistical Methods for Students in Education p. 233.
** McCall, W.A., How to Experiment in Education, Chapter VII.
* (page 65).
A comparison of the mean gains in Table VIII showed that all were statistically insignificant except those of the lower half of each group in the fundamental operations. In this respect there was found to be a small but significant difference, $1.26 \pm .27$, in favor of the experimental method.

Although the gains shown in Table VIII are very slight, it would seem that the more consistent gains were made by the lower half of each group regardless of the method used. On the basis of the gains made by the lower half of each group it would seem advisable to use the experimental method with pupils of poor mental ability. For the brighter pupils, the results of the experiment in question indicate that it makes no difference which of the methods is employed.
CHAPTER IV

CONCLUSION

As previously stated, this study was an attempt to evaluate the relative effectiveness of two teaching techniques in a scientific manner. An examination of the statistical results will lead to the following conclusions:

1. For the groups taught, it may be stated that the pupils learn equally well by either method.

2. The pupils learn to apply the mechanics of arithmetic to practical problems whether such application is made from the beginning of the learning period or whether the application is delayed until proficiency in the mechanics of the process is attained.

3. The gains were not sufficient to demonstrate a significant difference in favor of either method.

4. It is apparent, also, from the gains made by both groups in solving problems that great gains may be expected from definitely planned attacks upon and drills in problem-solving.

5. A combination of drill in the abstract processes with drill in solving concrete problems will, undoubtedly, produce good results.
Another fact which the statistics did not reveal but which became apparent to the teacher as the experiment progressed was the changed attitude of the students toward problem-solving. In the case of the experimental group there was a feeling on the part of the pupils which led them to attack problems and to choose the correct method very speedily. By the end of the experimental period, most of the pupils in this group had progressed beyond the stage where they had to select the correct procedure from several "trials" and were evidently able to select the appropriate procedure at once. The abstract process seemed to be associated with the concrete situation in such a way as to make it the immediate response of the pupil to the situation presented. The pupils of the control group, on the other hand, appeared to perform almost random calculations upon many of the problems before the correct solution was obtained. Whether this was due to the fact that solving problems of a specific type developed the general problem-solving ability of the experimental group, or whether it was due to the longer time devoted to problem analysis by that group is a question which may lead to further study. In either case it served to make the experiment worthwhile as far as this particular group was concerned for developing the ability of a class to attack concrete problems is an accomplishment greatly to be desired.

On the other hand, several factors tend to limit the conclusions drawn from the results of this very short experiment. In the first place, the number of cases was too small to give any great reliability. Secondly, the tests used for problem-solving, while carefully selected and objective, were not standardized. Thirdly, a second intelligence examination would
have been of value in equating the groups prior to the experiment. Fourthly, the time devoted to the entire experiment was very limited; an entire year's work taught to equated groups would measure more adequately the differences between the two methods.

Despite these limitations, the ability gained in handling concrete situations and the apparent growth of pupil independence in this respect proved that the experiment was of benefit both to the pupils and to the teacher of the groups involved.
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66. Osburn, W.J. Diagnosis and Remedial Treatment of Errors in Arithmetic Reasoning. State Department of Public Instruction, Madison, Wisconsin, 1922.


The thesis "A Study of Two Methods of Teaching Problem-Solving in Arithmetic," written by Nora Mary Carroll, has been accepted by the Graduate School of Loyola University, with reference to form, and by the readers whose names appear below, with reference to content. It is, therefore, accepted as a partial fulfilment of the requirements of the degree conferred.

Rev. Austin G. Schmidt, S.J. April 26, 1934

James A. Fitzgerald, Ph.D. May 7, 1934