Shifts In Persistence In The Turkish Real Exchange Rates

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1. Introduction

We have investigated the long-run behaviour of Turkish Real Exchange rates (RER) in three previous papers. In Erlat (2003) we made use of unit root tests that accounted for both multiple structural shifts in the deterministic terms and outliers, and found that more than one shift may exist. The series investigated were the RERs based on the German DM and US Exchange rates with the Consumer Price Index (CPI) and the Wholesale Price Index (WPI) used for the price variables.

In Erlat (2004), using the same data, we tested for unit roots against nonlinear stationarity generated by an Exponential Smooth Transition Autoregressive (ESTAR) model and found that, for the CPI-based DM series, unit root tests that took into account multiple structural shifts better explained the persistence in the RERs.

Finally, in Erlat and Ozdemir (2005), we considered treating the problem in terms of a panel of real exchange rates that showed very strong dependence and found that, due to this dependence, using panel approaches to testing for unit roots did not provide us with any new evidence that an RER based on the German DM may provide.

Hence, the findings in Erlat and Ozdemir (2005) lead us to return to the domain of Erlat (2003, 2004) and ask if the shifts observed in these studies may also indicate shifts in the nature of persistence; namely, whether they indicate shifts from I(0) to I(1) or vice versa. Since multiple shifts had been found in Erlat (2003), we used the regression-based method recently developed by Leybourne, Kim and Taylor (2007) as a generalization of Leybourne, Kim, Smith and Newbold (2003). Leybourne et al (2007) apply their approach to the logarithm of the yields on 10 year Government bonds for the UK, the USA, Canada and Australia, while Yoon (2008), in fact, applies it to the US/UK real exchange rate that covers the period 1791 to 1990. In Erlat (2008), we have applied the Leybourne et al (2003), single-shift test to monthly Turkish inflation rates. Shifts in persistence in inflation have also been considered by Chiquiar, Noriega and Ramos-Francia (2008) for Mexico and by Halunga, Osborn and Sensier (2008) for the UK and USA. They, however, use ratio-tests where the null hypothesis is that the series is I(0) for the full period.

In the present paper, we shall start by testing if the RER series have a unit root for the period as a whole. We shall use the $DFGLS$ statistic proposed by Elliott, Rothenberg and Stock (1996) since it is the statistic used by Leybourne et al (2003) and Leybourne et al (2007). We shall also use the test due to Kwiatowski, Phillips, Schmidt and Shin ($KPSS$) (1992), where the null hypothesis is stationarity, for corroboration. We shall next apply the

The plan of the paper is as follows. In the next section we describe the tests mentioned above. We then, briefly, describe the data, which happens to be the same used in Erlat (2003 and 2004). In section four we give the empirical results and, in section five, our conclusions.

2. The Test Statistics

We shall not describe the KPSS test since it is, now, quite well known. The same may also be said for the DGFLS test but since the subsequent tests are all based on it, a description would be useful.

The DFGLS test is based on first estimating

\[ y_t = \beta^* d_t + u_t \]

where \( d_t = 1, \ \beta = \beta_0 \) or \( d_t = (1,t)^\prime, \ \beta = (\beta_0, \beta_1)^\prime \), and \( u_t = \delta u_{t-1} + \epsilon_t, |\delta| < 1 \), by Generalized Least Squares (GLS), which involves regressing

\[ y_t^* = [y_1,y_2 - \bar{\delta} y_1, \ldots, y_T - \bar{\delta} y_{T-1}]^\prime \]

on

\[ d_t^* = [d_1,d_2 - \bar{\delta} d_1, \ldots, d_T - \bar{\delta} d_{T-1}]^\prime \]

We obtain \( \bar{\delta} \) by assuming that \( \delta \) takes on values in the neighborhood \( 1 + (c/T) \) where \( c < 0 \).

The choice of \( c, \bar{c} \), yields \( \bar{\delta} \). The residuals from this estimation, \( \hat{u}_t = y_t - \hat{\beta}_{GLS} d_t \), are used to form the equation, taking autocorrelation in the disturbances into account, as

\[ \Delta \hat{u}_t = \rho \hat{u}_{t-1} + \sum_{i=1}^{p} \gamma_i \Delta \hat{u}_{t-i} + \epsilon_t \]

where \( \rho = \delta - 1 \) and \( \rho = 0 \) is tested. The choice of \( \bar{c} \) differs depending upon the model the DFGLS test is applied to. In the present case, \( \bar{c} = -7.0 \) if there is only an intercept and \( \bar{c} = -13.5 \) if there are both an intercept and a trend term.

The test for a single shift in a time series, Leybourne et al (2003) apply the DFGLS statistic recursively. We may express their model as
The null hypothesis is that the series is I(1) throughout the sample and the alternative hypotheses may be a shift from I(0) to I(1) or from I(1) to I(0). Letting \( \tau = T_B / T \), \( T_B \) being an unknown shift point with \([\cdot]\) indicating the integer part of the argument, in the first case we test

\[
\begin{align*}
H_0 & : \rho = 0, \\
H_{01} & : \rho < 0, \\
\rho &= 0, \\
t & = [\tau T] + 1, \ldots, T
\end{align*}
\]

and, in the second,

\[
\begin{align*}
H_0 & : \rho = 0, \\
H_{10} & : \rho < 0, \\
\rho &= 0, \\
t & = T - [\tau T] - 1, \ldots, 1
\end{align*}
\]

To test \( H_{01} \), equation (1) is estimated recursively by GLS, which implies that the last transformed observations in (2) and (3) will now be \( y_{[\tau T]} - \tilde{\delta} y_{[\tau T]-1} \) and \( d_{[\tau T]} - \tilde{\delta} d_{[\tau T]-1} \), respectively and \( \tilde{\sigma} \) will be taken as -25.0. The residuals from these regressions are used to estimate (4) and recursive t-ratios of \( \rho \), \( t_\rho (\tau) \) are obtained. The test statistic is taken to be the minimum of these t-ratios. If the minimum value exceeds the appropriate critical value, then the switch point will be the \( \tau \)-value that corresponds to this minimum. To test \( H_{10} \) the series is reversed as \( z_t = y_{T-t+1} \) and the procedure described above is applied to the \( z_t \). We shall call the statistic to test \( H_{01} \), \( \text{min DFGLS}^F \) and the statistic to test \( H_{10} \), \( \text{min DFGLS}^R \).

When there is more than one shift in persistence, the coefficient being tested, \( \rho \), will be time-varying and will be denoted by \( \rho_t \). Supposing there are \( m \) shifts, then under the alternative hypothesis changes from I(0) to I(1) imply that \( \rho_t = 0 \) if \( \rho_{t-1} < 0 \) if and changes from I(1) to I(0) imply that \( \rho_t < 0 \) if \( \rho_{t-1} = 0 \).

Suppose \( m = 2 \) and that we want to test if the first shift is from I(1) to I(0) while the second shift is from I(0) to I(1). Leybourne et al (2007) suggest what they call a double-recursive procedure. Instead of a single trimming scalar, \( \tau \), in the single-shift case described
above, we now have two, $\lambda$ and $\tau$. $\lambda$ is assumed to lie in $(0,1)$ while $\tau$ is restricted to lie in $(\lambda, 1]$. Thus, the GLS regressions described by (2) and (3) will not only have the last transformed observations for the single shift case but the first observations will change from $y_1$ and $d_1$ to $y_{[\lambda T]}$ and $d_{[\lambda T]}$ so that the subsequent transformed observations become $y_{[\tau T]} - \delta y_{[\tau T]}$, etc. and $d_{[\tau T]} - \delta d_{[\tau T]}$, etc. respectively. Thus, the single-shift procedure for testing $H_{01}$ is applied recursively to the samples starting from $[\lambda T]$ for a given $\lambda$, the t-ratios of $\rho$ from (7), now denoted as $t_{\rho}(\lambda, \tau)$, are minimized over $\tau$, to yield

\[
\min_dfglsl(\lambda) = \min_{\tau \in (0,1)} t_{\rho}(\lambda, \tau), \quad \lambda \in (0,1)
\]

and then, these $\min dfglsl(\lambda)$ statistics are minimized over $\lambda$ to yield

\[
\min dfglsl(\lambda, \tau) = \min_{\lambda \in (0,1)} \min dfglsl(\lambda)
\]

\[
= \min_{\lambda \in (0,1)} \min_{\tau \in (0,1)} t_{\rho}(\lambda, \tau)
\]

Thus, in the $m=2$ case we are considering, $\lambda$ will be the point where the series shifts from $I(1)$ to $I(0)$ and $\tau$ will be the point where it shifts from $I(0)$ to $I(1)$. $\bar{c}$ will now be taken as -10.0.

When $m=2$ we end up obtaining three subperiods. In the first and last periods the time series is $I(1)$ while, in the middle period, it is $I(0)$. We may continue implementing this procedure to the two $I(1)$ subperiods to see if they contain further subperiods that are $I(0)$.

3. The Data

In order to compare our results with those in Erlat (2003, 2004), we use the same data set as in these two references. It consists of Turkish real exchange rates with the $\$US$ and the German DM. The CPIs and WPIs upon which the RERs are based, were obtained from the International Financial Statistics database in the case of the US and Germany and, in the Turkish case, they were downloaded from the Turkish Central Bank database. The two exchange rate series were also obtained from this database. We denote the natural logs of the resultant four RERs series as LRERUSCPI, LRERUSWPI, LRERDMCPI and LRERDMWPI.

These four series are monthly and cover the period 1984.01-2000.09. The Turkish prices indexes are 1987 based and the US and German indexes have been converted to this
base. No significant seasonality was found in any of the series. The US-based series are plotted in Figure 1 and the DM based series in Figure 2.

**Figure 1**

Plots of CPI and WPI Based Real Exchange Rates with The US

**Figure 2**

Plots of CPI and WPI Based Real Exchange Rates With Germany

4. Empirical Results

Table 1 contains the results of the \textit{DFGLS} tests applied to the period as a whole. We find that the \textit{DFGLS} test indicates a unit root in the DM-based series in models with and
without a trend term. For the US-based series, however, we find some weak evidence of stationarity for LRERUSCPI and LRERUSWPI in both models. The *KPSS* results corroborate these findings for LRERDMCPI for the intercept + trend model and for LRERDMWPI for both models. For the US-based RERs we find that there is corroboration only for LRERUSCPI in the intercept only case.

The strongest evidence of the series being I(1) throughout appears to come from the *KPSS* results. Coupled with the *DFGLS* results, it would be safe to say that the two DM-

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Unit Root Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p</td>
</tr>
<tr>
<td>LRERDMCPI</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>1</td>
</tr>
<tr>
<td>Intercept + Trend</td>
<td>1</td>
</tr>
<tr>
<td>LRERDMWPI</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>1</td>
</tr>
<tr>
<td>Intercept + Trend</td>
<td>1</td>
</tr>
<tr>
<td>LRERUSCPI</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>1</td>
</tr>
<tr>
<td>Intercept + Trend</td>
<td>1</td>
</tr>
<tr>
<td>LRERUSWPI</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>1</td>
</tr>
<tr>
<td>Intercept + Trend</td>
<td>1</td>
</tr>
</tbody>
</table>

Notes: 1. The critical values for the *DFGLS* test are from Elliott, Rothenberg and Stock (1996), Table 1.
2. The critical values for the *KPSS* test are from Kwiatowski et al (1992), Table 1.

Table 2 contains the single-shift-in-persistence results. We find that there are no shifts in persistence for LRERDMCPI, either from I(0) to I(1) or from I(1) to I(0). On the other hand, we find shifts from I(0) to I(1), in both models, for LRERDMWPI in 1987.05
Table 2

\textit{min DFGLS} Tests for a Single Shift in Persistence

<table>
<thead>
<tr>
<th>min DFGLS \textsuperscript{a}</th>
<th>Date</th>
<th>min DFGLS \textsuperscript{R}</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRERDMCPI</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-2.359</td>
<td>-1.622</td>
<td>-</td>
</tr>
<tr>
<td>Intercept + Trend</td>
<td>-2.610</td>
<td>-3.808</td>
<td>-</td>
</tr>
<tr>
<td>LRERDMWPI</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-3.028 \textsuperscript{**}</td>
<td>1987.05</td>
<td>-2.119</td>
</tr>
<tr>
<td>Intercept + Trend</td>
<td>-3.625 \textsuperscript{*}</td>
<td>1988.10</td>
<td>-3.680 \textsuperscript{*}</td>
</tr>
<tr>
<td>LRERUSCPI</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-2.525</td>
<td>-3.345 \textsuperscript{*}</td>
<td>1993.12</td>
</tr>
<tr>
<td>LRERUSWPI</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-2.569</td>
<td>-2.584</td>
<td>-</td>
</tr>
<tr>
<td>Intercept + Trend</td>
<td>-4.743 \textsuperscript{***}</td>
<td>1988.06</td>
<td>-3.364</td>
</tr>
</tbody>
</table>

Notes: The critical values (T = 200) for the \textit{min DFGLS} \textsuperscript{a} and \textit{min DFGLS} \textsuperscript{R} tests are from Leybourne et al (2003), Table 1.

\begin{tabular}{ccccc}
Intercept & 10\% & 5\% & Intercept + Trend & 10\% & 5\% \\
-2.72 & -3.02 & -3.43 & -3.72 \\
\end{tabular}

\ \textsuperscript{*}Significant at 10 percent \ \textsuperscript{**}Significant at 5 percent \ \textsuperscript{***}Significant at 1 percent

(intercept) and 1988.10 (intercept + trend). This series also shows some evidence of a shift from I(1) to I(0) in 1996.08, for the intercept + trend case.

For LRERUSCPI the shifts are apparently from I(1) to I(0) for both models and on the same date, 1993.12 and also from I(0) to I(1) in the intercept + trend case, in 2000.09, which happens to be the end of the period, implying that the series is I(0) for the full sample. In the case of LRERUSWPI, there appears to be no shift from I(1) to I(0) but a definite shift from I(0) to I(1) in the intercept + trend case, in 1988.06.

Finally, turning to Table 3, we find that there appears to be statistically significant multiple shifts in all but two cases; namely, in LRERDMCPI in the intercept + trend model and in LRERUSCPI in the intercept model. The longest I(0) subperiod is found for LRERDMCPI for the intercept model; 1991.02 to 1999.05. For LRERDMWPI, the I(0) period is a single observation in 1987.05 (for intercept only) and this corresponds to a single shift from I(0) to I(1) as was found in Table 2. The I(0) period is a little longer for the intercept + trend model but is still less than a year. Similar I(0) subperiods are also short for LRERUSCPI and LRERUSWPI.
### Table 3

*min DFGLS* Tests for Multiple Shifts in Persistence

<table>
<thead>
<tr>
<th></th>
<th>Intercept</th>
<th>Intercept + Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRERDMCPI</td>
<td>4</td>
<td>1991.02</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-4.504**</td>
<td>1999.05</td>
</tr>
<tr>
<td></td>
<td>-3.915</td>
<td></td>
</tr>
<tr>
<td>LRERDMWPI</td>
<td>13</td>
<td>1987.05</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>1991.06</td>
</tr>
<tr>
<td></td>
<td>-4.223**</td>
<td>1987.05</td>
</tr>
<tr>
<td></td>
<td>-5.280**</td>
<td>1991.09</td>
</tr>
<tr>
<td>LRERUSCPI</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-2.832</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-5.834***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1996.06</td>
<td>1996.11</td>
</tr>
<tr>
<td>LRERUSWPI</td>
<td>10</td>
<td>1993.04</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>1993.12</td>
</tr>
<tr>
<td></td>
<td>-4.102</td>
<td></td>
</tr>
<tr>
<td></td>
<td>-5.246**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1996.06</td>
<td>1998.05</td>
</tr>
</tbody>
</table>

**Notes:** The critical values (T = 200) for the *min DFGLS* test are from Leybourne et al (2007), Table 1.

In order to compare these results with those in Erlat (2003, 2004) we have constructed Table 4 from some of the results in Tables 3 and 4 in Erlat (2003) and Table 2 in Erlat (2004). The model and procedures from which these results are obtained are briefly described in the Appendix.

When structural shifts in the deterministic terms are taken into account based on the Augmented Dickey Fuller (*ADF*) statistic, as in Erlat (2003), we find evidence of stationarity in LRERUSCPI when there is a single shift in the deterministic terms and this is bolstered further when multiple structural shifts are taken into account. Evidence of stationarity is also found for LRERDMCPI and LRERDMWPI when there are multiple shifts. We also note that the shift date for LRERUSCPI is 1994.01, which is simply a month away from the single I(0)-to-I(1) shift in 1993.12. There does not appear to be any other proximity in the dates for the structural shifts and the shifts in persistence.

When the alternative hypothesis is the ESTAR model, as in Erlat (2004), then the nonlinear unit root test based on GLS detrending (*GLS-DT*) of the series indicates that, once again, LRERUSCPI and LRERDMWPI are stationary.
Table 4
Unit Root Test Results with Structural Shifts and Against a Stationary ESTAR Model

<table>
<thead>
<tr>
<th></th>
<th>Single Shift</th>
<th>Multiple Shift</th>
<th>ESTAR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>p  minADF</td>
<td>ττ ττ</td>
<td>p  minADF</td>
</tr>
<tr>
<td>LRERDMCPI</td>
<td>1  -3.660</td>
<td>-</td>
<td>1  -7.621***</td>
</tr>
<tr>
<td>LRERDMWPI</td>
<td>1  -2.905</td>
<td>-</td>
<td>1  -6.231**</td>
</tr>
<tr>
<td>LRERUSCPI</td>
<td>1  -4.897</td>
<td>1994.01</td>
<td>1  -6.730***</td>
</tr>
<tr>
<td>LRERUSWPI</td>
<td>1  -4.793</td>
<td>-</td>
<td>1  -5.893</td>
</tr>
</tbody>
</table>

Notes:
1. This table was compiled from Tables 3 and 4 in Erlat (2003) and Table 2 in Erlat (2004).
2. The critical values for the single structural shift min ADF test is from Zivot and Andrews (1992, Tables 2-4).

<table>
<thead>
<tr>
<th></th>
<th>0.10</th>
<th>0.05</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-4.820</td>
<td>-5.081</td>
<td>-5.570</td>
</tr>
</tbody>
</table>

3. The critical values for the multiple structural shift min ADF test is from Ohara (1999, Table 1).

<table>
<thead>
<tr>
<th></th>
<th>0.10</th>
<th>0.05</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-6.170</td>
<td>-6.400</td>
<td>-6.960</td>
</tr>
</tbody>
</table>

4. There is only one critical value for the GLS-DT test from Kapetanios and Shin (2002).

<table>
<thead>
<tr>
<th></th>
<th>0.05</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-2.930</td>
</tr>
</tbody>
</table>

* Significant at 10 percent  ** Significant at 5 percent  *** Significant at 1 percent

5. Conclusions

1. The unit root tests applied to the full period indicate that the DM-Based series are nonstationary but that the US-based series show some evidence of stationarity. When shifts in the deterministic terms are taken into account, we find that the evidence for stationarity in LRERUSCPI becomes stronger and continues to be so when tested against the ESTAR model. In the case of multiple shifts, one now finds the DM-based series to also show stationarity and that one of them, LRERDMWPI, also exhibits nonlinear stationarity.

2. We obtain further evidence of stationarity for LRERUSCPI from the test of a single shift in persistence from I(0) to I(1) as the shift date is simply the end of the period. Most single shifts in persistence are observed for movements from I(0) to I(1). This is the case for LRERDMWPI and LRERUSWPI, with the latter showing rather strong evidence of such a shift. The shift dates, for the intercept + trend case, are quite close for these two series.
3. In the case of multiple persistence shifts, we observe shifts for all series but the majority of them indicate rather short I(0) subperiods except LRERDMCPI. The shift in LRERDMWPI indicates a single date as the shift period, which practically implies an I(0) to I(1) shift and, as was found using the single shift test, on the same date.

4. Where does this analysis leave us? Taking shifts into account, either in the deterministic terms or in the nature of persistence, does lead to modifications of the results obtained from unit root tests applied to the full period. However, the shift dates in these two types of shifts rarely come close, let alone coincide and, in multiple shifts in persistence, the I(0) subperiods are rather short. Searching for further I(0) periods in the two I(1) periods did not give meaningful results.

References


**Appendix**

The results for the unit root tests involving a single shift in the deterministic terms were obtained from

\[
\Delta y_t = \beta_0 + \beta_1 t + \rho y_{t-1} + \alpha_1 DU_1(\tau) + \delta_1 DT_1(\tau) + \sum_{i=1}^{p} \gamma_i \Delta y_{t-i} + \sum_{r=1}^{m} \sum_{i=0}^{p+1} \psi_{i,r} D(T_{aro})_{t-i} + \epsilon_t
\]

where
\[ DU_i(\tau) = 0 \quad \text{for} \quad t = 1, \ldots, \tau \]
\[ = 1 \quad \text{for} \quad t = \tau + 1, \ldots, T \]
\[ DT_i(\tau) = 0 \quad \text{for} \quad t = 1, \ldots, \tau \]
\[ = t - \tau \quad \text{for} \quad t = \tau + 1, \ldots, T \]
\[ D(T_{\omega})_i = 1 \quad \text{for} \quad t = T_{\omega} \]
\[ = 0 \quad \text{otherwise} \]

The first two dummies account for the shifts in the intercept and trend, respectively. \( \tau \) indicates the shift point and is determined endogenously using a sequential procedure due to Zivot and Andrews (1992). The test statistic is the minimum value of the sequentially obtained t-ratio of \( \rho \), which we call \( \min ADF \), and \( \hat{\tau} \) corresponds to the shift point for which \( \min ADF \) is obtained.

The third dummy variable is included to account for outliers in the data. The way it is introduced in (A1) is due to Franses and Haldrup (1994) who show that the distribution of the unit root test is not affected when this is done.

The results for the unit root test involving, say, \( n \), multiple shifts in the deterministic terms were obtained from

(A1) \[ \Delta y_t = \beta_0 + \beta_1 t + \rho y_{t-1} + \sum_{i=1}^{n} \alpha_i DU_i(\tau_i) + \sum_{i=1}^{n} \delta_i DT_i(\tau_i) + \sum_{i=1}^{p} \gamma_i \Delta y_{t-\tau_i} + \epsilon_t \]

The procedure is again sequential with the \( \hat{\tau}_i \) indicating the shift points for which \( \min ADF \) is obtained. The details of this procedure may be found, e.g., in Ohara (1999).

The results for the unit root tests against an ESTAR model are based on estimating

(A3) \[ \Delta \hat{u}_t = \phi \hat{u}_{t-1}^3 + \sum_{i=1}^{p} \gamma_i \Delta \hat{u}_{t-\tau_i} + \epsilon_t \]

where \( \hat{u}_t \) is the GLS-detrended value of \( y_t \), as described in the main text above, and the test statistic the sequentially obtained t-ratio of \( \phi \). The trend equation contains both an intercept and a trend term and \( \tau \) is now taken to be -17.5. For the details of this procedure see, e.g., Erlat (2004).