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The Theory of Supposition: An Answer to Some of the Difficulties of Modern Logicians

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THE THEORY OF SUPPOSITION

AN ANSWER

TO SOME OF THE DIFFICULTIES OF MODERN LOGICIANS

BY

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Natural and accidental supposition. .... Propositions in necessary matter imply no actual existence. .... Propositions in necessary matter may be universal or particular. .... Propositions in contingent matter imply actual existence. .... Propositions in contingent matter may be universal or particular. .... Use of amplification and restriction to vary supposition. .... In subalternation, partial conversion, and certain syllogistic moods of inference the conclusion is legitimate. .... Mathematical procedure a difficulty to modern logic.

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Bibliography
A few words by way of preface perhaps will be of value for an understanding of the development of this thesis. The purpose of this work is not to make a comprehensive study of modern logic, or to discuss the relative merits and demerits of the new discipline. On the contrary, we wish to investigate a point that seems to be a source of many of the differences between modern and traditional logic, that is, the use of supposition.

The subject is approached by a brief history of the development of modern logic. It is not the aim of this brief history to express the contributions of the various exponents of modern logic, but rather to present the background and general trend of thought upon which the discipline is founded. The fundamental philosophic views and general thought upon which modern logic has been built are rather carefully presented by Frederigo Enriques in his work, The Historic Development of Logic. Mr. Enriques' history presents the development of modern logical thought by an investigation of the leading contributors to modern logic, and by an analysis of pertinent passages in their works. Hence, this work was found most useful in fulfilling the purpose of this part of the thesis, and frequently reference is made to it.
In treating the various questions in traditional logic it was found necessary to select one standard work. Many works of logic have been written by traditional logicians. Each, however, has its own particular approach, and some of them express different opinions on certain points. To eliminate any difficulty in this regard Maritain's work, *Formal Logic* has been accepted as the standard source of reference for the exposition of traditional logic. Here again, our interest does not lie in the history, or the precise origin of logical principles, but only in the traditional view itself expressed by Maritain.

Again, in approaching modern logic, Lewis and Langford's work, *Symbolic Logic*, which is commonly accepted as a standard text on symbolic logic, was selected as our chief source of reference. Our treatment of modern logic is limited merely to the calculus of classes, and its application to the syllogism. This procedure was followed because the difficulties that arise about the use of the supposition are clearly evident in the calculus of classes. Furthermore, since propositional calculus and functional calculus can be considered as developments from the calculus of classes, the same difficulties are present in both of them as are present in the calculus of classes. Hence, to eliminate repetition and to keep the paper within the scope of a Master's Thesis only the calculus of classes is considered. What is said, however, about supposition in regard to the
calculus of classes applies as well to the calculus of propositions and functional calculus in modern logic.

Finally, to the best of my knowledge, there has not been written any critical comprehensive study of modern logic from a scholastic point of view which has been published. I found Father John J. Wellmuth's unpublished manuscript, *The Logic of the Cambridge Logicians*, and articles in various periodicals a most helpful study of modern logic from the traditional logician's point of view. There is a great need for a critical analysis of modern logic from this approach. Perhaps the near future will see this lacuna filled.
CHAPTER I
AN INTRODUCTION TO THE PROBLEM

Much is being written today about modern logic. Its achievements are highly lauded, and, on the other hand, efforts have been made, and not futile ones either, to point out its various inaccuracies. But despite the many derogatory statements about it, it has been found to be exceedingly useful in the "development of experimental science."¹ This new discipline has its foundations in mathematics.² The precise name of the new study, remark Lewis and Langford, has not been absolutely determined.

The study with which we are concerned in this book has not yet acquired any single and well-understood name. It is called 'mathematical logic' as often as 'symbolic logic,' and the designations 'exact logic,' 'formal logic,' and 'logistic' are also used. None of these is completely satisfactory; all of them attempt to convey a certain difference of this subject from the logic which comes down to us from Aristotle and was given its traditional form by the medieval scholastics.³

Perhaps, it might be remarked that the lack of an absolute

name is due to the great interest most modern logicians have in
developing this new discipline without ever considering just
what it really is or is really doing. Father Wellmuth states it thus:

... the attention of modern logicians has been devoted rather to the elaboration of various systems of logic and the improve­ment of extant systems from a technical or 'intrasysemic' point of view, than to a careful study of the basic principles and philosophic doctrines implicitly involved, or explicitly contained in these systems.4

This same obscurity is evident again when a proper definition is sought. The notes that distinguish it from other sciences have not been adequately determined. Some writers characterize it as just a "deeper and wider study than the logic of tradition,"5 in other words, a development of a branch of traditional logic which was unknown to the Schoolmen. Other writers say "it has a wider meaning, that it is on the march to replace the tradition­al Aristotelian logic in all fields."6 This divergence of opinions on just what this new study is stems, I think, from lack of any close comparison between it and traditional logic. Is it just a development of traditional logic, or is it an essentially new discipline?

On first appearance it seems to be just a further develop­

4 John J. Wellmuth, The Logic of the Cambridge Logicians, An
unpublished manuscript, 1937, i.
5 Lewis and Langford, 3.
6 Reichenbach, v.
ment of traditional logic. However, recent Scholastic writers are pointing out notes of difference which make it an essentially different discipline. It is not the purpose of this paper to make a comprehensive study of this point. Here we wish to investigate a point that seems to be improperly used, or better, neglected in modern logic, namely, the use of the supposition of terms. But first let us trace briefly the history of this new study. Our intention in this regard is not so much to give a step by step analysis of the development of modern logic, but rather to mention the men who are greatly responsible for modern logic.  

It is difficult to point out in the history of logic the actual beginning of the modern technique. It would, perhaps, be impossible to say any one time saw its actual birth because so many elements were determining factors. Even though certain elements do not enter into symbolic logic itself, nevertheless, they do find a place in its history, because they gave stimulation and new insights into the field. The first impulse in the direction of modern discoveries, most writers agree, was given by Descartes. Descartes' life long ambition was to acquire mathematical clarity in all fields of knowledge. "The seven-

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7 If the reader is interested in a detailed development of modern logic, and wishes to see each man's personal contribution to this development, he is referred to the introduction in Lewis and Langford's work, Symbolic Logic. To treat this point here is beyond the scope of this paper.
teenth century," writes E.G. Salmon, "through Descartes as well as Bacon, gives voice to the cry for order and method. ... Descartes' discovery of Analytical Geometry was epochal not only in mathematics, but also in the development of physics." 8

Although the cry of Descartes for mathematical clarity stirred the minds of men, still his confinement of knowledge to the intuition of a clear and evident concept "cut him off from external reality, and transforms knowledge into a merely subjective process which does not measure itself on its object, but rather measures objects by itself." 9 This subjectivistic process found in Descartes' philosophy lived on and echoes and re-echoes through every succeeding era of philosophical achievement.

Although Descartes is credited with the initial impulse towards positivism, still, logical analysis or mathematical logic seems to have started with Leibniz. The subjectivistic aspect of Descartes' work is retained by Leibniz. The concept for Leibniz is not a criterion of existence, says Enriques, but of possibility. 11 The concept is a mental construct and is said to

have real existence because of the principle of sufficient reason.\textsuperscript{12} Thus it is, Enriques asserts, that Leibniz makes the distinction between being and existence. 'Being' is that which is distinctly able to be conceived, and, 'existence' is what is distinctly able to be perceived. By making being that which is able to be conceived, Leibniz cuts logic from reality outside the mind.\textsuperscript{13}

Working from this subjectivistic point of view, Leibniz began the development of logistics in his idea of a \textit{Characteristica Universalis}, and a \textit{calculus ratiocinator}. In the back of his mind Leibniz seems to have believed that a symbolic method could be formulated which would obviate thinking, and give results equal to those received in the science of mathematics.\textsuperscript{14}

The purpose of Leibniz's \textit{Characteristica Universalis} was to give a scientific universal language which would be common to all workers in the sciences and help in the circulation of new ideas. Besides this, the new method would substitute ideograms for the ordinary words of a language and help in analyzing and synthesizing scientific facts. The basic element of this procedure lay in his belief that all scientific concepts were capable of being defined in terms of a few. Such analysis would give a few simple ideas, he thought, which with their relations could be

\textsuperscript{12} Ibid., 77. Cf. Russell, 209.
\textsuperscript{13} Enriques, 78.
\textsuperscript{14} Russell, 169.
expressed with certain symbols, as a formula exhibits its ele-
ments. In a similar manner he believed the science of reasoning
could be carried out. The calculus ratiocinato was this
science of reasoning carried out. It was Leibniz's attempt to
develop an organon of reasoning in ideographic signs. This cor-
responds closely with our modern logic.

Leibniz's idea was, indeed, novel and gave rise to the la-
ter developments of symbolic or mathematical logic. The success
of Leibniz's work would have been greater, say most modern writ-
ters, if he had not been impeded by certain traditional laws in
regard to the intension and extension of the subject and pre-
dicate.

The new insights into logic that Leibniz developed gave
stimulation to some philosophers and scientists in Europe. In-
dependently of Leibniz's discoveries, however, such men as Sir
William Hamilton, Augustus De Morgan, George Boole, and other
Englishmen began making advancements in the new discipline from
a different approach. These new developments were taking place
in the early 19th century when mathematics was making its bid

15 Lewis and Langford, 5 - 6. Cf. Enriques, 82 - 83.
16 Lewis and Langford, 5 - 6.
17 Enriques, 84. Lewis and Langford have a note on this, and
express the precise point of the difficulty. "For example,
the conception that every universal proposition implies the
.corresponding particular; and the conception that the relation
of terms in extension are always inversely parallel to their
relations in intension." 7.
for independence from philosophy. And so it was from the mathematical aspect that logic was taking on a new color.¹⁸

The developments of this period were of a peculiar nature. The realism and metaphysics of Aristotle were being passed over for new modes of thinking, and, with the rise of the many psychological criticism of the theory of knowledge philosophy becomes more subjectivistic, and logic a purely formal science, nothing more than a "doctrine of mental processes."¹⁹ Because of this change in outlook in regard to the relationship of thought to reality, the mathematicians were beginning to feel that their science was free from all metaphysics and independent of external reality. The activity of the mathematicians, therefore, centered around constructs of the mind. They believed it was their work to formulate the framework, and the physicists' duty to arrange the real order of things according to this framework.²⁰ This, however, does not mean the mathematicians are free from all restraint. Enriques expresses this, and summarizes in a few lines the whole general trend of thought at this time. He writes:

But this does not mean that mathematical production goes on unbridled and unchecked and that all the water of the great river is scattered and lost in a thousand rivulets. The

¹⁸ Enriques, 115.
¹⁹ Ibid., 110.
²⁰ Ibid., 111.
development of thought obeys certain inner controlling forces, and in the different currents there can still be seen a reflection of traditional problems. The various specialized tendencies are in this way reunited in firm knots which give birth to higher doctrines. In short, that order which mind is not able to derive from external nature it finds in itself, in the full freedom of its activities. It is not, however, an order that is given; it is one that is progressively constructed.\textsuperscript{21}

One fact stands out clear at this point in the discussion, namely, that hand in hand with the new developments in mathematics the dichotomy of the processes of the mind from external reality was becoming wider. Modern mathematics possessed a different aspect from classical mathematics, which "held to the exigencies of quantitas interminata."\textsuperscript{22} Cassirer in speaking about mathematics expresses this very point. He says:

Here a field of free and universal activity is disclosed in which thought transcends all limits of the 'given'. The objects which we consider and into whose objective nature we seek to penetrate, have only an ideal being; all the properties which we can predicate of them, flow exclusively from the law of their original construction.\textsuperscript{23}

In the abstract science, then, of mathematics the whole construction flows from first principles which form the foundation of the mathematical science. Whether these first principles are in anyway connected with the physical sciences is arbitrary; the

\begin{itemize}
  \item \textsuperscript{21} Ibid., 111 - 112.
  \item \textsuperscript{22} Kocourek, 97.
  \item \textsuperscript{23} Ernst Cassirer, \textit{Substance and Function}, Open Court, Chicago 1923, 112, as quoted by Kocourek, 97.
\end{itemize}
important thing is, that the truth of the conclusion is independent of physical interpretation.\textsuperscript{24} The foundation of mathematics consists, it seems, not in real being but, as Larguier says, "simply of more or less autonomous postulate systems."\textsuperscript{25}

The truth of this seems proved when we find the mathematical logicians writing that one abstract theory can receive many different interpretations. George Boole states:

Those who are acquainted with the present state of the theory of symbolic algebra are aware that the validity of the processes of analysis does not depend upon the interpretation of the symbols which are employed but solely upon the laws of their combination. Every system of interpretation which does not affect the truth of the relations supposed is equally admissible.\textsuperscript{26}

Such a statement could not be made unless it was believed that the ultimate foundation was a construction of the mind which may or may not find an interpretation in physical reality.

The point that we are trying to bring to light is the nature of the principles on which the modern logic is based. It seems evident that nominalistic doctrine is predominant. Enriques also is of this opinion, for he writes:

But the English logicians, Boole and De Morgan,

\textsuperscript{24} Enriques, 126 - 127.
\textsuperscript{26} George Boole, \textit{Mathematical Analysis of Logic}, Cambridge, 1847, 3, as quoted by Enriques, 127.
are nominalists, at least after the manners of conceptualism and terminism. Symbolism indeed is for them a pure instrument for the analysis of thought, the process of thought being regarded by them above all from an inductive point of view, as proceeding from the particular to the general.27

The concept, it seems, has no reality other than that which the mind constructs for it. Since they do not attain real essences or natures through their concepts, the concept seems to be nothing more than a name that corresponds to nothing outside.

Keeping this fact in mind, namely, the nominalistic nature of modern logic, we can now investigate to somewhat greater extent the development of the discipline.

The greatest advancement in symbolic logic came from George Boole, although before him such men as Sir William Hamilton and Augustus De Morgan had rendered advancements to the science. Hamilton's chief work centered around the quantification of the predicate. Nothing of importance resulted from this, we are told, although, as Lewis and Langford remark, "it suggested a manner in which propositions can be treated as equations of terms."28

De Morgan, a follower of Hamilton, gave many new insights to the subject of logic. By explicitly quantifying the predicate

27 Enriques, 158.
28 Lewis and Langford, 7 - 8.
11.
he developed many new forms of propositions with the "rules for transformation and the statement of equivalents." 29 We will pass over the validity of these propositions at present, and merely state that his versatile mind made great strides in the development of modern logic. By investigating the modes of inference according to his principles he found new classifications, and presented principles to govern them. 30

Although much of importance had been done in the development of this new discipline by Leibniz, De Morgan, and others, nevertheless, it is George Boole who gave the basis for all future progress by successfully applying an algebra to logic. 31

Boole's work in the field of symbolic logic originated when he tried to give an explanation why ordinary language is not adequate to express thought. In order to give a more perfect expression of thought he began his study of symbolic languages. In this manner he believed all the operations of language could be carried out by a system of symbols. These symbols for Boole, as for Leibniz before him, were taken from algebra. 32 The work of Boole and De Morgan was essentially the same. De Morgan, however, did not work out as complete a notation as Boole, and so

29 Ibid., 8.
30 Ibid.
31 Ibid., 9.
32 Enriques, 159 - 160.
it is Boole who is remembered for his algebra of classes.33

Leibniz, many years before, had attempted an algebra of logic but failed because of certain difficulties. It was by surmounting these difficulties in his procedure that Boole succeeded. The chief difficulty with Leibniz, note Lewis and Langford, was solved when Boole considered logical relations in extension and not in intension as Leibniz had tried to do.34

Boole's algebra as applied to logic underwent various changes before it acquired the form it has today. In this transition of Boolean Algebra, W. S. Jevons, Charles S. Peirce and John Venn had influence. Jevons formulated a method independent of the algebraic technique of Boole. With this he was able to solve logical problems by the manipulation of his "logical alphabet." Peirce and Venn contributed their bit by adding to the Boolean Algebra a new relation of inclusion.35

With the developments of Peirce and Venn the Boolean Algebra of logic reached its climax. The next development to modern logic came when symbolic logic was united with "the methodology of rigorous deduction, as exhibited in pure mathematics."36

34 Lewis and Langford, 12.
36 Ibid., 16.
Up to this stage modern logic was concerned with the logic of classes. Now, however, it centers around the logic of propositions, propositional functions and logical relations. Here, in particular, Peano, Peirce, and Schroeder advanced the modern discipline. Peirce and Schroeder concentrated on the logic of propositions, propositional functions and logical relations until they arrived at a calculus which was assimilated to the operations of mathematics. Peano, on the other hand, arrived at a similar relationship between logic and mathematics by analyzing mathematics in its deductive form. By giving the relations symbols upon which the proofs of the defusions depended, he gave mathematics a "logistic form." 37

The next important contribution was made by Russell in his *Principia Mathematica*, in which he attempted to demonstrate the relation between mathematics and logic, or in other words, he attempted to show that mathematics is reducible to logic. 38 Russell was very critical in his analysis from a realistic point of view. "He comes to the conclusion," says Enriques, "that a proper understanding of symbolism tends naturally to bring us back to the Aristotelian position in the degree as we break with Boole's psychologism." 39 Russell's developments proceed in a

37 Ibid., 16.
38 Ibid., 22 - 23.
39 Enriques, 169.
different fashion, The calculus of classes for him is derived from the calculus of propositions. Peano, also, developed his work along this line. For him, however, the precedence of the calculus was merely a contingent matter, whereas for Russell it expresses a fundamental logical relation.40

Another point that Enriques makes in regard to Russell is worthy of note. Boole, in order to solve difficulties, restricted his considerations of logical relations to their extension only. Russell disagrees on this point, since the same relation taken in extension can have different meanings in intension. Enriques writes:

The extensional conception is above all unacceptable to him, because the couples connected by a relation are characterized by order; they cannot therefore be classes. For order itself also forms, for Russell, a certain relation among members of a class. Russell thus writes XRY in order to represent in general a relation holding between a certain domain X and a certain converse domain Y which form together the field (X,Y), where R assumes meaning. In this way he establishes a logic of relations, which exactly presents itself as a generalization of the logic of propositions. It is this which forms in the opinion of Couturat, the most original part of the work of Russell.41

The development of modern logic, historically at least, had its foundation in mathematics. The discovery of analytic

40 Ibid., 169.
41 Ibid., 170.
geometry and calculus started the search for a new universal science which could be used in all knowledge. Boole's algebra achieved this to some extent, and showed that some mathematical operations could be applied to logic. Russell and Whitehead developed the discipline further and defined all arithmetical ideas in terms of logical notions.\textsuperscript{42}

With that brief summary of the historic development of modern logic it might be well to present a working definition of the new discipline. As we already mentioned modern logic has not as yet been given any absolute name. Again, in regard to a standard definition little has been accomplished. The best understanding of the new discipline is given, I believe, by Father Wellmuth. Father Wellmuth first distinguishes logic from the other sciences. First of all, he notes that the principal manner of classification is according to the 'objects' of which the various sciences treat. A twofold division is made of objects, (1) ideal objects, and (2) real objects. The ideal objects are \textit{entia rationis} with no real existence outside of the mind. Real objects, on the contrary, make up the real existing world, and can be classified as sensible or non-sensible. Logic, then, is distinguished from the other sciences by having for its object "ideal and non-sensible objects to the exclusion of actual

\textsuperscript{42} Kocourek, 98.
The relation of logic to mathematics and to philosophy bears more serious consideration, since so many writers today disagree whether mathematics is a branch of pure logic, or whether pure logic is a part of philosophy. It is well to pass over this question at present until future investigation is able to establish the foundation of mathematics.

Father Wellmuth follows up his distinction between logic and the sciences with another valuable distinction; the distinction between logic and language. For any expression of our thoughts, a language of some sort is needed. If we concentrate merely on the expression of thoughts, our study is entitled "linguistics." Logic, on the other hand, is concerned about the thought behind the expression. However, for a full understanding of the thought an accurate language is supremely important. Yet it would be a misapprehension to suppose that it is possible to perfect language by means of symbols to such an extent as to eliminate all incorrect thinking. This seems to be an attitude, however, of modern logicians and especially of the semanticists. Perhaps if one supposes that the mind is limited to

43 Wellmuth, 5.
44 Ibid., 8.
46 Leonard J. Eslick, "Grammatical and Logical Form," New Scholasticism, XIII (1939), 233 - 244.
pure sense experience, there is a possibility that this might be true. But the mind has the power of thinking on its sense experiences, and as such, may make mistakes despite the perfection of the language. Language is of great importance in the accurate presentation of thought. However, a study of language is not logic. Logic, as Maritain remarks, "bears upon the act of reason itself," and therefore is concerned about the thought content.

After the distinction between logic and sciences, and between logic and language has been made, Father Wellmuth then discusses the nature of modern logic itself. Before we quote the definition which he expresses, let us mention a few of the various kinds of logic which are being studied today, and perhaps we shall have a better understanding of what we are treating. There are works today that go by such names as, 'formal' logic as opposed to 'material' or 'applied' logic, 'inductive' and 'deductive' logic, 'genetic' logic, 'general' logic, 'real' logic, 'transcendental' logic, and 'mathematical' or symbolic logic. It is with the last that we are concerned, and this, Father Wellmuth mentions, is nothing more than a progressive treatment of formal logic.

48 Wellmuth, 10.
49 Ibid., 11.
50 Ibid., 154.
What, then, is formal logic? Formal logic, Father Wellmuth writes, quoting from Lalande's *Vocabulaire de la Philosophie*, endeavors to establish "the necessary laws of thought, which hold whatever be the nature of the objects thought about." It has "nothing to do with the truth of the facts, opinions or presumptions, from which an inference is derived; but simply takes care that the inference shall certainly be true if the premise be true."\(^{51}\)

Formal logic, he goes on to add, "has always been regarded by its proponents as (in principle at least) a distinct non-philosophical science, independent of any particular system of metaphysics."\(^{52}\) Again later on he writes:

This modern formal logic is also called 'symbolic' or 'mathematical' logic: 'symbolic', because it is for the most part a collection of formulae made up of non-verbal symbols more or less like the symbols of mathematics; and 'mathematical' because, ... it is constructed according to the principles and methods of mathematical systems.\(^{53}\)

Alonzo Church gives a similar description of formal logic in the *Dictionary of Philosophy*.

Formal logic investigates the structure of propositions and of deductive reasoning by a method which abstracts from the content of propositions which come under consideration and deals only with their logical form. The distinction between form and content can be made definite with the aid of a particular language or symbolism in which propositions are expressed, and the formal method can then

\(^{51}\) Ibid., 11. Translator not mentioned.
\(^{52}\) Ibid., 14.
\(^{53}\) Ibid., 15.
be characterized by the fact that it deals with the objective form of sentences which express propositions and provides in these concrete terms criteria of meaningfulness and validity of inference. This formulation of the matter presupposes the selection of a particular language which is to be regarded as logically exact and free from the ambiguities and irregularities of structure which appear in English ... i.e., it makes the distinction between form and content relative to the choice of a language.\textsuperscript{54}

Again, when describing symbolic logic, Alonzo Church writes:

Symbolic logic, or mathematical logic, or logistic, is the name given to the treatment of formal logic by means of a formalized logical language or calculus whose purpose is to avoid the ambiguities and logical inadequacy of ordinary language. It is best characterized, not as a separate subject, but as a new and powerful method in formal logic.\textsuperscript{55}

With such a descriptive understanding of modern logic we might well proceed to ask, what is the formal definition that the modern logicians give for this new discipline? The student of modern logic would be somewhat disappointed if he sought such a definition in many of the modern books on symbolic logic, because, in general, such a definition is not given. However, Father Wellmuth comes to our aid and presents the definitions which he has found. He presents the definitions given by Professor Jørgensen and Professor Eaton. Professor Jørgensen, he

\textsuperscript{54} Alonzo Church, "Formal Logic," \textit{Dictionary of Philosophy}, 170. 
\textsuperscript{55} \textit{Ibid.}, 181.
says, defines logic as a "deductive science of deduction in
general." Professor Eaton presents a more elaborate definition.
"Logic is the science that exhibits all the relationships per-
mitting valid inference that hold between various kinds of pro-
positions considered merely in respect to their form." This
last definition Father Wellmuth analyzes, and shows that Eaton
supports Jørgensen's definition, that logic is a "deductive
science of deduction in general." This means:

... that the general statements by which it
fulfils its task of exhibiting those log-
ical relationships between propositions
which permit of valid deduction should be
so arranged as themselves to form a deduc-
tive system.

This definition expresses rather accurately, I think, the
general features and scope of modern logic. It is a deductive
science because of its similarity with mathematics. It starts
with certain "undefined primitive ideas" which are "symbolized"
and combined into "postulates", or "primitive propositions."
From these postulates, in turn, are deduced theorems and other
propositions by mathematical means, especially "by substitution."
Thus, modern logic is, it seems rather accurately defined as "a
deductive science of deduction in general."

It is true that the modern discipline is especially useful

56 Wellmuth, 156.
57 Ibid.
58 Ibid., 162.
in experimental science, but it must not be confused with tra-
ditional logic. Traditional logic also is a formal science. But 'formal' has a slightly different meaning according to the
traditional logicians. Traditional logic is formal in so far
as it "studies what reasoning is, and how it must proceed whatever its content or the use which mind makes of it." To stop
there in our description of traditional logic would leave plenty
of room for ambiguity. Although traditional logic has a formal
aspect it is not by any means purely a study of form. Maritain
writes: "Since logic is the art which enables us to proceed
with ease, order and correctness in the act of reason itself, it
must treat both the form and the matter of our reasoning."

Traditional logic is broken down into two divisions; minor
or formal logic and major or material logic, as Maritain points
out. But never is traditional logic considered just a study of
the logical form abstracted from the content.

Furthermore, a logic that is concerned merely with the
form of reasoning, if it is possible to have such a logic, could
never have as its object demonstration, since demonstration

59 Kocourek, 98.
60 Maritain, 9.
61 Ibid., 10.
62 Ibid., 9.
63 The use of the word 'formal' in modern logic has many and
varied meanings. Father Wellmuth in his manuscript makes a
rather detailed analysis of the term, 146-155. Such an an-
alysis is outside the scope of this paper.
would require a consideration of the content of propositions, that is, in so far as demonstration concludes to the true or false. Demonstration is had in a certain sense in formal logic, that is, in so far as formal logic sets forth the rules which must be observed if demonstration is to be correct in relation to the disposition of materials. But demonstration cannot be had in formal logic in so far as demonstration concludes to the true or false since reasoning attains to the true or false in virtue of the matter. For demonstration, as Aristotle says, est syllogismus efficiens scire, and John of St. Thomas adds to this, scire autem est cognoscere causam, ob quam res est, et illius causam esse, et fieri non posse, ut aliter res se habeat. But the object of traditional logic is demonstration, as John of St. Thomas explicitly states.

Respondetur, quod syllogismus, qui fit per tertiam operationem, est principale objectum Logicae, et inter syllogismos ratione materiae principalior est demonstratio. ... Cum ergo scientia Logicae proprie et essentialiter sit directiva rationis, illud est principale objectum, ubi principalius invenitur ratiocinatio et discursus cum certitudine sine errore. Contingit autem error vel ex defectu formae vel ex contingentia materiae. Ergo principale objectum ex parte formae est syllogismus recte dispositus, principale autem ex parte materiae est demonstratio, in qua invenitur processus rationis sine errore tam ex parte materiae quam ex parte formae.

65 Ibid.; 773.
66 Ibid., 265.
It seems to follow, therefore, that a logic that is concerned merely with the form of reasoning is different from traditional logic. Professor Eslick, in fact, maintains and gives strong arguments that modern logic, which is concerned only with the form of propositions, and traditional logic "differ as grammar and logic differ." He writes:

Modern logic has been primarily concerned with the symbolic exhibition of grammatical form. As such, it has been more interested in extension than intension, and in the grammatical relations of implication, disjunction, and conjunction, rather than in inference as such. ... and the distinction between implication and inference is a distinction between two essentially different arts.

Maritain expresses a similar view:

Logistics differs essentially from Logic. For, whereas the latter bears upon the act of reason itself in its progress towards the true, and thus upon the order of concepts themselves and of thought, Logistics is concerned with the relations between ideographic signs and therefore with these signs themselves which, once determined, are taken as sufficient.

What is called modern logic, then, is strictly a formal discipline concerned only with the form of propositions, and by some, it is said to be only a study of grammatical form rather than logical form. The question might be asked, why is it strictly a formal discipline which prescinds from the content? This

67 Eslick, 233.
68 Ibid., 235 - 236.
69 Maritain, 222.
might be answered by recalling that modern logic is said to have its foundation in mathematics,\(^{70}\) and is defined as, "a deductive science of deduction in general." This formal study is constructed, as Father Wellmuth mentions, according to the methods of pure mathematics. Now, pure mathematics is a system of theorems deduced from a set of undefined elements, properties, functions, and relations, and a set of unproved propositions by the methods of formal logic which abstracts completely from all reality.\(^{71}\) Thus, just as pure mathematics is concerned with the relations between the mathematical symbols which represent the postulates and definitions, so modern logic is concerned with the relations between "ideographic signs" which represent the expressions of our thought.\(^{72}\) Here, then, can be faintly seen the close relationship between modern logic and pure mathematics.

Modern logicians are concerned with the form, and believe that "considerations bearing exclusively on the form suffice to explain the entire discourse."\(^{73}\) Naturally, there are going to be many points of difference between a logic concerned with form alone and one concerned with the form and matter. A discipline


\(^{73}\) Maritain, 227.
that centers its attention merely on the form will ignore an essential point in traditional logic, -- the use of the supposition of terms. That is the point of this paper. We wish to attempt to show that some of the advancements which modern logicians claim they have made to traditional logic, and some of the errors which they claim to have found in it, are due to a lack of consideration of the use of supposition in traditional logic.
CHAPTER II
THE PURPOSE AND USE OF
SUPPOSITION IN TRADITIONAL LOGIC

It perhaps can be said that there is no thinking man today who would not admit that to express one's thoughts in clear, intelligible language is tedious work. Even after much effort and training the attempt at expression is sometimes rewarded only with embarrassment, because of inadequacies either on the part of the individual or on the part of the language. Maritain, in writing about language and thought, expresses a similar opinion.

Everything directly conceived or thought of by our intellect, everything of which we have a concept or "mental word" may be expressed or translated into language. But despite the flexibility, the docility, the delicacy of any system of language-signs, this expression is always more or less deficient in relation to thought. The loftiest intellectual knowledge, which reveals a world of consequences within a single principle, must, so to speak, be scattered and diluted in order to be orally expressed.74

The proper expression of one's thoughts, however, not only requires an effort on the part of the speaker but also an interpretative effort on the part of the hearer, and, unless this

74 Maritain, 58.
twofold effort is had there remains "but a radically insufficient system of lifeless symbols."\textsuperscript{75} In consequence of this Maritain continues:

Hence a twofold necessity accrues to philosophy: it must acquire a mastery over language by means of a whole technical apparatus of forms and verbal distinctions (terminology), and it must unceasingly exact from the mind an act of internal vitality such that[\textit{sic}] words and formulae can never replace, for they are there but to spur the mind to this act. ... Language, then, expresses or signifies as much of our thought as is necessary in order that another intellect, hearing the pronounced words, may present the same thought to itself.\textsuperscript{76}

But, if words and language cannot adequately replace the vital act of thought there must always remain a certain part of thought that goes unexpressed. Yet, when a concept of one's mind is expressed and understood by another this unexpressed "margin of thought" is also conveyed. Therefore, Maritain writes, "this unexpressed margin of thought ... is remarkably evinced by the diverse properties that affect the term considered, not by itself, but \textit{in the context of the proposition, as part of a proposition.}"\textsuperscript{77} It seems to be definitely implied here that the study of the proposition transcends the mere sign-expression of the proposition. In other words, the statement of a proposition contains more than what is sometimes explicitly

\textsuperscript{75} Ibid.  
\textsuperscript{76} Ibid., 59  
\textsuperscript{77} Ibid.
stated. Its full significance is only going to be understood by a study of the diverse properties that affect the term in the proposition.

Already logic has been defined as that which enables us to advance with order, ease, and correctness in the act of reason itself. Surely, then, these "diverse properties that affect the term," and through which the unexpressed margin of thought is conveyed to the hearer cannot be something foreign to logic, but must be worthy of serious study. Yet, it seems to be precisely the neglect of these properties by modern logicians in their study of traditional logic that makes them write, (1) that traditional logic is unsatisfactory because of certain ambiguities in the laws of intension and extension which affect inference, or, (2) that the only syllogisms are those with two universal premises and a universal conclusion, and those with one universal and one particular premise and a particular conclusion.78

The importance of these properties are immediately evident. Even Aristotle was conscious of this when he wrote, "It is impossible in a discussion to bring in the actual things discussed; we use their names instead of them."79 Maritain further remarks:

But we shall inevitably fall into a host of

78 Lewis and Langford, 49, 63.
errors unless we observe that not only may the same word have several different meanings but also that the same word, even while having the same meaning, ... and consequently even while signifying the same intelligible nature may, according to its use in the context, stand for very different things.\(^{80}\)

The ancient logicians were conscious of this difficulty, and had devised a way of rendering a solution. The theory of suppositio and ampliatio is this solution;\(^{81}\) and unless these are fully understood there is no need, as Javelli says, for anyone to attempt a criticism of the traditional moods of inference. *Tu autem adverte novitie, quod praedictas defensiones servare non poteris, donec intellekeris tractatum suppositionum et ampliationum et appellationum et probationum terminorum.*\(^{82}\)

Modern logicians are also conscious of this difficulty of language replacing thought, and it is for this reason that they make such valiant efforts to symbolize their logic to avoid ambiguities. Enriques writes:

> But the subtle disquisitions of scholastic language and especially the paradoxical conventions introduced from time to time into the formulations of mathematical theories, show how insufficient and inaccurate ordinary language is from the point of view of a complete analysis of thought. Hence the idea of replacing verbal analysis by symbolical

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\(^{80}\) Maritain, 59.

\(^{81}\) Joannes A S. Thoma, 28-42, 166-179. Cf. Javelli sub suppositio

\(^{82}\) Javelli, *Logicae Compendium Peripateticae*, Venetiis, 1541, 168
analysis and the shaping accordingly a new language after the fashion of algebra.  

It is, indeed, a commendable ambition to attempt to remove the inadequacies of language, but it is a gross error to say, as some writers do, that the ancient logicians were in ignorance about this difficulty, and, therefore, made rather grievous errors in inference.  

Before we attempt to respond to some of the statements of modern logicians, a brief survey of the use of the supposition and the amplification of terms should be made. The diverse properties of the term that determine its accurate use John of St. Thomas numbers as five: Proprietates partium propositionis, quae solum conveniunt illis, prout intra propositionem sunt, quinque sunt, scilicet suppositio, ampliatio, restrictio, alienatio, appellatio. We will pay particular attention only to the first three.  

Supposition is defined by John of St. Thomas as acceptio termini pro aliquo, de quo verificatur. Whenever a predicate is applied to a subject, attention must be given to the copula before the substitution of the term for the thing is valid. For  

84 Lewis and Langford, 62.  
85 Joannes A S. Thoma, 28.  
86 Ibid., 29.
example, I may say, "The dog is an animal," or "Dog is a three-letter word," or "Fido is a dog." In all three cases "dog" has the same meaning. Nevertheless, it would be incorrect to say that "Fido is a dog, therefore, he is a three-letter word." For although "dog" has the same meaning, still its use in the various sentences differs. Supposition of a term therefore, or its substitutive value, as Maritain notes, "is its function in discourse -- (while its meaning remains the same) -- of taking the place of a thing. To be legitimate this substitution (of term for thing) must answer the needs of the copula." 87

It is the latter part of this definition that deserves consideration. For, in it lies the explanation of many of the difficulties of modern writers. Here again Maritain is very helpful in giving us an understanding of the statement. To be legitimate this substitution must answer the needs of the copula; that is, if the copula expresses present time, before verifying the "conformity of predicate to subject, we must ascertain whether the subject itself exists in the manner required by the copula." 88 For example, if I say, "My horse is red," "my horse" stands for something which actually exists, because I can indicate by thought at the present something which I can say is my horse. If, however, I say, "F.D. Roosevelt will be president,"

88 Maritain, 61.
"F.D. Roosevelt" does not stand for anything because the copula is "will be," and there is no future being (in relation to the future time expressed by the copula) that can be indicated as "F.D. Roosevelt." Thus, when it is said that "suppositio is the property of a term by which it stands for, or takes the place of, a thing in discourse, this substitution being legitimate considering the copula," it does not mean "that this substitution is true in the nature of things, but only that the sort of existence -- actual (past, present or future), possible, or 'imaginary' -- denoted by the copula permits this substitution." If this is fulfilled then the proposition may be true; but, if the subject does not stand for anything the proposition is false. That is, it is false, as John of St. Thomas says, ex subjecto non supponente. However, this rule does not apply to negative propositions. By quoting Maritain's example this is made evident. "Richelieu is not a member of the Chamber of Deputies." This is true even though the subject does not meet the needs of the copula by the fact that Richelieu does not actually exist.

One further point in regard to legitimate substitution should be considered. Some propositions express eternal truths, that is, a proposition in which the predicate is essential to a subject. In such instances the subject always stands for some-

89 Ibid.
thing, and is said to have a natural supposition, because it is always connected with the predicate. In such a case "the copula denotes possible existence only, and is thus outside of time."⁹¹ For example, if I say, "The three angles of a triangle are equal to 180 degrees," I am stating a proposition in which the predicate is essential to the subject, and as such is always true even though a triangle does not actually exist here and now. When the predicate, however, is accidental to the subject, e.g. "My dog is white," it is necessary to know if the subject has a substitutive value before it can be said to be true, because the copula expresses actual existence, or "existence in time."⁹²

Perhaps more clarity will be added to our understanding of supposition if a distinction is made between supposition and signification. The signification of a term refers to the object from which the name comes, that is, "to the form or nature which it represents to the mind."⁹³ The supposition of a term, on the contrary, refers to that to which the name is given. The term, then, considered according to its supposition, stands in place of the thing, and is that to which the intellect "applies certain predicates." The signification of a term "relates to the natures that are the proper object of the first operation of

⁹² Maritain, 61.
⁹³ Ibid., 62.
the mind, whereas 'suppositio' relates to the subjects in which these natures are realized and which the second operation of the mind signifies as existing with such and such predicates." 94

An important point to be remembered is that the supposition in a syllogism must remain consistent. If the supposition in the major is taken in relation to ideal existence the supposition in the conclusion cannot be in relation to real existence. An example of this is Descartes' proof for the existence of God.

A perfect being is one that exists necessarily.
But God is a perfect being.
Therefore, God necessarily exists.

In the major premise the perfect being has only ideal existence, but in the conclusion real existence is indicated. 95

The substitutive value of a term in a proposition can be of various kinds. A study of these does not seem to be necessary in this paper. Whatever the substitutive value may be, the important thing is that it conform with the principles we have already discussed, namely, (1) the supposition of a term must meet the exigencies of the copula, (2) the supposition and the signification of a term, and, (3) the supposition must remain consistent in a valid inference.

Thus far we have considered only the substitutive value of

95 Maritain, 63.
the term as it stands in the proposition. Besides that property, others, which John of St. Thomas refers to as "amplification" and "restriction" of a term, must be understood. These are the properties of a term in a proposition by which the mind may vary the supposition, e.g. from past existence to present, or from actual to ideal existence, or ideal to actual existence. Amplification is defined as _extensio termini a minore ad maiorem suppositionem._ For example, the term "dog" is broader in this proposition: "Every dog (as a possible being) is a quadruped," than it is in the proposition: "Every dog (actually existing) is a quadruped." The first proposition includes all dogs, actual and possible. The second, only actually existing dogs. The operation whereby the mind enlarges the substitutive value of the term from the second proposition to the first, is called amplification. Let us consider the use of the amplification in a syllogism. For example, the syllogism:

No animal is incorruptible.
But every animal is a living being.
Therefore some living being is not incorruptible.

In each of these propositions the mind amplifies the existence to possible existence, and, therefore, by keeping the supposition consistent the mind arrives at a valid conclusion.

Restriction, on the contrary, is defined in just the oppo-

96 Joannes A S. Thoma, 37.
site manner: *Coarctatio a mairi ad minorem suppositionem*. The term "man" is broader in the proposition, "Every man (as a possible essence) is mortal," than in "Every man (as actually existing) is mortal." When the mind understands the term "man" in the proposition "Every man is mortal" as actually existing, it limits the term to a lesser supposition, and the operation is called restriction. The same operation is had in the case of a syllogism. For example, the syllogism,

Every visionary is a dangerous man.
But every Utopian is a visionary.
Therefore, some dangerous man is a Utopian,

would be invalid if we supposed ideal existence in the premises and actual existence in the conclusion. But the mind by means of the use of the restriction of terms, limits the understanding of existence to actual existence in the premises, and thus may validly conclude, "Some dangerous man is a Utopian."

With this brief understanding of several of the diverse properties of a term in a proposition or syllogism, we may now turn to the various difficulties presented by the modern logicians. It might be well to call attention once more to a point made earlier, namely, that no matter how flexible a system of language-signs is going to be, it will never be perfectly capable of representing or conveying the thoughts of man. The truth

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97 Ibid.
of this seems evident when we remember that language or words are merely a material instrument used to express the thought of a spiritual faculty -- the intellect --, which rises above material elements in its operations. Furthermore, the fact that a single word can have the same meaning and yet have different substitutive values in speech will always demand the operation of "distinguishing" in human discourse. As long as the human intellect is able to function, never will a system of signs be so perfect as to eliminate thought, and always will there remain a margin of thought in any discourse which will demand more than pure passivity on the part of the receiving mind.

Traditional logicians were conscious of the immaterial aspect of the mind and realized that material signs could never fully express immaterial thought. Logic, then, for them was a tool that served the intelligence and could never replace it. Traditional logicians accepted logic as a help to the mind, and were interested in perfecting this tool. Modern logicians accept logic as a substitute for thought, and are interested in perfecting this logic to eliminate much of thought itself. This, it seems, is one of the fundamental differences between traditional and modern logic. The object of modern logic as expressed here is also expressed, although unconsciously perhaps, by

98 Ibid., 232.
Professor Reichenbach. He writes: "Symbolic logic is the analysis of language." If we accept this statement at its face value it seems to follow that symbolic logic is nothing more than an analysis of a set of words. In symbolic logic the logician is interested, as Reichenbach puts it, in the "mechanical manipulations with symbols." These symbols, as he adds, "are distinguished only by geometrical shape," and "take the place of thought operations based on realizing the meaning of the symbols." 

The efforts of the modern logicians to analyze the means of expression with exactness and precision is to be highly commended, because the accurate presentation of concepts is of supreme importance. But, although they are extremely careful in the analysis of the means of conceptual expressions, still in their analysis and presentation of traditional logic they seem to be lacking in this very exactitude of which they are so proud. For, the traditional logicians, it seems, were not only aware of the difficulties which are troubling the modern logicians, but had a keener insight into the problems, and presented a much more subtle solution by understanding the use of suppositio and ampliatio.

99 Reichenbach, 2.
100 Ibid., 165.
101 Maritain, 226.
Since the objects of modern and traditional logic are so different, the two should, it seems, remain entirely diverse disciplines. Modern logicians, however, view the situation under a different light, and as a result, maintain that they are making additions in traditional logic, and even, in fact, finding certain principles of traditional logic erroneous.

The syllogism in general, subsalternation, partial conversion, and the square of opposition receive the brunt of their criticisms. A rather lengthy and summary quotation from Lewis and Langford, I believe, will bring the heart of the objection more clearly to light. They write:

Traditional logic is primarily a logic of terms. The laws of identity, contradiction, and the excluded middle, the dictum de omni et nullo, and the rules of the syllogism all tell us what must be or what cannot be true of the relations of terms. But terms have both intension and extension; they connote concepts or essential attributes, and they denote things or classes. The laws of intension and those of extension are analogous, ... the relation of a given set of terms in intension may not be parallel to their relations in extension. For example, "No trespassers are arrested" might be true in extension, meaning that the class of actually arrested trespassers has no members; but false in intension, meaning that the concepts 'trespassing' and 'being arrested' are mutually incompatible. ...

And a logic which is adequate to all propositions must, therefore, cover both intension and extension. The traditional logic

103 Lewis and Langford, 49.
is unsatisfactory on this point. The majority of its rules are valid both of intention and of extension. But some of them are ambiguous and are such that, whichever interpretation is chosen, they are partly incorrect. For example, the rules of the syllogism sanction EA0 in the third figure. But the argument,

No absentees are failed;
All absentees receive a grade of zero;
Therefore, some who receive a grade of zero do not fail;

is fallacious. ... The Boole-Schroeder Algebra unambiguously applies to the relations of terms in extension. When so applied it is not a complete logic of terms (though it is at least as adequate as traditional principles), but it is precise and completely accurate. ... Hence the Boole-Schroeder Algebra affords not only an exact logic but one having as wide a sphere of application as is possible without greater complexity.104

What is it that causes the modern logicians to criticize so severly certain parts of traditional logic? Are certain principles and rules of traditional logic actually invalid? An examination of the application of the Boole-Schroeder Algebra to various moods of the syllogism will reveal perhaps the main source of the difficulties, and make an explanation possible.

104 Lewis and Langford, 49-50.
CHAPTER III

AN APPLICATION OF THE BOOLE-SCHROEDER ALGEBRA

TO THE SYLLOGISM AND THE RESULTING DIFFICULTIES

To begin with, an understanding of a few symbols will be needed. In the application of the algebra the symbols, $a$, $b$, $c$, etc. represent classes of things denoted by a term.

- $a \times b$, or $ab$ signifies the actual common members of the classes $a$ and $b$.
- $-a$ negative of $a$, will represent the class of things not members of $a$.
- $a \bar{b}$ the class of things which are members of $a$ or members of $b$ or members of both.
- $l$, negative of $0$ will be the class in which everything is a member.
- $0$ null-class.
- $a ( b$ $b$ includes all the members of $a$.
- $a = b$ class $a$ has the same extension as $b$.

With this knowledge of the symbols it is now possible to proceed with the application of the algebra. Consider, first of all, the four standard propositions of logic, namely, A, E, I, O.

Writing these according to their symbolic form we have,

A. All $a$ is $b$: $a ( b$: $ab = a$: $a \bar{b} = 0$.
E. No $a$ is $b$: $a ( \bar{b}$: $a -b = a$: $\bar{ab} = 0$.
I. Some $a$ is $b$: $\bar{a} ( \bar{b}$: $a -b \neq a$: $\bar{ab} \neq 0$.
O. Some $a$ is not $b$: $\bar{ab} \neq a$: $a -b \neq 0$.

105 Lewis and Langford, 51.
106 Ibid.
It is interesting to note here the various equivalent forms\textsuperscript{107} for writing each of the propositions, and that each proposition can be symbolized by an equation or an inequation, e.g. \( a - b = 0 \). The universal propositions are symbolized by equations and the particular propositions by inequations. Thus by looking at the equations above it will be seen that universal propositions in extension assert a non-existence, and the particular affirm an existence. This point has particular significance and will be considered later. Lewis and Langford, however, explain its meaning more fully.

The last expression given in each case is an equation or an equation with one member 0, making them easily comparable. ... The two universals each affirm something is \( = 0 \); the two particulars, that something is \( \neq 0 \). That is, a universal proposition in

\textsuperscript{107} It is well, perhaps, to note here that the manner of writing the various propositions is unique to modern logic, and characterizes the relations of terms in extension rather than comprehension. Maritain notes this; "The ancients were neither exclusively 'extensivists' nor exclusively 'comprehensivists.' On the one hand, they emphasized the essential role played by the relations of 'extension,' in order to assure and guarantee the identification of the two extremes with the middle term, and in the theory of the syllogism they followed Aristotle in reflecting primarily upon the extension of terms. On the other hand, they said: *Prædicatum inest subjecto*, understanding thereby that the judgment has for its primary logical function the affirmation of the inherence of a Pr. in the comprehension of a S.; accordingly they called propositions *de inesse inasmuch as they attribute a Pr. to a S.* To indicate attribution, Aristotle says, not 'A is B,' but rather 'to A belongsB' ... indicating that for him, as for his scholastic disciples, the judgment and the proposition are to be understood first and foremost from the point of view of comprehension." p. 175.
extension asserts a non-existence: "All a is b," that a's which are not b's do not exist; "No a is b," that a's which are b's do not exist. And a particular proposition in extension asserts an existence: "Some a is b," that a's which are b's exist; and "Some a is not b," that a's which are not b's exist.108

Now, by making an application of the algebra to a syllogism, let us see to what conclusion this symbolization is to lead when the premises and conclusion are all universal propositions. For example the syllogism: All men are animals, but, all animals are mortal, therefore, all men are mortal. Writing this syllogism in symbols we have:

all a is b, or a -b = 0.

all b is c, or b -c = 0.

Therefore, all a is c, or a -c = 0.

Since this is an application of the algebra, the rules of the algebra are binding. Therefore, to arrive at the conclusion we add the two premises and obtain

a -b + b -c = 0.

The middle term of the syllogism is b, and to get the conclusion the middle term is suppressed according to theorem 5.51109 of the algebra (if Ax + B -x = 0, then AB - 0; p. 43). The

108 Lewis and Langford, 52.
109 In this paper we will accept the theorems as proved, and merely state them with a reference to Lewis and Langford for the proof. This reference will be placed in parentheses immediately after the theorem.
conclusion of the syllogism then appears.

\[ a - c = 0, \text{ or all } a \text{ is } c, \text{ or all men are mortal}. \]

Such a procedure, it is true, is an application of the algebra to the syllogism, and arrives at the same conclusion as the traditional logician will by syllogistic inference. Although the conclusion arrived at is the same, still the procedure widely differs. In the algebra the conclusion is obtained by the mechanical elimination of the middle term, and as such cannot be considered inference, even though it is a description of it from an "operational viewpoint." Professor Eslick clarifies this when he writes:

But syllogistic inference depends upon the communication of a universal to its subjective parts or instances. The middle term, as a universal, and not as a class, is vital to inference, and there is no real inference without it. This is why the dictum de omni et nullo is so important and why the position of the middle term in the premises of a syllogism has more than a material, accidental significance.

Hence, when the symbolic logicians arrive at a conclusion no real inference is made, since they ignore the importance of the universality of the middle term. Maritain explains this universality.

The essential force and merit of the syllogism lies, not in the passage from the universal

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110 Ibid., 55.
111 Eslick, 235.
to the particular but in the identification of the two extremes with a same third term. ... This third term must necessarily be universal if an inference is to be drawn by means of such an identification.\textsuperscript{112}

He adds further on:

If we were not sure of the legitimacy of the identification of the two extremes in virtue of their mutual identification with the middle term, that is, if the logical functions of the Fr. and S. in the propositions did not guarantee that the middle term, in being identified with T [major term] is no smaller than it is when it is identified with \( t \) [minor term] the syllogism would not be absolutely foolproof, and might sometimes lead us into error.\textsuperscript{113}

Yet according to the Boolean rules for syllogisms the middle term is not a universal, but only a class possessing only a "material, accidental significance," and is eliminated by mechanical manipulations.\textsuperscript{114}

Turning back again to the application of the algebra, we find that, despite the mechanical nature of symbolic logic, many times its conclusions are the same as those of syllogistic inference.

Let us consider now an application of the algebra to a syllogism in which one premise is particular. If both of the premises are universal, Lewis and Langford tell us that "the

\textsuperscript{112} Maritain, 206-207.
\textsuperscript{113} Ibid., 208.
\textsuperscript{114} Eslick, 235.
conclusion is always reached ... by the algebraic elimination of
the middle term."\textsuperscript{115} However, when one premise is particular
the process varies. For example, the syllogism: All horses are
quadrupeds. Some horses are red. Therefore, some quadrupeds
are red. In symbolic form this will read;

\[
\begin{align*}
\text{all } a & \text{ is } b, \text{ or } a - b = 0. \\
\text{some } a & \text{ is } c, \text{ or } ac \neq 0. \\
\text{Therefore, } & \text{ some } b \text{ is } c, \text{ or } bc \neq 0.
\end{align*}
\]

It is evident that the middle term cannot be eliminated by ad-
dition, since the middle term is positive in both premises.
Where an equation and an inequation occur the conclusion is de-
derived by expanding the premises until each contains all the terms.
This may be done in virtue of the Law of Expansion of the alge-
bra, theorem 4.6 \(ab = a(b + -b) = ab - a -b\); p. 37). Thus,
the first premise becomes

\[
\begin{align*}
a -bc & + a -b -c = 0,
\end{align*}
\]

and the second will be

\[
\begin{align*}
acb & + a -bc = 0.
\end{align*}
\]

Again there is present an equation and an inequation, but now
the two premises have one term in common \(a -bc\), and by another
theorem, 4.71, \((a + b = 0 \text{ is equivalent to the pair, } a = 0 \text{ and } b = 0); p. 39\), it follows that

\[
\begin{align*}
a -bc & = 0 \text{ and } a -b -c = 0,
\end{align*}
\]

\textsuperscript{115} Lewis and Langford, 56.
which makes the first premise become null. Then by theorem 6.32 (if \( a + b = 0 \) and \( a = 0 \), then \( b \neq 0 \), p. 47), the second premise becomes

\[
abc \neq 0,
\]
but by theorem 6.41 (if \( ab \neq 0 \), then \( a \neq 0 \), p. 47), if \( abc \neq 0 \) it follows that

\[
bc = 0,
\]
which reads some \( b \) is \( c \), or, some quadrupeds are red. And this is the conclusion of the syllogism. We see that by an algebraic process we have again suppressed the middle term, and arrived at the proper conclusion.116

Let us now consider several applications of the algebra to syllogisms that are known to be fallacious. The algebra in such cases, the writers say, will do one of two things; if a true conclusion is possible from the premises it will give the correct conclusion, if no conclusion is possible the algebra will present an equation that is incompatible, or an equation and an inequation which have no term in common. For example, let us take a syllogism in which the middle term is not taken universally at least once. All plants are living things, but, all animals are living things, therefore, all animals are plants. Written symbolically:

\[
116 \text{ Ibid., 56-67.}
\]
all \( a \) is \( b \), or \( a - b = 0 \)

all \( c \) is \( b \), or \( c - b = 0 \)

therefore, \( all \ c \) is \( a \), or \( c - a = 0 \).

According to the general rule, whenever the two premises are universal, the conclusion should follow by adding the premises and thus eliminating the middle term, but this will not work here because the middle term is not positive in one and negative in the other premise. By adding the premises we have

\[ a - b + c - b = 0, \]

which is an incompatible equation.\(^\text{117}\) This cannot be solved by expanding the premises either, since both will reduce to zero.

Again, let us take an illegitimate argument in which the first premise is universal and the second particular. No conclusion follows. For example: All dogs are animals, but, some animals are rational, therefore, some dogs are rational. In symbols we have

\[ all \ a \ is \ b, \ or \ a - b = 0 \]

\[ some \ b \ is \ c, \ or \ bc \neq 0 \]

therefore: \[ some \ a \ is \ c, \ or \ ac \neq 0. \]

According to the general rule of the algebra, when one premise is universal and the other particular, the conclusion is attained by expanding the premises. Thus,

\(^\text{117}\) Ibid., 59.
\[ a - b = 0 \text{ becomes } a - bc + a - b - c = 0, \text{ and} \]
\[ bc \neq 0 \text{ becomes } abc + -abc \neq 0. \]

Here we find, however, there is no common term, and a conclusion is not forthcoming.\(^\text{118}\)

Thus far in the application of the algebra to the syllogism we have found the algebra consistent with the syllogistic inferences of traditional logic, at least, as far as the conclusions of the syllogisms are concerned. In fact, the conclusions of the algebra will be in conformity with the syllogisms of traditional logic in all the moods of the syllogism in which both premises are universal and the conclusion is universal, or, when one premise is universal and one is particular and the conclusion is particular. The algebra, therefore, can be successfully applied to all the traditional moods of the syllogism except the moods EAO and AAI of the third figure. The algebra is accurate in all the other moods, and besides, will frequently not function when the syllogisms are invalid.

Why does the algebra not arrive at the same conclusion as traditional logic in these particular instances? Are these moods of traditional logic invalid? Let us investigate by an example. The syllogism EAO, No animal is rational, but, all animals are mortal, therefore, some mortals are not rational, is,

\(^{118}\) Ibid., 59.
according to modern logicians, invalid. In symbolic form it reads,

\[ \text{No } a \text{ is } b, \text{ or } ab = 0 \]
\[ \text{all } a \text{ is } c, \text{ or } a -c =0 \]

therefore: \[ \text{some } c \text{ is not } b, \text{ or } c -b \neq 0. \]

In this syllogism we have two universal premises, so according to the general rule a conclusion should be had by adding the premises thus,

\[ ab + a -c = 0 \]

but, since the middle term is not positive in one premise and negative in the other, no conclusion can be drawn. The modern logicians attempt to explain this by calling attention to the form of the syllogism. The two premises of the syllogism affirm that something is equal to 0, and the conclusion that something is not equal to 0, or in other words, the premises assert a non-existence, and the conclusion an existence. Thus the syllogism is invalid because it implies an existence which is said not to be implied in the premises. Lewis and Langford write in this regard that "universals affirm that something is = 0, and particulars that something is \( \neq 0 \). That is, a universal proposition in extension asserts a non-existence. ... and a particular proposition in extension asserts an existence."\(^{119}\)

As a result, modern writers of logic immediately conclude that

\(^{119}\) Ibid., 52.
certain modes of inference are invalid. Padoa, in his introduction to Peano's system, remarks that this discovery of the invalidity of certain traditional modes of inference "is one of the first and most remarkable results of the adoption of a logical ideography."120

Since, according to the modern logicians, a universal proposition never implies existence and a particular always implies existence, any inference from a universal proposition to a particular is going to be invalid. To strengthen this conclusion another argument is presented which centers around the notion of the null-class. Lewis and Langford write:

A particular conclusion cannot validly be drawn from a universal premise, ... . The simplest case in point is the traditional inference of "Some \( a \) is \( b \)" from "All \( a \) is \( b \)." This fails, as a fact, whenever no \( a \) exists. If \( a \) is an empty class \( (a = 0) \), then "All \( a \) is \( b \)" \( (a - b = 0) \) is true, but "Some \( a \) is \( b \)" \( (ab \neq 0) \) is false. If there are no centaurs, then "All centaurs are Greek" is true, but "Some centaurs are Greek" is false. In fact, if \( a \) is an empty class, every universal proposition with \( a \) as subject is true, but every particular proposition with \( a \) as subject is false. If there are no centaurs, then "All centaurs are \( x \)" will be true, no matter what \( x \) is: all the centaurs there are, will be anything you please. Also, "No centaur is \( y \)" will be true, whatever \( y \) may be. But "Some centaur is \( z \)" and "Some centaur is not \( w \)" will be false, for every \( z \) and \( w \).121

121 Lewis and Langford, 62.
If we ask, why must this be true? They respond, "Because the principles of the algebra require this."\textsuperscript{122} This, of course, is no answer at all, since the traditional logician would merely say that the algebra must be wrong because the principles of traditional logic require it to be otherwise.

Sometimes, however, the argument is stated in a different manner and rests, then, upon a principle of being. The argument when stated in such a fashion becomes a challenge to the traditional logicians. The case of the centaurs, which is considered and empty class or null-class, can be used as an example. The propositions "All centaurs are Greeks," and "Some centaurs are not Greeks" are said to be contradictory propositions, and therefore cannot be both true nor both false at the same time. Modern logicians say the particular proposition, "Some centaurs are not Greeks," could only be true if some centaur existed, but, since centaurs do not exist, it must be false. If, then, the proposition, "Some centaurs are not Greeks," is false, certain modern logicians say the contradictory must be true, namely, "All centaurs are Greeks," despite the fact that centaurs do not exist. The particular proposition could be true, it seems, only if there existed centaurs; and so, since the particular cannot be true, the universal contradictory must be true even though centaurs do not exist. Hence, they conclude from this, that the

\textsuperscript{122} Ibid., 62.
universal propositions do not imply existence, and particular propositions always imply it. About this last point a question at present will perhaps give the general trend of the solution which will follow. Does a particular proposition imply existence because it is a particular proposition, or, is it because it expresses a contingent truth?

Before we take up that point, let us turn back to the primary problem, which centers around the principle of contradiction. It is not hard, it seems, to see the fallacy of this reasoning. First of all, the principle of contradiction is a principle of being, and the principle, therefore, is going to hold only when applied to some form of existing being. Yet, all form of being is denied when it is said that centaurs have no existence, for, as St. Thomas remarks, Ratio autem entis ab actu essendi sumitur.\textsuperscript{123} Nothing more can be said about centaurs since no proposition, much less a contradictory one, can be mentioned, unless some type of existence is implied. This is even more evident from what Maritain points out, "all propositions affirm or deny the actual or possible, real or ideal existence of a certain subject determined by a certain predicate."\textsuperscript{124}

Therefore, there can be no universal proposition about a subject

\textsuperscript{123} Thomas Aquinas, \textit{Quaestiones Disputatae et Quaestiones Duodecim Quodlibetales, De Verit., I, 1, ad 3 in contr.}, Marietti, Taurini, 1941, III.
\textsuperscript{124} Maritain, 53.
that has no existence at all. The universal contradictory proposition of "Some centaurs are not Greeks" could be had only if some type of esse were present, since contradictory propositions require esse and non-esse. But if centaurs have no existence, there would be no esse of any kind in either the universal affirmative or particular negative proposition. Furthermore, neither truth or falsity can be had once existence has been denied, since truth follows existence. Thus, once centaurs are denied all form of existence there can be no question of contradictory propositions, or of truth and falsity.

The point of the argument will perhaps be somewhat clarified if we discuss the nature of existence. Whenever the verb to be is used, it signifies some type of existence, either ideal or real. The whole purpose of the copula in a proposition is to express the relation of identification between the subject and predicate -- two objects of thought, distinct as concepts but identified in the thing, whether this thing has actual, or possible, or ideal existence. Thus, whenever the verb to be occurs, it expresses the identification of two things which actually exist in the real physical order, or of two things that can exist in the real order, or of two things that can exist only in the mental order. For example, the proposition, "My horse is red," signifies the actual existence of "my horse," but the

125 Aquinas, De Verit., I, 1, c.
proposition, "A right triangle is a triangle with one angle equal to 90 degrees" signifies that the "right triangle" an object of thought exists with possible existence outside the mind with the form "one angle equal to 90 degrees." The copula in both of these cases signifies the real existence of the objects, in one case it is actual, and in the other, possible. In a proposition in which the object is expressed as not having real existence the same copula is used. Thus the proposition, "A chimera is unable to exist in reality," expresses an ideal existence, or an existence that can be had only in my mind.\footnote{Maritain, 52.}

Hence, the verb to be in a proposition always expresses some form of existence. This existence may be actual or possible real or ideal. Thus, for the clarification of the above difficulty presented by the modern logicians, two points should be stressed, (1) when we speak about existence, actual existence is not necessarily implied, and (2) if all existence is denied an object, it is no longer possible to speak about contradictory propositions, since the principle of contradiction is a principle of being and requires esse and non-esse. In the difficulty, then, a proper understanding of existence, whether it be real or ideal, and a proper understanding of esse and non-esse for contradictory propositions must be had. The whole purpose of the argument is to prove that universal propositions do not
imply existence, and particular propositions always imply existence. It is quite evident, therefore, that whatever the proposition is it must imply some form of existence. Whether it implies ideal or real existence is not going to be determined by the form of the proposition, that is, whether it is universal or particular, but by the matter, that is, whether it expresses a necessary truth or a contingent truth. When a modern logician begins to speak about existence being implied in propositions his science is no longer strictly formal in the sense in which he understands it to be formal.

Modern logicians, because they believe they have proved certain traditional moods of the syllogism to be invalid by their ideographic signs, immediately conclude that other traditional inferences are also invalid. Lewis and Langford enlarge upon this point.

These facts point to the invalidity of certain other traditionally sanctioned modes of immediate inference also; for example, the inference "Some b is a" from "All a is b"; and the inferences "Some a is not b" and "Some b is not a" from "No a is b." All of these are invalid, committing the same fallacy of inferring an existence from a non-existence. ... The same fallacy affects the traditional doctrine of the 'square of opposition,' according to which A and E, contraries, are supposed to be such that both may be false but both cannot be true; I and O, subcontraries, such that both may be true but both cannot be false; A and O, E and I, contradictories, such that both cannot be true and both cannot be false. I is supposed
to follow from A and 0 from E. None of these relations really hold except that of contradiction between A and 0, E and I. When \( x = 0 \), A and E are both true, I and 0 both false; the supposed relations of contraries, subcontraries, and subalterns all break down. ... Since a general principle of inference which sometimes fails is never really valid, this entire traditional doctrine -- except for contradictories--must be abandoned.127

Summing up the statements of the modern logicians these seem to be their chief claims. (1) Universal propositions imply no affirmation of existence, and, (2) particular propositions always imply existence. In consequence of these claims all inferences in traditional logic from a universal proposition to a particular are held to be invalid. Thus, the moods of the syllogism AAI and EAO, subalternation, and partial conversion are considered invalid.

What appears to modern logicians as invalid reasoning on the part of the traditional logicians, appears so, only, it seems, because the modern logicians are not fully acquainted with traditional logic, and the work of the scholastics. The fundamental problem, namely, that there is a certain margin of thought that frequently cannot be expressed by the oral sign in discourse, is not recognized by many modern writers. This point was recognized by the early logicians, and by means of the

127 Lewis and Langford, 63-64.
supposition, amplification and other properties of a term, they made allowance in order to solve the difficulty which some modern logicians still maintain is left unsolved in traditional logic. The fact that modern logic has difficulty with this point, where traditional logic has none, shows the limitations of an ideographic logic. Inferences that can be rightly held by traditional logicians, who hold that logic is an aid to thought, are condemned as invalid by modern logicians because their strict mechanical process, which is suppose to replace thought, has no way of coping with the intricacies of a spiritual mind. How, precisely, does the traditional logician explain the difficulties of the modern logicians?

129 Maritain, 226.
130 Ibid., 1.
132 Maritain 232-233.
CHAPTER IV

SUPPOSITION -- AN ANSWER

TO SOME OF THE MODERN LOGICIANS' DIFFICULTIES

Modern logic, it is frequently said, is a strictly formal science, and as such prescinds from the matter of propositions. This being true, it is impossible, it seems, for modern logic to investigate the existential or non-existential import of a proposition, which depends upon the matter of a proposition. "The fundamental error of the logisticians," writes Maritain, "lies in their failure to distinguish between the form and the matter of propositions and in their belief that considerations bearing exclusively on the form suffice to explain the entire discourse." 

The primary assertions of modern logicians that lead to the differences with traditional logic seem to be that universal affirmative propositions imply no affirmation of existence, and particular propositions imply an affirmation of actual existence. Is this true? On first appearance it seems to be true, since traditional logicians write that "propositions in necessary matter ... do not require the actual existence of the subject in

133 Church, 181.
134 Maritain, 226-227.
60.

order to be true." However, this only needs explaining to see its true meaning.

The subject in a proposition may have a natural or an accidental substitutive value. Natural supposition is had when the predicate is essential to the subject, in which case the proposition expresses an eternal truth, that is, the predicate essentially belongs to the subject. A proposition expressing an eternal truth, therefore, "does not require the actual existence of the subject to be true, and has not necessarily and of itself an 'existential' sense." Now, if what the modern logicians say is true, namely, that universal propositions imply no existence -- supposing here that existence means actual existence, -- then it follows that all eternal truths should be expressed in universal propositions. That is the point being investigated at present.

Although a proposition expressing an eternal truth "abstracts from time in its verification," nevertheless, Maritain adds, "it is false that [this] is realized only in universal affirmative propositions and is always realized therein." This he then proves by examples.

135 Ibid., 227.
136 Joannes A S. Thoma, 32.
137 Maritain, 227.
138 Joannes A S. Thoma, 32.
139 Maritain, 227.
Every man is mortal.
Some man is creable.
Some animal (viz., man) is rational. 140

All of these propositions express eternal truths, and do not necessarily imply an existential sense, yet, two of them are not universal propositions. Thus, there are particular propositions also that do not imply existence, because particular propositions as well as universal propositions may express eternal truths. This is true because in propositions which express an eternal truth the subject-term always stands for something, for in this instance "the copula denotes possible existence only, and is thus outside of time." 141 Such a situation may be had in particular propositions and in universal propositions. Whether I say, "Every man is mortal" or "Some animal is rational," "I can and always shall be able to show by thought a thing or an essence in the order of possibles of which I may truthfully affirm," 142 "This is a man," "This is an animal." Hence, whether a proposition is universal or particular, the existence or non-existence that is implied depends on the matter.

Moreover, there are universal propositions that imply existence as well as particular propositions. For, just as there are propositions in necessary matter, so there are propositions

140 Ibid., 227.
141 Ibid., 61.
142 Ibid., 61.
in contingent matter, that is, propositions in which the pre-
dicate is accidental to the subject. In such propositions the
subject has an accidental supposition. John of St. Thomas
writes:

Suppositio accidentalis est acceptio termini
pro his solum, de quibus verificatur iuxta
exigentiam verbi, seu alio modo: est acceptio
termini pro eo, cui praedicatum non intrinsece
sed accidentaliter convenit, ...\textsuperscript{143}

Such propositions, since they express contingent truths, require
the actual existence of the subject in order to be true, because
the predicate, since it does not follow from the essence of the
subject, could be verified only of a subject which actually
exists.\textsuperscript{144} This situation, however, is true not only for parti-
cular propositions but for universal propositions as well. For
example, the following propositions mentioned by Maritain ex-
press contingent truths and demand the actual existence of the
subject, yet they are not limited merely to particular proposi-
tions.

Some angel is damned.
Every man is born in sin.
All were taken prisoners.\textsuperscript{145}

It is evident, then, that there is a certain amount of truth
in the statements of the modern logicians, since some universal

\textsuperscript{143} Joannes A S. Thoma, 32.
\textsuperscript{144} Maritain, 227.
\textsuperscript{145} Ibid., 227.
propositions do not imply an existential sense, and some particular propositions imply actual existence. But it is actually an invalid inference to argue because "some" therefore "all." For we have seen that all propositions in necessary matter, whether universal or particular, do not imply actual existence, and that all propositions in contingent matter, whether universal or particular, do imply actual existence.

Furthermore, since logic is an art which facilitates the operations of the mind, it is only a means to an end and not the end in itself. Thus, when a proposition expresses an eternal truth, there is no reason why the mind cannot apply an existential meaning to the proposition even though the proposition in itself is concerned only with the possible existence of this subject with a certain predicate.146 This frequently happens in the experimental sciences, that is, the universals obtained by induction are given an existential implication. Maritain cites several examples of this:

Every acid makes litmus paper turn red.
Every mammal is viviparous.

He continues:

Taken in themselves these propositions would, no doubt, remain true even were there neither acids or mammals, but actually we never think of them without understanding that there are acids and that there are mammals.147

146 Ibid., 228.
147 Ibid.
Again, when a particular proposition has a predicate that is accidental to the subject, such a proposition implies the actual existence of the subject in so far as the supposition is accidental, and does not express an eternal truth. But even in such a case, the mind may vary the supposition, and give it merely an ideal or possible existence. The ancients were accustomed to this, and called the operation the "amplification" of the term, or, if the term was suppressed from a major to a lesser supposition it was called "restriction" of a term.

Secundo vero modo ampliatio et restrictio etiam termino singulari convenit; potest enim aliquod singulare verificari in pluribus differentiae temporum. Et tunc dicitur ampliari terminus quantum ad differentias temporis, quando verificari potest in pluribus temporibus seu differentiis temporum disiunctim seu divisim. ... in ampliatione tamen logica attenduntur quinque differentiae temporum, scilicet praesens, praeteritum et futurum, possible et imaginabile.

The point to be noted is that among the "differentiae temporum" are enumerated actual existence, possible, and ideal existence. Existence, then, is not limited just to actual existence; but possible existence and ideal existence are all special types of existence, and yet are existence in every sense of the word. "This dog is white" expresses actual existence, and the statement, "A myriagon is a ten-thousand-sided polygon" also

148 Ibid.
149 Joannes A S. Thoma, 37.
expresses existence, but a possible existence, that is, it is able to exist in the physical order. On the other hand the statement "The centaur is a fabulous creature" expresses existence, but ideal existence, that is, an existence that is only in the mind. It was not uncommon to vary the supposition from ideal to actual, or actual to ideal existence. The whole process did not require any special reference in the proposition itself, but the logician who had made a study of the diverse properties of the term was conscious of what took place, and was careful to keep the kind of existence, whether ideal or real, the same throughout his reasoning process. If he understood the premises to imply actual existence, he was consistent and maintained an actual existence in the conclusion. For example, the syllogism,

   All men are mortal,
   But Stalin is a man,
   Therefore, Stalin is mortal.

The major premise asserts an eternal truth, and in itself does not imply actual existence, but the traditional logician would be careful to understand actual existence, and, therefore, keep the supposition consistent throughout the reasoning. Again, the syllogism that the modern logicians keep presenting to show the ambiguities of traditional logic causes no trouble once the supposition of terms is understood. The syllogism,
No absentees are failed,
But all absentees receive a grade of zero,
Therefore, some who receive a grade of zero do not fail.
The syllogism arrives at a perfectly valid conclusion as long as it is remembered that the existence implied in the two premises is restricted to actual existence. This use of the supposition is frequent in traditional logic, and was clearly understood by traditional logicians before they began a study of syllogistic inference. It is in the light of this that Javelli remarked, Tu autem adverte novitie, quod praedictas defensiones servare non poteris, donec intellexeris tractatum suppositionum et ampliationum et appellacionum et probationum terminorum.\textsuperscript{150}

Thus, the assertions of the modern logicians about the implication or non-implication of existence in the universal and particular propositions of traditional logic seem to be nothing more than inaccuracies. First of all, there are universal propositions which imply actual existence, although the predicate is only accidental to the subject. And besides, there are universal affirmative propositions which in themselves do not imply actual existence, but, by a use of the supposition or amplification or restriction of terms, may be understood to imply real existence. Secondly, particular propositions, which modern

\textsuperscript{150} Javelli, 168.
writers say imply actual existence,\textsuperscript{151} may express eternal truths, and, in so far as they express eternal truths, they do not imply actual existence because a proposition that has an essential supposition or expresses an eternal truth "abstracts from time in its verification."\textsuperscript{152} Furthermore, a particular proposition in which the subject has only an accidental supposition and implies real existence, may be understood as having only an ideal or possible existence.

In consequence of this, it is immediately evident that subalternation, partial conversion, and the syllogistic moods AAI and EAO are valid if consistency in the supposition is observed. For example in subalternation,

if we say, in abstracting from actual existence in the A proposition:

\textit{Every man has imperfections (whether or not men exist)}

we conclude rightly that:

\textit{Therefore some man (even for example a saint) has imperfections, for the subalternate also abstracts from actual existence.}\textsuperscript{153}

Likewise, with partial conversion and certain syllogistic moods of inference we conclude legitimately if we keep the supposition of existence consistent.

The difficulties that modern logicians find with traditional

\textsuperscript{151} Lewis and Langford, 63.
\textsuperscript{152} Joannes A S. Thoma, 32.
\textsuperscript{153} Maritain, 229.
logic stem in part, it seems, from the mathematical procedure of modern logic. It was Descartes and Leibniz who dreamed of a universal mathematics which would be an instrument for all knowledge, 154 and it was Russell who finally declared mathematics and logic to be identified.155 But it is impossible for logic to have a mathematical clarity, and to function with the rigid mechanical procedure of mathematics. Logic is an art, a servant to man's mind, and as such, it cannot be a system that is rigid and mechanical. Maritain briefly summarizes the consequences of a system that fails to take into consideration the immaterial nature of the mind. A system that adopts too summary an ideographic logic, he writes,

reveals the fundamental falsity of every alleged Logic that aims at fixing the work of the intelligence once and for all in ideographic symbols and requires, not that these symbols signify the diverse inflections and the nice edge of thought more exactly than ordinary language -- a perfectly legitimate ambition -- but that they substitute a certain regulated manipulation of algebraic signs for the work of thought itself. An ideographic logic, thus conceived, could never be adequate to its object unless it were to replace the difficulties of rational labour by an infinite material complication. In truth it could not fix thought except as in the way that most stains used in Histology fix living matter -- by killing it.156

154 Kocourek, 98.
156 Maritain, 233.
CHAPTER V
THE USE OF THE SUPPOSITION OF TERMS
IN MODERN LOGIC

The discussion thus far has centered around an understanding of the supposition of terms in traditional logic as a refutation of certain arguments of modern logicians. It is evident that the use of supposition has a very significant place in traditional logic, so much so, that a neglect of it leads to many absurdities. Since it is of such importance in traditional logic, it will not be amiss to investigate its nature and use in modern logic. Perhaps the question might be asked at the outset: does modern logic have a place for the use of the supposition of terms? In other words, do the symbols in modern logic have some substitutive value? One would search in vain in modern symbolic logic books for an explicit treatment of this subject. Our treatment and conclusions, therefore, will be derived from the nature of modern logic and various statements of modern logicians and mathematicians.

It became evident in our treatment that modern logic has its roots in mathematics, or, as Father Wellmuth puts it, "it is itself a generalization of mathematics or (perhaps more accurately) an extension of pure mathematics: inasmuch as the system
form of pure logic is supposedly interpretable in terms of logical elements and logical relations from which the elements and relations of pure mathematics are derived.\textsuperscript{157} In consequence of this nature of modern logic, the method of development employed follows the technique of pure mathematics. Pure mathematics, we are led to understand in the early chapters of Lewis and Langford, is a science which has for its starting point certain assumptions or undefined terms and unproved propositions, which are chosen more or less arbitrarily. These undefined terms and unproved propositions are nothing more than class characteristics, which are to be fulfilled by certain objects. The objects which fulfill these conditions are then said to be contained in a certain class and are symbolized by various characters.\textsuperscript{158} Hence, since modern logic follows the technique of pure mathematics, it, too, has a similar beginning and development. Logic "is developed in the same deductive fashion," write Lewis and Langford, "from a small number of undefined ideas and a few postulates in terms of these."\textsuperscript{159} It follows, then, that a system-form of symbols is also necessary in logic.

But what are these objects, or classes in some instances, for which the symbols stand? Father Wellmuth in explaining

\begin{itemize}
\item \textsuperscript{157} Wellmuth, 477-478.
\item \textsuperscript{158} Lewis and Langford, 1-77.
\item \textsuperscript{159} Ibid., 23.
\end{itemize}
Bertrand Russell's understanding of a class gives us some inkling of what they are. He writes, "Each object in the totality must itself be a possible object of thought, a thinkable 'something-or-other'; it need not be a concrete existing individual, but may be a mere abstraction."\[160\] Hence, the objects contained in the various classes must be "possible objects of thought" which conform with the postulates and assumptions upon which the class is founded. Do these objects have any reality about them? That is, is there anything that corresponds to them in the real order? This question is of little importance to mathematicians. In fact, certain mathematicians criticize philosophers for sometimes demanding that all mathematics be made applied mathematics. Yet, mathematicians will not say that these objects which are represented by the symbols correspond to nothing in the real world, because they cherish a hidden hope that sometime an application will be found.\[161\] However, it seems that until this application is found these objects remain purely beings of the mind. Nevertheless, it is important to note here that there are present "objects" for which the symbols are substituted, even though the objects are beings of the mind.

The purely abstract or ideal nature of these objects is

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\[160\] Wellmuth, 220.
brought out when we consider the meaning of the symbols that are used in the logic. The symbols in mathematical or modern logic are constituted to signify the objects of thought, the thinkable "something-or-other." Some of them signify objects without any particular reference to any definite characteristic of those objects. Others, on the contrary, specifically signify definite characteristics of objects. Thus the symbols constitute a language of thought and are a medium of communication. As such, of course, these symbols are subject to the user's intention. But when the question is asked what does the symbol mean, Father Wellmuth remarks:

It will be noticed that the complex question 'what does a symbol mean?' reveals on analysis, not the slightest trace of such a question as, 'what is the something-or-other, or what is the definite characteristic, to which a symbol refers?' ... the symbol tells us absolutely nothing about 'the something-or-other as thought of' to which it refers, except that it is being thought of, or at least suggested to us to be thought of, by the user of the symbol.  

He then adds:

Neither does the meaning of a symbol raise any question about whether the object to which it refers exists or not. The most a symbol can do in this direction is to indicate that something-or-other is being thought of as existing, it is necessary that what it refers to should be a thinkable something-or-other, more or less distinguishable in thought from other 'thinkable somethings'. Further questions as to what that something-or-other is and

162 Wellmuth, 249.
whether it exists or not, have no bearing on the significance of the symbol itself.\textsuperscript{163}

George Boole reflects a similar opinion when he writes, "Those who are acquainted with the present state of the theory of symbolic algebra are aware that the validity of the processes of analysis does not depend upon the interpretation of the symbols which are employed but solely upon the laws of their combination."\textsuperscript{164}

From the above quotations it is evident that the symbol may stand for anything at all, as long as it is a "thinkable something-or-other." The important point to us at present is not what the "thinkable something-or-other" is, but that there is a "thinkable something-or-other" for which the symbol stands.

Before proceeding further a point might be expressed about the something-or-other for which the symbol stands. It seems to be true, as Boole remarks, that a set of symbols may receive any number of interpretations. But it must be remembered, however, that it is always assumed by all symbolic logicians that that "indefinite something-or-other" (1) remains definitely the same throughout the whole logical process, and (2) that when it is applied to a definite thing, then the former indefinite symbol is so applied as to mean always the same definite thing. In

\textsuperscript{163} Ibid.
\textsuperscript{164} George Boole, The Mathematical Analysis of Logic, 3, as quoted by Enriques, 127
a sense, the indefinite is always definitely determined. This still does not mean, however, that the interpretation given is without its ambiguities. A symbol is a symbol only in so far as it directs someone's attention to a thinkable something-or-other. But when an interpretation is given the symbol, two questions, as Father Wellmuth also notes, might be asked: (1) What object of thought does the user intend? And when the object of thought is recognized, (2) does the symbol represent the results of a correct logical analysis? Thus despite the exacting efforts of modern logicians in the representation of the objects of thought through symbols, they still have their difficulties, for the most accurate symbol in the most carefully written treatise cannot tell us the precise intended meaning as used by a particular user on a particular occasion. The difficulties that might arise from this are beyond the scope of this paper. It might be remarked, however, that all the difficulties that follow from an improper use of supposition will be present. The point that we wish to make at present is that although the symbols may receive many forms of interpretation, they still have a substitutive value, even though it is a purely ideal one. That is, whenever an interpretation is given, the symbols stand for a definite thinkable "something-or-other" in the mind.

Our analysis of modern logic, considering its origin from

165 Wellmuth, 289.
mathematics and its use of symbols, seems to indicate that each symbol has a definite substitutive value. A problem arises, however, which makes it difficult to see immediately the rectitude of our conclusion. This problem centers around the notion of the null-class.

The null-class is symbolized by the sign "0", and is the class which has no members. This class, we are told, has importance in modern logic and accounts for new positive relations. Hence it appears to give positive results, yet has no members. It is, apparently, a symbol which has no substitutive value; that is, it does not stand for a definite thinkable "something-or-other."

It will be necessary to investigate precisely what the modern logicians understand by the null-class. Mrs. Langer is rather clear on the point. She writes:

The word "the" means more than "all"; it also expresses the fact that there is at least one element in a certain class. If we speak of "The wife of King Arthur," we mean (1) that we refer to the whole extension of the concept "wife of King Arthur," and (2) that at least one element falls under this concept. So "the" really means "all", and "at least one." The singular noun "wife" then adds, "there is just one $x$, such that $x$ is a wife of King Arthur."

But suppose that King Arthur had remained celibate; would the form "$x \in \text{wife of King Arthur}\) define no class? Would the concept "wife of King Arthur" have no extension?

If we know what sort of thing we mean by
noun or a descriptive phrase, then there is a class of things defined thereby, for a "class" is exactly the same thing as a "sort." Now, there may be no wife of King Arthur; then there is simply nothing of that sort. That is the same as to say there is no element which is a member of that class. The class, therefore, is said to be empty, or to be a null class.

If a class is empty, or "null," then "all its members" means none at all.

Later on in her work she adds several other points of interest.

... the null class is the one and only class which may have incompatible properties, and more than that, it is the class which has all incompatibles properties which all absurd combinations of concepts define. It is the class of round squares, secular churches, solid liquids, and fellowmen without fellowmen. For to any of these we must say there is no such thing.

And again in speaking about null-classes she says:

... their extensions are all alike, namely "nothing," so all null-classes are identical, and we may speak of "the null class."

The null class is defined by any form that has no true values. It is the extension of any concept that has no application.

In writing about the null-class it may be noticed that Mrs. Langer makes no distinction between the empty class and the null class. This is not the opinion of all modern logicians. Bennett and Baylis write:

Again, other classes, although determined by

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167 Ibid., 128.
168 Ibid., 131.
self-consistent concepts, may as a matter of fact have no members. The class of my yachts is an example.

Classes which have no members are called empty classes. If they are determined by a self-inconsistent concept, so that they could have no members they are said to be not merely empty but also null. 169

From Mrs. Langer's remarks two points might be noted. First, she states, "If a class is empty, or null, then 'all its members' means none at all." Apparently, then, the null-class is a class that cannot be a class at all, since it can have no members. Without members there is no thinkable something-or-other which could be signified by it. Hence, such an understanding of null-class can be of no value to logic since there is no possible object of thought. Besides this, there is a second consideration that Father Wellmuth points out. If the null-class means a class with no actual members but one that might possibly have some, another observation might be made. Such a class is more properly called an "empty class." For example, the class "The wife of King Arthur," or the class of "my yachts," represent empty classes since they have no members. In this class would be included what the traditional logician would call "real possibles." This class has a class characteristic which distinguishes both the class and its members from all other classes.

and members thereof. This property is the property of "not actually existing," which again gives nothing in the way of what the symbol "0" stands for in a reasoning process.170 As Father Wellmuth observes, since every symbol must stand for a thinkable "something-or-other," "the question of whether the 'null-class' is useful for logic can hardly be raised, for in this sense 'null-class' does not refer to a possible object of thought at all."171

This conclusion seems to be absolute when one considers the understanding and definition that modern logicians give for the null-class, that is, a class of which "all its members" means "none at all."

The mathematicians, as mathematicians, however, view the question in a different light. The null-class for them does have a definite significance. It represents the point between 1 and -1. Such an understanding of the null-class is applicable to mathematics in which the negative numbers have some significance. This significance is expressed by Professor Young:

If numbers are interpreted geometrically by the points of a straight line in the familiar way by choosing an origin 0, and representing the positive numbers by points to the right of 0, the negative numbers by points on the left, it is at once seen that a number represents not

170 Wellmuth, 230.
171 Ibid., 229.
merely a distance (magnitude) from 0, but also one of two directions from 0.\textsuperscript{172}

The mathematicians as logicians attempt to understand the null-class in a similar fashion. Bennett and Baylis write:

The analogy between the number 0, introduced as an extension to the system of natural numbers, 1, 2, 3, ..., and the null class is doubtless obvious. ... The null class, no matter what its ontological status, performs a similar function in the logic of classes.\textsuperscript{173}

The analogy between the number "0", and the null-class is not as obvious as it may seem. In mathematics the symbol "0" has significance because it represents a point between 1 and -1. But in logic, what would be the meaning of a negative number? In logic to speak about a negative something-or-other means to speak about nothing at all. If, then, -1 would mean nothing at all, would a -2 mean twice a nothing-at-all, and more nothing-at-all than -1? Hence, although the mathematicians may have a place for the number 0, still the mathematicians as logicians cannot find a place for it in logic.

An interesting point arises now in relation to the expression of the principle of contradiction. For if the symbol "0" stands for a point between 1 and -1, as the mathematicians say it does, then the null-class cannot be the contradictory of 1,

\textsuperscript{172} Young, 108.
\textsuperscript{173} Bennett and Baylis, 105.
but only a contrary. Yet, the modern logicians write, "\( a - a = 0 \) = -aa states the Law of Contradiction: Nothing is both \( a \) and not-\( a \)." If the null-class in logic performs a function similar to the number 0 in mathematics, precisely what meaning can the above principle have? From what has been said it seems that the null-class, although perfectly useful in mathematics, has no meaning in logic, for if it follows the use of the number 0 in mathematics, the principle of contradiction cannot be stated, and if it does not follow the 0 in mathematics, it seems to be meaningless.

The null-class and its implications is worthy of more attention than is possible in this study. At this point, however, it might be well to point out that the null-class is not something that is new to modern logic, but long ago it had found a place in traditional logic. Modern logicians are not conscious of this fact, and make the boast that the traditional logicians were ignorant of the null-class, and, since they did not take it into consideration, modern logicians say that some traditional principles are false. It is true traditional logicians did not refer to the null-class as a null-class, but they were conscious of it although under a different terminology. To prove this, let us first of all summarize the chief characteristics about the null-class. From the understanding which we are given by

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174 Lewis and Langford, 30.
Mrs. Langer, these characteristics appear. (1) The null-class is the class that has no members. (2) It is the class which has all incompatible properties which all absurd combinations of concepts define. (3) It has no extension. The class of centaurs, for example, or the class of square-circles are null-classes; they have no members at all. In general, then, the class of centaurs the class of square-circles, or any other class which has all incompatible properties are classes which stand for nothing at all, or, as the traditional logicians would say, "they have no supposition." It is true traditional logicians did not speak about null-classes in modern terminology, but, were they not speaking about the same thing when they spoke de subjecto non supponente? Propositions, then, in which the subject has no supposition are nothing more than propositions about a null-class. For example, it seems to come to the same thing to say, "All square-circles are red," is a proposition about a null-class, or to say it is a proposition about a subject which has no supposition. John of St. Thomas gives example of such propositions. Angelus corporeus movetur, or Deus creatus intelligit. These are propositions about null-classes or classes which have incompatible properties. John of St. Thomas refers to them, however, as propositions about subjects that have no supposition.175 Such propositions, John of St. Thomas remarks,

175 Joannes A S. Thoma, 168.
are false, because they are affirmative propositions about subjects that have no supposition.\footnote{Ibid., 29.} And we might add, that they are false because they attribute an existence to a subject which can have no existence. Such propositions, it is evident therefore, have no supposition. But, do they have signification? Or, to state the question more broadly, do propositions in which the subject has no supposition have signification? John of St. Thomas seems to distinguish on this point. He makes it clear that propositions which have an intelligible subject, as for example, \textit{Antichristus fuit}, or \textit{Adam est}, have signification even though the subject has no supposition.\footnote{Ibid., 168.} Propositions, on the contrary, whose subjects are composed of incompatible properties seem, from what John of St. Thomas writes, to have no signification or supposition.\footnote{Ibid.} If they have signification it seems to come from the simple terms which are joined into a complex term in the proposition. The simple terms in themselves have supposition and signification, but as a complex term, the only signification seems to be that of incompatibility. For example, the term \textit{angelus}, and the term \textit{corporeus}, or the term \textit{square}, and the term \textit{circle} have supposition and signification outside the propositions \textit{Angelus corporeus movetur} and "A square-circle is red." In the propositions the subjects, \textit{angelus corporeus} and

\begin{footnotes}
\item[176] Ibid., 29.
\item[177] Ibid., 168.
\item[178] Ibid.
\end{footnotes}
square-circle, have no supposition or signification because the subjects are made up of incompatible properties. The only meaning the proposition might have, it seems, might be called a negative signification; that is, the meaning of the proposition seems to be its incompatibility.

The proposition about a null-class attributes an existence to a subject which the subject cannot have, and as a consequence the proposition is false. The proposition is not false primarily because it attributes a predicate to a subject which the subject does not have, although it is false on that count also, but, it is said to be false first of all, because it attributes an existence to a subject that has none. In such a case contradictory propositions are impossible, because there is no esse possible. And no esse is possible because of the lack of anything to realize esse.

There seems to be several reasons why modern logicians find a place for the null-class in their logic. (1) Modern logic is said to be strictly a formal science; because of this formal nature, modern logicians do not have a clear idea of esse, and its importance in logic. (2) Modern logicians forget a proposition about a null-class, or about a subject that has no supposition, if it is affirmative, is a false proposition. Traditional logicians saw that such propositions are false, and hence could have no place in a valid reasoning process. Thus,
it is clear that the traditional logicians were conscious of the nature of the null-class, but because they saw that propositions about a null-class were false, they realized that the null-class was useless in logic.

Mathematicians seem to forget what they say as mathematicians when they speak as logicians. Professor Young writes an interesting paragraph when treating some objections about the abstract point of view in mathematics. He writes:

Professor Klein has recently made a vigorous protest against this point of view. To regard the objects of mathematical study as mere empty symbols sounds the death knell of all science, he says. He recalls the witty though uncomplimentary characterization recently made by Professor Thomae of men who concern themselves exclusively with meaningless symbols and empty assumptions concerning them. Thomae dubbed such men "thoughtless thinkers." The axioms of mathematics are not arbitrary assumptions, Klein urges; but they are rather sensible statements ... . He seems to fear that the adoption of the abstract point of view will turn the attention away from the all-important possibility of concrete applications. This fear seems to us groundless. Anyone who should devote himself to the development of an abstract symbolism with no reference to its possible concrete applications would indeed deserve the epithet of Thomae.179

The opinions of various mathematicians as expressed in Professor Young's paragraph seem to reveal that everything is not

179 Young, 223.
perfectly satisfactory among the mathematicians in reference to abstract symbolism. About that, however, we are not at present interested. The point of interest is the last sentence of Professor Young, from which it would seem that all symbols for mathematicians have some definite intelligent purpose, and that even the symbol "0" means something. In modern logic, however, it seems impossible for the symbol "0" to have any value; yet the mathematicians as logicians use the symbol in logic, forgetting, it seems, what they said as mathematicians, namely, that symbols have some intelligent purpose.

Summing up, then, this perhaps can be said, that all the symbols do have some real substitutive value, and that those that do not have any value have no meaning, either in mathematics or in modern logic. A response to our first question now seems to appear, "Is there a place for the use of supposition in modern logic"? We can answer in the affirmative, that there is, and without it modern logicians become merely "thoughtless thinkers."

The necessity of this substitutive value of symbols is more obvious in logic than in mathematics. In mathematics the supposition is for the most part an intentional supposition, that is, the symbols have for their substitutive value beings of the mind or entia rationis. Such a supposition is perfectly legitimate since supposition may apply to entia rationis as well as
to real being, either possible or actual. The formal science that results from such an intentional supposition may be entirely satisfying from a mathematical point of view, but the facts which the principles of logic state are not, as Lewis and Langford would lead us to believe, simply facts of our own meanings in the use of language;\textsuperscript{180} that is, logic is not a strictly formal science, but the principles of logic have something to do with the character of reality. Even though a mathematical science can be built up without any apparent reference to reality true logic, it seems, cannot, since the logician is interested in more than a mere property or characteristic of a thing, which comes before the mind in order that the things may be arranged and mutually related and operated upon by thought; the logician is interested in what the things are.\textsuperscript{181}

Although the modern logicians are very careful to make it clear that their science is strictly formal, still in their treatment of logical principles they are forced to return to a material logic when their new technique is incapable of satisfying the "modalities of thought." Their science does employ, even though unconsciously perhaps, supposition in a general sense, that is, in so far as their symbols stand for some object of thought. This is shown by the fact that numbers or letters

\textsuperscript{180} Lewis and Langford, 212.
\textsuperscript{181} Wellmuth, 485.
or symbols in mathematical logic are not interchangeable, as they would be were there no supposition. Yet, because they lack a full knowledge of what supposition actually means, their ideographic signs cannot adequately express the different shades of thought. The truth of this is made evident by the rise of various semantic theories which aim at a proper determination of the meanings of the words as they are used in the different sciences.\textsuperscript{182}

The purpose of modern logic seems to be to integrate all our knowledge, as Professor Greenwood observes, "in a single rational perspective."\textsuperscript{183} The success of such an endeavor seems doomed to failure unless the operations of thought are founded on something outside the mind, "namely in the depths of a substantial ontology."\textsuperscript{184}

Mathematical logic is not without its good points. It has added many new ideas and concepts to the science of thought, and, in general, a wealth of details about logic. But, after an investigation of the modern technique, the analysis which modern logicians have accomplished is not as penetrating as some may suppose. Not only has there been negligence in the analysis of

\textsuperscript{183} Thomas Greenwood, "The Unity of Logic," \textit{The Thomist}, VIII (1945), 469.
\textsuperscript{184} \textit{Ibid.}, 470.
traditional logic, but a much deeper study must be made in modern logic itself before the methods of symbolization will achieve any lasting success. Until this deeper analysis has been accomplished modern logic can never replace traditional logic, and do the work which will need to be done as long as man does any non-mathematical thinking. Such conclusions are evident, it seems, from our discussion of supposition in modern and traditional logic.
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