A Comparative Study of Two Methods of Problem-Solving in Arithmetic

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A COMPARATIVE STUDY OF TWO METHODS
OF PROBLEM-SOLVING
IN ARITHMETIC

BY

MARY CECELIA MANGAN

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Arts in Loyola University 1933
VITA

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Member, Illinois Teachers Association and National Education Association.
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CHAPTER I

PROBLEM-SOLVING

This investigation is an inquiry into the relative effectiveness of two methods of solving arithmetical problems. The methods involved are the Conventional Formula Method and the Method of No Formal Procedure. The first of these two methods, as here understood, requires that the pupil should set down in writing, according to a certain prescribed formula, an analysis of the problem to be solved. This technique is in widespread use in the public schools of Chicago. In the Method of No Formal Procedure the pupil is left at liberty to work toward a solution in whatever manner he desires, being guided, however, by such instructions and suggestions as the teacher may have given. This second procedure is, so far as is known, the method more commonly employed in public-school classrooms.

The equivalent-groups method of experimentation has been used, and the subject of arithmetical problem-solving in a sixth-grade classroom has been singled out for the study. Will a detailed method of analysis in problem-solving prove more or less satisfactory than a technique involving no written analysis? Practice in which method will decrease variability in ability in
The report of this study is divided into the following parts:

1. A brief historical sketch of the development in the content and teaching of arithmetic and problem-solving.
2. Definitions of "problem."
4. Attitudes and abilities developed through arithmetical problem work.
5. Review of related studies.
6. Report of this investigation.

Early Teaching of Arithmetic in America

In the colonial schools arithmetic was a study to which little attention was given. It was not a subject familiar to teachers, the majority of whom were unfitted for giving instruction in it. By many instructors arithmetic was considered too difficult to teach, and as a result it assumed no place or prominence in the school curriculum. In some of the Latin schools arithmetic was studied in the fifth and sixth years only. The time allotted to such teaching was but one hour a week.

Usually, when arithmetic was given, it was taught principally for its commercial value. In Boston and other towns where interest in commerce and trade was prevalent this practice was quite noticable. Much more time was given to arithmetic in the
trade and commercial schools of that day, but even in those schools the procedure consisted in working difficult and often tedious problems and in memorizing long rules. Students desiring to enter the employ of merchants engaged in foreign trade, entered trade schools. Provisions were also made for adults who wished to learn or review the needed arithmetical content of the time. Thus we read that in 1718 "Mr. Browne Tymms Living at Mr. Edward Oakes Shopkeeper in Newbury Street at the South End of Boston" taught "Young Men Arithmetick and Merchants Accounts" (47:372).

The need for mathematical knowledge due to territorial expansion in the eighteenth century brought the teaching of the subject into prominence. With the increase in trade and commerce, arithmetic found a place of great importance in the curricula of the elementary schools of the nation.

Little was known in those days about teaching methods. Scarcely any recitation in arithmetic took place in the schoolroom. Ciphering books were used by the scholars who could afford them and the other pupils copied the problem and rule given by the teacher on paper. The pupils worked the problem at their desks and offered the finished product to the instructor when accuracy had been reached. The books or quires of paper sewed together served as permanent records for the problem and rules. Because of the widespread practice of placing the arithmetical content in notebooks, this early period
until 1821 is called the Ciphering Book Period.

In the schools of that time many of the problems given to the pupils to solve related to commerce and trade. The writer was fortunate to secure a copy of Scholarl's (sometimes spelled Scholar's) Arithmetic published in 1827 by Daniel Adams, and somewhat widely used by the teachers of the day. This important book contained scores of commercial problems.

Breslich, in *Arithmetic One Hundred Years Ago*, reproduces eight photographic copies of examples given in a schoolroom in this state between 1804 and 1808. He found that "no formulas were developed and all problems were solved by applying rules" (6:165). A rule followed the statement of the problem and the explanations of new terms. The student was expected to apply the rule to content of a similar type. The task of locating the correct procedure was not difficult because the rules were placed with the problems.

Over a century ago the method of the Swiss educator, Pestalozzi, greatly influenced a change in teaching. He taught that presentation of objects in class instruction helped the child to understand the content. This advocate of inductive instruction favored the study of the psychological development of the mind. Joseph Neef opened a Pestalozzi school in Philadelphia in 1809. A study was made of number relationships in the classroom. This technique was used by the advocates of the Pestalozzian manner of teaching. The country's extension during
this period gave an important place to arithmetic in the school curriculum because trade had increased.

Colburn, following the objectives of Pestalozzi, rationalized the number relationships in his mental arithmetic. Rapid changes in procedures followed the appearance in 1821 of his book entitled *First Lessons in Arithmetic on the Plan of Pestalozzi*. Many accounts by educators of the period have been left in the first two volumes of Barnard’s *American Journal of Education* which emphasize the effect of Colburn’s rationalization of arithmetical processes for the child’s educational welfare. He gave no rules in his *First Lessons*, but his second book, the *Sequel*, had rules for the work outlined in the first volume.

Referring to the standards endorsed by Colburn, Monroe, in his bulletin on the Development of Arithmetic as a School Subject, stated (34:70):

> In his analysis of the subject matter of arithmetic, Colburn distinguished between the processes of arithmetic which he calls "principles," and the application of arithmetic which he designates as "subjects." To him the "principles" mean arithmetic and the applications merely a field for the exercise of these principles.

Colburn’s arithmetics were interesting departures in methods of presenting the content to the school children of that period. His methods met with a cordial reception; one teacher, writing in Barnard’s *American Journal of Education*, published in
1856, Volume I, after over ten years' use of his *First Lessons*, declared it the most worthy school book in circulation in the nation (3:302).

Recently the present writer located a copy of Ray's *Practical Arithmetic, Third Book, by Induction and Analysis*, stored among old school books. It was published in 1857, and had served some relatives in Indiana. Listed on page 8, under "Observations to Teacher," the following instructions appear (43:8):

> When the solutions are completed, let some one be called on to explain the process, giving the reason for each step of the operation. Exercises thus conducted animate the class and by requiring the learner to explain every process, and assign a reason for every step, he learns to rely on his own reasoning powers.

As in the arithmetics published twenty-five and thirty years before Ray's *Practical Arithmetic*, so in this book articles (explanatory notes), rules, and "cases" (type problems explained in full) offered assistance to the pupil. Under "Promiscuous Examples" (43:194) we find: "What will the cost of 7 hogshead 23 gallons of wine be, $49.00 per hogshead?" A problem listed under division of fractions selected from many of the same type, "At 2-2/5 dollars for 1 yard of cloth, how many yards can be bought for $6.00?" (43:165), resembles in content one taken from Durell's *New Day Arithmetic* (15:168): "A baker uses 4/5 pounds of flour to make each loaf of bread."
How many loaves can he make with 20 pounds of flour?" In this modern arithmetic, under the caption, "Multiplying and Dividing Fractions," only ten problems printed in large type are put on a page. The boys and girls of seventy-six years ago had to struggle through many problems, completed through rules and cases, printed in much smaller type than in the modern arithmetics. In the textbooks and practice exercises of the present day, concrete material with vocabulary within the range of the children is plentiful.

Scientific investigations began in earnest during the nineties, when Stanley Hall developed his psychological studies of children, and so commenced the child-study movement. In 1895 Dewey published with McLellan a book The Psychology of Number. It contained a statement of significance (32:32): "Number is the product of the way in which the mind deals with objects in the operation of making a vague whole definite."

In the same year the Report of the Committee of Fifteen of the National Educational Association declared that too much importance was placed on arithmetic in the elementary school. Teaching procedures underwent a change, and psychological studies of children at the elementary-school age level were advocated.

During the first period of testing in arithmetic Rice, in 1902, issued a report on measurements in that subject. Unstandardized tests of the period were comparisons of school ratings
After Stone published his results on reasoning tests in 1908, the arithmetical testing movement gained momentum. In 1909, during the second period, the Courtis tests began to appear. After a series of trial tests, he administered tests in the New York City schools in 1912. Wide variations in the scores of pupils in the same grade were found, and overlapping averages among children of lower and upper grades were shown. Bonzer had published reasoning tests in 1910.

The tests appearing after 1915 constitute the third period in the test movement. Starch published tests in 1916, Monroe in 1918; and Stone issued a revised group of two reasoning tests in 1927. Speed (57:275) and power tests (57:294) in arithmetic have been recorded at various times. Trabue (57) has listed a number of them. Worth While source material on two types of survey tests is mentioned by Kelley (28:43).

Many changes in problem content have taken place since the Ciphering Period. Since 1918 investigations have been made centering around locating techniques for presenting the content in an understandable manner. Experiments dealing with locating difficulties in reasoning have been conducted. Studies have recorded the types of errors in fundamentals and in reasoning. Detailed accounts have been given of the results which follow the introduction of unfamiliar words in the content.
Definitions of "Problem"

Thorndike, in the Psychology of Arithmetic, declares (56:9-10):

The aim of the elementary school is to provide for correct and economical response to genuine problems, such as knowing the total due for certain real quantities at certain real prices, knowing the correct change to give or get, keeping household accounts, calculating wages due, computing areas, percentages, and discounts, estimating quantities needed of certain materials to make certain household or shop products, and the like. Life brings these problems usually either with a real situation (as when one buys and counts the cost and his change), or with a situation that one imagines or describes to himself (as when one figures out how much money he must save per week to be able to buy a forty-dollar bicycle before a certain date).

A problem is a situation seeking through its question a solution. In the solving, some method of thinking must occur. The course followed by the individual depends on his training and reasoning ability. Burton (9:119) thus defines the difference between good and bad thinking in problem-solving:

The poor thinker is satisfied with the first information he meets or with a haphazard solution which brings him to grief later. The careful thinker suspends his judgment,
even in the face of seeming certainty, until the analysis of the problem and the search for information are completed. He may even wait until verification by experiment is under way before fully accepting an answer.

Since the law of effect demands that ideas which secure a permanent place in the mind must be satisfying to us, teachers should use only problems which in some way appeal to the child. Whether this appeal is to be found only in problems useful in the daily activities of the pupils is a mooted point. The verbal portion of the problem must be within the child’s understanding. Despite the story movement in problem solving, some teachers still cling to old-type problems outside the experience of the pupils. Many real problems can be used in the class work and uninteresting types can without loss be omitted.

Brueckner states (7:264): "Teachers should aim to state their problems in arithmetic so that mental reactions and activities of the pupil will be similar to those used in life itself." Activities from the construction of a box to the building of a schoolhouse require problem-solving ability. Klapper (29:269) has explained the term problem in the following way: "A problem is a situation coming naturally into the life or experience of an individual and capable of arousing his effort for its solution."

The trend at present in problem-solving content is to have the story portion of each situation suggest the processes nec-
necessary for a solution. Morrison (37:248), writing on the use of proper assimilative material in mathematics, states: "Every feature of the examples given for study, however, must itself be within the comprehension of the pupils, for otherwise it will not focus upon the unit." The child should realize that the ability to diagnose any problem as to method will be a help in his daily experiences.

Youthful surf-board riders observed by the writer on the beach at Waikiki, evidently derived satisfaction from the mathematical calculations necessary to allow their boards to dodge the white surf of the Pacific, and glide them safely to the security of the sand. The calculations were mentally accomplished with apparently no displeasure by those tiny Americans.

Some students display weakness in ability whenever any situation demanding solution is present. Boys and girls have been observed shopping at various times. They were grimacing, frowning, and emotionally upset because their reasoning power in problem-solving was limited. Their faces showed fear lest the change received from their purchases would bring rebuke from their parent if a deficit was revealed. The atmosphere around those children sparkled with unreliability and instability. Are the deductions right or wrong?

The explanation for such situations supplied by E. R. Hamilton (19:138) is this: "A problem is a situation that can only be reacted to intelligently, even though he gets the right
answer; he has not solved a problem." Again, when referring to the concrete quality of a problem, he wrote (19:139):

it must refer to a situation which is sufficiently familiar to the pupil for him to be able to realize the full significance of the data and to see clearly what it is that he has to find out.

Psychological Processes Involved in Problem-Solving

The pupil's manner of thinking in arithmetical problem-solving must be considered in attempting to analyze the steps which the youthful problem-solver meets. How does the pupil carry on his work and what mental pictures must he focus clearly? In the thought procedure, the student must have the ability to arrive at a conclusion with the quantities present in the situation. A computational knowledge is also necessary in reaching a definite solution. Thorndike has said (56:20): "Reasoning is essentially the organization and control of habits of thought."

When content known as a problem in arithmetic is given, what method does a mind follow? The teacher must attempt to understand the mental processes through which a child must delve when problem material is faced. The director must remember that in reaching a problem solution, however interesting it is, two fields of educational psychology are represented. They are the native equipment of the child and the psychology
of learning how to solve. Leo J. Brueckner's articles, published in the October, 1931, *Journal of the National Education Association* (8:241), states:

The psychological function of arithmetical instruction is concerned with the development in the pupils of the power to think accurately and precisely and the ability and disposition of the individual to think quantitatively about aspects of the environment which, to be dealt with effectively, must be dealt with quantitatively.

The child, confronted with a situation, must follow definite procedures between the reading of the problem and the arrival at the answer. These procedures are as follows:

1. in perceiving a problem, reading with understanding should be the logical first part of the process. In this reading a clear understanding of the conditions of the problem should be sought. The problem will be read with a question in mind and a desire to see the work through. Again Brueckner (7:266) writes that

   The first step is a complex process involving eye movement, perception, association of a meaning with symbols and combining the several elements of meaning into an understanding of the problem.

   Every problem includes at least three quantities, two present and the remaining one to be located. Many children begin to solve the problem without a careful reading. They lack a clear picture of the information supplied and what is
to be found. Overman concluded after a study of this question (41:257):

If left to their own devices
the pupils' tendency is to start
doing something before they really
know what the problem is or have
any definite plan for its solution.
They jump blindly into the middle
trusting to luck that they will
come out safely at the other end.

2. An analysis of the problem into its parts should follow careful reading. The divisions can be sorted to prepare for a solution. Mathematical relationships must be analyzed in this procedure, by recalling previously studied situations which suit the story of the moment. A careful reading will help to recall a similar setting. If memory is thus active, the individual's interest is aroused. With interest and recalled relationships working toward the problem solution, reflective thinking should come. The clue is located and a procedure will be tried.

3. The student will form a tentative hypothesis. The plan of procedure will be based upon the recalled relationships and the arithmetical meanings deducted from the reading with understanding.

4. After a tentative hypothesis has been decided on, a test of that hypothesis should follow. The operations selected as necessary by the individual will
depend on the recalled relationships and procedures.

5. Then a decision will be reached, by putting together all the parts of the problem that have been analyzed. A solution to the carefully studied situation is ready for presentation. The individual has been able to think through a method of proceeding. He not only knows what to do, but why he is doing it. In *How Children Learn*, Freeman said (16:218-19)

> When this higher type of learning which is called problem solving reaches the stage in which we definitely and consciously pass through a number of steps in order to reach a solution, and clearly recognize that these steps are dependent upon one another because they lead in the direction of the solution, we call the mental process reasoning.

6. A decision will be executed when the problem has been clearly studied, and a decision concerning procedure reached. Purely mechanical ability in computation is involved at this stage. However, memory is often used. For instance, if in the placing of a decimal point, the student would remember the rule he had learned, and then apply it to the problem story at hand, more correct solutions would be found. 

7. The last step in problem-solving should be the checking of results. The solution of arithmetic problems shows two kinds of errors:
(a) Errors in the thinking procedure necessary to bring an accurate conclusion.

(b) Errors in the mechanical calculations.

In 1845, W. M. Gillespie writing in Barnard's American Journal of Education undoubtedly had both of the great causes for incorrect solutions in mind when he stated (17:535):

Another remark we think important. It is of no use to arrive at a numerical result, if we cannot answer for its correctness. The teaching of calculation should include as an essential condition, that the pupils should be shown how every result, deduced from a series of arithmetical operations may always be controlled in such a way that we may have all desirable certainty of its correctness; so that, though a pupil may and must often make mistakes, he may be able to discover them himself, and never to present at last, any other than the exact result.

In considering results, reasoning again enters the solution. Many errors can be detected if, after the results are obtained, the pupils will stop to think what the answer means. If it seems logical when reading with understanding is repeated, then the pupil can be reasonably sure that the thought processes are correct. If the result is entirely out of reason, then the pupil sees that the thinking in the problem was wrong, and decides it must be solved again. The only way to check mechanical errors is to work the problem over again. Again reasoning plays a large part in the checking of results.
in summing up the writer would say that the psychological processes involved in problem-solving are:

1. The recognition of certain facts, called perceiving a problem.

2. Analysis of the facts, which leads to the separation of known relationships and unknown quantities.

3. The use of recall to place these facts in their correct relations and so form a tentative hypothesis.

4. Orderly thinking, or reasoning, which recombines the parts which have been broken up by analysis, into the correct solution of the problem, and

5. Judgment, which is necessary for the purpose of seeing whether the answers are proportionate to the numbers used.

Concerning reasoning Starch wrote (48:445)

Reasoning, even of the most original and inventive type, probably consists fundamentally in starting with a certain idea, desire, or problem, in short, with a stimulus, and in waiting for associations to arise and then in following out in turn by trial and error, one link after another, and in waiting for each one to bring up its links until a chain of successful links arises which satisfies the desire or which meets no opposition and which is then selected.

Judd (27) has contributed valuable evidence of the activities which take place in the child's mind before he can plan the solution of a problem. His study is a contrast in analysis of counting in the abstract, and counting of objects and sounds.
in his laboratory analysis he has compared the method in which
children count with the procedure followed by adults.

After citing some cases to reveal how reflective thinking
must be accomplished by the problem-solver between the stimulus
and the arrival at the answer, Dewey distinguishes five steps
in the process of solving a problem (14:72):

Upon examination, each instance
reveals, more or less clearly, five
logically distinct steps: (i) a felt
difficulty; (ii) its location and
definition; (iii) suggestion of possi­
ble solution; (iv) development by
reasoning of the bearings of the sug­
gestion; (v) further observation and
experiment leading to its acceptance
or rejection; that is, the conclusion
of belief or disbelief.

Attitudes and Abilities Necessary in Arithmetic

Attitudes

The teacher should work to instill in her pupils certain
attitudes that are conducive to reasoning.

1. Neatness, which makes for carefulness of form.

2. Carefulness of form, which makes for accuracy of
thought.

3. Accuracy of thought brings satisfaction, which tends
toward thorough work.

4. Confidence, which should develop in problem-solving,
through the successful choice ways of solution.
Morrison (37:19), stating the objectives of teaching, writes: "We shall think of attitudes as being always either attitudes of understanding, where reflection and rationalization have been involved -- found typically in the field of the sciences; or attitudes of appreciation."

The attitudes necessary in aiding reasoning are:

Neatness

The pupil's written work should be neat, orderly in the method of arrangement, and have a general appearance of cleanliness. The student will learn, through studying data in a logical manner, that neatness and legibility are worth while and desirable. The systematic arrangement of the work will help the pupil to develop habits of cleanliness.

Carefulness of Form

Carefulness of form developed through the presentation of neat work written in logical order, will bring about, to some degree, an accuracy of thought. The child will desire to discriminate between the true and false. He will develop a responsibility for logical reasoning and accuracy of thought. By following a definite method in presenting problem work, the student will learn to search for relevant data and to eliminate unnecessary material. He will appreciate the value of examining the content in order to seek the relations between quanti-
ties and present them in an understandable way.

Accuracy of Thought

Accuracy of thought brings satisfaction which tends toward work of an understandable variety. Thorndike believed (52:14):

Almost all children like to have their tasks definite so that they can know what they have to do and when it is done, and enjoy the sense of action, achievement, and mastery.

By a definite statement the pupil knows what is expected of him, and goes to work and does it. When his task is completed, and he realizes that he has done a piece of work well, there is a feeling of just pride and honest satisfaction. At the same time a feeling of dissatisfaction with unreliable results should be present. A too easily satisfied pupil is not the one who achieves success in arithmetic. The pupil should have a sensitiveness for the correct solution that he will be forced to say to himself, "What is the trouble with this? My answer must be wrong, because six books cost less than one. I will work it over and find out what is wrong with it!" May we not hope that this searching, questioning attitude will carry on in life, after school tasks are finished?

Pupils should be interested in developing their skill in mathematics. In teaching, we should develop a spirit similar to that which leads a boy to take pride in his skill in basket
ball. This skill may help to develop such qualities as stick-to-it-ive-ness and perseverance. An eager, alert attitude of mind which keeps the pupil wide-awake to the world around him is of great value in other lines of work. The interested student of mathematics is constantly seeking for applications of it in his daily life. It will be a continual surprise to him to locate the involved relationships in problems encountered in the store, at home, and at play.

Successful Choice of Ways of Solution Brings Confidence

Problem-solving should develop self-confidence. In the problem there is always a choice of ways of solution. Training should develop an ability to choose the correct process and disregard all of the other processes. A pleasure arises from the correct solution. The correct solving of one problem develops ability to solve another. Deductive thinking follows, and the power to use the correct association. This is the foundation of all reasoning.

Parker, considering reasoning, said (42:326):

For example, we suggested that pupils should come to esteem open-minded, impartial, suspended judgment as an ideal, as a personal attribute which they desire to possess. Similarly, we suggested that they learn to be on their guard to hold the question under discussion.

The desire to do a piece of work neatly, systematically,
and correctly will always encourage the pupils to do their best work. The completed, intelligently finished work should fill their minds with pleasure in their achievement. In fulfilling their requirements for correct problem-solving, the children have been called upon to look over a situation as a whole, to pick out the necessary facts, and to combine them to reach a correct solution. To do this they may have to weigh, consider, and reweigh.

Abilities

Developing in children the ability to solve problems in arithmetic is one of the teacher's hardest tasks. The children must be prepared to act competently in familiar and in unforeseen situations. Only through the development of the power of reasoning can the student meet either the unexpected occurrence or the frequent happening successfully. In the development of logical thinking, computational ability is necessary if reasoning power is desired. Does the student who is accurate in the four fundamental processes display the same efficiency in problems involving reasoning? Does the pupil who is accurate in addition of fractions show the same proficiency in division?

Morton stated (38:295):

The end is the ability to solve the problems which one meets and the fundamental skills are the tools with which one works. The fundamental skills are important but we should not
our enthusiasm for training pupils in the fundamental skills blind us to the fact that the principal purpose of arithmetic instruction yet remains to be accomplished. Skill in solving problems is the main thing.

Many studies have been conducted to show that a relationship is present between ability in reading, and ability in reasoning in problem-solving. The Twenty-Ninth Yearbook of the National Society for the Study of Education consisting largely of the report of the Society's Committee on Arithmetic, describes an experimental study by Stone (52:589-99). The purpose of the work was to determine how diagnostic and practice exercises can help in improving reasoning ability. The tests were given in schools of Spokane and other cities. Initial and final scores from equivalently graded pupils were used. After difficulties in reasoning had been located through the preliminary test, practice exercises followed. The problem material stressed buying and selling situations. Survey tests were given before and after the experiment and again a year later. The superiority of the experimental group gave evidence that the practice exercises had helped. Stone's contribution is listed also in Buswell's 1930 bibliography on arithmetic investigations (10).

Bonser (4) conducted a series of experiments in reasoning problems with fourth-, fifth-, and sixth-grade pupils. His results showed that younger pupils possessing greater ability
received higher scores than older pupils in arithmetical reasoning.

Overman wrote that mastery in problem-solving must involve the plan of solution, and the successful rendition of that plan. He states (41:235):

This ability to plan the solution does not come through the blind following of rules or directions; it can only come from meeting many different kinds of problems and reasoning each through in terms of the relationships involved.

Rosse (46) used two groups of eighteen sixty-grade pupils to study gain in reasoning ability. The Otis Arithmetic Reasoning Test and the National Intelligence Test were used to equate the pupils. One group used practice sheets which offered drill in problem reasoning. An arithmetic textbook was used by the other group. After fifty-eight days of problem practice, the same form of the Otis Arithmetic Reasoning Test was given to both groups. The slight difference in achievement favored the group adhering to the practice sheet method.

In 1922 Banting (2) conducted an investigation in the Waukesha Public Schools. Children of elementary-school age were used in the study. Difficulties in arithmetical reasoning were under consideration. The Monroe and Buckingham Reasoning Tests were given, and the results scored. The daily records of the individuals selected for the work were observed. The review listed many causes for failure to reason in arithmetic.
CHAPTER II

REVIEW OF RELATED STUDIES

In this chapter other investigations not included in the studies mentioned in the previous division will be considered. Many distinct techniques and methods are available for review. Error studies bear upon the question of finding effective methods for problem solution, since they give insight into the difficulties encountered by children. Each study reveals some angle of the general problem by which the instructor can gain guiding information desirable in increasing control over learning techniques.

In order to encourage the development of problem-solving ability among pupils of elementary-school age, worthy material is essential. The content in a problem situation must be within the understanding of the students. Hall-Quest found (18: 316):

While it is true that skill in fundamental operations will occupy a very large place in the practical application of arithmetic, the ability to solve the variety of problems demanded of us each day is equally important. To know how to proceed in such situations is obviously invaluable.
Problem-reading must be emphasized if the productive corollary "R's"—reasoning and results—are expected. In the last decade numerous methods employing varied techniques have been reviewed, discussed, and applied throughout the nation. Many reasons for hesitation and often failure on the child's part to respond to problem content have been noted. The solution of arithmetical material involves activity on the individual's part. Dewey's reflection is true (13:353):

Only by a pupil's own observations, reflections, framing and testing of suggestions can what he already knows be amplified and rectified. Thinking is as much an individual matter as is the digestion of food.

Experiments and studies have recorded concerning:

1. Vocabulary Difficulties.
2. The influence of Terminology on Solving Problems.
3. Reading Trends.
4. The Error Factor and Its Influence on Accuracy.

Vocabulary Difficulties

Many difficulties confront children when unfamiliar words are used. The child's limited knowledge of language should be kept in mind by the problem writer.

Monroe, in an article in the September, 1918, issue of
School and Society, explains that there are two kinds of words in the statement of a problem. Some words describe the setting of the problem and other words affect the relationships and are called "technical words." He illustrates this point as follows (35:297):

What is the value of sugar obtained at a Vermont sugar camp, if it is worth ten cents per pound and 6 pounds are obtained on an average from each of 1,275 maple trees? Words in this problem, such as "Vermont," "sugar," "maple," and "camp" describe the setting. They have nothing to do with the solution of the problem. The technical words are such as "value," "per pound," "are obtained," and "each." They define the relationships which exist between the quantities and are cues for formulating the hypothesis or plan of solution which is another step in the process.

Osburn and Drennan (40) reported an experiment conducted in an elementary school in Wheeling, West Virginia. They had teachers of two classes of third-grade children teach a list of problems with emphasis on the language content or cues of the problems. A test consisting of twenty verbal problems containing new language "cues" with no additional vocabulary difficulties, was given after six weeks of instruction. On the following day, another test containing vocabulary difficulties, through the medium of such words as "chemist," "excavating," "sulfuric acid," "tortoise," and "gypsum," was administered. The data indicate that acceptable scores were made by the
children on both tests. Since only nouns were changed when vocabulary difficulties were introduced in the second test, the arithmetical difficulty remaining the same, the investigators concluded that the pupils sensed the meaning of the words. The vocabulary changes in the cues were not significant factors in the results. The tests following each other on succeeding days probably helped the pupils to sense the similarity of the two tests.

Children do not always understand as much about the meaning of words as their instructors give them credit for knowing. In an investigation conducted by Stevenson (50:98) a group of fifth-, seventh-, and eighth-grade pupils were asked to define and illustrate the word "average." The following are some of the definitions supplied by the children:

Average --

1. The answer to an addition problem 4 plus 6 equals 10.

2. The answer to a subtraction problem 1361 minue 146 equals 1221.

3. The answer to a multiplication problem 24 times 2 equals 48.

4. The answer to a division problem 42 divided by 6 equals 7.

5. The answer to any problem.

6. The amount of anything like 24 plus 35 equals 59.

7. The number right.
8. Is a grade or something.

9. Means your grade like 80 or 90.

16. The amount of the bill.

18. "So many things."

The present writer agrees with this author that many children have no adequate concept for the word (50:99).

Influence of Terminology on Solving Problems

Investigations show that the response of a pupil to problems differs according to the content of the exercise. He may read the problem carefully and yet, through lack of word knowledge, be unable to solve the situation.

Monroe (36) conducted an experiment to test the responses of children to verbal problems. A test was administered to 775 sixty-grade, 5902 seventh-grade, and 2579 eighth-grade pupils in over forty cities of Illinois. The pupils were divided into four groups, and equivalence was obtained by random sampling. The four tests were later onated so that in the distribution of them to the pupils, the first, thirty, ninth, would receive Test A, and the second, sixth, tenth would receive Test B. Tests C and D distributed in similar ways would make Test D the fourth, eight, and so forth. The tests given to the four groups differed only in the terminology used in the statements.

The questions for which solutions were sought were of the
following types:

1. Is there a difference in result between stating a problem concretely or abstractly?

2. How does the response of pupils carry to problems stated in technical words and to that content given in simple language?

3. Is there any difference in the answers of children to problems where unnecessary data is included and to those situations introducing only essential material?

Examples of the variations introduced into the tests follow:

The second problem in Test A was given in simple language with relevant content.

In Test B the data was relevant but technical terminology was presented.

While in Tests A and B the setting was concrete, in Test C it was abstract with relevant data.

In Test D abstract setting again was used with unrelated data and technical terminology.

It was found that the introduction of unnecessary data made little difference in the results. There was a slight improvement in the number of pupils trying problems when concrete material was used. When a familiar expression, "amount of a bill," was used with irrelevant data worthwhile results came. A familiar terminology is easiest for the pupils. Mon-
roe concludes (36:19) that:

A large percent of seventy-grade pupils do not reason in attempting to solve arithmetic problems. Instead, many of them appear to perform almost at random calculations upon the numbers given. When they do solve a problem correctly, the response seems to be determined largely by habit.

In the investigation of Wheat (61) the solutions offered by pupils to problems of the conventional and imaginative types were compared. The tests were given to two thousand fifth-, sixth-, and eighth-grade pupils in several towns in various parts of the country. In Problem Test 1, ten pairs of two-step problems were given, one conventional and one imaginative in each pair. The situation, the operations required for solution, and the time remained constant. Only the manner of stating the problem varied. The results showed only slight variation in the achievements of the pupils between the conventional and imaginative types. Less time was required for the conventional type.

Yet Washburne and Morphett (59) report that fifth-grade pupils get better results with problems containing familiar elements than with those stressing unfamiliar language. In the investigation 441 fifth-grade pupils in six different towns were used. All the children received a test of eight pairs of problems. The data and results report gains when familiar terminology is understood by the solvers.
In Bowman's study (5) pupils' preferences for different type problems were investigated. Children of junior high school level in schools of Sedalia, Missouri, were the subjects. One group contained pupils with I. Q.'s of 115 and above, the second group ranged from 114 to 88, and the third had I. Q.'s of 88 or below. Fifty problems, grouped five to a page, were given. Some problems discussed children's activities; others, adult's work. There were puzzle problems and some with computational difficulties only. After solving a page the pupil reported his preference.

Pupils with lower ability showed greater power to solve problems dealing with pure computation, and reported preference for problems containing no complicated situations. In the higher intelligence groups less difference in choice of settings was noted.

According to Thorndike, life experiences will supply more adaptable settings for satisfactory results in problem accuracy, than content beyond the comprehension of the reader. He states (55:127):

Many of the difficulties of pupils in learning and of the teachers in teaching problem-solving, are due to the use of problems described in words. With imposed tasks in no real setting the pupil is much less likely to know what the question is, or to have any strong interest in obtaining its answer. And these difficulties are, to a certain extent, unprofit-
able, since in life the question will commonly be his own and come in a real setting that helps to guide him to its answer. Life problems are thus easier than book problems.

Reading Trends

Several studies reveal that reading difficulties are important causes of error to many pupils in solving problems. Wilson (59) used a group of thirty-four sixth-grade pupils of low intelligence. The students were given the Stone Reasoning Test at the beginning of the investigation and again when the experiment was finished. Problem reading by a questioning procedure for twelve minutes, three times a week, was the method followed. The readings continued for five weeks. She concluded that reading drills centering on the meaning of problems brought considerable gain in the final Stone Reasoning Test scores.

Claude Mitchell (33) asked in effect in his experiment: (1) Will the pupils gain the ability to solve a general problem through the solution of a specific one? (2) Will the reaction of the individual be the same to situations of a general nature without numbers and to specific problems? He used two lists of problems. List A contained problems with numbers, and List B a set without numbers. Seventy eighth-grade and sixty seventy-grade pupils were tested. One-half of each class received Test A, followed by List B. The order was reversed in
the other group. The pupils in all the groups reached higher scores when the problems contained definite and specific numbers.

An experiment recorded by Lorena Stretch (51) measured the results obtained by special instruction in problem-solving over a period of thirteen weeks. The study was conducted in the fifth grade of a public school in Waco, Texas. There were thirty-two children in each group, control and experimental. The groups were equated on a composite score basis of tests in arithmetic reasoning, reading comprehension, and general intelligence.

The control group was taught forty minutes a day and five days a week. A four-minute exercise in fundamentals was given first, followed by the teacher's explanations of new procedures. Next, pupils demonstrated the explanations; then followed exercises in which all the students practiced the procedures. A review of the work of previous lessons followed, and then a drill in the fundamentals.

The experimental followed the control group for the second forty minutes of the morning session. This group received six-minute exercises in problem-solving, followed by instruction by the teacher. The students chose the operations in problems without performing the actual solving. A problem in analysis, followed by work in the fundamentals, was given, and finally a drill.
Final tests were the Stanford Reading Tests, the Otis Arithmetic Reasoning Test, and the Compass Diagnostic Problem Analysis Test. Greater gains were made by the experimental group. The gains showed that there is a relationship between the problem-solving ability and the ability of the pupils to comprehend the reading.

Locating the factors of difficulty in the understanding of problems was the essence of an investigation conducted by Hydle and Clapp (24). Problems were paired according to the types of difficulties studied. There was just one element of difference in the paired examples. Right elements of difficulty which disturb the accuracy of arithmetic problems were included in the five pairs of problems. Over seven thousand city and village children were tested. Two sets of problems were given to the two groups in each classroom, one set to each group. When the cues were easy to visualize, the percentage of pupils who succeeded was greater. The investigator believed that the reasoning ability of pupils depended on visualization, which was induced by the relationships expressed in the situation.

A study reported by Robertson (45) compared the ability of children to solve a set of problems read aloud by the teacher and a set read by the children. The children recorded the answers on paper. The test selected contained forty problems. The odd-numbered problems in the test were read aloud by the
instructor. Each group was given one test, and was allowed ten minutes. The Otis Reasoning Arithmetic Test, Form B, was administered at the close of the study. The investigator believed that more instruction in the two types selected for the instruction periods would produce gratifying results.

Stevens (49) found that ability in the fundamental operations was more closely allied with problem-solving ability than with reading ability. In this controlled experiment, training in problem analysis helped the slower pupils, but the class as a whole did not show higher records. The Stevenson Arithmetic Test was used in measuring reading related to problem-solving.

Lazenta, in a study (30) with an envelope test in solving arithmetical problems, reached the following conclusion:

The evidence suggests that pupils are not guessing as much as we sometimes think, when they are attempting to solve problems. Their success in attempts to write solutions parallels their performance in reading, in analysis and in thinking about the methods that should be employed.
The Error Factor and Its Influence on Accuracy

Lenore John's thesis (25) reports a study in problem difficulties. She followed the individual observation technique. She sought to gain evidence on the type of errors made by pupils in the intermediate grades, and whether errors made by children in Grades IV, V, and VI differ. The experimenter observed the oral work of each subject, alone. How the individual's mind reacted to the problem content was recorded. Pupils in the University of Chicago Elementary School and a near-by public school were tested on fifteen two-step problems. She reached the conclusion that errors decreased from grade to grade, although a child with a tendency to make errors frequently did not always decrease the number. Wrong processes and omissions of parts of problems were found to occur. Workers in the field of improving the child's ability to solve problems in arithmetic can well consider her conclusion:

Most of the errors which children make in solving arithmetical problems may be classified as errors in reasoning, in performing the fundamental operations, and in reading. These classes are given in the order of their frequency of occurrence (26:100).*

*This study was reported also in the November, 1930, Elementary School Journal.
Chase (11) studied some of the difficulties met in arithmetical problem-solving. In 1927, children in the Fordson High School who had a normal intelligence but were unsatisfactory in arithmetic, were tested. Blanks were filled in by teachers, giving information about the causes of difficulty. Seventeen of these were selected, and tests were given to that number of pupils. One subject scored low because of frequent change of schools. The author concluded that reasoning difficulties can be lessened through a case study of individual weaknesses.

Method Investigations

Lutes conducted an experiment (31) in 1925 in Sixth B classes of twelve elementary schools of Des Moines, Iowa. About six hundred pupils participated. A comparative study of three techniques was made. They were:

1. A computational method comprising drill in the fundamental operations.

2. The selection by the pupil of a correct operation from several present.

3. The selection by the pupil of a solution from many offered.

Each method was used with a separate group of children. In addition to the three methods used in the study, there was a control group taught by the technique required by the course of study. The groups were equated by measuring general in-
intelligence and arithmetical attainment. After twelve weeks of practice the Stanford Achievement Test was given. The author concluded that drill in computation does increase the students' ability to solve problems. The groups using the computational method showed the greatest gain.

Washburne and Osborne (60) experimented with three groups of sixth- and seventh-grade children in eighteen schools. The following procedures were used:

1. No suggestion as to method for the solution of the problems,
2. The analysis method,
3. The use with the same pupils of easy oral and difficult written problems for training in analogies.

The first and second group each contained over 300 pupils, and the third had 134 students. These groups were equated with respect to intelligence and problem-solving ability. First, one-step problems were given. Two sets of tests used included pictures which helped the pupils to visualize the problems. Most of the pupils had no difficulty with that type. In the analysis method, children would sometimes analyze correctly and solve incorrectly or analyze incorrectly and solve correctly.

The conclusion was reached that "ability to make the type of formal analysis frequently taught in school was practically no relation to solve problems" (60:22). The group following
the analysis method did as well as the group following no special technique. In the third part of the experiment, the average was only seven per cent higher when problems with familiar situations were given.

The experiment continued for six weeks, when a special problem test was given. The net change in achievement did not show significant differences in any one method of procedure.

Newcomb (39) used two control and four experimental groups differing in size from fourteen to thirty-six each. The groups of seventh- and eighth-grade pupils were equivalent in arithmetical reasoning ability according to the Stone Reasoning Test. The experimental group followed an analysis method as outlined by means of sheets of general directions. They solved one problem a day, while the control pupils were taught the same problems in the customary manner. After twenty days of practice the Stone test was again given. The experimental group showed superiority in speed over the control group, but were only slightly better in accuracy.

Washburne (58) in his experiment used two groups of second-grade pupils, two groups of fourth-grade pupils, and two groups of sixth- and seventh-grade pupils of sixteen cities of northern Illinois. The groups were equated on a composite-score basis. One group was taught the fundamentals in connection with verbal problems, while in the other group the pupils were taught verbal problems and the fundamental processes separately.
Final tests were given after six weeks. The difference in achievement did not favor either procedure. His findings show that the fundamental processes may be studied separately and then problems studied, or the two processes may be discussed throughout the period of learning.

In a study made by Clark and Vincent (12) a comparison between the formal-analysis method and the method of graphic representation was made. Two groups of forty seventh- and eighth-grade children in one school were equated on an arithmetic and a mental ability basis. Eight days of practice were given to the study of the two methods. The scores on the practice tests favored the dependencies method. To test gains in achievement the arithmetic part of the Stanford Achievement Test, Form A, was used as the close of the experiment. Gains made on the final test over the initial one showed the analysis method slightly more effective.

This dependencies method, called also the procedure of graphic representation, was one of the three experimental factors used in an extensive study conducted by Hanna (20,21). Children of the fourth and seventh grades were drilled on two- or multiple-step problems for a period of six weeks. The gains in arithmetic ability resulting from the study involving three methods of solving problems are recorded.

The groups were equated and initial arithmetic scores secured through a series of four standardized tests. About one
thousand pupils, distributed in twenty-four classes of the New York City schools, took part. Twelve seventh-grade and twelve fourth-grade groups were used. These classes of twelve in each grade were placed in three experimental groups, with four classes using each of the three experimental methods.

The teachers participating in the work received written directions for conducting the experiment. The children were given practice sheets, each containing seven problems. For the first seven days the pupils worked the problems on one practice sheet. On the eighth day and alternate days, until the study was finished, the problems were worked by the students without the help of the teacher. The experiment lasted six weeks, and in that time twenty practice sheets had been worked. The final tests given consisted of the identical test forms used in the initial test. These were the Stone and Stanford tests.

The results showed that the greatest gain with the dependencies method (Method A) was found in the fourth grade, especially with children below the grade standard in arithmetic. In the seventh grade the greatest gains were made by the pupils using the individual and dependencies method.

The children of superior ability in this grade favored the dependencies and the individual methods. With pupils of average and inferior ability the individual method proved the most effective. The conventional-formula method was found to be inferior to the other two methods used.
Adams (1) employed commendable techniques in the three experiments conducted by him in Pennsylvania. The first, known as the Philadelphia Experiment, took place during 1927-28. Pupils from third and fourth grades of ten public schools were selected. Over eight hundred pupils were taught by the analysis method, almost the same number followed the Course of Study in Arithmetic used in that city, and over five hundred followed the methods usually employed by teachers. Eight weeks were used in the experiment. The scores showed that the greatest gains in achievement were made in the experimental classes—that is, the classes using the analysis method.

In his second experiment, conducted with third- and fourth-grade classes in Reading, Pennsylvania, over 1,000 pupils were placed in the experimental classes and 1,065 control pupils were used. The teachers supervising the methods were paired according to their teaching ability. The final test came after seven week's work. The data secured did not show superiority for any of the methods used.

In the third report nearly 2,000 experimental and 1,836 control pupils were used. All of the ninety-six classes taking part in the work were paired on an initial arithmetic test. The analytical method of solving problems was used by the experimental groups and no urging of analytical reasoning was expected in the teaching of the control pupils. Each instructor or followed one procedure of teaching in her class.
Again, in this experiment teachers with approximately the same training and experience were used. After eight weeks the final tests were given. Detailed analysis was found to help in the lower grades, especially in the third. In the fourth grades, the method of solving many problems without the analysis technique increased the gain.

A study conducted by Hazer and Harap (22) and appearing in 1930 brought the conclusion that problem ability can be increased through the arithmetic activities, both in problem work and in the fundamentals.
CHAPTER III

THE PRESENT EXPERIMENT

In order to compare the relative effectiveness of the Conventional Formula (analysis) Method and the Method of No Formal Procedure in the solving of arithmetical problems, the writer set up a controlled experiment. The pupils were divided into experimental and control groups. The experimental group used the analysis procedure, while the control group solved the problems without a written technique. Both of these methods are described in detail on pages 51 to 53.

The pupils were tested at the beginning and at the close of the experiment on the identical form of the Stone Reasoning Test. An attempt was made to equate the groups so well that the differences in accomplishment of the subjects in the two groups might be attributed to the variation in the instruction plan.

The Experimental Set Up

The Subjects

The subjects for the experiment were pupils in the first half of sixth grade in a Chicago public school. The subjects
numbered twenty for each group. All the pupils remained in the group for the entire period of the study. Subject 10 B was absent six different days during the experimental period, but made up the missing exercises and tests.

EQUATING THE GROUPS

The subjects were equated on a basis of chronological age, scores in the Otis Intelligence Tests, and scores in a test of fundamentals. The Otis Self-Administering Test of Mental Ability, Intermediate Examination: Form A was given to measure ability to learn. The scores in this test were transmuted into intelligence quotients by means of the table supplied in the manual which accompanies the test.

The New Stone Reasoning Test, Form 1, has no time limit, but sixty minutes was the longest time required by any subject to try the twenty-one problems. This test yields scores for both reasoning and accuracy. Both scores were used for equating the groups. An informal computational test prepared by the writer, and complying with regulations of the Chicago course of study in arithmetic for fundamentals mastered at the close of the fifth grade, was given.

A tabulation of the pupils was made showing chronological age, Otis intelligence quotients, Stone reasoning scores, and results on the fundamentals. The pupils were then matched as nearly as possible in ability and chronological age. The groups
A and B, were built up by assigning a subject alternately to Group A or B. The subjects in the experimental group were designated Al, A2, commencing with the highest and descending to the lowest. In the paired group, called the control, the pupils were sorted in the same manner and called Bl, B2, down to B20.

All the work of equating the groups was done before the study began. The experimental procedure lasted ten weeks. In the eleventh week the final test scored for reasoning and accuracy was given.

Table I gives the distribution of subjects into two paired groups, experimental and control. Pupils were paired according to chronological age, intelligence quotients, Stone reasoning and accuracy, and a test on fundamentals. Means and standard deviations for the tabulations were found. The chronological age mean is given in months. The standard deviation formula used is given by Holzinger (23:108): "S.D. =
### TABLE I

Pairing of Pupils for Experimental
And Control Groups

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<td>11-7</td>
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<td>7</td>
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<td>12</td>
<td>79</td>
<td>3.2</td>
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</tbody>
</table>

Mean 142.6 102.15 5.11 4.6 6.65 142.65 102.2 5.0054.7 6.7

S. D. 11.04 1.11 1.2 1.5 11.10 1.0112

C. A. Chronological Age

I.Q. Intelligence Quotient

S.R. Stone Reasoning

S.A. Stone Accuracy

F. Fundamentals
Description of Methods

The Conventional Formula Method

The analytical procedure known as the Experimental Method in this study is based upon the theory that a measure of complete analysis of the elements in a problem in arithmetic should bring about a mode of reasoning that will be effective in similar situations. The pupil follows a definite procedure for each step of the work.

The outline advocated has five steps:

1. What is given in the problem?
2. What is asked in the problem?
3. The process or processes required to reach a solution.
4. The computational work.
5. The answer obtained.

The steps may be classified in the following manner: (a) statement, (b) question, (c) symbol to show process, (d) work, (e) result.

The following problem, solved by the Conventional Formula Method, will show the application of the technique:

If from a piece of pongee silk containing 10 1/2 yards, 7 3/4 yards were cut, how many yards remained?
1. Given.
   From a piece of silk containing 10 1/2 yards,
   7 3/4 yards were cut.
2. Question.
   How many yards remained in the piece of silk?
3. Process, -
   \[
   \frac{10\ 1/2 \text{ yards}}{7\ 3/4 \text{ yards}} = 2\ 3/4 \text{ yards}
   \]
5. Result.
   2 3/4 yards were left on the piece of goods.

The Method of No Formal Procedure

In the Method of No Formal Procedure the children were allowed to follow whatever method of solution they wished to use. However, the teacher impressed upon the members of the control group the desirability of following the thought or analysis procedure required by the other group without writing out the facts.

The same problem will serve for an illustration:

If from a piece of pongee silk containing 10 1/2 yards, 7 3/4 yards were cut, how many yards remained? The child read the problem with understanding, and tried to get a mental picture of a bolt of goods from which some yards were to be taken. On the paper
would be written:

\[
\begin{align*}
10 & \quad \frac{1}{2} \text{ yards} \\
- & \quad 7 \quad \frac{3}{4} \text{ yards} \\
\hline
2 & \quad \frac{3}{4} \text{ yards left on piece of silk.}
\end{align*}
\]

In the short-cut method used by some children who would not wait to acquire a mental picture, the word "remained" would settle the procedure they needed to complete the work. They would get the answer as hurriedly as possible.

Units of Problem Work

The next major step in this study was to write out units of work suitable for use in testing the progress of the groups. Each unit of work contained five problems. Thirty units at the rate of three a week were given on Tuesday, Thursday, and Friday for ten weeks. The units of problem work used in the study were written by the experimenter. The material organized was similar in content to problems given in arithmetic textbooks.

The content of the unit material presented a gradation of work from integers, dollars and cents, and fractions and decimals in the four fundamental processes and of an understandable variety. The work was motivated by utilizing in the units everyday language within the understanding of normal children of intermediate-school age. In this gradation of units some were introduced for the purpose of recall in the upper third of the set, which repeated in new arithmetical terms previously studied units.
During the first week of the experimental study, the control group had its arithmetic time from 9:15 A.M. to 9:45 A.M. daily. The following week the experimental group was given the first period for its work, followed by the control group. The groups alternated weekly during the experiment so that each class had the fifty thirty-minute period for an equal length of time. The same unit of work was given to each subject in the study on Tuesday, Thursday, and Friday.

The arithmetic periods on Monday and Wednesday of each week were used to clear up difficulties relating to the units under consideration, and to teach the new processes in decimals necessary in 6B arithmetic. The practice problems given and studied during the two days mentioned each week stressed the same type of situations found in the units for the week. Frequently during the informal teaching period, problems were written on the board. After discussion each group, during their instruction period, would solve the problems, using the technique assigned to it.

On the other three days of the week, during the respective arithmetic periods, each member of the group worked the five problems contained in a unit. Thirty minutes were allowed for the work period. Some children, especially in the control group, worked more rapidly than others and all time records were kept. Since the pupils following the analysis technique had considerable writing to do, full time was consumed by many
of them for several units. Each pupil had a mimeographed copy of the problems in a unit during the work periods.

The pupils following the method where written analysis was not required, timed themselves and entered on the top of the papers the number of minutes taken by them to complete a unit. An added check was made by the teacher, who recorded the time on each paper as it was completed. When a pupil finished the five problems in a unit and was satisfied that the best work possible had been offered, he brought the problem unit to the desk and could attend to any other matter until the thirty minutes were completed.

Since accuracy is stressed in problem work in the Chicago public schools, only problems correct in answer were marked correct by the teacher. The scores made on the units containing the five problems each are therefore, 100 - 80 - 60 - 40 - 20. If the pupil comprehended the problem, but had the wrong answer due to incorrect computation, or any one of the numerous causes of errors in problem-solving, no credit whatsoever was given. The units were marked on a right or wrong basis.

The identical form of the Stone Reasoning Test given at the beginning and end of the study to measure achievement was scored for accuracy and for reasoning.

Table II gives the processes contained in the five problems of each of the thirty units of exercises given to the group during the ten weeks of study and testing. As stated
heretofore, instruction and practice were given each week on the first and third days of each week, and the units containing the processes shown in Table II were given on the second, fourth and fifth days of each week.
### TABLE II

**Processes Involved in Each Unit**

<table>
<thead>
<tr>
<th>Processes</th>
<th></th>
<th>Processes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit 1</td>
<td>1 2 3 4 5</td>
<td>Unit 1</td>
</tr>
<tr>
<td>1 F+</td>
<td>F+ F- M-</td>
<td>16 Mx-</td>
</tr>
<tr>
<td>2 F+</td>
<td>F+ F- F+</td>
<td>17 D-</td>
</tr>
<tr>
<td>3 Fx</td>
<td>F- Fx+</td>
<td>18 I+</td>
</tr>
<tr>
<td>4 Fx</td>
<td>Fx- F:</td>
<td>19 Mx+</td>
</tr>
<tr>
<td>5 F:</td>
<td>I-Fx Fx-</td>
<td>20 Dx</td>
</tr>
<tr>
<td>6 Fx:</td>
<td>Fxx F++</td>
<td>21 Mx</td>
</tr>
<tr>
<td>7 I.Fx</td>
<td>M-- D:</td>
<td>22 I+</td>
</tr>
<tr>
<td>8 M: x</td>
<td>F: M+ M Mx</td>
<td>23 D+</td>
</tr>
<tr>
<td>9 M: x</td>
<td>M- F: x</td>
<td>24 D+</td>
</tr>
<tr>
<td>10 Mx</td>
<td>Mx F- Dx</td>
<td>25 D+</td>
</tr>
<tr>
<td>11 M- x</td>
<td>Dxx F-</td>
<td>26 D+</td>
</tr>
<tr>
<td>12 Dx</td>
<td>Dx Dx Dxx M-</td>
<td>27 D+</td>
</tr>
<tr>
<td>13 Fxx-</td>
<td>Dx- F+ Dxx M+</td>
<td>28 Mxx+</td>
</tr>
<tr>
<td>14 I-</td>
<td>Mx+ F+ Fx</td>
<td>29 D+</td>
</tr>
<tr>
<td>15 M+</td>
<td>M- Mx M Mx</td>
<td>30 Mxx+ F+</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>F. Fraction</th>
<th>D. Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Integer</td>
<td>M. Money</td>
</tr>
</tbody>
</table>
The following are typical examples of the units of work used in the experiment.

Unit 1

1. Mr. Eastman has three apple trees. He gathered 5 1/4 bushels, 3 3/5 bushels, and 6 1/2 bushels. How many bushels does he gather from all?
2. A dealer bought three car loads of coal, weighing as follows: 27 5/8 tons, 17 3/5 tons, and 31 3/4 tons. How many tons in all were there?
3. If, from a piece of pongee silk containing 18 2/3 yards, 7 3/4 yards were cut, how many yards remained?
4. If you had $45.50 in the Savings Bank and drew out $12.75, how much money would you than have in the bank?
5. A man bought some shares in the ownership of a business paying $86 3/8 for each share. He was compelled to sell them later for $74.25 a share. How much did he lose on each share?

Unit 5

1. If 3.8 of a yard of lace will trim the sleeves of one dress, how many dresses can be trimmed with 3 3/4 yards of lace?
2. A newsboy earns $3/4 every day for a month. He did not work on Sundays. How much did he earn during the month of March which had four Sundays?
3. In a certain school room having 48 pupils, \(\frac{3}{8}\) were girls. How many boys were there?

4. What is the cost of \(1\frac{3}{4}\) pounds of tea, when \(1/2\) pound is 25¢?

5. If you buy goods to the amounts of $6.50 and $7.25, how much change is due from a ten and a five dollar bill?

Unit 7

1. Mrs. Stone bought \(\frac{3}{4}\) dozen rolls at 24¢ a dozen, and 1 1/2 dozen buns at 30¢ a dozen. How much change did she receive from a dollar bill?

2. John made $55.25 and he spent $20.25. How much more does he need to make a $50 payment on a Ford car?

3. The rainfall in a certain city was 2.25 inches in June, 2.8 inches in July, and 1.15 inches in August. What was the average rainfall per month?

4. Eunice used \(\frac{3}{8}\) of a yard of ribbon in trimming a hat. How many hats could she trim with 3 yards?

5. Betty saw a red raincoat in a store window which was marked $5.50. She saw some galoshes costing $1.50. Betty had saved $4.75. How much more money must she get before she can buy the raincoat and galoshes?
Unit 18

1. If it is 85 miles from Chicago to Milwaukee, and 315 miles from Milwaukee to St. Paul, how far is it from Chicago to St. Paul?

2. Tom bought two books of stamps each containing 48 stamps. He used them all except 9, so he must have sent out how many Christmas cards?

3. Chester's father said, "The coal bin is almost empty. It costs more to heat this house every year. This year I bought 14 tons of coal at $10.50 a ton. Last year my bill was only $100." Chester said, "I'll find out how much more it cost." What did Chester find?

4. A boy worked 3 1/3 hours on Monday, 2 1/4 hours on Tuesday, and 2 1/2 on Thursday. He was paid 24 cents an hour. How much did he receive for his work on the three days?

5. John's home is 1.5 miles from school. He walks to and from school five days a week. How far does he walk going to and coming home from school each week?

Unit 25

1. A large family found that its household expenses for food during a certain 31 day month had been $196.25. What were the average expenses per day in dollars and cents?
2. The sales of a grocer during six days were $219.80, $258.25, $198.65, $278.95, $259.19, and $350.58. What was his average of sales per day?

3. James has his picture taken. His mother bought 1 1/2 dozen at $8 a dozen. What did she pay for them?

4. Joe's car read 22, 176.5 miles before a trip, and 23, 426.7 after it. How long was the trip?

5. When George started on the bicycle trip, his cyclometer read 91.5 miles. When he returned it read 102.8 miles. How far did he ride?
CHAPTER IV

ORGANIZATION, COMPARISON, AND INTERPRETATION

OF DATA

Analysis of Results on Problem Units

The number of pupils in the groups selected for experimentation remained the same throughout the time of the study. Twenty in each group worked the units and took the final test. The experimental factor, the written analysis of each problem, was completed satisfactorily by the members of the group following that technique.

As stated heretofore, the class was paired into two groups on the basis of the initial tests. For further comparison of abilities, each group was divided into three classes - the high-, medium-, and low-ability groups.

The first six pupils from A1 and including A6 according to their record on the initial testing, have been grouped and called high. From A7 through A14 is named the medium group. The low group included those subjects from A15 to the end or A20. The high and low groups each had six pupils, and in the medium group eight pupils were placed.
When the scores for the entire thirty units were completed, the mean for the high group on Unit 1 was found. An average was obtained for the six pupils in that group for each succeeding unit until thirty means had been secured. The same procedure was followed with the eight subjects in the medium group for each of the units from one to thirty. An average was made of the scores received by the six pupils of the low-ability group on Unit 1. Thirty means were found for this group for that number of units.

Then the pupils in the control group, consisting of twenty and rated from Bl to B20, were divided into ability groups. The six who were highest on the initial testing were placed in the high-ability group, the next eight in the medium-ability group, and the six lowest pupils in the low-ability group. The mean scores of each group on every unit from one to thirty were obtained in the same manner as the averages for the pupils following the experimental method.

Tables III and IV on pages 62 and 63 are a comparative record of the mean scores made by the pupils of the experimental and control groups in solving the problems contained in the units from one to thirty. A1 to A6 are the high-, A7 to A14, the medium-, A15 to A20 the low-ability group. B1 to B6 are high, B7 to B14 are medium, B15 to B20 are the low-ability group.
TABLE III

Scores Made on Units 1 to 15 by Experimental
and Control Groups of High, Medium,
and Low Abilities

<table>
<thead>
<tr>
<th>Unit</th>
<th>High</th>
<th>Medium</th>
<th>Low</th>
<th>Unit</th>
<th>High</th>
<th>Medium</th>
<th>Low</th>
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<td>73.67</td>
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<td>77.5</td>
<td>70</td>
</tr>
<tr>
<td>8</td>
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<td>76.67</td>
<td>8</td>
<td>86.67</td>
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<td>73.33</td>
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<td>15</td>
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<td>80</td>
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<td></td>
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</table>
TABLE IV

Scores Made on Units 16 to 30 by Experimental and Control Groups of High, Medium, and Low Abilities

<table>
<thead>
<tr>
<th>Experimental</th>
<th>Control</th>
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</thead>
<tbody>
<tr>
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<td>29</td>
<td>100</td>
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<tr>
<td>30</td>
<td>100</td>
</tr>
<tr>
<td>Total</td>
<td>90.02</td>
</tr>
</tbody>
</table>

Mean
In Method A, the mean scores for the high group range from 76.67 on Unit 1 to a perfect average of 100 on Units 29 and 30. Five of the six high-ability pupils had no difficulty in following the written-analysis technique required of the experimental group. Subject 5A found it very troublesome during the first third of the study to adhere to the written analysis. When the high group had reached Unit 8, the six pupils had attained an average score of 90. They held that mean until Unit 22 with two exceptions. On Units 16 and 20 they reached an average of 93.33. From Unit 22 the scores progressed upward. The average score for the high-ability group on the entire thirty units was 90.02.

The average score of the medium group of Method A ranged from 55 to 95. An average gain of 15 was made on Unit 2 over Unit 1. Fluctuating averages were found in the medium-ability group. The gain was not constant because the averages were retarded several times. This group did not reach a mean score of 90 until Unit 25. The total mean for the 30 units made by the medium ability group was 82.5.

The low-ability group in Method A received an average of 46.67 on Unit 1. Progress was hindered at first by the written analysis of the problems. When the written-analysis method had been followed for several units, the group began to make consistent gains. On Unit 30 the six pupils of this group reached 96.67. This score was 1.67 higher than the average
made by the medium-ability group on the same unit.

Pupil 19A, one of the six of the group, was an exceedingly slow writer and thinker. Until she had completed twenty units of work, her paper never contained five finished problems. The written analysis of the problems in each unit was very difficult for her to comprehend.

A greater range of means in this low-ability group was noted than in either of the other two groups of Method A, and also greater than any range of means in Method B. The total mean made by the group for the thirty units was 80.44.

The resulting scores show greatest gains for the low-ability group. The analysis technique hindered some of the pupils from completing the five problems in a unit during the allotted time, but the group progressed rapidly after Unit 22. They equalled the scores made by the medium-ability group on Units 25, 26, and 27 and exceeded their averages on Unit 24 and also on the last three units. The pupils of the group more than doubled the average score from the first to the thirtieth unit.

The gain in achievement favored the low group in this method, since the scores increased more consistently than in the other two groups. Fluctuations did not appear with this group. Only one is recorded in Figure 1 on Unit 12.

In Method B, also shown in Tables III and IV, where written analysis of the problems was not asked, the mean score for the high-ability group on Unit 1 was 73.67, and on Unit 30
an average of 93.33 was obtained. The gains on scores by the pupils at the high-ability level show constant fluctuations. The group made a general average of 86.025 on the thirty units.

Pupil B1 was exceedingly quick in reaching an accurate solution to the problems in a unit, but his papers contained very little writing. His rate of progress probably would not have been so rapid if written problem analysis had been required. The high-ability group following the technique where less writing was required included the six pupils to B6 who were rated highest on the initial score.

The medium-ability group included from B7 to B14. The average scores in this group ranged from 55 on Unit 1 to 90 on the last unit. Fluctuations appeared after Unit 9 was worked and continued until Unit 23. The general average for the medium-ability group, based on all scores for the thirty units, was 79.77. The difference between the general average for the medium-ability group of Method B procedure is smaller than the differences found in either of the other ability groups. Total means for the ability groups are recorded in Table IV.

The six lowest pupils from B15 to B20 (Control Group) in the initial grouping were placed in the low-ability group. The mean scores for the group ranged from 46.33 on Unit 1 to 83.33 on the final unit. This score on Unit 30 lacked 13.34 of meeting the mean average made by the low-ability group of the experimental method. Pupil B17 was absent a few times, as stated
heretofore, but made up all the work.

The low-ability group of Method B made a mean score of 74.89 on the thirty units. The difference in scores between the low-ability groups in Methods A and B is larger than the difference in mean scores between the medium- or high-ability groups in the two methods.

Many fluctuations in ability to solve the units were found. The written-analysis method brought more accurate results for the low-ability group than did the technique of solving the problems without a formal written procedure. A definite objective was missing for the low group when formal written analysis was not required.

Figure 1 illustrated graphically the progress made by the pupils of the high, medium, and low abilities following the analysis method. The low-ability group made the greatest progress in reaching high scores, exceeding the records made by the medium-ability group in several units. The low group began with an average of only 46.67 and commenced an upward climb immediately.

The high-ability level group maintained a high standard of work throughout the study. Their progress fluctuated downward in only two instances. The six of the group all received a perfect score on Units 29 and 30. The graphical representation given in this figure is based on scores for the three ability groups shown in Tables III and IV.
Figure 1
Comparison of Three Abilities of Experimental Group on Units 1 to 30
A graphical comparison of the records made by pupils of the three ability levels following the method of No Formal Procedure, is given in Figure 2. This illustration shows that each ability group made progress during the study, but that the scores of the three levels fluctuated from time to time. The low group did not overtake the medium group in the way the experimental low-ability pupils exceeded their medium-ability group.

The pupils of Method A, particularly the high and low levels, maintained a gain for several units. The high group retained the same average from Unit 8 to Unit 15. The record of the three levels in Method B shows many fluctuations in each group.
Figure 2
Comparison of Three Abilities of Control Group on Units 1 to 30

High
Medium
Low
All ability scores were averaged and a mean score obtained for each unit from 1 to 30. Table V records the average scores for each of the thirty units when all mean averages were collected. In Method A the scores range from 59.44 for Unit 1 to 97.22 for Unit 30. In Method B the averages extend from 58.44 on Unit 1 to 88.88 on the last unit.

The greatest progress was made in the beginning units of both methods, but this fact was due apparently to the Law of Diminishing Returns. For the first eight units the averages of all unit scores for both the Conventional Formula Method and the Method of No Formal Procedure retained the same general trend. From that unit the experimental scores began to ascend faster than the control averages.
### TABLE V

Mean Scores of Three Abilities Made on Units 1 to 30 by Pupils of Experimental and Control Groups

<table>
<thead>
<tr>
<th>Unit</th>
<th>Experimental Mean</th>
<th>Unit</th>
<th>Control Mean</th>
<th>Unit</th>
<th>Experimental Mean</th>
<th>Unit</th>
<th>Control Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>59.44</td>
<td>16</td>
<td>87.22</td>
<td>1</td>
<td>58.44</td>
<td>16</td>
<td>81.94</td>
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<tr>
<td>2</td>
<td>68.88</td>
<td>17</td>
<td>86.11</td>
<td>2</td>
<td>63.16</td>
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<td>85.27</td>
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<td>69.16</td>
<td>18</td>
<td>81.11</td>
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<td>74.82</td>
<td>19</td>
<td>85.27</td>
<td>4</td>
<td>75.27</td>
<td>19</td>
<td>82.22</td>
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<td>87.21</td>
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<tr>
<td>6</td>
<td>79.16</td>
<td>21</td>
<td>86.11</td>
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<td>78.33</td>
<td>21</td>
<td>81.94</td>
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<tr>
<td>7</td>
<td>78.33</td>
<td>22</td>
<td>88.33</td>
<td>7</td>
<td>76.94</td>
<td>22</td>
<td>82.22</td>
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<tr>
<td>8</td>
<td>81.38</td>
<td>23</td>
<td>89.16</td>
<td>8</td>
<td>80.00</td>
<td>23</td>
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<td>91.11</td>
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<td>25</td>
<td>84.38</td>
</tr>
<tr>
<td>11</td>
<td>84.44</td>
<td>26</td>
<td>91.11</td>
<td>11</td>
<td>80.83</td>
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<td>30</td>
<td>88.88</td>
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</tbody>
</table>
Figure 3 shows graphically the scores contained in Table V. Although the two groups began at the same level and remained in the same general trend for eight units, the conventional-formula group gorged ahead and retained the lead during the remainder of the experiment.
Figure 3
Comparison of Scores Made by Experimental and Control Groups
Table VI records the percentage of time saved through use of the two techniques. In the high group of each method there were six pupils. Each subject was allowed 30 minutes to complete a unit, making 180 minutes required by the six, and 5,400 minutes allotted for the same group to complete 30 units. Since the medium group contained eight pupils, the time allotment was 7,200 minutes for the central group in each method. The low group had the same number of pupils as the high group, and therefore the same number of minutes for the thirty units.

The pupils recorded the time taken on each problem unit. This report was approximately accurate because the subjects knew how to check the minutes.

However, the teacher kept a time sheet, and the minutes were reported. The amount of time saved by the pupils in the experimental group was negligible. Every pupil used the full thirty minutes through the first ten units. In the completion of the thirty units, the high group following the analysis method saved 4.6 per cent, or 248.4 minutes. The medium group using the same method saved 1.9 per cent of the time, or 136.8 minutes, and the low division 1.7 per cent of 91.8 minutes. Because of the analysis technique which required much writing, many pupils needed the full time for a number of the units.

The high group using the control method saved many minutes. Subject Bl was unusually quick in reaching a finished paper and saved much time. He took thirty minutes only on the first unit.
and finished Unit 30 in fifteen minutes. The high group saved 16.1 per cent of the time allotted - 869.4 minutes, or three and one-half times the per cent saved by the high group following the analysis method. The medium group in the control method saved 8.9 per cent, or 640.8 minutes, and the low group saved 7.2 per cent, or 388.8 minutes.

The high, medium, and low groups following the analysis technique saved 1.2 per cent of the allotted time. The three groups using the control method where very little writing was required saved 32.2 per cent. The written analysis of the problems required considerable time, so that the pupils using Method A saved very little time out of the minutes allotted for the entire thirty units.
| Time Allotment | Experimental | | | Control | | |
| | High | Medium | Low | | High | Medium | Low |
| Minutes Saved | 5400 | 7200 | 5400 | 5400 | 7200 | 5400 |
| | 248.4 | 136.8 | 91.8 | 869.4 | 640.8 | 388.8 |
| Per Cent of Time Saved | 4.6 | 1.9 | 1.7 | 16.1 | 8.9 | 7.2 |
| Per Cent of Time Saved by Group | 8.2 | | | 32.2 | | |
In the manual which can be secured with the New Stone Reasoning Tests, a table records the grade scores for correct answers on the test (53:12). Table VII records the grade equivalents for the Stone Accuracy Test based on the initial and final scores made by the pupils in this study. The mean score was computed for the two tests given to the groups following Method A and Method B.

At the beginning of this experiment the Stone Accuracy Tests were used in equating the groups. When the thirty units of work were completed, tests were again administered. The scores made by the two groups in the final tests were compared with those made on the initial tests. Table VIII records these scores and the standard deviations.
TABLE VII

Grade Equivalents on Initial and Final Scores for Stone Accuracy Test

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### TABLE VIII

Initial and Final Scores Made on Stone Tests

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<td>3</td>
</tr>
<tr>
<td>20</td>
<td>3</td>
<td>8</td>
<td>20</td>
<td>3.1</td>
</tr>
</tbody>
</table>

Mean: 4.85, S.D.: 1.13  
Mean: 8.14, S.D.: 1.46  
Mean: 4.85, S.D.: 1.08  
Mean: 6.99, S.D.: 1.53
Table IX shows the results found by measuring the probable error of the difference between the two means. It was first necessary to find the mean difference between the scores made by the two groups in the final tests. This was found to be 1.15. Computations of the probable errors of the means of the final tests were made, using the formula \( P.E.m = \frac{S.D.}{\sqrt{N}} \). The finding of the probable error of the difference followed, with the aid of formula \( P.E.m_1 - m_2 = (P.E.m_1)^2 + (P.E.m_2)^2 \). (23:234-5).

The mean change for the experimental group (a) was computed to be 3.29 and for the control group 2.14, making a difference of the final scores of 1.15 = .320.

This indicates that the method followed by the experimental group was superior to the one used by the control group, if arithmetical problem ability can be measured by these tests. The chances are 99 in 100 that the Conventional Formula Method is superior to the Method of No Formal Procedure. To be significant, the chances should be practically 100 in 100 that the difference will always be greater than zero. As the findings approach this significance, the indicate that it is much more than a pure chance that the Conventional Formula Method is superior to the Method of No Formal Procedure.
TABLE IX

Probable Error of the Difference
Between Two Means

<table>
<thead>
<tr>
<th></th>
<th>Experimental Group</th>
<th>Control Group</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Pupils</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Mean Intelligence Score</td>
<td>102.15</td>
<td>102.2</td>
</tr>
<tr>
<td>Mean Score on Initial Tests</td>
<td>4.85</td>
<td>4.85</td>
</tr>
<tr>
<td>Mean Score on Final Tests</td>
<td>8.14</td>
<td>6.99</td>
</tr>
<tr>
<td>S.D. for Final Tests</td>
<td>1.46</td>
<td>1.53</td>
</tr>
<tr>
<td>Probable Error of Mean</td>
<td>.221</td>
<td>.231</td>
</tr>
<tr>
<td>Probable Error of Difference</td>
<td>.320</td>
<td></td>
</tr>
</tbody>
</table>

\[
\frac{D}{P.D.\text{diff}} = 3.6
\]

Number of chances in 100 | 99
CHAPTER V

CONCLUSIONS

A brief summarization of the results of this experiment is given in this chapter and a comparison of the findings with the results of other closely related experiments is made.

A class of forty beginning with sixth-grade pupils took part in this experiment. By equating, this number was divided into two groups of twenty pupils each and these pupils were carried through an experiment of ten weeks' duration. In recording the progress made on Units 1 to 30, which were administered three times a week, each group was subdivided into ability levels. Comparisons of the growth of the two groups, as well as the growth of the three ability levels of each group, were made.

Chapter IV, which records these results, shows that the experimental group which followed the Conventional Formula Method surpassed the control group which followed the Method of No Formal Procedure. The results also show that greater progress was made in the low-ability level of the experimental group. This sub-group excelled the medium group in several
units of work. In the control group it was found that the Medium-ability level excelled the high level in only one unit, while the lower ability level lagged behind. From this, a conclusion may be reached that the Conventional Formula Method brings better results in problem-solving than the Method of No Formal Procedure, and that it works most effectually in the lower level of the group.

As the time element also entered into this experiment, some recognition of the same should be made. The experimental, which excelled the control group, took a longer period of time to solve the problems which were contained in Units 1 to 30. By the experimental group 8.2 per cent of the time was saved, while 32.2 per cent of the time was saved by the control group. This was as expected, since extra time was needed for the written analysis.

Two factors must be considered when interpreting results. Accuracy alone was counted when recording the scores of Units 1 to 30. The marking was carried on in this order because this is the system followed by teachers in the elementary schools of Chicago. If the time allotted for solving the problems had been shortened, the results would, in all probability, be reversed. The pupils of the experimental group were required to write out the analysis. The pupils of the control group had to write no analytical statement. A shortening of the time would
give the experimental group less time for the actual workings of the problem, and thereby might lower its score.

The statistics from which the probable error of the difference between two means are drawn is presented in Chapter IV. The chances are 99 in 100 that the true difference is greater than zero. In making observations from this statistical computation of the difference, it is the writer's opinion that the number of pupils taking part in this experiment was too small for any definite conclusions. The briefness of the time duration is another factor which may have affected the result.

The findings of this experiment conflict, in some instances, with those of other studies of similar purpose. Washburne and Osborne (60) found that the ability to make the type of formal analysis had practically no relation to solving problems. Hanna (20) conducted an experiment similar in some degree to the writer's. The results showed that the conventional formula method was less desirable of three experimental methods used. Adam's (1) findings agreed somewhat with the findings of this experiment. He found that detailed analysis helped, especially in the lower grades.

In reviewing related experiments, consideration should be given to the teacher factor, which was not constant. Different teachers were employed to teach the Conventional Formula Method
than were employed in teaching the other methods compared in the various experiments. In the present experiment the teacher factor was constant and this may, in some manner, have affected the results. An experiment following the same procedure, but carried out with a much larger group of pupils, would in all probability give results from which more definite conclusions might be drawn. Conclusions would be even more indicative if such an experiment were carried on for a greater duration of time. This time might extend over several semesters of work, the teacher factor being kept constant, that is, the same teacher being permitted to teach the two methods over the entire period of time. As the Conventional Formula Method is advocated by many visiting superintendents and principals of our Chicago Public Schools, it is pertinent that its real value be known.

The superiority of the Conventional Formula Method is the most consistent result of this experiment. It is the opinion of the writer that there is a relationship between the ability to formulate a written analysis and ability in problem-solving. That the variability of the abilities of the experimental group decreased through practice is also evident. The writer believes that the practice of directing the pupil to arrange his thoughts in a logical form is a step toward logical reasoning for problem-solving. That this practice tends to reduce varia-
bility of abilities, high, medium, and low, is also evident.

Briefly summarizing the findings:

1. The Conventional Formula Method is superior to the Method of No Formal Procedure.
2. The Conventional Formula Method is superior for decreasing the variability of abilities.
3. The Method of No Formal Procedure is superior in speed.
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The thesis "A Comparative Study of Two Methods of Problem-Solving in Arithmetic," written by Mary Cecelia Mangan, has been accepted by the Graduate School of Loyola University, with reference to form, and by the readers whose names appear below, with reference to content. It is, therefore, accepted as a partial fulfillment of the requirements of the degree conferred.

Austin G. Schmidt, S.J. 
July 6, 1933

James A. Fitzgerald, Ph.D. 
July 11, 1933