1933

The Mathematical Processes Found in Representative High School Texts in Chemistry

Arthur Peter O'Mara
Loyola University Chicago

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THE MATHEMATICAL PROCESSES FOUND IN REPRESENTATIVE HIGH SCHOOL TEXTS IN CHEMISTRY

BY

ARTHUR PETER O'MARA

A Thesis Submitted in Partial Fulfillment Of The Requirements For The Degree of Master of Arts in Loyola University 1933
VITA

Arthur Peter O'Mara

Born March 9, 1904 in Piper City, Illinois. Was graduated from Piper City High School, May 1920.


Degree Conferred: Bachelor of Arts in Education, Valparaiso University, August 1926.
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PREFACE

High-school chemistry teachers have been complaining that high-school students have an insufficient knowledge of mathematics to master high-school chemistry. The purpose of this study is to investigate the problems used in chemistry textbooks and in the quantitative experiments of the accompanying manuals and to determine the types of mathematical terms and processes found in the high-school chemistry texts and manuals and the frequency with which they occur.

I gratefully acknowledge the valuable assistance rendered by Doctor W. H. Johnson, Lecturer on Education at Loyola University, Dr. James A. Fitzgerald, Professor of Education, and Dean Austin G. Schmidt, in guiding the thesis through its development.
CHAPTER I

THE MATHEMATICS OF HIGH SCHOOL CHEMISTRY

Statement of the Problem

A complaint frequently encountered among high-school chemistry teachers today is that their students have an insufficient knowledge of mathematics to grasp the essentials of the classroom and laboratory work in chemistry.

In this thesis an attempt will be made to evaluate this contention of these teachers by determining the nature of the mathematical abilities that are involved in the study of high-school chemistry.

Chemistry is defined as the science that treats of the composition of substances and the transformations which they undergo.¹ For example, nearly all metals tend to rust, losing their bright luster and changing into material that looks more like earth than metal; coal and wood burn, producing gases and ashes; in the motor car, gasoline burns, producing great quantities of gases. In the latter case, however, we are usually more interested in the motion of the car than in these changes.

¹. Webster’s New International Dictionary
in the gasoline. Growing plants take up materials from the soil and form from these the vast variety of things we get from plants and trees; our bodies digest the food we eat, using a part as fuel and changing a part into bone, muscle, and other structures of the body.

In all these examples the materials have been transformed into others of entirely different composition; as the metals into oxides and coal into gases and mineral ash. In other words chemistry treats of the laws of nature. Herbert Spencer defines a law of nature as a proposition stating that a certain uniformity has been observed in the relation between certain phenomena. Therefore, every physical law can be represented in the form of a mathematical equation (16:1-2). For example, the law that if the temperature of any gas is kept constant the pressure varies inversely as the volume can be expressed in the mathematical equations: \[ P = \frac{k}{V} \quad \text{or} \quad \frac{P}{P'} = \frac{V}{V'} \]

While chemistry concerns itself primarily with the transformation of matter, of necessity is also involves a study of the physical properties of various substances, for example: melting points, boiling points, specific gravities, solubilities, and the like, as well as numerous chemical properties such as reactivity, ionization, atomic weight, and molecular weights.

Like every other investigator who collects large numbers
of widely dispersed facts, the chemist seeks to classify his data in the form of theories and laws. The chemist makes these generalizations to aid him in remembering facts already obtained and to assist him in forecasting other probable generalizations, such as the Mendeljeff Periodic Law, laws relating to valence, changes in gas volume due to changes in pressure or temperature, laws concerning ionization, solubilities, etc.

The chemist's theory may be only a rule of thumb - merely an aid to memory - but he seeks always to make it something more - a law which he can express mathematically as the dependence of one thing on another (11:71-78).

Mathematics is a concise language in which relations may be exactly expressed, hence, it is a very satisfactory method of expressing scientific facts. It is an indispensable instrument in the hands of scientists today (4:1293). A knowledge of mathematics enables one to correlate and interpret the results of his own work. One function of mathematics is to express as clearly and concisely as possible what cannot be concisely expressed in any other way. A mathematical statement is already correlated and digested. For example, Boyle's law states that a volume of gas varies inversely as the pressure, the temperature remaining constant. This can be expressed in a much clearer and concise form: \( \frac{V}{V'} = \frac{P}{P'} \). This statement shows us at once that the product \( VP = V'P' \) and that as \( V' \)
increases $P'$ must decrease in order to maintain the equilibrium of the equation, and as $V'$ decreased $P'$ must increase to restore the equilibrium of the equation. This is one of several laws with which a high-school student comes in contact.

This present writer has attempted to show the value of mathematics to the chemist, and also that mathematics is obviously in part a desirable prerequisite to the study of chemistry. High-school chemistry students are in an entirely different class from the chemist regarding the necessity of mathematics in every day life. To determine whether mathematics should be a factor in the high school chemistry we may well consider some of the aims of a course of this nature.

A high school course in chemistry should:
1. Enable the student to gain such a mastery of the fundamentals of the subject that he may make further progress in that subject without the aid of a teacher if he so desires.
2. Prepare the student for further work in the subject in college.
3. Give a fund of information which will broaden the student's outlook, and be a source of value and pleasure even though he never does any further work in the subject.
4. Instill in the mind of the student the principles of the "Scientific Method."

Chemistry has a language of its own. It has a method of recording results in this language which is peculiar to itself, and which is different from all other methods. It deals with unfamiliar materials and processes. This means that the pupil beginning the study of chemistry
finds himself in a strange land, confronted with a multitude of new ideas, new terms, and a new way of saying things. If he is to be successful in his endeavor to orient himself in this new environment, and not become bewildered by the number and strangeness of the ideas and terms it is necessary that his first work, especially, should be clear and definite (13:159).

That mathematics is essential to the study of chemistry is exemplified clearly in each of the aims given above. (1) To enable the student to gain such mastery of the fundamentals that he may further progress - certainly implies mathematics, which is in itself a valuable aid to the study of chemistry. (2) To prepare for further work in college - truly affirms that the student shall have mastered the fundamental principles of chemistry, which are frequently expressed in mathematical equations. (3) To give a fund of information which will broaden the students' outlook and be a source of value and pleasure - surely involves mathematics, since it is primarily in the application that the value of chemistry is evident, and the applications often depend upon quantitative determinations which are entirely mathematical. (4) To instill in the mind of the student the principles of the "Scientific Method" - since the scientific method presupposes truth and accuracy, it may frequently involve mathematical statements and laws.

To this point this present writer has attempted to show that mathematics is a useful instrument in the hands of the
The purpose of this study is to determine the various types of mathematical terms and processes and the frequency with which they occur:

A. In the exercise material of the two most widely used texts mentioned below,

B. In the quantitative experiments of their accompanying manuals.

As a means of securing the data, problems were taken from the two most frequently used high school chemistry text books in sixteen central and western states. All the problems at the end of the various chapters and in the appendices were solved. No problems were taken from the text material.

The text books were selected from a list secured from Ginn and Company, Chicago, Illinois. The list is compiled from the data furnished to the home office by the firm representatives in the district. The texts with their frequency of use for the years 1930-1931 and 1931-1932 are indicated in Table I and Table II, respectively.
## TABLE I

Chemistry Texts and Their Frequency of Use 1930-1931

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*State Text
# Bruce
\* Text numbers are listed on page 8.


II. Brownlee, Raymond B.; Fuller, Robert W.; Hancock, William; Sohon, Michael D.; and Whitsit, Jesse E. First Principles of Chemistry. Boston: Allyn and Bacon, 1931. 777p.


I. Various Texts.

f. State Text.


The texts finally selected for the present study are:

(1) Brownlee, Raymond B.; Fuller, Robert W.; Hancock, William J.; Sohon, Michael D., and Whitsit, Jesse E. First Principles of Chemistry. Boston: Allyn and Bacon, 1931 vii+777p.


The books chosen were selected because:

(a) both are new editions of popular old texts

(b) the texts are the two most frequently used in the high schools of sixteen of the central and western states.

Table I shows text 1 to have a frequency of 926 out of 2247, or a frequency of 41.3 per cent. Text 2 has a frequency of 618, or a frequency of 27.5 per cent. The two texts show a frequency of 68.8 per cent of all texts used in the sixteen states.
**TABLE II**

Chemistry Texts and Their Frequency of Use 1931-1932

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Total: 576 1001 128 103 63 90 45 39 52 99 2196
High Schools Using State Text ........... 90

*State Text
#Bruce
1.Ibid.
Table II shows text 1 to have a frequency of 1001 out of 2286, or 43.83 per cent. Text 2 has a frequency of 276, or a frequency of 25.2 per cent. The two texts have a frequency of 1577, or a frequency of 69 per cent of all the texts used in the sixteen states.

Throughout the discussion the texts will be distinguished as:

No. 1. Brownlee, Raymond B; Fuller, Robert W.; Hancock, William J.; Sohon, Michael D.; and Whitsit, Jesse E. First Principles of Chemistry. Boston: Allyn and Bacon, 1931. vii+777p.

CHAPTER II

A SURVEY IN THE FIELD OF THE PRESENT STUDY

In surveying the literature in the field of this investigation, reviews were made of related studies in high-school chemistry, college chemistry, physics, general science, popular science, auto-mechanics, newspapers, magazines, and ordinary life situations.

Studies were included that applied to this problem, whether or not they referred to the study of chemistry. For example, studies in physics and in general science were reviewed because they are in a field closely related to chemistry. Studies, such as a survey of the mathematics encountered in newspapers and periodicals, were reviewed because the problem was one of analysis of mathematics. Studies of life problems, such as the work of Woody (25:505-520), in which were analyzed the bills of sale from wholesale and retail hardware stores, a department store, and a wholesale and retail grocery store for the purpose of discovering the mathematics needed by the clerk in selling the goods, and by the purchaser in buying them, were reviewed.
Studies of the above types were reviewed for the following reasons:

(1) To find out what has been done in the field of the present study,

(2) To discover an adequate method of approach to the problem of this thesis.

Studies which related simply to chemistry and did not involve mathematics were omitted. All investigations of the questionnaire type were omitted because the present study involves only the analysis of textbooks. In selecting the studies to be reviewed, two criteria were kept in mind:

(1) Is the study mathematical?

(2) Is the study an analytical type?

With the above as criteria the following is a summary of the available studies relating to mathematics in the fields of chemistry, physics, general science, popular science, auto-mechanics, magazines, periodicals, and life situations.

Study Number 1

A study by Rendahl (19:683-89) in high school chemistry was reviewed. The purpose of the study was to determine what mathematics is used in solving quantitative problems of high school chemistry.
As a means of securing the data three chemistry textbooks which had been chosen at random from an approved list of textbooks for North Dakota high schools were examined. Only problems at the ends of the chapters were chosen. Supplementary problems found in the appendices and problems in the text material were not included in the study.

The textbooks examined were:


The problems were selected on the basis of two criteria:

(1) They must require computation.

(2) They must have numerical answers.

A total of 396 problems was found in the three textbooks. In several cases a single statement involved two or more problems. Each was handled as a separate problem.

As far as possible the problems were solved in the simplest way. In cases where the textbook gave directions or presented examples the methods suggested were followed. Thus similar problems from different texts were not necessarily
solved in the same way. The attempt was made to solve the problems in the way that students using the text would solve them.

Of the total of 396 problems found, text number 1 had 173 problems, or an average of 40 for each hundred pages; text 2 had 144 problems, or an average of 26 for each hundred pages; and text number 3 had 79 problems, or an average of 10 for each hundred pages. This indicates that there is considerable difference in the number of mathematical problems found in the various textbooks in high school chemistry.

The results on the fundamental operations of Rendahl's study were expressed in Table III.

It is to be noted that of all the fundamental operations, multiplication is the most used, and subtraction the least. There were no cases of addition, subtraction, or division of common fractions. Multiplications involving common fractions were found eighty-eight times.

In text number 1 most of the cases of multiplication of common fractions were found in connection with problems based upon chemical equations. In text number 2 the problems on gas laws furnished most of the cases of multiplication of common fractions found in that book.

The denominators of the fractions were of various sizes and degrees of complexity. Only eight of the denominators
TABLE III

Fundamental Processes in the Three Texts

<table>
<thead>
<tr>
<th>Type</th>
<th>Frequency</th>
<th>Per Cent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition</td>
<td>232</td>
<td>19.02</td>
</tr>
<tr>
<td>Subtraction</td>
<td>24</td>
<td>1.96</td>
</tr>
<tr>
<td>Multiplication</td>
<td>604</td>
<td>49.50</td>
</tr>
<tr>
<td>Division</td>
<td>360</td>
<td>29.52</td>
</tr>
<tr>
<td>Total</td>
<td>1220</td>
<td>100.10</td>
</tr>
</tbody>
</table>

were one-place numbers. These included the denominators two, three, four, and eight. Six denominators were two-place numbers. There were thirty-four denominators involving three-place numbers. Three had one decimal place, twenty-five had two decimal places, and twenty-six had three decimal places, making a total of fifty-four decimal denominators.

Of the numerators forty-nine were decimals, forty-five were whole numbers of two or more digits, and eight were whole numbers of one digit.
All the cases of fractions with numerators and denominators containing decimals were found in text number 1. Fractions with numerators and denominators over one hundred were frequent in text number 2. In text number 3 all the denominators were under ten.

All the numbers in text number 1 were tabulated according to the number of digits and the number of decimal places. Sixty-one per cent of the numbers used in addition were decimals. The highest number of digits was seven. Eighty-nine per cent of the addition problems had two or three addends. Fifty-six per cent of the numbers used in subtraction were decimals. The largest number used was one of five digits. Forty-two per cent of the numbers used in multiplication were decimals. The largest number used was one of eight digits. The greatest number of decimal places was five. Ninety-seven per cent of the numbers used in multiplication had five digits or less. In the division operations seventy-one per cent of the numbers were decimals. Sixty-one per cent had two or more decimal places. The largest numbers had eight digits. These numbers were of sufficient size to cause some difficulty in division.

Examples of the three cases of percentage were found in each of the three books investigated. In seventy-three problems the base and rate were given, in forty-six problems
the base and percentage were given, and in twenty problems the rate and percentage were given.

The proportion with one unknown is found sixty-nine times. Text number 1 avoids the use of proportion entirely in solving chemistry problems. Although the method of proportion is frequently used by chemists, it does not necessarily follow that the process is essential to the student of high school chemistry.

In the 396 problems solved, the square of a number, the cube, and the cube root of a number were each found once.

Fundamental operations on minus numbers are infrequent. Addition was found eight times, all in connection with thermometer readings. In no case was subtraction used; multiplication appeared twice; division, once.

Chemical equations were found 124 times.

Numbers were substituted in formulas in fourteen cases.

Fractional equations of the first degree occurred thirty-three times.

Only a little direct use was made of measurement. For example:

(1) Finding the dimensions of a cubical box when the volume was given.

(2) Finding the volume of a rectangular box when the dimensions were given.
(3) Finding the volume of a cylinder.

Forty-five instances of reduction of denominate numbers were found in the three books. The most frequent reduction was from tons to pounds, found fourteen times. Other reductions were from: liters to cubic centimeters, kilograms to grams, inches to centimeters, pounds to ounces, and quarts to pints.

A total of 82 different mathematical terms were found in the vocabulary of the 396 problems in the three books. The following were used most frequently:

- gram 119 times
- weight 118 times
- per cent (%) 83 times
- liter 70 times
- volumes 68 times
- pound 60 times

Thirty of the mathematical terms were not found in Thorndike's list of ten thousand most common words and sixty percent of these were units of measurements.

From the findings Rendahl (29:688-89) draws the following conclusions:

(1) The fundamental operations on whole numbers and decimals are slightly complicated by large numbers and numerous decimals.
(2) Multiplication is the only operation used upon common fractions. Fractions with large numerators and denominators further complicated by decimals are found in problems of some texts.

(3) The three variations of the percentage problem are found in all the texts examined.

(4) It is possible to avoid the use of proportion in solving chemistry problems.

(5) The more important uses of algebra are in solving simple and fractional equations of the first degree and with one unknown, and in substituting in a formula.

(6) The use of geometry is negligible.

(7) Only thirty-four different units of measure are used in the three books. Thirty-eight per cent of the units are based on the metric system.

(8) Not over fifty-four different mathematical terms are found in the vocabulary of the problems from any one book. Sixty-two per cent of them are among the 10,000 commonest words.

(9) There is considerable variation in the texts examined as to number of mathematical problems, the use of large fractions, the use of proportion, fractional equations and the metric system.
Study Number 2

Leo L. Boles and Hanor A. Webb (3:539-46) made an investigation to determine:

(1) The type of mathematics found in high school and college textbooks on inorganic chemistry,

(2) The quantity of mathematics found in high school and college textbooks on inorganic chemistry,

(3) In "what chemical connections" the mathematics was used,

(4) Whether the "mathematical demands of college chemistry" were greater than those of high school chemistry.

The material for examination consisted of fifteen inorganic chemistry texts of the high-school level and eighteen of the college level, all published within approximately ten years. These were minutely examined page by page for instructions and problems that involved any mathematical operation. The data were recorded on cards in such a manner that the cards could be selected and arranged for any desired grouping.

The space devoted to mathematics in proportion to the space of the entire text was a minimum in a high-school text with but .3 per cent, and a maximum in a college text with 4.1 per cent. The median proportion of space given to mathe-
matics in comparison to the space devoted to all material of all high-school and college texts was 1.2 per cent. For practical purposes one might report that the attention to mathematics is equal in both high-school and college texts in proportion to their size.

The dominant type of mathematics used in inorganic chemistry is proportion. In two aspects the data which are expressed in Tables IV and V are significant in that they show:

1. The relative amount of space given to each type of mathematics, and

2. The relative number of occurrences of each type of mathematics.

This information follows in Tables IV and V.

A study was also made to determine the relative percentage of space devoted to chemical topics; the relative percentage of occurrence of the chemical topics; the relative percentage of space devoted to chemical elements and compounds in mathematical topics, and their relative percentage of occurrence in mathematical topics; but, as the present study is one of mathematics rather than one of chemistry, the writer did not include this material in the present review.
### TABLE IV

The Mathematical Topics: Their Relative Percentages of Space

<table>
<thead>
<tr>
<th>Topics</th>
<th>Per cent in High School Texts</th>
<th>Per cent in College Texts</th>
<th>Per cent in Both Types of Texts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Propportion</td>
<td>88.3</td>
<td>84.8</td>
<td>86.2</td>
</tr>
<tr>
<td>Addition</td>
<td>4.4</td>
<td>4.6</td>
<td>4.6</td>
</tr>
<tr>
<td>Multiplication</td>
<td>3.8</td>
<td>6.7</td>
<td>5.7</td>
</tr>
<tr>
<td>Division</td>
<td>2.6</td>
<td>1.6</td>
<td>1.9</td>
</tr>
<tr>
<td>Algebra</td>
<td>.5</td>
<td>.2</td>
<td>.3</td>
</tr>
<tr>
<td>Subtraction</td>
<td>.4</td>
<td></td>
<td>.1</td>
</tr>
<tr>
<td>Logarithms</td>
<td></td>
<td>.4</td>
<td>.2</td>
</tr>
<tr>
<td>Equations</td>
<td></td>
<td>1.7</td>
<td>1.0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>100.0</strong></td>
<td><strong>100.0</strong></td>
<td><strong>100.0</strong></td>
</tr>
</tbody>
</table>
TABLE V

The Mathematical Topics: Their Relative Percentages of Occurrence

<table>
<thead>
<tr>
<th>Topics</th>
<th>Per cent in High School Texts</th>
<th>Per cent in College Texts</th>
<th>Per cent in Both Types of Texts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proportion</td>
<td>87.5</td>
<td>77.3</td>
<td>82.1</td>
</tr>
<tr>
<td>Addition</td>
<td>5.3</td>
<td>6.4</td>
<td>5.9</td>
</tr>
<tr>
<td>Multiplication</td>
<td>3.6</td>
<td>10.3</td>
<td>7.3</td>
</tr>
<tr>
<td>Division</td>
<td>2.1</td>
<td>2.6</td>
<td>2.3</td>
</tr>
<tr>
<td>Algebra</td>
<td>0.1</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>Subtraction</td>
<td>1.0</td>
<td></td>
<td>0.4</td>
</tr>
<tr>
<td>Logarithms</td>
<td></td>
<td>0.4</td>
<td>0.2</td>
</tr>
<tr>
<td>Equations</td>
<td></td>
<td>2.6</td>
<td>1.4</td>
</tr>
<tr>
<td>Total</td>
<td>99.6</td>
<td>100.1</td>
<td>100.0</td>
</tr>
</tbody>
</table>
The conclusions as presented by Boles and Webb were as follows:

(1) There is comparatively little mathematics presented in our present texts of general inorganic chemistry.

(2) Such mathematics as is present is concentrated to a high degree in the first chapters of the texts.

(3) Simple proportion is the principle of mathematics used almost to the exclusion of other forms in general inorganic chemistry. This being true, it would appear that such foundation of drill and clear understanding might be laid that no student should suffer mathematical handicaps in his beginning chemistry course.

(4) The question "How much?" is the vehicle on which most of the chemical mathematics is brought into a course; the "gas laws" are close behind. Chemistry and Physics, therefore, are each represented.

(5) The most popular chemical elements for mathematical treatment are oxygen, hydrogen, and chlorine; the compounds that serve mathematics best are potassium chlorate, water, and sulphuric acid.

(6) The treatment of mathematics in high school and in college textbooks of inorganic chemistry is strikingly similar in relative amount, type, and application (3:546).
Study Number 3

Lewis W. Williams (22:654-65) made a study in an effort to discover whether the criticism that high school students entering college with a year of algebra and a year of geometry did not have sufficient mathematics to handle freshman chemistry, was justified.

Williams selected the text book, "A Textbook in Chemistry," by Professor W. A. Noyes, which was required in freshman chemistry at the University of Illinois. The material was classified as exposition and as problem material. Under exposition were listed:

(1) Mathematical concepts - for example, the words "add," "absolute zero," "negative," "number," "weight," etc. A total of one hundred twenty-four mathematical concepts were encountered 1156 times. Per cent and volume comprised nearly one half of these.

(2) Denominate numbers such as ampere, day, gallon, ohm, etc., totaling seventy-four different expressions in all.

(3) Common fractions such as one-half, one-millionth, 1^2, 3^4, etc. Fifty-five different types were listed.

(4) Mixed decimals and integers were classified as to number of decimal places. There was only one expression of five decimal places and none above that. Seven hundred and
eight decimals were listed.

The problems were solved and the processes and expressions were classified under integers, written both as words and figures, and also tabulated as to the number of decimal places. For example, a two place integer was encountered eleven hundred fifty times in figures and thirty-nine times as words.

In the processes used arithmetical multiplication with integers was most frequent (149 times), chemical equations were used 134 times, arithmetical multiplication with decimals 118, equations of the first degree involving one unknown 7 times, involving two unknowns twice. There were no other algebraic equations found, nor were there any algebraic subtractions, multiplications, or divisions. Ratio and proportion played a prominent role, being used 77 times.

There were twenty-nine cases of percentage found among the problems. Nine were cases of base and rate given, to find percentage; eight of percentage and rate, to find base; twelve of percentage and base, to find rate.

Among the 139 cases of division, 46 consisted of the division of one integer by another. These varied from one to twelve place integers. Most of them were one-, two-, or three-place integers. There were 25 cases where a decimal was divided by an integer. In all but one case the decimal was mixed.
In conclusion Williams (22:665) states:

(1) The study shows a rather limited use of mathematics. Algebra and geometry have a very limited use, though it is possible to use algebra occasionally when arithmetic might not be quite as direct.

(2) A good working knowledge of the important principles of the following processes is desirable: addition, subtraction, multiplication, division, fractions, graphs of simple curves and proportions, percentage, surface measure, volume measure, metric system, simple progression, ratio, and proportion, and square root.

(3) For solving problems given, a thorough knowledge of the fundamental processes of arithmetic is necessary. This should hold, no matter whether integers, decimal fractions, or common fractions are used. Simple equations of one unknown and simple fractional equations should be mastered. The pupil should have a thorough training in percentage, denominate numbers, and ratio and proportion.

(4) A knowledge of the fundamental processes in algebra seems desirable though by no means as essential as might be inferred. Aside from their use with algebraic equations, themselves seldom found, they are scarcely ever used.
Study Number 4

B. C. Bradshaw (4:1293-96) states that mathematics is the only language by which relations may be exactly expressed; hence, it is the most satisfactory method of expressing scientific facts. Thus it is an indispensable instrument in the hands of scientists today. He illustrates the wide use of mathematics by examining typical publications of the *Journal of the American Chemical Society*, *Industrial and Engineering Chemistry*, and "two foreign journals," and listing the total number of articles involving mathematics, including arithmetic, algebra, calculus, etc. The conclusion of his analysis was that although a knowledge of arithmetic and elementary algebra may suffice for the intelligent reading of most articles, it was altogether inadequate for research work. However, as useful as mathematics is, it is not absolutely necessary. Many non-mathematical chemists have done brilliant work, and, as the tables show, many articles contained very little mathematics.

Study Number 5

Brinkley (5:1283-88) states that it is generally agreed that solving problems impresses on the minds of the students
the law of definite proportions, and that the formulas and symbols are not just abbreviations, but that they represent the definite and exact weight of substances.

If this is the sole purpose and function of calculations, the desired result can probably be obtained most satisfactorily when problems bear a close relation to the class work under consideration, instead of being merely for the purpose of furnishing something to calculate. The solving of numerical exercises may, however, serve other useful purposes. The significance of many of the topics discussed in the course can be understood more clearly through the medium of problems than in any other way. For example, gas laws, laws of multiple proportions, determination of formulas, etc.

Study Number 6

H. P. Ward (21:568-70) discussed the many aims of chemistry instruction with great stress on applied chemistry. He also discussed the desire of some people to omit mathematics in chemistry, because it means nothing to the individual. His answer is that only simple grammar grade arithmetic, including ratios, proportions, and the fundamental operations of addition, subtraction, multiplication, and division is needed for high school chemistry.
Study Number 7

W. Conrad Fernelius (11:71-72) discusses the importance of mathematics to chemistry and attempts to show that it is essential to this study. For example, a chemist writes an equation $2H_2 + O_2 = 2H_2O$, which tells in a concise and clear way the elements reacting, the product formed, the law of definite proportions, and the law of conservation of matter, all of which are mathematical. He advises that the student should come to college better prepared in mathematics, and that he should have mastered the elements of calculus and analytic geometry, which he recommends placing in the high school curriculum.

Study Number 8

A study in a closely related field by Kilzer (14:360-62) to determine what mathematics is needed in solving problems in high school text books of physics, was reviewed.

The texts for examination were selected by the results of a questionnaire to 500 high schools in the state of Iowa to ascertain the names of the high school physics texts used. On the basis of the replies it was found that only five text books were used in 2 per cent or more of the schools. These
textbooks, in alphabetical order, are: Black and Davis; Carhart and Chute; Dull; Fuller, Brownlee and Baker; Millikan and Gale.

"An attempt was made to solve each problem in every way that one would reasonably expect high school pupils to solve it." (10:215). The investigator's solutions were checked against the author's in the answer books, and processes used by the authors not already included in the investigator's list were incorporated.

A few of the conclusions that are indicated by this study will be listed:

(1) The mathematics needed in solving the problems of high school physics involves a considerable body of information usually taught in arithmetic, algebra, and plane geometry. Not much trigonometry is needed.

(2) Most of the mathematics needed in solving high school physics problems is not very difficult.

(3) Most pupils who are eligible to take high school physics use their mathematics poorly.

(4) There is definite need for maintenance drills covering the items and processes needed in physics.

(5) There is considerable difference in the preparation of individual pupils on certain items and processes. The difference in preparation of different schools is nearly as great (14:362).
Study Number 9

Harold (12:1-98) investigated the field of auto mechanics to discover the amount and kind of mathematics that is actually needed by one engaged in the field of general automobile engineering.

For this study, seven correspondence school text books, five popular hand books, and two issues each of three non-technical magazines were chosen. Four of the textbooks were from the American School of Correspondence, Chicago, Illinois, and three from the International School of Correspondence, Scranton, Pennsylvania. The periodicals were such as a repair man or a car owner would read as a means of keeping in touch with the general automobile news.

The method of procedure was simply a matter of going through the various sources of material, and carefully tabulating every figure, term, or mathematical concept under proper headings.

Harold concludes:

(1) If this study may be taken as a criterion, much of the arithmetic which is now being taught in the schools may well be eliminated from the course of study for the individual in training for this occupation. Apothecaries, weight, cube root, complex fractions,
and troy weight are among the operations that do not seem to function in this field, and, therefore, may be considered as an unnecessary part of the training course.

(2) One engaged in the field of auto mechanics should be well trained in the use of the four fundamental operations of arithmetic. He should be thoroughly familiar with the use of common and decimal fractions as a means of quantitatively expressing facts and relationships as well as their use in solving problems.

(3) The auto mechanic has considerable use for geometry and its application.

(4) One engaged in the field of auto mechanics should be well trained in using and solving algebraic equations of at least the second degree. The school should see that he is trained in the use of logarithms and trigonometric ratios. There is nothing in the data of this study that would indicate any necessity for studying mathematics beyond the level of trigonometry (12:96-98).

**Study Number 10**

McCluskey (15:1-75) made a study, the purpose of which was to discover two things:

(1) The types of mathematics that are used by the woman in the home,—the types she uses in cooking, in canning and preserving, in household hygiene, in household finance, in home decorating, in laundering, in the care of
the children and in general reading,

(2) The types that are used by the student of home economics in preparation for the more technical work.

An investigation of magazines and books that women use was made to discover the type of mathematics needed. These books were selected on the recommendation of several authorities on home economics in the University of Chicago, and also on the result of the demand for magazines in home economics at the Chicago Public Library.

The magazines were read and all mathematical terms and processes were tabulated and from the results McCluskey (15: 73-75) concludes:

(1) There are many mathematical concepts, a knowledge of which is necessary for understanding any reading material.

(2) The reader not only meets problems of a mental type, but some which require pencil and paper solution.

(3) The study of home economics, like other studies, uses fewer processes than the traditional arithmetic text contains.

(4) Unlike other studies denominate numbers are well represented.

(5) Arabic numerals are used to eleven places; money is used to ten place dollars; simple and mixed fractions, most of whose denominators are the first ten digits; percentage of first case only; decimal fractions to
five places; Roman numerals to first twenty-five numbers; ratio; graphs which can be taught with no knowledge of algebra.

(6) A few have little use and may as far as present evidence goes be omitted from the curriculum. For example:
1. Foreign money.
2. Linear rod.
3. Square rod.
4. Hundred weight (16:73-75).

Study Number 11

Adams (1:1-23) analyzes twenty widely read newspapers and magazines to discover the mathematics employed in the news, editorials, advertisements, legal notices, market reports, and sporting pages. All pages of one issue of the following newspapers and magazines were analyzed:

(1) The Chicago Herald and Examiner
(2) The Chicago Evening American
(3) The St. Louis Post-Dispatch
(4) The Springfield Missouri Republican
(5) The Springfield Leader
(6) The Pathfinder
(7) The Furrow
(8) The Dearborn Independent

(9) The Springfield Laborer

(10) Cosmopolitan

(11) The Woman's Home Companion

(12) The American

(13) The Household

(14) The Pictorial Review

(15) The Woman's World

(16) The Literary Digest

(17) The Modern Priscilla

(18) McCall's

(19) The National Geographic

The frequencies were tabulated under column headings: dates, addresses, phone numbers, numerals, Roman numerals, money, common fractions, decimals, percentages, ratio, mathematical terms, graphic representations, problems, and higher mathematics.

Adams (1:76-80) states:

The first and perhaps the most far reaching conclusion drawn is that so far as the problems presented to the public through the reading of newspapers and periodicals is concerned there is nothing to do but read and comprehend. There are no problems presented which require a pencil and paper solution.
Study Number 12

Boabo (2:1-4) made an analysis of the mathematics found in books on popular science. The books examined were written expressly for the layman by Henry Smith Williams. The Story of Modern Science consists of ten volumes:

(1) Charting the Universe
(2) Exploring the Atom
(3) Analyzing the Man
(4) Conjuring with Plants
(5) Juggling with Animal Life
(6) Wonder Working by Machinery
(7) Bettering the Race
(8) Super Engines of War
(9) Radio Mastery of the Ether
(10) Man and the Magic of Medicine

Six other books on popular science written by other authors were also chosen and analyzed. The books selected were:

1. Science Remaking the World by Caldwell and Slocson
2. The Story Book of Science by Jean Henri Fabri
3. Everyday Mysteries by Charles Abbott
4. The Romance of Air Craft by Leonard Smith
5. The Book of the Microscope by Frederick Collins.
The purpose of Bobo's study was to determine what mathematical terms and processes were encountered in reading popular science, in addition to the frequency with which they occurred.

The mathematical terms were classified under the several general headings: numerals, expressions of time, ordinals, geometrical concepts, fractions, decimals, U. S. money, mixed numbers, ratios, problems, graphs, and algebraic terms. The material under each classification was arranged in a frequency distribution. In concluding Bobo (2:72-74) states:

The mathematics encountered by the reader in books of popular science is largely of the type that needs to be read and comprehended rather than performed. If use, as indicated by the frequency of occurrence in the readings, may be taken as the criterion then the mathematics that is used in the books which are read should be emphasized in one's study of mathematics in the school room.

A large part of the mathematics which one finds in the reading is of the vocabulary type. The terms are mathematical and are used with a mathematical meaning. As such one needs to understand them. It is a question, though, whether these mathematical terms will be learned more efficiently through the mathematical activities or by meeting them as one finds them in everyday life, or, in other words, as one naturally acquires a vocabulary. A method must be found to make these quantitative terms have a very definite and full meaning for the individual.
One must understand the four fundamentals, with much emphasis on multiplication since it seems to function more frequently than the other three. One finds there is little problem solving to do. One should understand how problems are solved in order to comprehend the content of the reading. The mathematics is of comprehension rather than performance.

One must read the cardinal numbers up to fifteen or sixteen places, sometimes even farther, and comprehend their size if possible. He must recognize the ordinals and their relationships, and understand functions and decimals and their uses.

He should be acquainted with United States money in terms of dollars and cents, and with several kinds of foreign money. He needs to know the tables of weights and measures as well as some of the less definite quantitative terms.

Although graphs are not encountered frequently in books on popular science the individual should understand the interpretation of simple graphs.

Geometry occupies a place of importance in the mathematics activities. In the main it is of the vocabulary type. The concepts found convey very definite meanings. Whether the understanding of these terms can be gained most expeditiously from a study of geometry or in some other way is an open question. The frequency with which geometry terms are encountered entitles geometry to a place in the curriculum until some better, more efficient method of presenting the necessary understanding is secured.
A few samples of algebra and other types of higher mathematics were found. They seem to indicate some slight use in the human activities for these more advanced studies.

The frequency of ratios and proportions appearing in the reading as such, and their frequency when written as fractions give them an important place in the mathematics studies.

**Study Number 13**

Wise (26:116-136) tested over a greater area the Wilson Survey method of determining the social needs of the arithmetical processes.

The problems of adults, 7345 in number, were gathered from Texas, Iowa, Illinois, Wisconsin, California, and Missouri. The classification used showed the type of process and its difficulty.

The results of the investigation are:

1. Differences in the classifications of problems from the city and rural districts and also differences in the classifications of problems from different parts of the country were negligible.

2. Eighty-five per cent of all problems classified involved only the four fundamental operations, or combination of fundamental operations.

3. The fractions commonly used were $1\frac{2}{5}$, $1\frac{3}{4}$, $2\frac{3}{4}$, $1\frac{4}{5}$, $3\frac{4}{5}$, $1\frac{5}{8}$, and $1\frac{8}{9}$. These constituted 93.9 per cent
of all fractions which occurred.

(4) Problems involving common weights and measures occurred very frequently.

(5) There were very few problems in compound interest, compound proportion, insurance, plastering, printing, masonry, and bank discount.

(6) No problems were received involving taxes, investments, stocks, bonds, equation of payments, foreign exchange, apothecaries' weight, alligation, annual interest, compound and complex fractions, folding paper, troy weight, or the metric system \(18:49-50\).

**Study Number 14**

Woody (25:505-520) investigated in an attempt to gain a reliable index of the type of arithmetic needed by the clerk in selling goods and by the purchaser in buying goods. The problem consisted in the analysis of 4,661 bills of sale, representing a total value of $41,560.67, from three large stores in the city of Seattle, Washington - a wholesale and retail hardware store, a department store, and a wholesale and retail grocery store. The bills from the hardware store represent the total sales for three consecutive days; the bills from the department store, the "wash delivery" sales for seven consecutive days; the bills from the grocery, a random selection from the files in which the accounts are kept.
In analyzing these bills of sale, everything was considered a problem which involved a calculation.

Woody (25:520) summarizes:

(1) The amount of arithmetic actually used in the selling of the goods as listed on the bills of sale is very small. The problems most commonly met in the four fundamental operations consist of adding three place or four place numbers having two-, three, or four addends; subtracting numbers with four places or less in the minuend; multiplying a three place number by a one or two place number; and dividing a three place number by a two or three place number. The most common fractions are halves, fourths, twelfths, and sixths.

(2) Multiplication and addition are used much more than subtraction and division.

(3) Denominate numbers as such are not used in the buying or selling of goods.

(4) Decimals and percentage are used a great deal.

(5) From the facts disclosed by this investigation it is evident that the school is emphasizing much arithmetic that is unessential in meeting the situations confronted by the salesman and the consuming public.
CONCLUSIONS

The survey was undertaken as mentioned at the beginning of the chapter for two reasons:

(1) To find out what has been done in the field of the present study.

(2) To discover an adequate method of approach to the problem of this thesis.

Study number 1, by Rendahl, is a study very similar to the present investigation. It involves an analysis of high-school texts. These books, however, are different texts from those of the present study, one, in addition, being an earlier edition of one of the texts of this study. The study of Rendahl's is an excellent example of the method for the present investigation.

Study number 2, by Boles and Webb, is also a study closely related to the present study, being an analysis of high-school and college chemistry tests. Study number 3, by Williams, an analysis of a college test, is closely related to Study number 2 and to the present one. The two studies were fine examples on classification of material and evaluation of the findings.

Other studies as those of Bradshaw, Brinkley, Ward, and
Fernelius in the field of Chemistry were excellent dissertations on the value of the mathematics and the mathematical content of the texts and chemical journals.

The study by Kilzer in physics is the same as the present one, except for the difference in the field of application. Because high-school physics and high-school chemistry are so closely related the method especially is of great interest to the writer.

The studies of Harold, McCluskey, Adams, and Bobo are interesting to the writer because they are all master's theses, and for that reason have proved very helpful in the arrangement of the material of the present thesis.

The studies of Wise and Woody are investigations of life situations, and for that reason, and because of the fact that all are trained workers the method of investigation, tabulation, and the treatment of the findings were considered to be of value. The several studies mentioned and others excluded because of their nature have proved helpful in guiding the progress of this investigation.
CHAPTER III

THE METHOD

The investigation described in the following pages is an attempt by a statistical method to determine the types and frequency with which the mathematical terms and processes appear in the problems of the texts and the quantitative experiments of the laboratory manuals mentioned in Chapter I.

The investigation is divided into two parts:

(1) Analysis of the textbooks
(2) Analysis of the laboratory manuals.

The analysis of the textbooks consists in the solution of all problems at the ends of the chapters and in the appendices. An exercise in order to be classified as a problem must meet two criteria:

(1) The exercise must require computation
(2) The exercise must have a numerical answer.

The problems are of various types and similar problems in the two textbooks may be solved by different methods. In order to be consistent in the method used, teacher's manuals were secured from Ginn and Company, Chicago, Illinois, and from Allyn and Bacon, Chicago, Illinois. The solutions of the
authors were used for these reasons:

(1) In order that the solutions of the problems will be the same as the method for solution suggested in the text-
books.

(2) In order that the method of solution for all problems of the same type would be more consistent.

(3) In order to obtain more reliable data, that is, so that the problems might be solved as they would be by a class using the respective texts.

The mathematics involved in solving the problems in the text books will be classified under:

1. Mathematical concepts;

Mathematical concepts includes such terms as volume, area, height, weight, temperature, add, absolute zero, etc.

2. Denominate numbers;

A denominate number is a number specified in the concrete as opposed to the abstract; thus 7 feet is a denom-
inate number, while 7 is a mere abstract quantity or number.¹ Denominate numbers include terms such as calories, c.c., cm., cubic feet, etc.

¹ Webster's New International Dictionary

page 595
3. Addition.
   a. Integer
   b. Decimals
   c. Mixed decimals
   d. Common fractions.

   All additions will be classified as to the number of addends; the number of place integers in case of addition of integers; the number of place decimals in case of addition of decimals; and the number of place integers and number of place decimals in the case of addition of mixed decimals; and the number of digits in numerator and denominator in the case of addition of fractions.

4. Subtraction.
   a. Integer
   b. Decimals
   c. Mixed decimals
   d. Common fractions

   All subtractions will be classified as to the number of place integers in the minuend and subtrahend in the case of subtraction of integers; the number of place decimals in the minuend and subtrahend in the case of subtraction of decimals; the number of place integers and number of place decimals in the minuend and subtrahend in the case of subtraction of mixed decimals; and the number of digits in numerator and denomina-
tor of the minuend and subtrahend in the case of subtraction of fractions.

5. Multiplication.
   a. Integers
   b. Decimals
   c. Mixed decimals
   d. Common fractions

All multiplications will be classified as to the number of place integers, number of place decimals, number of place number of place integers, and number of place decimals in the multiplicand and multiplier, depending upon whether the example is one involving integers, decimals, or mixed decimals. Multiplication of common fractions will be classified as to the number of digits in the numerator and denominator of the multiplicand the multiplier.

Multiplication is by its nature an addition process. However, all multiplications are tabulated as multiplications only, and all additions necessary are omitted from the tabulation.

6. Division
   a. Integers
   b. Decimals
   c. Mixed decimals
   d. Common fractions
All divisions will be classified as to the number of place integers, number of place decimals, number of place integers, and number of place decimals in the dividend and divisor, dependent upon whether the example is one involving integers, decimals, or mixed decimals. Division of common fractions will be classified as to the number of digits in the numerator and denominator of the dividend and divisor.

In performing a long division problem, subtraction and multiplication are involved, but throughout this investigation a division process will be classified only as a division example. Subtraction and multiplication will not be tabulated when used in performing a division.

7. Ratio and proportion.

8. Algebraic fundamental operations.
   a. Addition
   b. Subtraction
   c. Multiplication
   d. Division

   a. Base and rate given
   b. Base and percentage given
   c. Percentage and rate given.

10. Equations.
    a. 1st degree, 1 unknown
b. 1st degree, 2 unknowns

2. 2nd degree, 1 unknown

d. Fractional

11. Lowest common denominator.


   a. Length
   b. Area
   c. Volume

14. Roots and powers.

15. Symbols.

16. Formulas.

17. Chemical equations.

18. Substitution.

The three terms—chemical symbols, chemical formulas, and chemical equations—all have a very extensive use, and, therefore, exert great influence upon the mathematics involved. With these are bound up the question of valence, atomic weights, and molecular weights. A symbol stands for the name of an element, for one atom of that element, and for one gram-atom of that element. More than one atom is indicated by a number placed at the right and a little below the symbol. A formula stands for the name of a compound, for one molecule of that compound, for one gram-molecule, or for
one gram-molecular volume, if a gas. Two or more molecules are represented by a number places before the formula. An equation gives the substances which react with each other and the products which are formed, the number of grams of each substance, or, if the substances are gases, the number of gram-molecular volumes.

The mathematics used here is relatively simple. Representing more than one atom or more than one molecule involves the simple principle of algebraic coefficients, the location of the figure in the case of the symbol being a mere matter of convenience.

With atomic weight, the problem is similar. Here the numbers are generally not simple, but larger and often mixed decimals, as 35.457 for chlorine. The molecular weight is obtained by adding the atomic weights of the elements in the formula (or the multiples) of the atomic weights.

Chemical equations have some things in common with algebraic equations, but they differ materially in several respects. The writer is concerned with the mathematical phases only. The use of the equality sign, the transfer of terms from one side of the equation to the other, and general handling of the equation, all conform to mathematical principles. Balancing a chemical equation is, however, a distinct process. It involves a simple numerical relation, keeping the number of
atoms of each kind on one side of the equation the same as the number on the other. The success of this, of course, depends upon knowing the products of the reaction and the use of coefficients in expressing the formulas of products and their quantities.

Some reactions represented by chemical equations involve a great amount of mathematics in the explanation of the theory; for example, the reaction of copper with nitric acid is represented thus:

\[ 3\text{Cu} + 8\text{HNO}_3 \rightarrow 3\text{Cu(NO}_3)_2 + 2\text{NO} + 4\text{H}_2\text{O}. \]

The above and similar equations involve what is known in chemistry as the processes of oxidation and reduction. The balancing of the equation is usually limited in high school chemistry texts to the mere selection of coefficients of the elements expressed as atomic weights, and to coefficients of the molecular weights that will balance the members of the equations, without any attempt to explain the balancing by any algebraic process. Therefore, the writing of an equation and its balancing in any exercise will consist only of the selection of the proper coefficients, and for this reason the algebraic processes that are usually involved in more advanced work can be considered outside the scope of the present problem.

The problems of the texts are of various types; for
example, exercise 16, Chapter VI, page 94, text number 1. - "How much mercury can be obtained from a kilogram of mercuric oxide?"

Solution

(1) \( \text{HgO} \)  \( \text{Hg} + \text{O}_2 \)
(2) \( 2\text{HgO} \)  \( 2\text{Hg} + \text{O}_2 \)
(3) \( \text{Hg} = 200 \)  \( \text{Hg} = 200 \)
    \( 0 = 16 \)
    \( \text{HgO} = 216 \)
    \( 2\text{HgO} = 432 \)
(4) \( 432: 400: 1000: x \) grams
(5) \( 432x = 400000 \)

\( x = 925 \) grams of mercury.

The above example involves as the first step the use of a chemical equation which is a mathematical expression of the law: mercuric oxide yields mercury and oxygen. The second step involves the balancing of the equation, which consists in selecting the proper coefficients for the symbols and formulas so that each side will have equal number of life atoms. The third step involves an addition of two integers, one of three digits and one of two; also, two arithmetic multiplications, the multiplicands being integers of three digits and the multipliers integers of one digit each. The fourth step makes use of proportion, for we know from the law of conser-
vation of matter that if 432 parts of mercuric oxide give 400 parts of mercury, 1 kilogram or 1000 grams of mercuric oxide will yield $x$ grams of mercury, which written as it is in text number one is:

$$432 : 400 :: 1000 : x$$

The writer has considered all proportions of this type, also, as algebraic equations of the first degree with one unknown.

The fifth step involves solving the proportion or equation for the unknown $x$, which consists in multiplying the means and also the extremes. Therefore, 432 times $x$ is an algebraic multiplication of an unknown with a coefficient of 1 by an integer of three places. There is also another arithmetic multiplication of integers in which the multiplicand has 4 places, and the multiplier 3 places. The sixth and final step is to divide both sides of equation 5 by 432, which consists of one algebraic division of a monomial by its numerical coefficient which is an integer of 3 places. There is also an arithmetic division of a five place integer by a three place integer giving the answer $x = 9.25$ grams of mercury.

The denominate numbers used in the solution are kilogram (1), gram (2). Symbols were used 16 times; Formulas, 4 times; and chemical equations, 2 times.

An example of a different type is exercise four, Chapter I, page 133, of text No. 2 - "Find the percentage
composition of each of the following compounds: (2) $\text{H}_2\text{O}_2$; 
(b) $\text{KClO}_3$; (c) $\text{ZnSO}_4$; " This example involves three parts but the writer has listed such examples as one problem.

**Solution**

(a) **Formula**

Substituting the atomic weight $\text{H}_2\text{O}_2$

$2.016 \times 2(16) = 34.016$

Per cent of hydrogen $\frac{2.016}{34.016} \times 100 = 5.92$

Per cent of oxygen $\frac{32}{34.016} \times 100 = 94.07$

(b) **Formula**

Substituting the atomic weights $39.10 + 35.457 + 3(16) = 122.557$

Per cent of potassium $\frac{39.10}{122.557} \times 100 = 31.90$

Per cent of chlorine $\frac{35.457}{122.557} \times 100 = 28.93$

Per cent of oxygen $\frac{48}{122.557} \times 100 = 39.16$

(c) **Formula**

Substituting atomic weights $65.38 + 32.06 + 4(16) = 161.44$
Per cent of zinc \[= \frac{65.38}{161.44} \times 100 = 40.50\]

Per cent of sulphur \[= \frac{32.06}{161.44} \times 100 = 19.86\]

Per cent of oxygen \[= \frac{64}{161.44} \times 100 = 39.64\]

The solution of the above problem involves the process of percentage in which the base and percentage are given to find the rate. The mathematical concepts used are percentage once, and per cent eight times.

Addition is used three times, once with two addends and twice with three addends. Three addends are integers, all of two digits; five addends are mixed decimals, three of two place integers and two place decimals, one of one place integer and three place decimal, and one of two place integer and three place decimal.

Multiplication of integers is used three times, in all of which the multiplicand consists of two digits and the multiplier consists of one digit. Multiplication of decimals is used eight times. The multiplicands are all decimals, one of three places and seven of four places. The eight multipliers are the same, all being 100 or, in other words, consisting of three digits.

Division is used eight times, five of the dividends and
all the divisors being mixed decimals. Three dividends were integers of two digits, one was a mixed decimal of one place integer and three place decimal, three were mixed decimals of two place integer and two place decimal and one was a mixed decimal of two place integer and three place decimal.

Symbols were used eight times; formulas, three times; and substitution, eight times.

An exercise of another type is example 25, Chapter XXIV, page 411 of text No. 1 - "How many liters of oxygen are required for the complete combustion of 10 liters of acetylene gas?" How many liters of air are required,"

Solution

(1) \( \text{C}_2\text{H}_2 \oplus \text{O}_2 \) \( \text{CO}_2 \oplus \text{H}_2\text{O} \)

(2) \( 2\text{C}_2\text{H}_2 \oplus 5\text{O}_2 \) \( 4\text{CO}_2 \oplus 2\text{H}_2\text{O} \)
   2 vol.  5 vol.

(3) 2 volume: 5 volume: : 10 liters : x

(4) \[ 2x = 50 \]

(5) \[ x = 25 \text{ liters} \]

(6) Since air is but 1.5 of oxygen, there would be required:
   5 \times 25 \text{ liters or 125 liters of air.}

The solution of the above problem uses the mathematical concept of volume four times, the denominate number, liter
three times.

Arithmetic multiplication is used twice with integers, both multipliers being two digit integers, and each multiplier is a one digit integer. Algebraic multiplication is used once - 5 times x.

Arithmetic division is used once, the dividend being a two place integer, and the divisor, a one place integer. Algebraic division is used once - 5x divided by 5.

Proportion is used once, and algebraic equation of one unknown of the first degree is used once. Symbols are used 14 times, formulas, six times.

Still another type of problem is the determination of a formula. For example, exercise 6, Chapter X, page 133, text No. 2. "A compound was found on analysis to contain 5.93 per cent of hydrogen and 94107 per cent of oxygen. Experiments prove its molecular weight to be approximately 34. Calculate (2) its simplest formula and (b) its correct formula."

Solution

\[
\begin{align*}
5.93 & \div 1.008 = 5.88 \\
94107 & \div 16.00 = 5.88
\end{align*}
\]

Simplest ratio: 1:1. Simplest formula H₂O but if this is the correct formula the molecular weight would be 1. 008 \div 16 = 17.008. Experiment shows the molecular weight to be 34.
hence the correct formula must be twice the simplest formula or \( \text{H}_2\text{O}_2 \).

Addition is used once involving two addends, one being a mixed decimal of one place integer and three place decimal, the other, an integer of two places. Division is used twice; the dividends are mixed decimals, one of one place integer and two place decimal, and the other two place integer and two place decimal. One divisor is a mixed decimal of one place integer and three place decimal, the other an integer of two places.

The various problems of the texts are treated in the manner shown above. The tabulation of the results will be explained in Chapter IV, which treats of the findings of the present investigation.

The method of analyzing the laboratory manuals is different from the analysis of the textbooks, because in the laboratory manuals practically all the computation depends upon data obtained by experiment that will vary according to the experimenter. Therefore, it is impossible to classify the data any farther than the mere processes of addition, subtraction, multiplication, division, proportion, etc.

The experiments, like the exercises, are of various types. Some are qualitative and some are quantitative, the latter involving mathematics. Therefore, all mathematical
terms and processes will be tabulated that are used in the working of experiments.

The laboratory manuals selected are:

(1) Brownlee, Raymond B.; Fuller, Robert W.; Hancock, William J.; Sohon, Michael D.; and Whitsit, Jesse E. *Laboratory Exercises to Accompany the Brownlee-Fuller-Hancock-Sohon-Whitsit Chemistry*. Boston: Allyn and Bacon, 1932. 282 p.


The types of experiments are illustrated by the following examples. Example seventeen, page 45, manual No. 1 "The Verification of a Chemical Calculation" is a laboratory experiment to verify the theoretical value determined by a calculation. First, a test tube and some manganese dioxide are weighed, then some potassium chlorate is added, and all are weighed again. The quantity of potassium chlorate is determined by subtraction. The object of the experiment is to determine what is the weight of the potassium chloride which remains after the oxygen has been driven from the potassium chlorate by heat. Table VI represents the outline of the experiment.
TABLE VI

Outline of Experiment

(a) Weight of test tube and manganese dioxide .......... g.

(b) Weight of test tube, manganese dioxide, and potassium chlorate ........................................... g.

(c) Weight of potassium chlorate \(b-a\) ................. g.

(d) Weight of tube and contents after heating .......... g.

(e) Weight of tube and contents after reheating ....... g.

(f) Weight of tube and contents after another heating ... g.

(g) Final constant weight ......................................... g.

(h) Weight of potassium chloride calculated from the equation .................................................... g.

(i) Weight of potassium chloride found experimentally \(g-a\) ............................................................. g.

(j) Variation between observed and calculated weights \(h-i\) ........................................................... g.
The value \( \bar{w} \) in Table VI is calculated from a chemical equation. For example, supposing the value of \( c \) to be 10 grams then (1) by equation:

\[
2\text{KClO}_3 + \text{heat} = 2\text{KCl} + 3\text{O}_2
\]

(2) by substitution: \( 2(39 + 35.5 + 48) = 2(39 + 35.5) \)

\[
245 = 149
\]

(3) by proportion: \( \frac{245}{149} = \frac{10}{x}; \quad x = \frac{10 \times 149}{245} \)

\[
x = 6.08 \text{ gms.}
\]

An example of a different type is Experiment twenty H, page uu, manual No. 2. This is an experiment to determine the volume and weight of hydrogen displaced from an acid by the metal magnesium. The results obtained are used to calculate the valence of the metal magnesium in the following way:

Having learned by experiment what weight of hydrogen is displaced by any given weight of magnesium, one can calculate the weight of hydrogen that would be displaced by one gram-atomic weight of the metal. This weight of hydrogen will be one-, two-, or three-gram-atomic weights of hydrogen according to whether the metal is univalent, bivalent, or trivalent. Hence, if we determine the weight of hydrogen liberated by one gram atomic weight of the metal and divide this weight by the gram atomic weight of hydrogen, namely 1.008; that quotient will be approximately a whole number, and this will equal the valence
of the metal.

Experiment: Volume of hydrogen liberated by magnesium; valence of magnesium.

Arrange the apparatus as shown in the figure below.

![Apparatus diagram]

A represents a wide-mouthed bottle of about 150 cc. capacity. The funnel B is connected with the glass tube D by a short rubber tube which may be closed by the screw clamp C. The lower end of the tube D is drawn out slightly to a jet. E is an ordinary 5-pint acid bottle. The end of the exit tube F must extend up into the bottle E, as shown in the figure, so that when the experiment is completed, it will be well above the surface of the water in E. The apparatus must be air-tight; so it is necessary to use a good cork in bottle A.

Next select a piece of clean magnesium ribbon (polish the surface with a piece of emery paper or scrape it with a knife) weighing about 1.8 g., weight is accurately to 0.01 g.; and
record the weight in the table shown at the close of the experiment. Place the magnesium in bottle A, cover it with 10cc. of water, and set the bottle aside. Close the screw clamp C and nearly fill the funnel B with the dilute hydrochloric acid. Place a beaker under tube D, open the screw clamp, and allow the acid to flow into the beaker until tubes C and D are entirely filled with the acid; then close the screw clamp. Now insert the cork air-tight into bottle A. Fill bottle E completely with water, invert it in the through G, and bring the end of the exit tube F well up into the bottle, as shown in the figure. Now open the screw clamp C slightly and allow a few drops of the acid to flow into bottle A. Hydrogen is liberated, passes out of tube F, and collects in bottle E. Continue to add a few drops of acid from time to time, enough to cause a gentle evolution of hydrogen, taking care to keep the funnel partly filled with acid so that no air will be drawn into bottle A along with the acid. Continue until all the metal is dissolved; then allow the apparatus to stand until bottle A and contents (which have been heated by the action of the acid on the metal) are cooled to room temperature. Time may be saved if at the beginning of the experiment bottle A is placed in a beaker of cold water. This will keep the temperature down. Finally withdraw the exit tube F from bottle E.
Now raise or lower bottle B, taking care to keep the mouth of the bottle below the surface of the water, until the water inside and outside the bottle is at the same level; then, holding the bottle in this position, tightly stopper it with a good cork and quickly bring the bottle and contents to an upright position on the desk. Now measure the volume of the gas in the bottle by pouring water from a graduated cylinder into the bottle until filled. Record the volume thus found in the table provided for this purpose. Also measure the volume of liquid in bottle A and record it in the table.
TABLE VII

Outline of Experiment

1. Weight of magnesium taken ........................................... g.
2. Volume of gas collected in bottle E ............................... cc.
3. Volume of liquid in bottle A at the end of experiment ............ cc.
4. Total volume of hydrogen evolved, measured under laboratory conditions (difference between volumes 2 and 3 less 10 cc., which is the volume of the water in bottle A at the beginning of the Exp.) ......................... cc.
5. Temperature of laboratory ...........................................
6. Barometric pressure ...................................................
7. Total volume of hydrogen evolved measured under standard conditions (calculate from equations found in Appendix C of text, using data given under 4, 5, and 6) ......................... cc.
8. Weight of this volume of hydrogen (1 liter of hydrogen = 0.08987g. under standard conditions) ................................................................. g.
9. Weight of hydrogen evolved by 1 gram-atomic weight of magnesium (namely, 24.32g.) calculated as follows: Weight of Mg. taken: weight of H liberated = 24.32: x .............................. g.
10. Valence of magnesium calculated by the following equation: Valence of Mg = weight of H evolved by 24.32 g. of Mg. ............................ 1.008
The combining weight of an element is the weight of that element which will combine with, or replace, 1 g. of H.

Combining weight of Mg. = \[
\text{weight of Mg. (l)} \div \text{weight of volume of hydrogen}
\]

The above experiment involves the processes shown in Table VIII.

**TABLE VIII**

Processes Involved in Experiment

<table>
<thead>
<tr>
<th>Processes</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition</td>
<td>2</td>
</tr>
<tr>
<td>Division</td>
<td>4</td>
</tr>
<tr>
<td>Multiplication</td>
<td>6</td>
</tr>
<tr>
<td>Subtraction</td>
<td>2</td>
</tr>
<tr>
<td>Percentage</td>
<td></td>
</tr>
<tr>
<td>Proportions</td>
<td>4</td>
</tr>
</tbody>
</table>

Total ........ 18

The mathematical concepts used in the above experiment are: calculate (l), calculated (l), cylinder (l), equation (l), pressure (l), table (l), temperature (l), volume (7), weight (5).
The denominate numbers used are: c.c. (7), gram (8), gram-atomic weight (1), liter (1), mm. (1), pint (1).

As in all cases of laboratory work, the metric system is used. The operations are in the use of decimals, which in this case involves decimals as high as the fifth place, but no higher. The difficulty of this experiment does not lie in the mathematical processes, but in the comprehension of the laws of physics and chemistry.

The method has been outlined and several type examples are given to explain the method of solution and also the method of analysis. The problem is to determine the mathematical processes and terms and tabulate the frequency with which they occur. The findings of this investigation with tables to illustrate the material are included in Chapter IV.
CHAPTER IV

THE FINDINGS

The mathematical terms and processes used in solving the problems of the texts consist of various types and of various degrees of difficulty.

As mentioned before, the problems were selected from the exercises at the end of the various chapters and in the appendices. The exercises at the ends of the chapters are by no means all mathematical. Some are of the problem type, others are of the question type. For an example of the question type we may take exercise 9, Chapter XVII, page 268 of text Number 1 -- Why will substances often react in the dry state?. Another example is exercise 4, Chapter XI, page 143 of text Number 2 -- How do the diamond and graphite differ in (a) color? (b) density? (c) hardness? (d) uses?

The total number of processes and total number of exercises in the texts are shown in Table IX.
### TABLE IX

Total Number of Exercises in Texts

<table>
<thead>
<tr>
<th></th>
<th>Text Number One</th>
<th>Text Number Two</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exercises in text</td>
<td>2094</td>
<td>1184</td>
</tr>
<tr>
<td>Mathematical exercises in text</td>
<td>157</td>
<td>90</td>
</tr>
<tr>
<td>Percentage of mathematical exercises</td>
<td>7.48%</td>
<td>7.60%</td>
</tr>
</tbody>
</table>

Table IX indicates that the ratio of the number of mathematical exercises to the total number of exercises in text Number 1 is very nearly equal to the ratio of the number of mathematical exercises to the total number of exercises in text Number 2.

The total number of exercises in text Number 1 exceeds the total number in text Number 2 by 910, while the number of problems in text Number 1 exceeds the number of problems in

1. Brownlee, Raymond B.; Fuller, Robert W.; Hancock, William J.; Schon, Michael D.; and Whitsit, Jesse E. *First Principles of Chemistry.*
text Number 2 by 67. This tends to indicate that text Number 1 is more mathematical than text Number 2. This is due to a great extent to the insertions at the ends of Chapters VIII, XVI, XXVII, and XLI of text Number 1 of tests which consist of questions of the true and false type, of the completion type, and of the problem type. Each test consists of 100 exercises, making 400 in all, of which 44 are problems. These tests were tabulated because they were in the text, and because in final analysis the number of problems do not cause the difficulty, it is the difficulty of the problems themselves.

As to the number of mathematical exercises, text Number 1 has a great many more. As to the difficulty of the problems themselves, the analysis of the various problems will show whether the mathematics of text Number 1 is more difficult than the mathematics of text Number 2.

The analysis will be considered as outlined in Chapter III.

1. Mathematical Concepts

In this division all terms of mathematical meaning that are not classified under denominate numbers are listed.

Table X shows 50 different concepts used 371 times in text Number 1, and only 20 concepts used 166 times in text Number 2. The terms weight, percent, and volume were most frequently used. Several terms as area, cost, cubes, density,
sum, and thickness were used only once in text Number 1, and not at all in text Number 2. The only words used in text Number 2 that were not used in text Number 1 were the words formula and one-half.
### TABLE X

#### Mathematical Concepts and Their Frequency

<table>
<thead>
<tr>
<th>Concept</th>
<th>Text No. 1</th>
<th>Text No. 2</th>
<th>Concept</th>
<th>Text No. 1</th>
<th>Text No. 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altitude</td>
<td>1</td>
<td>2</td>
<td>Percent</td>
<td>54</td>
<td>29</td>
</tr>
<tr>
<td>Atomic-weight</td>
<td>2</td>
<td></td>
<td>Percentage</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Area</td>
<td>1</td>
<td></td>
<td>Pressure</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>Calculate</td>
<td>8</td>
<td></td>
<td>Price</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Cost</td>
<td>1</td>
<td></td>
<td>Profit</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Cubes</td>
<td>1</td>
<td></td>
<td>Proportion</td>
<td>6</td>
<td>1</td>
</tr>
<tr>
<td>Double</td>
<td>1</td>
<td>2</td>
<td>Quantity</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Dividing</td>
<td>1</td>
<td>1</td>
<td>Ratio</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Decreases</td>
<td>2</td>
<td></td>
<td>Selling</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Divisions</td>
<td>1</td>
<td></td>
<td>price</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>5</td>
<td></td>
<td>Smaller</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Equal</td>
<td>3</td>
<td>13</td>
<td>Specific</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Formula</td>
<td></td>
<td>9</td>
<td>gravity</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>Formula weight</td>
<td>6</td>
<td>2</td>
<td>Sum</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gram-molecular</td>
<td>1</td>
<td>3</td>
<td>Table</td>
<td>6</td>
<td>9</td>
</tr>
<tr>
<td>One-half</td>
<td></td>
<td>2</td>
<td>Temperature</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Heavier</td>
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</tr>
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<td></td>
<td>Times</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Larger</td>
<td>2</td>
<td></td>
<td>Total</td>
<td></td>
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</tr>
<tr>
<td>Lighter</td>
<td>4</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Minus</td>
<td>18</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Molecular weight</td>
<td>22</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>Negative</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One and</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>one third</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One fifth</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One third</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>One thousandth</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Brownlee, Raymond E.; Fuller, Robert W.; Hancock, William J.; Schoen, Michael D.; and Whitsit, Jesse E. *First Principles of Chemistry.*
### TABLE XI

Denominate Numbers and Their Frequency

<table>
<thead>
<tr>
<th>Denominate Number</th>
<th>Text No. 1</th>
<th>Text No. 2</th>
<th>Denominate Number</th>
<th>Text No. 1</th>
<th>Text No. 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute degree</td>
<td>24</td>
<td></td>
<td>Inch</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>CC.</td>
<td>79</td>
<td>20</td>
<td>Kilogram</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>Cubic Meter</td>
<td>1</td>
<td>2</td>
<td>Liter</td>
<td>103</td>
<td>51</td>
</tr>
<tr>
<td>Calorie</td>
<td>1</td>
<td>10</td>
<td>Meter</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Cent</td>
<td>9</td>
<td></td>
<td>Mile</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Cubic feet</td>
<td>13</td>
<td>3</td>
<td>Millimeter</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Cubic inch</td>
<td>2</td>
<td></td>
<td>Minute</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Degree</td>
<td>38</td>
<td>11</td>
<td>Ounce</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Dime</td>
<td>3</td>
<td></td>
<td>Pound</td>
<td>47</td>
<td>35</td>
</tr>
<tr>
<td>Feet</td>
<td>1</td>
<td>1</td>
<td>Second</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Gallon</td>
<td>1</td>
<td>1</td>
<td>Square inch</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Gram</td>
<td>234</td>
<td>123</td>
<td>Ton</td>
<td>20</td>
<td>17</td>
</tr>
<tr>
<td>Hour</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total: 625 297

1. op.cit.
2. ibid.
II. Denominate Numbers

Table XI shows that 24 different denominate numbers are used 625 times in the solution of the problems in text Number 1, while 17 different denominate numbers are used 297 times in text Number 2. One denominate number (second) is used twice in text Number 2 and not at all in text Number 1. The denominate number "gram" is one of the most frequently used in both tests, "liter" is next in frequency while "cubic meter," "calorie," "feet," "gallon," "hour," "meter," and "mile" are used only once in text Number 1. "Feet," "gallon," "hour," "mile," and "minute" are used only once in text Number 2.

III. Addition

All addition problems consisted of 2 addends, 3 addends, or 4 addends; 49 had 2 addends, 27 had 3 addends, and 7 had 4 addends. The various addends are divided into integers, decimals, and mixed decimals, a number consisting of an integer and a decimal, as 73.265.

The type of integers used in the solution of the problems in the two texts are listed in Table XII.

Text No. 1 exceeds text No. 2 greatly in the number of integers used in addition. In fact, over 92% of the additions
in text Number 1 involved the addition of integers. In the
addition of integers no addends in either text had more than
seven digits and there was only one of seven digits. It was
involved in the solution of problem 4, page 75 of text Number
2. One hundred pounds of ice was placed in a refrigerator.
It melted, and the resulting water flowing from the refriger-
ator had a temperature of 10 degrees. How many calories of
heat were absorbed from the interior of the refrigerator?

Solution

One hundred pounds of ice equals 43,359 g. The heat ab-
sorbed in melting this equals \(45,359 \times 80 = 3,628,720\) cal.;
to raise the temperature of the resulting water from 0 degrees
to 10 degrees would require \(45,359 \times 10 = 453,590\) cal. Hence
the total heat absorbed is \(3,628,720 + 453,590 = 4,082,310\) cal.

From the solution it is seen that 3,628,700 (the largest
addend in either text) is added to 453,590 equaling 4,082,310.

No addend of a pure decimal type appeared in either
text.

Text Number 2 exceeds text Number 1 in the number of
mixed decimals used. Text Number 2 employs the use of mixed
decimals over 61\% of the time in addition while text Number 1
uses mixed decimals only 8\% of the time. No mixed decimals
used in text Number 1 consisted of more than one decimal, while
## TABLE XII

Addition of Integers

<table>
<thead>
<tr>
<th>Place Integers</th>
<th>Text Number 1&lt;sup&gt;1&lt;/sup&gt;</th>
<th>Text Number 2&lt;sup&gt;2&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>38</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>198</td>
<td>69</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td>6</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

**Total ...........** 260 80

1. op. cit.
2. ibid.
TABLE XIII
Addition of Mixed Decimals

<table>
<thead>
<tr>
<th>Place Integer</th>
<th>Place Decimal</th>
<th>Text Number 1</th>
<th>Text Number 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td></td>
<td>30</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>19</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td></td>
<td>45</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td></td>
<td>29</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td></td>
<td>7</td>
</tr>
</tbody>
</table>

Total .......... 22 127

1. op. cit.
2. ibid.
text Number 2 used mixed decimals consisting of three place decimals in 52% of all cases listing mixed decimals.

Throughout the investigation text Number 2 was found to use decimals more frequently than text Number 1, while text Number 1 used integers more frequently than text Number 2.

The use of fractions was very small, so all operations with fractions will be grouped together and treated later in the Chapter.

IV. Subtraction

The number of subtraction processes is small in comparison with the other processes, being used 13 times in text Number 1, and 14 times in text Number 2. Table XIV represented the types of arithmetic subtractions. There were no cases of decimals in subtraction; three minuends and five subtrahends of text Number 1 were mixed decimals. Eight minuends and nine subtrahends were mixed decimals. Ten of the minuends and eight of the subtrahends of text Number 1 and six of the minuends and five of the subtrahends of text Number 2 were integers.

V. Multiplication

Text Number 1 employs the use of 234 multiplications, while text Number 2 employs 207 multiplications. Table XV shows the use made of integers in multiplication, Table XVI,
the use of decimals, and Table XVII, the use of mixed decimals.

**TABLE XIV**

Subtraction of Integers and Mixed Decimals

<table>
<thead>
<tr>
<th>Place Integer</th>
<th>Place Decimal</th>
<th>Text Number 1$^1$</th>
<th>Text Number 2$^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Minuend</td>
<td>Subtrahend</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

| Total .......... | 13 | 13 | 14 | 14 |

1. op. cit.

2. ibid.
TABLE XV

Multiplication of Integers

<table>
<thead>
<tr>
<th>Place Integer</th>
<th>Text Number 1</th>
<th>Text Number 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Multiplicand</td>
<td>Multiplier</td>
</tr>
<tr>
<td>1</td>
<td>26</td>
<td>164</td>
</tr>
<tr>
<td>2</td>
<td>111</td>
<td>21</td>
</tr>
<tr>
<td>3</td>
<td>28</td>
<td>15</td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>178</td>
<td>205</td>
</tr>
</tbody>
</table>

1. op. cit.
2. ibid.
TABLE XVI

Multiplication of Decimals

<table>
<thead>
<tr>
<th>Place Decimal</th>
<th>Text Number 1</th>
<th>Text Number 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Multiplicand</td>
<td>Multiplier</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
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<tr>
<td>4</td>
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</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>14</td>
</tr>
</tbody>
</table>

1. op. cit.

2. ibid.
### TABLE XVII

Multiplication of Mixed Decimals

<table>
<thead>
<tr>
<th>Place Integer</th>
<th>Place Decimal</th>
<th>Text Number 1</th>
<th>Text Number 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>v2</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
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<td>2</td>
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<td>1</td>
<td>23</td>
</tr>
<tr>
<td>2</td>
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<td>2</td>
<td>35</td>
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<tr>
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<td>4</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
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<td>3</td>
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<td></td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td></td>
<td>2</td>
</tr>
</tbody>
</table>

Total: 42, 12, 127, 24

1. op. cit. 2. ibid.
Integers were used in the multiplicand 176 times in text Number 1 and 71 times in text Number 2. They were used 205 times as multipliers in test Number 1 and 176 times in text Number 2. Only two multiplicands involved integers greater than 7 places and only two of 7 places were found, one in each text. The solutions of the problems in text Number 1 involves the use of two multiplicands of ten place integers. One exercise is number 6, page 459, Chapter XXVII, text Number 1.

If a cubic inch of glass were cut into cubes each .001 inch along an edge, what would be the total surface exposed?

**Solution**

There would be $1000 \times 1000 \times 1000 = 1,000,000,000$ cubes; there would be $6,000,000,000$ faces, each having a surface of $.000001$ sq. in. Hence the total area would be 6000 sq. in.

No multiplier involved integers greater than 5 places and only three of 5 places were found - one for text Number 1 and two for text Number 2.

Decimals were used in text Number 1 as multiplicands 14 times, and 17 times as multipliers, and in text Number 2, 9 times as multiplicands and 9 times as multipliers. No decimal was found that exceeded 6 places, and only three were found of 6 places, one as multiplicand in text Number 1, one as multiplier in text Number 1, and one as a multiplicand in text
Number 2.

Mixed decimals were used from a one place integer, and one place decimal to a six place integer and 2 place decimal. The largest decimal place used was four. Two multiplicands of text Number 2 consisted of four place integers and four place decimals.

VI. Division

The solutions of the problems of text Number 1 involves 112 divisions and of text Number 2, 103 divisions.

Integers are used to 9 places. A 9 place integer was used in the dividend twice in each text. The exercise in which it was used in text Number 2 is exercise 2, page 555, Chapter XLVI, text Number 2. "A clap of thunder travels at the rate of about 1100 ft. per second. Compare the speed of the sound with the speed of an alpha particle."

Solution

The alpha particle travels at the rate of 20,000 miles per second. 20,000 x 5280 (number of feet in mile) = 105,600,000 feet per second. \(\frac{105,600,000}{1100} = 96,000\) times as fast as sound.

Text Number 1 makes use of no decimal above two places; text Number 2 makes use of three decimals of 5 places in the
division.

Text Number 2 gives the true atomic weights of elements and true weights of liters of gases to three or four places. It gives the weight of a liter of hydrogen as 0.08987 grams, hence the problems are solved using this as the constant. Therefore, the decimals are larger.

Text Number 2 uses mixed decimals as dividends as high as those having 7 place integers and 3 place decimals. Text Number 1 uses only one mixed decimal higher than 4 place integer and 4 place decimal, and it is one of an 8 place integer and 1 place decimal. Generally speaking, text Number 2 uses higher decimals than text Number 1 in the process of division.
### Table XVIII

Division of Integers

<table>
<thead>
<tr>
<th>Place Integer</th>
<th>Text Number 1</th>
<th>Text Number 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dividend</td>
<td>Divisor</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>30</td>
<td>41</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
<td>32</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

| Total         | 87            | 89            | 28       | 33      |

1. op. cit.
2. ibid.
**TABLE XIX**

Division of Decimals

<table>
<thead>
<tr>
<th>Place Decimal</th>
<th>Text Number 1</th>
<th></th>
<th>Text Number 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dividend</td>
<td>Divisor</td>
<td>Dividend</td>
<td>Divisor</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
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</tr>
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<td>3</td>
<td></td>
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</tr>
<tr>
<td>4</td>
<td></td>
<td>1</td>
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<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Total .... 2</td>
<td>7</td>
<td>2</td>
<td>4</td>
</tr>
</tbody>
</table>

1. op. cit.

2. ibid.
<table>
<thead>
<tr>
<th>Place Integer</th>
<th>Place Decimal</th>
<th>Text Number 1&lt;sup&gt;1&lt;/sup&gt;</th>
<th>Text Number 2&lt;sup&gt;2&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Dividend</td>
<td>Divisor</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
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<td>5</td>
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</tr>
<tr>
<td>1</td>
<td>4</td>
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<tr>
<td>1</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>6</td>
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</tr>
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<td>4</td>
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</tr>
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<tr>
<td>4</td>
<td>1</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

<sup>1</sup> L. op. cit.
<sup>2</sup> 2. ibid.
<table>
<thead>
<tr>
<th>Place</th>
<th>Place</th>
<th>Text Number $1$</th>
<th>Text Number $2$</th>
</tr>
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<td>Divisor</td>
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<td>6</td>
<td></td>
</tr>
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<td></td>
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<td>1</td>
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<td></td>
<td></td>
</tr>
<tr>
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<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>3</td>
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<td>4</td>
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<tr>
<td>5</td>
<td>5</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>1</td>
<td></td>
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<tr>
<td>7</td>
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<tr>
<td>7</td>
<td>2</td>
<td>1</td>
<td></td>
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<tr>
<td>7</td>
<td>3</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

**Total .............** 23 16 73 66

1. op. cit.         2. ibid.
VII. Fractions

There are no additions or divisions of fractions. There are three cases of subtraction in which the subtrahends are fractions of two digits in the numerator and two in the denominator, the minuends consist of two types, one is an integer of three digits, the other is an improper fraction of 4 place numerator and 2 place denominator. All cases of subtraction of fractions are found in text Number 1.

Multiplication of fractions is found several times in text Number 1, but only three times in text Number 2. Most of the multiplications of fractions in text Number 1 are found in the last chapter which is introduced in case the students have not had physics and are not familiar with the gas laws of Boyle and Charles. There are 30 fractions - 28 have 3 place numerators and 3 place denominators; two of the others are mixed fractions as 738 2/13; one fraction has a mixed decimal in the numerator and 3 in the denominator and one has 2 places in the numerator and 3 in the denominator.
VIII. Ratio and Proportion

Ratio and proportion are very closely linked together. Everytime a proportion is used, two ratios are understood because a proportion is the expression of equality between two equal ratios. However, ratios are used otherwise.

A use of ratio separate from proportion is in the calculation of a formula. It is necessary to get the ratio of the number of atoms of one element to the number of atoms of another as: in sulphuric acid, H : S : O (2 : 1 : 4). Therefore, the formula is $\text{H}_2\text{SO}_4$.

In text number 1 proportions are used 70 times and in text Number 2, 47 times. Ratios are used 9 times in text Number 1 and 10 times in text Number 2.

Proportion is used inversely; for example, Boyle’s law of gases expresses that a volume of gas is inversely proportional to the pressure, the temperature remaining constant. Proportion is also used directly; for example, Charles’ law expresses that the volume of a gas is directly proportional to the absolute temperature, meaning that if the temperature is doubled the volume is doubled, or if the temperature is decreased by a half the volume is decreased by a half.

Various other problems in proportion are encountered, but they are of the direct type; for instance, what weight of HCL
does it take to dissolve 10 gms. of zinc?

**Solution**

Zinc + Hydrochloric Acid = Zinc Chloride + Hydrogen

\[ \text{Zn} + 2 \text{HCl} = \text{ZnCl}_2 + \text{H}_2 \]

65 \( 2(1 \div 35.5) \)

65 \( 2(36.5) \)

From the equation we see that 73.0 parts of HCl react with 65 parts of Zn; therefore, \( x \) parts of HCl will react with 10 parts of Zn which is written in a proportion thus:

\[
\frac{73.0}{65} = \frac{x}{10}
\]

**IX. Algebraic Fundamental Operations**

Algebraic addition and algebraic subtraction are not used in the solutions of the problems in either text. Algebraic multiplication, that is, multiplication involving letters, is used 63 times in text Number 1 and 47 times in text Number 2. The type of multiplication in every case is finding the product of an unknown as \( x \) or \( y \), by an integer, decimal, or mixed decimal as shown in Table XXI. The type of equation to be solved is:

\[
\frac{x}{332.064} = \frac{10}{253.864}
\]

\[ 253.864 \times x = 3320.64 \]

\[ x = 13.08 \]
Whether the process is classified as proportion or an algebraic equation in which clearing of fractions is to be performed does not alter the number of algebraic multiplications.

Algebraic division is in every case the reverse of the algebraic multiplication, because after the product of the unknown and the other term of proportion is found it is necessary to divide by the coefficient of the unknown, therefore, the algebraic division is classified with multiplication in Table XXI.

**X. Percentage**

There are three types of percentage:

1. To find the percentage, given the base and rate,
2. To find the rate, given base and percentage,
3. To find the base, given the percentage and rate.

Under case No. 1 are listed such problems as 5 per cent of a certain mixture weighing 10 grams is salt. What is the weight of the salt in the mixture? Under case No. 2 we find this type - What is the per cent of hydrogen in ethyl alcohol C₂H₅ OH? No. 3 includes such problems as: 11.11 per cent of a certain compound is hydrogen and the weight of hydrogen in the molecular weight of the compound is 2.016. What is the molecular weight of the compound?
### TABLE XXI

**Algebraic Multiplications and Algebraic Divisions**

<table>
<thead>
<tr>
<th>Place Integer</th>
<th>Place Decimal</th>
<th>Text Number 1</th>
<th>Text Number 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>22</td>
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<td>3</td>
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<td>1</td>
<td></td>
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<tr>
<td>1</td>
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<td>1</td>
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<td>1</td>
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</tr>
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<td>2</td>
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<td>11</td>
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<td>3</td>
<td>3</td>
<td></td>
<td>7</td>
</tr>
</tbody>
</table>

**Total .............** 63 47

1. op. cit.

2. ibid.
The frequency of use of the various types of percentage is as follows:

<table>
<thead>
<tr>
<th>Type of Percentage Use</th>
<th>Text Number 1</th>
<th>Text Number 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Given base and rate</td>
<td>4</td>
<td>12</td>
</tr>
<tr>
<td>(2) Given base and percentage</td>
<td>27</td>
<td>16</td>
</tr>
<tr>
<td>(3) Given percentage and rate</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

XI. Equations

Equations were used 63 times in text Number 1 and 47 times in text Number 2. All equations were of the first degree with one unknown.

XII. Lowest Common Denominator

There were two cases of lowest common denominators used in the subtraction of fractions.

XIII. Clearning of Fractions

In text Number 1 the proportions are treated as common proportion, but in text Number 2 the author speaks of clearning fractions. For example, \(\frac{60}{127.14} = \frac{x}{159.14}\), means the same as the product of means equals the product of the extremes, because the common denominator is needed to clear the equation \((127.14)(159.14)m\), but after each numerator is multiplied by the common denominator and the denominator divided into the
product the resulting equation is the same. However, in listing the processes all solving of proportion in text Number 2 have been listed as clearing of fractions but the proportions in text Number 1 are treated as the author does only as a proportion. There are 47 cases of clearing of fractions.

XIV. Mensuration

Measurement of length, and calculation of areas and volumes are few. In text Number 2 one exercise involved finding the volume of a room in cubic meters. It is necessary to measure the length, width, and height of the class room and then find the volume as $10m \times 12m \times 15m = 1800 \text{ cu.m.}$ In text Number 1 in one example it is necessary to find the total area exposed if a cubic inch of glass is cut into cubes $0.001$ inches on a side. Solution: $1000 \times 1000 \times 1000 = 1,000,000,000$ cubes; there would be $6,000,000,000$ faces each having a surface area of $0.000001 \text{ sq. in.}$; hence the total area would be $6,000,000,000 \times 0.000001 = 6000 \text{ sq. in.}$ In text Number 1 there are three cases of finding area, and two of volumes. In all cases the dimensions are obtained from the problem.

A course in geometry is certainly not necessary to obtain the necessary knowledge to deal with the measurement of area and volume as found in text Number 1 and text Number 2.
XV. Roots and Powers

No use is made of the knowledge of roots and powers.

XVI. Symbols

All symbols were chemical in nature and appeared 176 times in text Number 1 and 48 times in text Number 2.

XVII. Formulas

All formulas were chemical formulas as \( \text{H}_2\text{O} \), \( \text{H}_2\text{SO}_4 \), and \( \text{H}_3\text{C}_6\text{H}_5\text{O}_7 \). Text Number 1 has 253 formulas and text Number 2 has 165.

XVIII. Chemical Equations

Chemical equations were used 56 times in text Number 1 and 44 times in text Number 2.

XIX. Substitution

Substitution was used 292 times in text Number 1 and 22 times in text Number 2.
Laboratory Manuals

The experiments of the laboratory manuals were classified as to the qualitative and quantitative experiments. Table XXII indicates the total number of experiments and the number of quantitative experiments of each text.

The total number of processes appearing in the laboratory manuals are listed in Table XXIII.

It has been mentioned previously in this study that it is impossible to give the nature of the various processes because the computation depends on data that is obtained by the experimenter in performing the experiment. Therefore, the data for each experiment would be different because of the different quantities used and the apparatus employed. The altitude of the laboratory, humidity and other variable factors would also affect the results of some of the experiments.
### TABLE XXII

Total Number of Experiments in Laboratory Manuals

<table>
<thead>
<tr>
<th></th>
<th>Text Number 1</th>
<th></th>
<th>Text Number 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Experiments</td>
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<td></td>
<td>113</td>
</tr>
<tr>
<td>Mathematical Experiments</td>
<td>12</td>
<td></td>
<td>14</td>
</tr>
<tr>
<td>Percentage of Mathematical Experiments</td>
<td>14 %</td>
<td></td>
<td>12 %</td>
</tr>
</tbody>
</table>

### TABLE XXIII

Total Number of Processes in Laboratory Manuals

<table>
<thead>
<tr>
<th>Processes</th>
<th>Text Number 1</th>
<th>%</th>
<th>Text Number 2</th>
<th>%</th>
<th>Total</th>
</tr>
</thead>
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<td>0</td>
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<td>28</td>
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<td>Multiplication</td>
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<td>36.1</td>
<td>34</td>
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<td>77</td>
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<tr>
<td>Division</td>
<td>22</td>
<td>18.5</td>
<td>34</td>
<td>26.8</td>
<td>56</td>
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<td>Subtraction</td>
<td>10</td>
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<td>14.9</td>
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<td>9.4</td>
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<td>Proportions</td>
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<td><strong>Total ...119</strong></td>
<td><strong>99.9</strong></td>
<td><strong>127</strong></td>
<td><strong>99.9</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. op. cit.  
2. ibid.
CHAPTER V
CONCLUSIONS

On the analysis of the findings of this investigation as summarized in Chapter IV, the following conclusions seem to be justified:

(1) The mathematical terms and denominate numbers used are found in either elementary school or previous high school training or can be taught early in the regular course in high school chemistry.

(2) The operations of addition, subtraction, multiplication and division on integers, decimals, and mixed decimals of great number of places. In the use of decimals it can be inferred from the findings that decimals are used greatly in text 2.

The reason for this is that the authors desire accurate answers and desire that the atomic weights to three decimal places be used. For example, text 1 gives the atomic weight of chlorine as 35.5, while text 2 gives the atomic weight of chlorine as 35.457.
(3) The use made of fractions is practically negligible except in multiplication, and in multiplication most of the cases involve large fractions of three places as \( \frac{273}{299} \) or \( \frac{770}{760} \).

(4) Percentage is used in all three cases and is extremely important in mastering the material of the texts.

(5) While it is possible to solve all problems by proportion that may employ algebraic equations, text 2 does not speak of the solution as being done by proportion, but the writer believes that many teachers using the text would probably do so.

(6) Algebraic fundamental operations used are wholly of multiplication and division, the knowledge of which could be easily taught in the chemistry class without the aid of a formal course in algebra.

(7) The algebraic equations that are used could be classed as proportions and all are of the first degree with one unknown.

(8) The use made of measurement of length, area, and volume is of such a nature that a course in geometry is not essential for the solution of the problems.

(9) Symbols, formulas, and equations are extremely important, for without these there could be no chemistry.
(10) Substitution is used frequently, but it is unnecessary to have previous training in the use of the process since all substitutions consist of placing numbers for symbols.

To conclude, it appears evident from the data herein presented that high school chemistry consists chiefly of arithmetic. The algebra and geometry involved in the solution of the problems is meager. The knowledge of algebra and geometry necessary for the solution of problems has been taught during the elementary grades or could readily be taught during the high-school chemistry course.

Mathematics cannot be said to be a simple learning process because it consists mainly of arithmetic. It is a simple process for most high school students to solve a proportion, but every teacher and every student realizes the difficulty many pupils experience in the obtaining of the correct proportion. To perform the calculation may be easy for him, but to obtain the proper equation, the balancing of a chemical equation, may be beyond his mental powers. The difficulty may be due to mathematics, and it may be due to chemistry.
The matter of mental difficulties in high school chemistry can only be answered by a psychological study of the mental difficulties.

This is outside the scope of the present study.
BIBLIOGRAPHY


