A Comparative Study of Two Types of Elementary Algebra Textbooks

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A COMPARATIVE STUDY OF TWO TYPES OF ELEMENTARY ALGEBRA TEXTBOOKS

By

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A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of Master of Arts in Loyola University

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Vita

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CHAPTER I
INTRODUCTION

Several years ago, while the writer was teaching mathematics in a Jesuit high school, a meeting of all the mathematics teachers of the eleven Midwest Jesuit high schools was held. The usual complaints about the poor performance of high school students in mathematics were voiced; the poor arithmetic foundation of the pupils was discussed; the inadequate achievement in elementary mathematics of college students was condemned. All this negative criticism of the mathematics program was not a matter of surprise to the teachers. What did create considerable astonishment was the remedy suggested by a group of teachers from one of the schools.

This group had experimented rather boldly with the mathematics program at Rockhurst High School in Kansas City. The teachers from Rockhurst reported that they began their new program by disregarding one of the fundamental dogmas of the traditionalist school of mathematics. This dogma is that understanding comes first and worthwhile mathematical achievement comes only as a result of such an understanding. The Rockhurst group claimed that the beginning of a remedy lay in getting the students to work mechanically and blindly and thereby, after working through a large number of problems, to
come gradually to an understanding of the processes.

In addition to discarding this major premise that understanding proceeds problem solving, the Rockhurst teachers discarded much of the traditional content material of traditional high-school mathematics courses. Claiming that nearly sixty percent of the content of algebra is of no consequence, the teachers omitted in their experimental course the usual exercises in addition and multiplication of polynomials, limited almost to the vanishing point the traditional exercises in factoring, and the drills in exponents and radicals. Almost the whole section of the drill material and related explanatory material on the fundamental processes, which is the substance of most elementary algebra texts, was omitted in the new course. All the processes were introduced, not through the traditional drill in fundamental operations, but through exercises of equations. The substance of the new course from beginning to end was equation work in one, two, and three unknowns. Another feature of the course was the fractional answers of a large number of the equations which had to be solved. Many of these fractions were large and all of them had to be completely checked. The purpose of this was to provide a constant check against the usual deficiencies of arithmetic, to train the student in computational skills, and to provide a more satisfactory approach to algebra through arithmetic.
To the teachers from the other schools this report of the Rockhurst group was a revelation. Some of them pointed out that even after they had spent months in teaching and explaining the laws of signs, the fundamental operations, factoring, and the rest, many students were still not able to perform these operations accurately when they did finally come to equation work. How then could a student be expected from the very beginning of the course to solve equations involving the use of signs, and the other operations, without first having been drilled in exercises which were aimed at developing an understanding of the operations used in equations? The teachers greeted the new program with great uncertainty and doubt precisely because they could not accept the solution of the Rockhurst teachers.

Most of the teachers present had, after all, accepted the basic premise of the traditionalists that understanding comes first. The structure in fact of the typical high-school textbook in mathematics, especially algebra, is built upon such a premise.

This premise has been most authoritatively explained and defended by the National Committee on Mathematical Requirements in its 1923 report entitled The Reorganization of Mathematics in Secondary Education. The report will be discussed in full in a later section of this paper. Suffice it to remark here that the National Committee’s word is considered final by most
high-school educators, and that the report has been and is pretty much the norm for all high-school mathematics texts and teaching technique.

Only within recent years has this key premise of the traditionalists been challenged. Irwin A. Buell has expressed the challenge well in articles in the mathematical journals. Typical of the newer trend of thought are his remarks in the November, 1944, *Mathematics Teacher*:

> Article after article has been printed in which the author has said that the pupil in mathematics must consciously understand each time he performs a definite operation just the reason why he is permitted to do as he does. I should like to take exception to such statements.1

Taking exception to the report's claim that children should "see meaning in the numbers which they use and operate," Buell replies: "Just what meaning do they see in the number 4 that they did not see ten years ago?" And to the report's recommendation that the new arithmetic should be "meaningful rather than mechanical," Buell counters with:

> I say lets make it increasingly mechanical and then go on to something more abstract. Let us continually make the difficult into the mechanical, and go on

---

1 Irwin A. Buell, "Let Us be Sensible About It," *Mathematics Teacher* 37:306-7, November 1944.
to something more difficult.
Let us not look for "meanings"
that are not there. Let us
not become too mystical about
this whole business. And please,
let us not load up our minds and
the childrens' minds with round­
about philosophies, methods, pro­
cedure, "meanings," etc., etc.
Let us teach him that 8 and 6
are 14; that is all there is to
it.\(^2\)

The purpose therefore of this paper will be to discuss
these two theories of mathematical study. We will review the
major documents describing the different theories and we will
examin at length two textbooks in elementary algebra which are
considered representative of the different views. In addition,
in an attempt to provide some evidence which may help to ans­
wer the fundamental question of whether students can learn the
fundamental operations of algebra without drill work in the
same, the results of a modest testing program will be reported.

\(^2\) Ibid. 306-307.
CHAPTER II
REVIEW OF IMPORTANT RELATED LITERATURE

The literature relating to the teaching of mathematics in the high school is very considerable. Several periodicals appearing monthly, or even more frequently devote much of their space to the various phases of the subject. The books, special studies, yearbooks, and articles on the subject are so numerous that a staff of investigators would take considerable time even to catalog the material. The present writer, confronted with such a vast literary storehouse, was faced with the initial problem of limiting himself severely to the material which was most pertinent to the content of the thesis.

In making this selection several pertinent questions were kept in mind: What determines the content of the average or typical elementary algebra textbook? What are the objectives related to this content matter? What is the essential pedagogical procedure resulting from these objectives? With these questions in mind, the present investigator was thus able to limit and select the materials which he thought appropriate to this review.

The reviewer soon learned that there were several outstanding and highly authoritative sources. It is this source of material and related literature which will be the concern
of this chapter. This material is contained in the four following national and semi-national mathematical reports. These reports are entitled:

1. The Reorganization of Mathematics in Secondary Education, a report by the National Committee on Mathematical Requirements under the auspices of the Mathematical Association of America, Inc.


The four reports represent the different schools of thought of Educators concerned with mathematics in the high school. They have been chosen as the main concern of this section of the study because they contain the general principles and important applications of mathematical instruction in the secondary school. These principles and applications are pertinent for an understanding of the problems to be discussed in the principal part of this study where an analysis and comparison is made by two radically different elementary
algebra textbooks. These reports have been and continue to be the guide and standard from which both writers and teachers take inspiration and appraise their mathematical techniques. They are the source and inspiration for a large part of the periodical literature concerned with mathematics in the secondary school. In fact it would be impossible to read with adequate understanding the vast amount of matter being published today on elementary mathematics without a knowledge of the recommendations of these reports. For these reasons the present writer has made these four reports the substance of the review.

In addition to the above reports and in connection with their interpretation considerable use was made of the various yearbooks of the National Council of Teachers of Mathematics, of articles in the mathematical and educational journals, of specialized studies such as those published in the various educational monographs, and of selected reports and books relating to elementary algebra.

Unquestionably, the most important study made in relation to mathematics in American secondary schools is the report published in 1923 by the National Committee on Mathematical Requirements, entitled, The Reorganization of Mathematics in Secondary Education.1 This report has been and is the bible

1 The Reorganization of Mathematics in Secondary Education, A report by the National Committee on Mathematical Requirements under the auspices of the Mathematical Association of America, Inc., 1923.
of textbook writers, teachers, and critics interested in the mathematics of secondary education. The reference to it in the periodical literature, in subsequent reports and studies are countless. It takes up practically every phase of the various mathematical subjects traditionally included in American high schools.

Probably the most unbiased estimate of the report's significance has been made by the group which objects most vehemently to its teachings. Thus the Progressive Education Association in its own report says:

In recent years, the report of the National Committee on Mathematical Requirements ..., published in 1943, has been widely recognized as authoritative. ... The influence of this report on point of view and practice of teachers can hardly be overestimated. Authors of mathematics texts have almost invariably claimed that their books conform with its recommendations. Teachers in training have studied it as they have studied no other pronouncement in the field, and writers on methods of teaching continue to devote many of their pages to a discussion of its views.2

The declaration of the National Council of Teachers of Mathematics in its First Yearbook is fairly indicative of the general tone of the comments which have been made regarding this report:

It is not too much to say that the advance in the teaching of mathematics in our secondary schools in the last decade has been due in large part to the work of this Committee.3

The report is very explicit in its expression of the objectives of mathematics on the secondary level. It divides these objectives into the three traditional categories: practical, disciplinary, and cultural. But lest one or the other of these aims be given an emphasis not intended by the Committee, the report carefully declares:

The primary purposes of the teaching of mathematics should be to develop those powers of understanding and of analyzing relations of quantity and of space which are necessary to an insight into and control over our environment and to an appreciation of the progress of civilization in its various aspects and to develop those habits of thought and of action which will make these powers effective in the life of the individual.4

In the light of this primary objective of all mathematical study at the secondary level, the report proposes and explains the practical aims of mathematics. It defines a practical or utilitarian objective in a rather narrow sense, declaring:

4 Reorganization of Mathematics, 10.
... we mean then the immediate or direct usefulness in life of a fact, method, or process in mathematics.\footnote{Ibid.}

The following is a summary of the report's practical aims:

1. Accuracy and facility in numerical computation, together with an increased understanding of the nature of the fundamental operations, and the use of common sense in computation with approximate numbers.

2. Understanding of the language of algebra and the ability to use this language intelligently and readily.

3. Understanding and use of elementary algebraic methods through study and drill in the fundamental laws and technique of algebra, resulting in a better understanding of arithmetic processes.

4. Ability to understand and interpret graphic representation, including the representation of statistical data and quantitative variables.

5. Familiarity with geometric forms necessary for mensuration, space perception and spatial imagination.\footnote{This last mentioned objective together with the other practical aims are given in full on pages 6-8 of the report.}
It should be noticed that the practical aims recommended by the report place considerable emphasis upon the maintenance and improvement of arithmetic skills. The Committee was aware of the unhappy state of arithmetic learning in the elementary schools, and was also anxious to provide a kind of bridge between the study of arithmetic and algebra. Thus, in recommending work in algebraic technique, the report justifies a "certain minimum of drill," because this will lead to a better understanding of similar processes of arithmetic. In this connection the report includes the observation that,

The essence of algebra as distinguished from arithmetic lies in the fact that algebra concerns itself with the operations upon numbers in general, while arithmetic confines itself to operations on particular numbers.\(^7\)

In recommending certain disciplinary aims, the report defines these as "aims which relate to mental training." This training "involves the development of certain more or less general characteristics and the formation of certain mental habits."\(^8\) These mental habits are considered as applicable not alone to mathematical fields but to other related areas. The Committee was so anxious about the problem of transfer involved in its statement of objectives that an entire chapter of the report is devoted to explaining theory

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\(^7\) Ibid. 7
\(^8\) Ibid. 8
of transfer.

The following disciplinary objectives are recommended by the Committee in the report:

1. The acquisition of those "ideas or concepts in terms of which the quantitative thinking of the world is done." This means ratio and measurement, positive and negative numbers, and dependence of one quantity upon another.

2. The ability to think clearly in terms of the above concepts in the

   a. analysis of complexities into simple parts,
   b. recognition and expression of logical interrelations of parts,
   c. generalization, i.e., discovery and formulation of general laws.

3. The acquisition of mental habits and attitudes. This means more precisely, "a seeking for relations and their precise expression; an attitude of enquiry; concentration and persistence; a love for precision, accuracy, thoroughness, and clearness, and in a distaste for vagueness and incompleteness; a desire for orderly and logical organization as an aid to understanding and memory."

4. Training in "functional thinking."

The last-mentioned objective is considered the key aim of all mathematical study on the secondary level. So important did the Committee consider it that an entire chapter is devoted to explaining its meaning, and its application to all the

9 Ibid.
10 Ibid. 9
branches of high-school mathematics. The unifying principle according to the recommendations of the report is not to be any superficial generalized course, or even a course vitalized by problems from science and industry, but rather this idea of functional relationship. The report declares that:

The one great idea which is best adapted to unify the course is that of the functional relationship. The concept of a variable and of the dependence of one variable upon another is of fundamental importance to everyone. ... The primary and underlying principle of the course should be the idea of relationship between variables, including the methods of determining and expressing such relationships. The teacher should have this idea constantly in mind, and the pupil's advance should be consciously directed along the lines which will present first one and then another of the ideas upon which finally the formation of the general concept of functionality depends. 11

Since many teachers are apt to misunderstand the precise meaning of the report in regard to "functional relationship," the Committee is very careful to explain the meaning of the recommendation:

It will be seen that ... there is no disposition to advocate the teaching of any sort of function theory. A prime danger of misconception ... is that teachers may think it is the notation and the definitions of such a theory that are to be taught. Nothing could be further from the intention of the Committee. ... What is desired is that the idea of relationship or dependence between variable

11 Ibid. 12
quantities be imparted to the pupil by the examination of numerous concrete instances of such relationship.\textsuperscript{12}

Having clarified the meaning of functional relationship, the Committee insists over and over again on its prime importance in secondary mathematics. Indeed, the Committee feels that for the majority of high-school students there is little of permanent value in the study of mathematics except this residue or habit of functional thinking. Very detailed are the instructions to the teachers on how to get the highest possible functional value from each course in mathematics. Thus the report makes the following recommendation:

To attain what has been suggested the teacher should have in mind constantly not any definition to be recited by the pupil, not any automatic response to a given cue, not any memory exercise at all, but rather a determination not to pass any instance in which one quantity is related to another, or in which one quantity is determined by one or more others, without calling attention to the fact, and trying to have the student "see how it works.\textsuperscript{13}

Perhaps no part of the report has aroused greater comment, both favorable and unfavorable, than this recommendation and explanation of functional thinking. That the recommendation has been taken seriously by the authors of mathematics textbooks and by the compilers of objective tests in mathe-

\textsuperscript{12} Ibid. 64
\textsuperscript{13} Ibid. 65
matics is evident from even a superficial inspection of the texts and tests in common use. Thus questions like the following occur in any typical objective test for algebra: Given the formula, \( I = Prt \), if \( p \) is doubled, what will happen to \( I \)?

Functional thinking is given this unique emphasis because it tends so readily to give meaning and understanding to mathematical endeavor. More than anything else perhaps, the basic philosophy of the report is that students be educated to an understanding and meaning of the mathematical operations they perform. Purely mechanical skill and manipulative dexterity prior to understanding or without understanding is anathema to the Committee. Hence the emphasis upon functional thinking.

The report is not very much concerned with what it calls the cultural aspects of high-school mathematics, but it does list three such objectives while at the same time admitting that the "realization of some of these aims must await the later stages of instruction."\(^{14}\) The cultural objectives recommended by the report are:

1. Appreciation of beauty in the geometrical forms of nature, art, and industry.

\(^{14}\) Ibid. 10
2. Ideals of perfection as to logical structure, precision of statement and of thought, logical reasoning, discrimination between the true and the false.

3. Appreciation of the power of mathematics.

Having described and explained at considerable length the objectives of mathematics in the high school, the report shows their application in the various mathematical subjects. Over and over again the report expresses strong disapproval of the formal or technical aims so prevalent in mathematical courses. Thus in regard to algebra, the report declares that:

The excessive emphasis now commonly placed on manipulation is one of the main obstacles to intelligent progress. On the side of algebra, the ability to understand its language and to use it intelligently, the ability to analyze a problem, to formulate it mathematically, and to interpret the result must be dominate aims. Drill in algebraic manipulation should be limited to those processes and to the degree of complexity required for a thorough understanding of principles and for probable applications either in common life or in subsequent courses which a substantial proportion of the pupils will take. It must be conceived throughout as a means to an end, not as an end in itself.15

This clear condemnation of the overemphasis of the formal or technical aspect of algebra is something to which the report returns more than once. Perhaps the temper of the report on this point is most clearly shown in Part II where, in an

15 Ibid. 11
investigation of the use of standardized tests, the following objectives for the "ideal" algebra course are given:16

1. The development of the patience and willingness to go through a mass of details, keeping each item in mind.

2. The ability to do close and detailed thinking.

3. The ability to give sustained attention.

4. The ability to weigh the significance of each word in a sentence.

5. The ability to know whether a sentence has meaning or not.

6. The ability to think rigorously and soundly rather than carelessly.

7. The inquiring or questioning attitude of mind.

8. The ability to subject a statement to a severe test of its truth or validity.

9. The ability to discover whether sufficient data have been given.

10. The ability to analyze.

11. The ability to draw conclusions.

As can be seen from these eleven objectives of the ideal algebra courses, the Committee expects that objectives which give meaning and understanding be foremost, while aims looking towards manipulative skill be given as little attention as possible. This point is important in the analysis of the two

16 Ibid. 369
textbooks which follows in a later section of this study. It will be shown there how one author, attempting to follow the report's recommendation on giving priority to understanding, prepared a text which contains more of the mechanical than another text written by an author who rejects the fundamental principle of the report.

Having laid down the general principles and objectives which should govern the teaching of mathematics, the report proceeds to outline the content of high-school mathematics courses in the light of these objectives. The following summary is the recommended content for the elementary algebra course:

1. The formula - its construction, meaning and use.

2. Graphs and graphic representation in general - their construction and interpretation in (a) representing facts, (statistical, etc.); (b) representing dependence; (c) solving problems.

3. Positive and negative numbers - their meaning and use, (a) as expressing both magnitude and one or two opposite directions; (b) their graphic representation; (c) the fundamental operations applied to them.

4. The equation - its use in solving problems.
   a. Linear equations in one unknown.
   b. Simple quadratic equations with formulas and problems.
c. Equations in two unknowns, with concrete illustrations.

d. Simple applications of ratio and proportion, and simple cases of variation.

5. Algebraic technique:

a. The fundamental operations. Their connection with the rules of arithmetic should be clearly brought out and made to illustrate numerical processes.

b. Factoring. These cases only: common factors of the terms of a polynomial, the difference of two squares, trinomials of the second degree that can be easily factored by trial.

c. Fractions: They are to be simple and are to illustrate similar arithmetical processes. Constant use is urged to gain accuracy and facility.

d. Exponents and radicals. This section should be confined to the simplest material required for formulas. The laws of positive integral exponents should be included, as well as the square root of a number, but not of a polynomial.

e. Solutions are to be checked.

The report strongly recommends that the algebraic technique be taught in such a manner that:

Drill in these operations should be limited strictly in accordance with the principle mentioned in chapter II, page 11. In particular, "nests" of parentheses should be avoided, and multiplication and division should not involve much beyond monomial and binomial multipliers, divisors, and
quotients.\footnote{Ibid. 24}

The report recommends that the following matter be excluded from the elementary algebra course:

1. Highest common factor and lowest common multiplier, except the simplest cases.

2. Theorems of proportion relating to alternation, inversion, composition and division.

3. Literal equations, except for common formulas.

4. Radicals except as indicated above.

5. Square root of polynomials.


8. Simultaneous equations in more than two unknowns.

9. The binomial theorem.

10. Imaginary and complex numbers.

11. Radical equations except such that are needed for elementary formulas.

It should be noticed here that the report recommends algebraic content which is largely formal or technical in nature: graphic representation, positive and negative numbers, fundamental operations, factoring, exponents and radicals. The aim however is not to teach this formal aspect of algebra.
in order to train for mechanical or manipulative skills, but rather to bring the student to an understanding of the laws and operations inherent in this matter. A relatively small part of the total content is given over to equation work, which is after all the prime mathematical work of algebra. The formal aspect of the subject is but an analysis of the parts of an equation. Hence, implicit in the report's suggested content matter for elementary algebra is the recommendation that the equation be taught deductively. This means that the student should first learn and understand the operations involved in solving the different parts of an equation, and then apply this knowledge and skill to equation work.

This arrangement is logical in view of the report's insistence on understanding as a fundamental principle. And, as will be shown, it is the fundamental assumption justifying the arrangement of content matter in the typical elementary algebra text.

In resume, therefore, the report outlines in detail the objectives, methodology, and content for high-school mathematics. In the field of objectives, the disciplinary aims are stressed; in regard to methodology, the purely manipulative processes are to be given as little emphasis as each subject will allow; functional thinking is to be stressed and is to be the unifying principle; much of the content material traditional to the older texts is eliminated, and new material such as graphic representation and statistics, is added. In
algebra the aim is predominantly to develop thought processes such as broad functional thinking. Slight emphasis is to be given to manipulative processes of a mechanical nature. Algebra is to be taught deductively, not inductively. The content is more intimately connected with what the Committee feels are more important needs for life.

So important have the effects of this report been that there is hardly an American mathematics textbook that does not follow its recommendations, especially as to content material faithfully. In fact, it is impossible to understand the purpose of the content and teaching methods of the typical mathematics textbook of high school without reference to this report.

The respect in which it is held is well expressed in the First Yearbook of the National Council of Teachers of Mathematics:

Never was an investigation in this field better organized, more adequately financed and more painstaking in the effort to impartially weigh all phases of the questions involved. Probably no report of this kind ever received more open-minded consideration of its various recommendations by so large a proportion of the constituency.18

A review of the vast periodical literature which grew out

18 A General Survey of Progress in the Last Twenty-five Years, 187.
of the discussion of the report is impossible in this present paper. One study, however, deserves particular mention because it shows in a factual way the precise effect of the report's recommendations regarding content material for elementary algebra. The study referred to is an analysis of thirty textbooks of elementary algebra made by Ruth Olson and published in the Mathematics Teacher.\(^{19}\) The author selected the algebra textbooks on the basis of frequency of use in schools and chose ten texts for each of the different periods from 1895-1900, from 1910-1915, from 1924-1929. The following digest of Miss Olson's study reveals that the changing content of elementary algebra texts is following rather closely the recommendations of the report. The following topics have been given largely the same emphasis during all three periods:\(^{20}\)

1. Nature of algebra.
2. Fundamental operations with integers.
3. Fundamental operations with fractions.
4. Simple equations.
5. Evaluation.
7. Special products.
8. Linear equations.


\(^{20}\) Ibid. 309
9. Simultaneous equations.
11. Arithmetical square root.
12. Reviews.

The following topics are being given a decreasing emphasis:\textsuperscript{21}

1. Quadratic equations.
2. Literal equations.
3. Radicals and surds.
4. Ratio and proportion.
5. Variation.
6. Highest common factor and lowest common multiple.

The following topics are being entirely eliminated from elementary algebra texts:\textsuperscript{22}

1. Nested parentheses.
2. Complex factoring.
3. Complex fractions.
5. Square root of polynomials.
6. Cube and higher roots.
7. Various theorems.

\textsuperscript{21} Ibid. 310
\textsuperscript{22} Ibid. 310
9. Imaginary and complex numbers.
11. Series and progressions.

The following topics have been introduced into elementary algebra texts and are being given increasing emphasis:

1. Formula.
2. Illustrations.
3. Historical and biographical notes.
5. Trigonometry.
7. Tables (trigonometric).
8. Significant figures.
10. Intuitive geometry.
11. Slide rule.

The above tabulation indicates that the recommendations of the report have been taken seriously by the textbook writers. In all cases the topics which were recommended to be eliminated by the Committee are being gradually dropped; on the other hand topics which the Committee suggested for inclusion in the elementary course are being included in the new texts or are

23 Ibid. 310
being given increased emphasis. There is a tendency away from the formal technique and algebraic manipulation, and towards the inclusion of topics which the Committee considered more meaningful, such as graphs, statistics, and the like.

A somewhat similar study of general trends in high-school mathematics was made by Orlando E. A. Overn and published in the *Journal of Experimental Education*. This study was an analysis of the examinations of the College Entrance Examination Board for the period 1901–1935. The analysis showed that, where in 1901–1904, 71 percent of the questions were on algebraic technique, in the period 1924–1935 this percentage had been reduced to 29 percent. Evidently the report's strong recommendation to reduce to a minimum the purely technical aspects of elementary algebra has been effective, for the examinations are indicative of general trends in the teaching of high-school mathematics. Furthermore, the same study showed that an increasing emphasis in the examinations was being given to numerical equations and especially to the verbal problems in one or two unknowns. Between 1901 and 1935 the percentage of verbal problems rose from 6 percent to 22 percent. Problems on graphs and graphic representation together with prob-

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25 *Ibid*.

26 *Ibid*.
lems in numerical trigonometry, topics entirely absent in the examinations for 1901-1924, rose to 21 percent of the entire material during the period 1924-1935.27

There are two other reports of considerable importance in regard to mathematics in secondary education. The first of these, The Place of Mathematics in Secondary Education, 28 is essentially an amplification and continuation of the 1923 report. The other report, Mathematics in General Education, 29 published by the Progressive Education Association, is representative of the far left in secondary education and in many ways challenges the views and principles of the 1923 report and its successor.

The educational and mathematical journals predicted that these new reports, which were both issued in 1940, would have a far-reaching effect on mathematics in secondary education. Fairly typical of the spirit with which the reports were received at the time of their publication is the remark by Harl R. Douglas in an article in the Mathematics Teacher:

The report of the Joint Commission is the most important document in this subject since the report of the National Committee

27 Ibid.
29 Mathematics in General Education.
... which appeared in 1923 ... and will probably be even more influential.30

The report of the National Council agrees substantially in its statement of objectives with the 1923 report. However, where the earlier report was content to give these objectives a precise expression and point out their connection with the various mathematical courses, the new report goes into a rather lengthy discussion of the meaning of the objectives from both a mathematical and psychological viewpoint. In addition it relates them to the larger aims of education in general.

In many ways the report is an apologia for the study of mathematics in the secondary schools. The Committee which prepared the report was aware of the vast pressure on school administrators either to abolish mathematical courses entirely or to lower their present standards. For this reason the report is very clear in urging that mathematical courses be maintained at highest possible level. The following recommendation speaks for itself:

The Commission believes strongly that educators not resign themselves to the doctrine of "minimum education" as a norm. It believes that we should by all means require as ideals and standards

something definitely superior to the small amounts of this or that subject which some people "get along with" ... the schools should be unremitting in their efforts to raise the general standards of American culture. 31

In an effort to suggest every possible means which will tend to maintain these high standards, the report recommends that there be no mathematical requirement for those "whose rebellious distaste for the subject seems firmly entrenched." 32 On the other hand the commission is anxious that administrators be not too lenient in giving in to those with such "rebellious distaste" for mathematics. It recommends that:

A strong and corrective influence should be exerted upon those boys and girls who are capable of doing fair work with secondary mathematics but who ... think it unnecessary and yield to what is easiest. 33

The report does not recommend strict homogeneous grouping for the maintenance of standards. However, there is a strong implicit approval of such an arrangement, both in the fact that the report goes to some length in answering the usual objections to homogeneous grouping, and in the fact that it gives in detail a special course for retarded pupils. Although it is never mentioned explicitly, the tone of the whole document gives the impression that the Committee felt that mathematics

31 Mathematics in General Education, 36-37.
32 Ibid. 36
33 Ibid. 44
was for the upper half of the secondary-school population. It seems to sense that much of the pressure for eliminating mathematics from the secondary-school curriculum comes from those who are convinced that normal mathematical courses cannot be carried readily by the lower half of the secondary-school population.

The result of this apologetic tone on the part of the Committee is evident in the more recent texts where considerable effort is made to "sell" the subject to the students. "Heart to heart" talks on the value of the subject abound in such texts, and the book itself is made attractive by the use of pictorial illustrations, modernistic graphs, and the like. The result in some of the texts is a considerably expanded volume containing page after page of printed explanation and interesting descriptions of mathematical applications. And for the less able (or perhaps less industrious) student, many of the exercises contain fairly simple problems, while the more difficult problems are marked "honor work" and are not considered essential or required matter. Such an arrangement lends itself to a more or less homogeneous grouping of the pupils, as no doubt the report intended.

One of the notable differences between this report and the 1923 report is that the former is more explicitly concerned with attitudes necessary or desirable for the effective learning of mathematics. Much space is given in the recent
report to an explanation of mathematics as a "mode of thinking."
The report is explicit in insisting that significant results come only as a product of understanding, and, like the earlier report, it frowns upon a course in mathematics depending too much upon memory or manipulative skill.

In recommending particular courses, the report gives two different plans. The essential feature of both plans is that they provide for continuous mathematical instruction for grades seven to twelve, and that they make provision for the normal pupil, and the retarded and advanced pupil. Except for the retarded pupil, the report recommends that the study of mathematics be extended through as many years as is consistent with sound educational practice. More and more mathematics for the able student is the platform of the report:

One must, however, face the fact that in many school systems the requirement of mathematics does not extend through as many years as, in the opinion of the Commission, are justified by the best ultimate interests of the boys and girls. Hence, there is the urgent practical problem of bringing about an extensive election of further mathematical courses beyond the required subjects. ... The Commission believes that mathematics should be required through the ninth school year, and beyond the ninth year in the case of competent students...34

The course in elementary algebra is essentially the same as that recommended by the 1923 report: there is the same in-

34 Ibid. 74
sistence that drill in algebraic manipulations be limited, that functional thinking be constantly used, that the four fundamental operations and the formal technique be taught with a view to the students understanding the basic algebraic laws.

In recommending the details of the course in elementary algebra the report points out that most of the texts bearing the title algebra contain not only strictly algebraic material but also arithmetic, graphic representation, and trigonometry. The following detailed outline is given here because of its importance to an understanding of the report's concept of the recommended course in elementary algebra. The course for ninth-grade mathematics, as outlined by the commission in the report, is as follows:

**Arithmetic**

I. Continuing the study of arithmetical concepts, skills, and principles, primarily in connection with the work in algebra.

II. Solving applied problems, as suggested by the work in algebra; also problems involving approximate computations; the slide rule (optional).

**Geometry**

I. Reviewing and applying the geometric training suggested for grades 7 and 8, in connection with the work in algebra.

II. In classes of superior ability, making a beginning in the study of demonstrative geometry.

35 The outline of the matter for ninth-grade mathematics is quoted exactly as printed in the report, pages 88-91.
Graphic Representation

I. Reviewing and applying the technique acquired in grades 7 and 8, in the representation of statistical data demanding greater maturity, mainly from the fields of business, economics, the social studies, and science.

II. Making graphs representing algebraic formulas, such as \( p = 3s, i = .04p \), and so on.

III. Using graphs in the study of linear equations.

IV. (For superior pupils) using graphs in the study of simple cases of quadratic equations.

Algebra

I. Basic Concepts.

(1) The acquisition of the basic vocabulary.

(2) Developing the ability to explain clearly the meaning of key concepts, such as exponent, positive, negative, ratio, proportion.

II. Fundamental Skill and Techniques.

(1) The four fundamental operations involving

(a) Positive and negative numbers.

(b) Algebraic monomials or simple polynomials.

(c) Algebraic fractions, mainly with monomial denominators.

(2) Special products and factoring as follows:

(a) Squaring a binomial.

(b) Finding the product of the sum and the difference of two terms.
(c) Factoring a polynomial the terms of which contain a common monomial factor.

(d) (Optional). Factoring trinomials of the form $x^2 + bx + c$.

(e) Factoring the difference of two squares.

(3) Powers and roots.

(a) Laws of exponents and their use.

(b) Square roots of positive numbers.

(c) Fundamental operations involving radicals, mainly of the monomial type.

III. Fundamental Principles.

(1) A study of the principles governing the fundamental operations, such as the rules of order and grouping, the rules of signs and of exponents.

(2) A study of the principles used in the solution of equations, such as the rules of equality and of transformation.

IV. Study of Relationships and of Dependence.

(1) By tables.

(a) Interpreting tables of related number-pairs.

(b) Making tables based on formulas.

(2) By graphs.

(a) Graphs as means of illustrating quantitative relationships.

(b) Making graphs based on tables of related number-pairs.

(c) Making graphs based on algebraic expressions or formulas.
(d) Using graphs in the study of equations.
(e) Using graphs in the solution of problems.

(3) By formulas.
(a) Formulas as a means of expressing relationship or dependence.
(b) Making formulas based on verbal statements; on geometric figures; on tables.
(c) Evaluating a formula.
(d) Transforming a formula (only simple cases.)

(4) By equations.
(a) Equations as means of expressing quantitative relationships.
(b) Solving equations of the first degree in one unknown.
(c) Solving pairs of equations of the first degree.
(d) Solving fractional equations.
(e) Solving equations of the form $a x^2 = b$.
(f) Solving simple radical equations.
(g) Using equations in the study of proportion and of variation.
(h) Using equations in the solution of problems stated in verbal form.

V. Using Algebra in Life Situations and in Problem-Solving.

(1) Learning to translate quantitative statements into the language of algebra.
(2) Learning to make generalizations suggested by the techniques and principles of algebra, particularly with relation to the precise way in which definitely related, changing quantities will influence each other under given conditions.

(3) Solving general verbal problems, using as a means of solution the table; the graph; the formula; the equation.

(4) Applying the techniques of algebra in problem situations arising in business; in the shop; in science; in everyday life.

(5) Interpreting the solutions of equations, including negative values, where they have significance.

**Trigonometry**

I. Reviewing the necessary concepts and skills.

II. Finding heights or distance indirectly by scale drawing; the Pythagorean relation.

III. Finding heights, or distances, or angles, indirectly by using the natural trigonometric functions (sine, cosine, tangent.)

IV. Using a table of natural functions. (Interpolation should be regarded as optional.)

The committee no doubt felt that the above outline of the elementary algebra course would be considered too extensive by many algebra teachers. The report points out, however, that it can be adapted to local circumstances, and it also makes the observation that the requirements of the course can be attained if the teacher gives but slight emphasis to technical processes.
Here again it should be remarked that although the report recommends that slight emphasis be given to manipulation processes, the recommended course is largely of a technical nature. Exercise in such material must necessarily be through simple manipulative drills. Yet relatively slight emphasis can be given to equation work in the course outlined by the report. This deductive approach to the equation, based as it is on the notion that understanding comes first, is precisely the point at issue between the authors of the two textbooks to be analyzed in this study. On this important point the report is essentially the same as the 1923 report. Further, this deductive approach is something that is never explained or justified in either report; it appears to be an assumption very much taken for granted. Yet it is precisely this assumption which the Progressives attack with their programs of incidental learning, and it is the point most disputed by the proponents of "inductive" mathematical texts.

Very much unlike the report of the Joint Commission is the document of the Progressive Education Association. This report, entitled Mathematics in General Education, is not only a study of aims and methods in mathematics, but more essentially a rehashing of much of the educational doctrine of the Progressives. It boldly suggests the elimination of the entire present organization of high-school mathematics.
but it offers no substitute curriculum.

Mathematical work would be retained in the curriculum but the attack would be along the incidental or instrumental pedagogy of the Progressives. In general its aim is:

... To provide rich and significant experiences in the major aspects of living, so directed as to promote the fullest possible realization of personal potentialities and most effective participation in a democratic society.36

Thus the mathematics course would in some obscure way contribute to developing personalities essential to democratic living, social sensitivity, esthetic appreciation, tolerance, cooperativeness, self-direction, creativeness, and the disposition to use reflective thinking in the solution of problems.

Unlike the report of the Joint Commission is the Progressive's report in that it suggests a mathematical program of studies suitable to the needs of all pupils in the secondary schools. Mathematics is explicitly related to general education, and as the Progressive report declares:

... general education ... is not specialized or restricted to any particular group. It emphasizes meeting the educational needs of each student, and the group with which it is concerned is the secondary-school population as a whole.37

36 Mathematics in General Education, 43.
37 Ibid. 13
Since this is the case, the report admits that it is impossible to recommend any logically organized course of mathematics, or even any particular curriculum in such a course of general education. It declares that:

In such a program the curriculum for a given school or group must, in the last analysis, be determined in the light of the needs of the particular individuals who make up the group to be taught. Since students differ widely as to their needs, capacities, and interests, it would not be consistent with the purpose of this report to outline a detailed course of study to be followed by all, or even to propose a number of alternative courses.38

In any classroom the teacher is to follow his own desires and arrange in an informal way whatever mathematics course he deems suitable:

For these reasons, instead of recommending a single more or less formal course of study, the report outlines a set of fundamental concepts and guiding principles designed to serve as a basis upon which teachers may so organize their own work as to make it appropriate to the possibilities and limitations of individual schools, or classes, or, ideally, individual students.39

The resulting chaos in mathematical studies at the secondary level should be obvious. That a serious attempt to put in practice the recommendations of the report might actually lead to some confusion, the Progressives admit:

38 Ibid., 13
39 Ibid., 13
In formulating the outlines of a program through which mathematical education may advance during the next few years, the Committee had of necessity to be idealistic. To make the proposed program effective and to supply innumerable details of possible content and organization will require experimentation, both extensive and intensive, over a period of years. In the light of such experimentation certain suggestions of this report will almost certainly need to be modified or even rejected.40

A brief description of this report would seem to suffice for the purposes of this present study. Rejecting as it does the fundamental principle of the two reports already described, namely the deductive approach to mathematics, this report represents an essential break with this traditional position. It holds a position far to the left in mathematical instruction not because it denies the basic premise of the traditionalists, but because it recommends a method of instruction both revolutionary and detrimental to mathematics in secondary education. Thus it not only disputes the basic notion of the other two reports, but in so far as it reduces mathematical standards, it tends to undermine the very purpose of these reports. The importance of this report, and its relation to this study, lies in this challenge to the fundamental principle of deductive instruction and not in its own positive doctrine of incidental learning.

40 Ibid. 14
Part I of the Progressive report gives a general outline of Progressive educational philosophy, and the role of the mathematics teacher in achieving these aims. Part II is a description of how the teacher is supposed to make these ideals practical and living in the actual mathematical instruction. Part III is an excellent treatise on the nature of mathematics. This section is quite abstract and appears to be but a summary of much of the matter usually found in books on mental hygiene.

Organized mathematical courses together with the traditional methods of mathematical instruction are condemned by the Progressives. Mathematical concepts and skills are of course to be taught, but in relation to and incidental to the problems of every day living that come up for discussion in the Progressive classroom. The report feels that the only thing necessary now is to find out exactly what these problems are in detail, and how they can be taught in such a way as to develop mathematical abilities.

The report is almost fanatically interested in promoting experimentation in the secondary schools of America. This is not surprising in a Progressive Education document. What does cause some surprise is the unusual tone given to even strictly mathematical discussions. Thus, in describing the educational
values in the formulation and solution of problems the report claims that this purely mathematical work helps considerably in "fostering the ideals of democracy."41 Thus the report declares:

In the judgment of the Committee the connections are immediate as well as fundamental. They become clear when the ways in which problems are formulated in countries where there is less democracy (as in Germany, 1939) are contrasted with the solution and formulation of problems in a country where there is more democracy (as in the United States, 1939.) In the first place, the difference lies in the very possibility of formulating problems at all. In dictatorships like Germany (in 1939) the man on the street is not permitted even to formulate many problems, ...42

This peculiar "democratic" virtue involved in problem solving is even limited in the United States because of opposition to the free exercise of Marxist theories:

The United States is called more democratic at least partly because in this country the situation with reference to the formulation and solution of problems is different. In far fewer instances is the possibility of formulating our pressing problems restricted. True, there are groups that do not wish certain questions to be asked, and these groups are sometimes powerful (advertisers exert a large measure of control over all the widely circulated media.

41 Ibid. 63
42 Ibid. 63
of expression, for example, the so-called "patriotic" groups try to forbid the objective study of Marxist theories of society.) 43

The report wants a mathematical course which will sharpen the appetite of students for experimentation not only in regard to numbers, but essentially in social and economic areas. No abstract or organized mathematical curriculum can contribute, according to the report, to this urge for experimentation. Thus the report remarks that:

It may be remarked in conclusion that the fundamental operations of arithmetic, the formal solution of algebraic equations, the memorization of Euclidean propositions are taught in Germany (in 1939) possibly as well as they are taught anywhere. It is clear that mathematics taught as an abstract science contains little that is anti-authoritarian or pro-democratic; the same is not true for the kind of teaching of mathematics urged in this report. 44

It is not too easy to unravel the exact mathematical content of this very wordy and suspiciously propagandish report. It is clear, however, that the report is opposed to the functional concept as the unifying principle recommended by the 1923 report and its successor, the 1940 report of the Joint Commission. It is also apparent that the report is against the logical organization of subject matter into specific fields such as algebra, geometry, etc. Mathematics is to be taught as

43 Ibid. 64
44 Ibid. 69
something incidental to the solving of problems connected with life. Perhaps the following excerpt describes as well as any the position of the Progressive report:

The position of the Committee, briefly stated, is essentially this: A mathematics curriculum may be built by locating and studying concrete problem situations which arise in connection with meeting needs in the basic aspects of living. The major concepts here emphasized play a fundamental role in the analysis of these problems. They help to clarify the method of attack, and they tend to recur systematically in diverse problems. This recurrence in itself provides for the development of a sense of unity in mathematics as a method of dealing with problems.45

It is evident that the Progressive report rejects the notion that understanding of all the parts of a mathematical problem comes before any attempt at its solution. For the Progressives understanding comes as a result of or, at least, along with the continued effort to solve the problem. Hence, the approach is inductive in the sense that it first attempts the solution of problems and from these goes on to the knowledge and understanding of general laws. Its weakness would not seem to be in this inductive approach but rather in the disorganized and incidental manner it has of selecting these problems.

Although this report was received with considerable attention in the school journals, and although its influence

45 Ibid. 73
may be considerable, at least in those educational circles where Progressive Education ideals hold forth, its basic approach to the selection of mathematical material has been ably criticised by William J. Betz, a leading authority in the field of American secondary education, in articles in the Mathematics Teacher. His position represents the sane reaction of American educators to this somewhat dangerous report. His general comment expresses this sane American viewpoint clearly:

The underlying melody of the PEAR [the Progressive report] after all, is unmistakably the one which has become identified with the functionalism and instrumentalism of the Dewey-Kilpatrick group of educators. If one is in general agreement with Dewey's theory of knowledge and his naturalistic doctrine of experience, all is well. But if one is forced, on the basis of classroom experience, to reject a large part of that educational gospel - as essentially unworkable under conditions of mass education such as we face today - one is led to a contrary appraisal.  

Betz gives the following reasons why it is impossible to build a mathematics course around life situations rather than around a body of scientifically organized mathematical prob-

lems:

(1) Life situations are NOT a substitute for thorough mathematical training. They do NOT guarantee the learning of mathematical ideas, principles and techniques.

(2) It has been found impossible, even in the field of arithmetic, to arrange life situations sequentially in such a way that mathematical concepts, principles and processes can be built up, with their aid, in the cumulative manner which is essential in mathematics.

(3) In secondary mathematics an adequate solution seems even more difficult. A generation ago the Perry movement in England, with its passionate devotion to practical applications and its contempt for "mere theory," failed completely. ... A similar fate overtook all attempts to vocationalize secondary mathematics.

(4) In each of these cases, failure was due to the same basic causes, as follows: (a) the absence of a coherent program; (b) an almost total neglect of real understanding, insight and mastery; (c) a shocking disregard of the principles of learning; (d) a bunching of too many unfamiliar elements in a single learning situation; ... 

In preparing this review of the literature dealing with mathematics in secondary education the writer has been guided by what has influenced or is apt to influence the content, methodology, and objectives of mathematics in secondary education and particularly elementary algebra. Thus, the 1923 report of the National Committee on Mathematical Requirements determined to a very considerable extent the content and

47 Ibid. 352
methodology of high-school mathematical texts, and particularly algebra. The 1940 reports of the Joint Commission and the Progressive Education Association will have a marked influence on high-school mathematics teaching for some time to come.

One further report needs to be reviewed to complete the purpose of this study. This is the report of the Committee on Mathematics of the Missouri Province of the Society of Jesus. This report is important because it contains the principles which to some extent determine the pedagogical practices in mathematics in Jesuit high schools of the Midwest, and because it has been the primary inspiration for one of the textbooks to be discussed in a later section of this thesis.

This Jesuit report is very much unlike the other reports mentioned in this study. It is much older than the others, having been issued in 1914. It aims not at the whole American secondary-school system, but is concerned only with Jesuit high schools in the Midwest. It is by far the most intelligible of all the reports and vastly more concise in its explanation. While the other reports contain more than three hundred pages, this report is only fifty-eight pages long. In this report there is no special pleading for such things as the functional concept, or for the developing of special educational and social attitudes, or for widespread experimentation. It discusses plainly and simply the usual mathematical subjects of secondary education, algebra, geometry, and trigonometry, and
sets forth the particular objectives and teaching methods with which these courses are to be taught in Jesuit high schools of the Missouri Province.

The report mentions and defines the threefold nature of educational objectives: disciplinary, cultural, informational. There is an admission that these values overlap and are found to a greater or less extent in all high-school subjects. In relating these objectives to Jesuit high schools, the report strongly declares that the cultural aim should predominate, and only

... as much of 'informational' studies will be admitted as is absolutely necessary; moreover even these must be taught as to bring their own proper 'cultural' effects to a maximum. This is the old ideal of the Society of Jesus; and it is now universally admitted as the true ideal of what a High School should be.  

Relating this educational creed to algebra, the report admits that there is some meritorious disciplinary effect in mathematics and little informational value. Of the latter, the report feels that for the average run of students in Jesuit high schools the informational effect is negligible: "We may safely put down zero as the informational value of algebra to the average boy."  

Admitting then that algebra, if it is to

49 Ibid. 10
be of any cultural value in the Jesuit high school, must contribute this in virtue of the manner in which it is taught, the report shows precisely how this cultural objective can be effected. Thus the report declares that:

1. A boy, in solving exercises or problems, learns to muster his resources, and use them independently; he acquires self-confidence in attacking or circumventing difficulties, learns to think correctly, learns how to check and test his conclusions when in doubt; he gains faith in his own powers, and his originality develops.

2. The symbols form a new language, very condensed and idiomatic; he learns to translate to and from these, and again acquires self-confidence and power.

3. The habit of generalizing gradually grows on him, and gives him broader views, and the power of grasping and subordinating details.

4. Incidentally he is acquiring habits of mental accuracy, and of concentration of mind upon one point.

5. The sense of satisfaction in discovering new things or solving hard problems makes him eager to try his powers on other difficulties, independently; he will say, "Don't tell me - I'll get this myself;" it becomes a game in which he is intensely interested, and his self-reliance thus continually grows day by day.\footnote{Ibid. 10}

The report admits that textbooks in elementary mathematics written from the informational or disciplinary point of view will differ widely from those written from a cultural point of view.
view. The textbook used in Jesuit high schools ought not to be didactic or formal, according to the report. For it asserts that:

... induction and not deduction, must be the keynote of a correct High School textbook on algebra. And this last remark shows why it is that algebra gives mental culture of a kind quite different from that given by the study of the classics.51

In suggesting classroom methods whereby the cultural objective may be obtained, the report is emphatic that the teacher never use the lecture method. Understanding is admittedly important, but the view of the report is that it comes inductively; that is, as a result of applied and concentrated work in mathematics, and not as a result of lengthy explanations and analysis by either the textbook or the teacher. Thus the report declares that:

The teacher should never use the lecture method; this belongs not to the High School but to the College. It is a matter of experience that a teacher who explains and talks continually in the classroom while the boys are passive and receptive, is unsuccessful; they may say and think that they understand the matter perfectly; the teacher may think that now everything is obvious and clear; but they are both mistaken. Things do not sink in when a boy is passive; he sheds the knowledge thus communicated. ... These methods rest on the false idea that the acquirement of new knowledge is the main thing; on the contrary, as we have seen, it is

51 Ibid. 12
the by-product; the main thing is that the boy learn how to think originally and how to study new problems with what he already has.52

Finally, in order that this cultural objective be achieved to the greatest possible degree, the report urges that a kind of laboratory method of teaching be employed. In the mathematics classroom the teacher is to do a minimum of explaining or lecturing, and is to spend his time going from desk to desk dealing individually with the pupil:

The teacher should deal with his pupils individually, as much as possible; for this reason, some part, say one-half, or one-third, of each class hour - and very frequently the entire hour - should be devoted to desk-work.53

This report for Jesuit schools is unique in the emphasis it places on the cultural objective of high-school mathematics. While the other reports specifically included the cultural aims of mathematics, their primary purpose was either to teach a great deal of mathematical information and skill, or to develop through mathematics the "ideals of democracy," which in the Progressive Education report means an advanced leftish spirit of experimentation. Further the Jesuit report insists that mathematical subjects be taught inductively: that is, from the numerical concrete to the abstract general principle. It urges also that the lecture method be eliminated. Stress is placed on the laboratory method of teaching and individual in-

52 Ibid. 16
53 Ibid. 18
struction is favored over class instruction.

It is important to notice that the Jesuit report agrees with the 1923 report and with the 1940 report of the National Commission on insisting upon the scientific organization of content into the usual mathematics courses of algebra, geometry, and trigonometry. Nevertheless, it holds with the Progressives for inductive instruction and consequently rejects the deductive teaching methods of the other two reports as being out of harmony with its own cultural objective. But in insisting upon inductive instruction the Jesuit report does not hold to the incidental selection of mathematical problems from life situations, as the Progressives recommend, but relies rather upon the traditional problem material of the high-school mathematical courses.

In conclusion, therefore, these reports are the basis and the source of almost all the important mathematical discussion in regard to secondary education. In importance there is no doubt that the 1923 report has been and is the most influential. Indeed, it is difficult to discover a modern textbook of high-school mathematics that does not follow to a very exact degree the recommendations of this report. Just how important the 1940 report of the National Commission will be remains to be seen. As was mentioned, this report is a sequel and a development of the 1923 report. Those who have commented upon it in the mathematical and educational journals predict that it will
be even more influential than the earlier report. The Progressive Education report is not strictly a mathematical study at all, although it professes to be such, but rather a restatement of the philosophy of the Progressives. An attempt, not too clear it would seem, was made to twist mathematics into this instrumentalist point of view. Finally, the Jesuit report has to some extent been influential in giving emphasis to mathematical objectives peculiar to the liberal code of education which is professedly the mark of American Jesuit secondary schools.
CHAPTER III

A COMPARISON OF THE CONTENTS, OBJECTIVES, AND TEACHING PROCEDURE OF FIRST COURSE IN ALGEBRA AND EXERCISES IN ALGEBRA

Two algebra textbooks are in current use in the high schools of the Missouri and Chicago Province of the Society of Jesus. The first of these is entitled, First Course in Algebra,¹ by A. D. Theissen and L. A. McCoy, and has been in use in the two provinces for several years. The other text, Exercises in Algebra,² by W. C. Doyle, S. J., was introduced in one school (Rockhurst High School, Kansas City) more than five years ago as an experimental text, in mimeographed form. In the year 1945-46 this book was published by Loyola University Press and introduced into several of the schools of the two provinces. A few of the schools used the new text in all the freshman classes; many of the schools preferred to experiment with the new book in one, two, or three of the first-year classes, while teaching the First Course in Algebra in the remainder of the classes.

Naturally, there has been considerable discussion on the relative merits of each book. Some of the teachers strongly criticise the new text and faithfully teach the older

one. There are others who just as fervently defend *Exercises in Algebra* and claim considerable success in teaching it.

This study will describe in detail the new book, *Exercises in Algebra*. The basis of comparison will be the older text, *First Course in Algebra*. The general and specific objectives of the two texts will be compared, the content matter of each described, the particular mode of presentation of content matter of each book will be discussed, the teaching technique required by the different texts will be described.

*First Course in Algebra* by Theissen-McCoy is a traditional algebra book. This means that it follows the usual run of content matter which is presented in nearly all first-year algebra books. The sequence of matter and the methods of instruction are pretty much of the same pattern which is followed in the usual elementary textbook in algebra. The authors appear to have followed, as do most textbook writers of algebra, the recommendations of *The Report of the National Committee on Mathematical Requirements of the Mathematical Association of America*, published in 1923. This report is the standard guide for writers of mathematic textbooks and has probably influenced the content matter of present day algebra texts more than any other single factor. The following list of topics from the Table of Contents³ of this text is considered typical algebraic

³ Theissen-McCoy, ix.
material for first-year high school:

The Formula and the Simple Equation
Symbols and Indicated Operations
Addition and Subtraction
Multiplication and Division
Equations and Problems
Special Products and Factors
Fractions
Linear Equation in Two Unknowns
Graphical Representation
Linear Fractional Equations
Powers and Roots
Exponents and Radicals
Quadratic Equations
Ratio and Proportion
Numerical Trigonometry
Supplementary Review Exercises

This is standard first-year matter in elementary algebra. That this material is representative of what is given in the usual first-year algebra course is shown from a study of fourteen currently used textbooks in elementary algebra made by Joseph J. Urbancek, and published in School Science and Mathematics. The following table gives the distribution of problems of fourteen typical texts as tabulated by Urbancek,4 and the distribution of problems in Theissen-McCoy.5 The percent in each case refers to the ratio between the total number of problems in the text, and the number of problems given to each topic. The substantial agreement between the typical text and Theissen-McCoy is clearly seen from a study of this table:

5 The distribution table for Theissen-McCoy was prepared by the author of this paper.
## PROBLEM DISTRIBUTION OF TYPICAL ALGEBRA TEXT

### TABLE I

<table>
<thead>
<tr>
<th>Topic</th>
<th>Distribution of problems in 14 typical texts</th>
<th>Distribution of problems in Theissen-McCoy</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.23</td>
<td>Equations</td>
<td>15.46</td>
</tr>
<tr>
<td>11.33</td>
<td>Review Exercises</td>
<td>9.91</td>
</tr>
<tr>
<td>10.32</td>
<td>Fundamental operations with signed numbers</td>
<td>15.78</td>
</tr>
<tr>
<td>8.51</td>
<td>Verbal problems</td>
<td>6.41</td>
</tr>
<tr>
<td>7.06</td>
<td>Fractions</td>
<td>8.66</td>
</tr>
<tr>
<td>4.45</td>
<td>Factoring</td>
<td>12.13</td>
</tr>
<tr>
<td>4.32</td>
<td>Supplementary material</td>
<td>0.00</td>
</tr>
<tr>
<td>4.04</td>
<td>Radicals</td>
<td>6.19</td>
</tr>
<tr>
<td>4.02</td>
<td>Trigonometry</td>
<td>1.95</td>
</tr>
<tr>
<td>3.94</td>
<td>Arithmetic</td>
<td>0.00</td>
</tr>
<tr>
<td>3.28</td>
<td>Formulas</td>
<td>2.66</td>
</tr>
<tr>
<td>3.03</td>
<td>Parentheses</td>
<td>1.12</td>
</tr>
<tr>
<td>3.02</td>
<td>Exponents</td>
<td>2.56</td>
</tr>
<tr>
<td>2.92</td>
<td>Roots</td>
<td>1.57</td>
</tr>
<tr>
<td>2.55</td>
<td>Algebraic Expressions</td>
<td>3.14</td>
</tr>
<tr>
<td>2.52</td>
<td>Special Products</td>
<td>5.10</td>
</tr>
<tr>
<td>2.24</td>
<td>Graphs</td>
<td>2.88</td>
</tr>
<tr>
<td>1.92</td>
<td>Evaluation</td>
<td>1.31</td>
</tr>
<tr>
<td>1.38</td>
<td>Fundamental operations, (positive sign only)</td>
<td>0.00</td>
</tr>
<tr>
<td>.85</td>
<td>Ratio</td>
<td>.93</td>
</tr>
<tr>
<td>.82</td>
<td>Powers</td>
<td>1.09</td>
</tr>
<tr>
<td>.72</td>
<td>Proportion</td>
<td>.77</td>
</tr>
<tr>
<td>.61</td>
<td>Variation</td>
<td>.38</td>
</tr>
</tbody>
</table>

A study of these texts, as well as Theissen-McCoy, shows also that there is general agreement in the approach to the algebraic material. Thus, Theissen-McCoy follow what has been called the symbolical approach to the study of algebra. This means that the four fundamental operations, factoring, special products, exponents and powers, etc., are taught as separate learning units entirely independent of the context in which they are used in mathematics. The process of teaching, for instance, the addition of signed numbers is to give many exercises in the
addition of signed numerical and literal groups. Thus, drills in combining such quantities like the following abound not only in Theissen-McCoy but in most of the traditional algebra texts:

\[ 3a^2 + 5c^2d^2 - 8a^2d^2 + 9c^2d^2 - 2a^2 \]

The purpose of exercises of this type is to teach the process of algebraic operations in such a way that the student will have a good understanding of the algebraic laws involved. The ultimate mathematical purpose, of course, is to prepare the student to use these operations in real mathematical problems, such as equations. This amounts to teaching the fundamental laws and operations of algebra out of algebraic context. It is equivalent to the English grammar textbooks which give a large amount of drill work in the use of phrases and clauses and other grammatical structures in order that ultimately the student may use these devices correctly in sentence and paragraph writing. It is interesting to note, however, that while Theissen-McCoy give approximately 159 pages to the formal teaching of these processes, they devote only about 35 pages to the equation where these processes are used. This means that 78 percent of the total space of the book is devoted to teaching what will be used in the remaining 22 percent of the book.

Another typical element in this teaching process, found in Theissen-McCoy and in similar texts, is that many of the symbolical exercises in the fundamental processes are much more complicated than they are ever found in equation work,
even in advanced college textbooks. This note of complexity in teaching the fundamental operations appears to be inconsistent with what is required for a working knowledge of the laws of algebra as well as for an adequate skill in solving equations, especially of the relatively simple kind found in Theissen-McCoy and similar texts. For instance, one would have to investigate very widely to discover an equation in a first-year algebra text with variables like the following:\(^6\)

\[7a^2c + 8xyz - 2ab + xyz + 9ab^2 - 8a^2b\]

The point is that expressions like the above are never found in elementary equation work, and rarely in advanced equations. If the purpose is solely to teach the fundamental operations of combining like quantities, certainly the complex expression above is apt to confuse the young beginner's mind and consequently obscure his understanding of an algebraic law which he is supposed to be mastering by exercises like the above. The authors, however, justify such complex expressions on the ground that they help to inspire the student's confidence in his use of the technical operations of algebra.

In general, then, the content matter of Theissen-McCoy is substantially the same as that found in the typical ninth-grade algebra text; the division of the topics and the emphasis given to the various topics is much the same. Above

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\(^6\) Cf., Theissen-McCoy, p. 42–43, for examples of similar problems.
all the teaching process is similar. The fundamental laws and operations of algebra are taught as learning units outside any real mathematical context. The overwhelming emphasis is on the teaching of these mathematical laws as such, with very little emphasis on their purpose or use in real mathematical situations such as the equation. Again, the exercises are very often more complicated than necessary to teach the fundamental laws, and more complicated than they will be encountered in realistic mathematical equation work. The primary purpose is an analytical understanding of the laws as such. The method used is drill work in mathematically "nonsense" problems in the fundamental operations. The authors of the text give the following justification:

They [the authors] have not forgotten that algebra must be built on the student's knowledge of arithmetic, and that algebra should render more perfect and intelligible the knowledge of arithmetic already possessed. Moreover, they regard as a truism the fact that a mastery of the equation, with its power as a means of solving word problems, should be the goal to be attained even in the first year of algebra. But equation work requires not only the ability to think in terms of the unknown - to 'set up' the equation - but also the beginnings, at least, of a mastery of technique. Students must have a sufficient mastery to inspire confidence in their ability to solve the equation.
All the while, however, the authors are safeguarding the student from acquiring a merely mechanical knowledge of processes. There is a constant effort to explain underlying principles in such a manner as to cause them to be intelligently applied.

The authors indicate that, in addition to the mathematical objectives mentioned above, they also wish to achieve the traditional disciplinary and cultural objectives. With these objectives undoubtedly in mind the authors remark in the Foreward that

It is well not to underrate the power of the student to think and to analyze, nor to underrate his eagerness to think, and to discover reasons and logical connections. Algebra furnishes one of the very best opportunities to satisfy this eagerness. Everyday problems of life furnish an incentive to start this process of clear thinking, but the student needs an intelligent familiarity with technique to put the solution of such problems within his reach.

Algebra is a symbolic language... This intelligent use of symbols directing the student to very definite operations corrects the habit of vague, indefinite reasoning and leads him to acquire more mental accuracy and precision.

The effort of the authors, then, has been to explain fundamental principles, to suggest better methods of work, to provide an abundance of drill, and to correlate algebra with life and with other subjects of the curriculum.

7 Ibid., vii.
8 Ibid., vii-viii
This statement of objectives on the part of the authors of *First Course in Algebra* reveals that they were well aware of and understood the recommendations of the various national mathematics committees which have been active in this country for more than fifty years. They have followed these recommendations fairly closely in their statement of objectives, avoiding the different extremes of current educational theory, which tends either to favor strongly the traditional disciplinary values of mathematics or to insist almost exclusively on the so-called progressive "vitalized" objectives which place the emphasis on the present needs and interests of the student. Theissen-McCoy have given attention to both these types of objectives and claim value in both.

However, since these objectives are to a large extent different from the objectives of *Exercises in Algebra*, the following outlined summary of the objectives of *First Course in Algebra* is given in order that this difference will be made clear.

The purely mathematical, or more precisely, the algebraic objectives are:

1. To "render more perfect and intelligible the knowledge of arithmetic already possessed."

9 *Loc. Cit.*
2. "The mastery of the equation."

3. The "solving of word problems."

4. The "mastery of technique;" i.e., the four fundamental operations, factoring, special products, exponents, etc.

5. The avoidance of "merely mechanical knowledge of processes" by a "constant effort to explain underlying principles in such a manner as to cause them to be intelligently applied."

The cultural and disciplinary objectives are:

1. To satisfy and develop the students "eagerness to think and to discover reasons and logical connections."

2. The development of "mental accuracy and precision" on the part of the student.

3. To "correlate algebra with life and with other subjects of the curriculum."

The extent to which Theissen-McCoy adhere to these objectives, especially the mathematical objectives, will be made clear by the following complete analysis of the book. We should bear in mind the authors own statement that "they regard as a truism the fact that a mastery of the equation, with its power as a means of solving word problems, should be the goal to be attained even in the first-year of algebra."\(^{10}\) This statement is in full agreement with a large

\(^{10}\) Ibid., vii.
part of the literature on elementary algebra. Thus, Rugg and Clark, in *Scientific Method in the Reconstruction of Ninth Grade Mathematics*, make the point that

> We shall state repeatedly that the core of our mathematical creed is that we teach mathematical subjects in the public schools to develop in the pupil the ability to use intelligently the most powerful devices of quantitative thinking: the equation, the formula, the graph, and the properties of the more important space forms. This really means the equation.\textsuperscript{11}

A mere glance at *Exercises in Algebra* will show that Doyle embraces the idea that the equation is the central operation of algebra. In fact there is little of anything else except the equation.

With this idea in mind, that the equation is the core of first-year algebra, the following division of the problems in Theissen-McCoy has been made:

\textsuperscript{11} Harold O. Rugg, and John R. Clark, *Scientific Method in the Reconstruction of Ninth Grade Mathematics*, (Supplementary Educational Monographs, ii.), University of Chicago Press, 1918, 31. The frequency with which Rugg and Clark appear in the various mathematical journals, as well as the number of references to their above mentioned book, are evident signs that they are considered representative of expert opinion on current mathematical thought.
(1) formal problems, (2) equation problems, (3) verbal problems, (4) graphs. Under "formal problems" are those techniques covered in the four fundamental operations; i.e., addition, subtraction, multiplication, division, as well as special products, factoring, radicals, roots, exponents and powers, fractions. These are problems which are not taught in any mathematical "setting" or context such as the equation, but as means of developing a manipulative technique which will bring about an understanding of those algebraic laws necessary in the solution of equations. Under (2), equation problems, are literal and numerical problems in one or two unknowns leading to a simple numerical or literal solution of the unknown(s). Verbal problems under (3) refer to those which are stated in the form of a written expression and which usually require the setting-up and solving of an equation for complete solution. The verbal problems are thus distinguished from the formal problems in that the latter are expressed in symbolic form and are of a mechanical or drill type. Under (4), graphs, are placed all problems in the reading and construction of graphs, expressed in either symbolic form or in the form of equations. The following table contains the detailed inventory of all the problems in First Course in Algebra:
SUMMARY OF INVENTORY DATA OF FIRST COURSE IN ALGEBRA

TABLE II

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of problems</td>
<td>1879</td>
<td>482</td>
<td>200</td>
<td>90</td>
</tr>
<tr>
<td>Percentage distribution</td>
<td>62.85</td>
<td>16.12</td>
<td>7.02</td>
<td>3.01</td>
</tr>
</tbody>
</table>

Total number of pages ........................................ 352
Number of pages of explanation ............................... 118
Number of required problems\(\text{12}\) .................. 2990
Total number of problems\(\text{13}\) .......................... 3116
Number of problems in formal processes .................... 1879

The divisions of the formal processes:

1. Fundamental operations ................................. 492 problems
2. Special products ...................................... 159
3. Factoring ............................................. 378
4. Powers and roots ...................................... 83
5. Exponents and radicals ................................ 273
6. Fractions .............................................. 271
7. Parentheses ............................................ 35
8. Algebraic expressions ................................. 98
9. Evaluation .............................................. 41

Number of equations ............................................. 482
Number of verbal problems .................................... 200
Number of formal problems .................................... 1879
Number of graph problems ..................................... 90
Number of formal processes taught ......................... 58

The above tabulation of First Course in Algebra shows that 2990 problems were counted in the inventory. Actually there are a few problems which were not counted because they did not fit accurately into the above system of inventory.

These problems are in the chapters on ratio and proportion

\(\text{12} \) Excludes trigonometry, ratio, proportion, variation.
\(\text{13} \) Includes trigonometry, ratio, proportion, variation.
and numerical trigonometry. However, the purely verbal problems in these sections were counted because their solution requires a somewhat similar attack to that necessary in the solution of the other verbal problems scattered throughout the book. What was omitted in these two chapters were a few formal numerical problems which are aimed at explaining and teaching the technique. In a sense they are formal problems, but they are aimed at very specific items and not generally at the fundamental laws of algebra.

The inventory shows that there are 1679 purely formal problems in the text. These problems together with the generous amount of explanation preceding each new section take up 159 pages of the entire book. Six general items make up this formal problem section of the book: problems in the four fundamental operations, special products, factoring, powers and roots, exponents and radicals. This work is in turn broken up into the following learning units of work: oral and written exercises in the use of symbols, vertical and horizontal addition of signed numbers, vertical and horizontal subtraction of signed numbers, (under these operations the authors give exercises in combining monomial and polynomial terms); multiplication and division of signed numbers, both horizontal and vertical, and from simple integral numbers to complicated polynomial terms; removal and introduction of parentheses. Five different types of exer-
cises in "Special Products" are given with drill problems in each of the types. Under factoring the authors include the usual six different cases with drills and exercises for each case. The section on fractions, containing 271 problems, involves work in the reduction of numerical fractions, simple and complex algebraic expressions, which have to be factored and then reduced; addition, subtraction, multiplication and division of everything from simple numerical fractions to very complicated algebraic expressions. There are also several pages of problems in the solution of complex fractions. The section on powers and roots is made up of 83 problems in which the exercises involve raising quantities to higher powers, extracting the principal roots of quantities, and finding roots by factoring. The exercises in the section on exponents and radicals contain 273 problems. There are drills in simplifying expressions, with positive and negative exponents, both integral and fractional; drills in the identification, combination and reduction of surds and irrational numbers; problems in rationalizing the denominator of radical terms, in changing "a mixed expression to an entire surd," in the addition and subtraction of dissimilar radicals, in the multiplication and division of radicals.

A careful analysis of Theissen-McCoy shows that there are drills in 58 different formal processes. When this figure
is checked against the amount of time provided in the syllabus for the mastery of this formal technique, it is significant to note that 78 days is the time assigned. When we exclude the indefinite time assigned for review work at the conclusion of the first and second semesters, there are 140 days of work indicated, 70 for each semester. In the first semester, 58 days are given over to the mastery of the four fundamentals, special products, factoring, and addition and subtraction of fractions. The remaining 12 days are to be devoted to equations and verbal problems. In the second semester, a total of 20 days is given over to the formal processes of multiplication and division of fractions, powers and roots, exponents and radicals; the remaining 50 days of the second semester are taken up with equations, verbal problems and graphical representation. The requirement for the entire year is to spend 78 days mastering the formal technique in order to solve intelligently the equation and verbal problem work of 40 days.

14 The syllabus referred to here is the allotment of days for classes using First Course in Algebra. The syllabus is the official directive of the Chicago and Missouri Provinces of the Society of Jesus to Jesuit schools in these provinces. It was issued in September 1941, and is still (1946) in force.
The detailed inventory, therefore, of *First Course in Algebra* shows that the text follows in its general features the content and logical development of the typical first-year course in algebra. It compares favorably with the fourteen texts examined by Urbancek in the distribution and relative emphasis given to the various topics. It contains the topics suggested by the National Commission of 1923. The text attempts to avoid too great an emphasis upon purely manipulative skill, and also makes a determined effort to teach an understanding of the basic algebraic laws. The many pages of explanation for each topic, the careful italicizing of rules, the multiple and carefully explained model problems, the graduated drill work in the different topics, the logical sequence of topics— all this is intended by the authors to result in the student's understanding the laws of algebra.

There are some aspects of the book, however, which merit further discussion. Let this discussion be based on the authors' own principle that the equation is the goal in first-year algebra. In their own words, "... they regard as a truism the fact that a mastery of the equation, with its power as a means of solving word problems should be the goal to be attained even in first-year of algebra."¹⁵ The teaching of the formal processes is mathematically and educationally justifiable only in so far as it leads to the

¹⁵ Theissen-McCoy, vii.
mastery of the equation. It appears that there is a very heavy proportion of formal work in comparison with the equation work, even when we count in the word problems. The substance of the textbook can be considered the work in equations. The teaching of the formal processes is, after all, but an attempt to anticipate the difficulties found in the solution of equations. In the textbook it appears that the anticipation of difficulties takes up far more time and energy than the actual meeting of the difficulties in equation work. There are 1879 problems in the formal processes, problems, that is, in anticipating difficulties and only 482 equations, that is, problems in actually meeting the difficulties. When we include the 200 verbal problems, the solution of which involves difficulty of reading and interpretation, more than difficulties in the use of the laws of algebra, the total equation work is only 682 problems, or 23 percent of the problems in the entire book.

The heavy emphasis on the formal processes — that is, the anticipating of difficulties — becomes even more apparent from a study of syllabus referred to previously. The ratio of 58 days for formal processes as against only 12 days for equation work in the first semester would appear to give precious little time for the authors' "mastery of the equation." In the entire year only 40 days out of a total of 140 days are given over to equation work. Furthermore, when it
is recalled that a total of 200 verbal problems has to be solved during the course of these 40 days for equation and verbal problem work, and since it is true that the greater difficulty by far, and the element most consuming of time, is the understanding and interpretation of verbal problems, rather than the algebraic solution of the equation required, the amount of time that can be given to equation work is even further diminished.

Another element which probably decreases even further the time which can be spent on equations is the large amount of formal processes involved in the mastery of certain key techniques. Thus, the syllabus gives 10 days to the formal mastery of powers and roots, and exponents and radicals. During these 10 days the teacher has to take up 16 different but related drills involving 356 problems. Each drill represents a subtopic in the formal process under consideration. The following outline gives the various topics and the number of problems for each topic:

<table>
<thead>
<tr>
<th>TOPIC</th>
<th>NUMBER OF PROBLEMS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Powers and Roots</strong></td>
<td></td>
</tr>
<tr>
<td>1. powers of monomials</td>
<td>34</td>
</tr>
<tr>
<td>2. roots of monomials</td>
<td>33</td>
</tr>
<tr>
<td>3. roots by factoring</td>
<td>16</td>
</tr>
<tr>
<td><strong>B. Exponents</strong></td>
<td></td>
</tr>
<tr>
<td>1. simplifying expressions with fractional, zero, and negative exponents</td>
<td>26</td>
</tr>
</tbody>
</table>
B. Exponents

2. simplifying and finding value of fractional, zero, and negative exponents  
3. eliminating negative and fractional exponents

C. Radicals

1. identification of rational and irrational numbers  
2. addition and subtraction of similar surds  
3. reduction of radicals by factoring  
4. rationalizing denominators  
5. reduction of radicals by reducing index  
6. changing a mixed surd to an entire surd  
7. changing radicals with different indices to radicals with same indices  
8. addition and subtraction of dissimilar radicals  
9. multiplication of radicals  
10. division of radicals  
11. assorted review problems

All the above work must be covered, according to the syllabus, in 10 days. This means, of course, that the teacher must take a large amount of work in a relatively
brief period, and that he will have to keep up a decidedly fast pace. It seems likely that, even with the most capable first-year class, such a rate is altogether out of the question with work of this type, admittedly some of the most difficult work of the first year course. In practice, therefore, the teacher must either abandon the directive of the syllabus to give more time to this section, and consequently in the long run neglect other required matter, or he must disregard the authors' development and pick the few topics and drills he can cover in the specified time. Investigations reveal that teachers of mathematics in high-school generally follow pretty closely the prescribed text. If this is the case, and the teacher decides to follow out the authors' scheme for the development of this topic in a formal way, he will have to spend even more than the 78 days indicated in the syllabus for formal processes and consequently encroach even further upon the 40 days set aside for equations and verbal problem work. The total result must be that actually very little time during the entire year can be given to the mastery of the equation, what the authors refer to as the "goal of first year."

First Course in Algebra is a textbook devoted wholeheartedly to the teaching of the formal processes. The overwhelming amount of work is on teaching the fundamentals and related topics as such and not as necessary steps in the
solution of an equation. Relatively little time is left to spend on the mastery of the equation. The authors have devoted such a large amount of space, and consequently of the time available to the teacher, in anticipating difficulties, that the actual difficulties are hardly ever met in their true and meaningful setting; i.e., the equation. When we judge the book by the authors' own standard these points stand out:

1. "Algebra should render more perfect and intelligible the arithmetic already possessed." 16

COMMENT: Actually there is not one specific drill in strictly arithmetical processes and very few problems of purely arithmetical nature. There is a considerable effort to teach underlying principles common to both arithmetic and algebra through drills in the formal processes. It is difficult, however, to see how these principles are actually related and applied in the student's mind to arithmetic because they are taught deductively in the course and because very little opportunity is given to apply them in purely arithmetical work.

2. "They regard as a truism the fact that the mastery of the equation ... should be the goal ... in the first year of algebra." 17

COMMENT: If mastery of the equation means a formal under-

16 Theissen-McCoy, vii.
17 Loc. Cit.
standing of all the difficulties which can be anticipated in equation work, then every effort has been made by the authors to realize this prime objective. If it means the actual encounter and solution of equations in its manifold forms, simple and complex, then the objective is scarcely realized because of the slight time which can be devoted to the mastery of the equation. If mastery in working equations is understood, the text would seem to be a poor means of insuring this mastery. If mastery in understanding theoretical principles implicit in equation work is meant, every effort is made to achieve this goal. But it must be remembered that an understanding of the underlying principles of an equation is not the ability to work the equation, no more than knowledge of the laws of machines means the ability to drive an automobile.

3. "The beginnings, at least, of a mastery of technique."18

COMMENT: If the authors mean the ability to work through numerous drills which exemplify the technical aspect of algebra, the objective is apparently carefully adhered to in the text. If they understand the use of these processes in an equation, the objective can hardly be realized, because of the formal makeup of the book.

18 Loc. Cit.
4. "All the while, however, the authors are safeguarding the student from acquiring a merely mechanical knowledge of processes. There is a constant effort to explain underlying principles in such a manner as to cause them to be intelligently applied." 19

COMMENT: While there is ample explanation— in fact, space equaling 118 pages of the 352 pages of the book— it appears that the end result will be a "merely mechanical knowledge of processes." This is likely because of the large number of formal processes, (58 in fact), given in the book, and the consequent speed with which the various drills must be taken. Students learn by doing, not by listening to or reading the most carefully thought out explanations of technical processes.

First Course in Algebra is a text, therefore, which teaches or aims to teach the formal processes of algebra. It does little else. When the text is measured according to the authors own standard as contained in the Foreward, it is seen to be very inadequately arranged to satisfy the prime objective of first-year algebra. What Rugg and Clark mention in the Reconstruction of Ninth Grade Mathematics as indicative of the typical elementary algebra text would appear to apply to the text under consideration:

Careful analysis of the actual use of ninth-grade mathematics in the other

19 Loc. Cit.
high-school subjects and in occupational subjects shows that it is almost impossible to defend the large amount of attention to this material, (i.e., the formal processes) and absolutely impossible to defend this emphasis upon special products and factoring. The only use which is made of the latter is a very meager one found in connection with the solution of quadratics — and that forms less than two percent of all the algebraic material called into play — even in this further academic instruction in the high school. 20

This general indictment would seem to be true in relation to First Course in Algebra. Since specific attention has been called to special products and factoring as an example of the overloading of elementary texts with formal processes, notice the space devoted to these topics in Theissen-McCoy: 625 problems in 11 different drills are given for the mastery of this technique which will be used in solving quadratic equations. This work extends over 36 pages and it is to be taken in 25 days according to the direction of the syllabus. On the other hand there are only 78 equations which have to be solved by the factoring method. This work covers 5 pages, including the lengthy explanations, and is to be taken in 7 days.

The following remarks by Rugg and Clark would seem to be applicable to First Course in Algebra.

20 Rugg and Clark, 33.
Evidently textbook writers have not practiced what they preach concerning the use of the equation as the core about which the material shall be organized. In these fundamental operations a great deal of material has been included which has no relation to equational work. For example we find no instance in which polynomial multiplication or division (involving more than two terms) has any relation to the solution of equations. And yet they abound in the textbooks. Clearly a sane judgment on the matter would lead to an immediate elimination of such routine material.\textsuperscript{21}

Since Theissen-McCoy make a point in the Foreward of "constant effort to explain underlying principles," and to formulate these explanations into rules, the following criticism by Rugg and Clark of this teaching method is revealing:

A study of the attempts made by the textbooks to aid the pupil through the development and statement of rules and principles shows first, that it is common practice to formulate rules for most of the processes in algebra, though such statements are not always labeled as rules; secondly, that these rules are developed inductively; and thirdly, that they are intended to serve as a model whose perfection in form the pupil should try to reach in the statement of and direction of his mathematical thinking. Nothing illustrates better the formalism in our teaching, at least from the psychological point of view than this practice of using rules to govern skill in the manipulation of formal material. Psychological analysis shows for example that the mental processes included in

\textsuperscript{21} Loc. Cit.
memorizing a rule and in setting up a habit of controlling a particular skill by facility in bringing the right rule to bear on the automatic reaction desired are quite different. Furthermore, the response is not automatic at all, if it involves the use of the rule. Rules ... are of service in interpreting habit systems which we have developed, not in learning the system itself.22

Exercises in Algebra, by William C. Doyle, S. J., is in many ways a unique type of first-year algebra. In no sense does it come under the heading of typical ninth-grade algebra text. It is as dissimilar to First Course in Algebra and the other traditional elementary algebra textbooks as any two textbooks could be which aim to teach the same course to the same students at first-year high level. The content of the book is different from the typical text: the omission of content material generally required for first-year is very considerable; the inclusion of material rarely seen in any elementary algebra text is equally unusual. The objectives of the book sound strange to the supporters of the typical text, the teaching procedure is unorthodox to the proponents of such texts as First Course in Algebra. In a word, Exercises in Algebra, whatever else may be said in its favor or against it, is an altogether different type of first-year algebra text.

The inventory of Exercises in Algebra shows a tremendous 22 Ibid., 37.
disparity between it and Theissen-McCoy's text. The latter book gave a total of 2990 problems in the required matter. *Exercises in Algebra* contains but 1244 problems in the required matter. And of these 1244 problems there are several in almost every exercise which the author specifies as 'honor problems,' to be assigned as extra work to only the more capable students. When the content material is separated into the four divisions mentioned in the inventory of *First Course in Algebra*, namely, (1) formal problems, (2) equations, (3) verbal problems, and (4) graph problems, the following table reveals some of the fundamental differences of the two texts:

**COMPARATIVE DISTRIBUTION OF PROBLEMS IN THEISSEN-McCOY AND DOYLE**

**TABLE III**

<table>
<thead>
<tr>
<th></th>
<th>DOYLE</th>
<th>THEISSEN-McCOY</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Formal Processes</td>
<td>Number of Problems</td>
<td>306</td>
</tr>
<tr>
<td></td>
<td>Percent</td>
<td>26%</td>
</tr>
<tr>
<td>2. Equations</td>
<td>Number of Problems</td>
<td>534</td>
</tr>
<tr>
<td></td>
<td>Percent</td>
<td>48%</td>
</tr>
<tr>
<td>3. Verbal Problems</td>
<td>Number of Problems</td>
<td>227</td>
</tr>
<tr>
<td></td>
<td>Percent</td>
<td>20%</td>
</tr>
<tr>
<td>4. Graph Problems</td>
<td>Number of Problems</td>
<td>66</td>
</tr>
<tr>
<td></td>
<td>Percent</td>
<td>6%</td>
</tr>
</tbody>
</table>
The above table, while showing in a general way the distribution of the problems in the two texts, does not reveal the entire picture. The table, for instance, shows that where Father Doyle gives 26 percent of the problems to formal processes and Theissen-McCoy 63 percent, it does not reveal the considerable difference in the concepts of "formal processes" in the two texts. It was seen that in the latter text formal processes meant 1879 problems in the four fundamental operations, in special products, factoring, powers and roots, exponents and radicals, and fractions. In Exercises in Algebra formal processes mean something quite different. For instance, there are no drills whatsoever in the four fundamentals, no space devoted to special products, only eight problems exemplifying factoring. Formal processes in Exercises in Algebra mean 134 problems in concrete arithmetical processes divided thus: 35 problems in factoring integers, 52 problems in the addition, subtraction, multiplication and division of fractions, 14 problems in multiplying and dividing decimal numbers, and 20 problems in arithmetical square root. The only agreement between the two texts in this matter of formal processes is the multiplication and division of polynomial terms, and in exponents and radicals. Actually, therefore, there are only 180 problems, or a percentage distribution of 15 percent of the total problem material, devoted to formal processes. The
syllabus for *Exercises in Algebra* shows even further this difference in emphasis on the formal processes. Where the syllabus for *First Course in Algebra* prescribed a total of 78 days for the formal work, the time given over to formal work in the syllabus for Father Doyle's text is 30 days. During 12 of these 30 days the time is spent in a review of the arithmetic processes already referred to; 10 days is prescribed for exponents and radicals and 8 days to multiplication and division of polynomials. For purposes of an accurate comparison with *First Course in Algebra*, Father Doyle's text requires, according to the syllabus, a total of 18 days to the formal symbolic processes. On a percentage basis this means that 56 percent of the time has to be spent with the formal processes in using the former text, and 13 percent of the time when *Exercises in Algebra* is the text used. One further point which would seem further to reduce the time spent on formal processes, in the use of the latter text, is the relatively few subtopics within the formal processes. Thus where Theissen-McCoy cover 13 subtopics in the section

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23 This is the official syllabus for Jesuit Schools of the Chicago and Missouri Provinces of the Society of Jesus. It was issued in August 1945 as a directive for those classes in which *Exercises in Algebra* was used. Upon the publication of the revised text of *Exercises in Algebra* in September 1946, a new syllabus was issued. The new syllabus provides the same time for formal processes as the older one.
on exponents and radicals, Father Doyle takes only 3 sub-
topics. The time given by the syllabus for both courses is
10 days for this material. Where the teacher of the former
course has to cover 13 different drills including 356 prob-
lems, the instructor in the latter course must cover 3 dif-
ferent drills which contain 112 problems. The teacher of
the first course would probably be forced to extend the per-
iod beyond 10 days, while the teacher of the second course
could readily cover the matter in less than 10 days.

One of the characteristic features, therefore, of Exercises in Algebra is the relatively slight emphasis given to
teaching the formal processes and the very heavy stress on
equation work. This is perhaps the most striking thing about
the text, the fact that it departs in a very pronounced de-
gree from what has been universal practice in first-year
algebra texts, and omits entirely a very large section of the
work on formal processes.

Another noteworthy feature of Exercises in Algebra is
the paucity of printed explanation. Again a comparison with
the text by Theissen-McCoy will bring this point out clearly.
The older text gives a total of 118 pages to explaining the
algebraic processes used in the text. Father Doyle's text,
on the other hand, gives a total of only 4 pages of explana-
tion. The different points of view of the authors of the two
texts stand out in sharp relief in regard to explanation of the
problem material of algebra. Theissen-McCoy remark in the Foreward that "There is a constant effort to explain underlying principles ... "24 Contrast this with Father Doyle's statement in the Preface to *Exercises in Algebra*: " ... discussion and long explanations only tend to confuse if the student has not been prepared by extensive drill."25 Again in the Teacher Manual sent out in mimeographed form to the schools using the text this refrain is often repeated: "The teachers should never use the lecture method;"26 and, "Mathematics must be learned by the student and not just talked about by the teacher."27 "It seems, too, that all classroom discussion, even the asking of questions, has to be completely suppressed especially during the brief explanation."28

In addition to the characteristic marks already referred to, the following points make *Exercises in Algebra* a decidedly nontypical first course in algebra:

1. **Anticipating student difficulties is avoided.**

   In marked contrast to Theissen-McCoy's stand that "Students must have a sufficient mastery [of the formal technique] to inspire confidence in their ability to solve the equation,"29

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24 Theissen-McCoy, vii.
25 Doyle, 3.
26 William C. Doyle, *Teacher Manual*, 3. This is an unpublished mimeographed set of directions for *Exercises in Algebra*, distributed to the teachers using the new text.
27 Loc. Cit.
28 Loc. Cit.
29 Theissen-McCoy, vii.
Father Doyle holds that "Understanding is important, but it must come as a result of experience [with the equation]. This is true because the reasoning processes involved are inductive, not deductive." Hence, the elimination of most of the formal processes.

2. **Computational ability is stressed.**

This does not mean drill work in the fundamental operations of algebra, but exercise, constant exercise, in arithmetical processes. In addition to the work already referred to, nearly every problem in the book requires a check in which the arithmetic processes with both integral and fractional numbers are used. Evidently the author of *Exercises in Algebra* agrees with Theissen-McCoy that "Algebra should render more perfect and intelligible the knowledge of arithmetic already possessed." The important difference is that Theissen-McCoy stress "knowledge of arithmetic," while Father Doyle emphasizes the use of arithmetic.

3. **Exercises are complex.**

Here again Father Doyle's text is in many ways unique. Where the typical texts give a large number of drills with relatively easy problems and attempt to develop speed in completing these drills, *Exercises in Algebra* offers relatively few problems, but nearly all the problems are long and complex.

31 Theissen-McCoy, vii.
Father Doyle's defense is that:

Mathematics is essentially complex; each detail is easy to understand. Transfer of knowledge nearly always has to do with concepts that require a great deal of detail work. Students drilled in brief problems that test knowledge of but one or two simple items will forget all that was learned in a short time.  

One reason why there are fewer problems in Father Doyle's text, in comparison with the typical text, is this very fact that the problems are nearly always long and complex. This is especially true of the equation problems. There are 534 equations, or 48 percent of all the problems of the text. This does not tell the whole story. The solution of each equation requires on the average 5 to 10 steps in which practically all of the fundamental operations are used. In addition, each equation is not considered solved until it is checked. Since the answer is very often fractional, the checking of an equation means the use of practically all the arithmetic functions with fractions, as well as the use of the fundamental algebraic laws. For purposes of illustration, the following problem with its solution and check is given. This problem has been chosen because it is typical of more than 60 percent of the mathematical work in *Exercises in Algebra*, and because it is among the essentially required problems in the text. It is problem number 32.

15, of Form XII, on page 16. According to the syllabus this Form is to be taken in the sixth week of school in an average class. The problem with its solution and check is as follows:

\[ (3 \mu - 5)^2 - (3 \mu + 5)^2 = 40 \]
\[ (3 \mu - 5)(3 \mu - 5) - (3 \mu + 5)(3 \mu + 5) = 40 \]
\[ 3 \mu (3 \mu - 5) - 5(3 \mu - 5) - [3 \mu (3 \mu + 5) + 5(3 \mu + 5)] = 40 \]
\[ 9 \mu^2 - 15 \mu - 15 \mu + 25 - (9 \mu^2 + 15 \mu + 15 \mu + 25) = 40 \]
\[ 9 \mu^2 - 30 \mu + 25 - 9 \mu^2 - 30 \mu - 25 = 40 \]
\[ -60 \mu = 40 \]
\[ \mu = -\frac{2}{3} \]

\[ (3 \mu - 5)^2 - (3 \mu + 5)^2 = 40 \]
\[ \left[3\left(-\frac{2}{3}\right)-5\right]^2 - \left[3\left(-\frac{2}{3}\right)+5\right]^2 = 40 \]
\[ (-2-5)^2 - (-2+5)^2 = 40 \]
\[ (-7)^2 - (3)^2 = 40 \]
\[ 49 - 9 = 40 \]
\[ 40 = 40 \]
The complexity of the problem is evident from the manifold operations involved in solving and checking this equation. The following analysis of the problem will point out the different algebraic laws used in solving and checking the problem, as well as the different formal techniques used. It should be kept in mind that in *First Course in Algebra* these techniques are part of the formal processes taught in anticipation of their use in the equation. In Father Doyle's text, these processes have not been formally taught, but nevertheless the student is expected at the end of the sixth week of class to be able to solve and check this problem with a fair amount of speed.

1. \((3x-5)^2 - (3x+5)^2 = 40\)

The student would have to recognise this as an equation; attacking the problem also presupposes a knowledge of what a binomial is, i.e., \((3x-5)\), and what it means to have a binomial squared. He must also have an idea of the significance of parentheses.

2. \((3x-5)(3x-5)-(3x+5)(3x+5) = 40\)

Here the student reexpresses the problem by a correct interpretation of the squared binomial and by the use of parentheses.

3. \(3x(3x-5)-5(3x-5) - \sqrt{3x(3x+5)+5(3x+5)} = 40\)
The ability to make this step involves the knowledge of how to go about multiplying a binomial, how to interpret correctly the minus sign before the second binomial, and how to use the brackets and parentheses.

4. $9x^2 - 15x - 15x + 25 - (9x^2 + 15x + 15x + 25) = 40$

Several important skills were used in making this step. The use of signs in multiplication was made 8 times; the student must know how to multiply quantities with like and unlike signs. In addition, the student would have to have at least an elementary knowledge of the use of exponents in multiplying the quantities $3x \cdot 3x$; further, the ability to multiply a literal quantity with its coefficient by an integral is used in multiplying $3x \cdot 5$. In resolving the parentheses system the student correctly reduces the bracket to a parentheses and removes by multiplication the interior set of parentheses. The most important step, and the one where error would most likely happen, is in properly interpreting the minus sign before the second system of parentheses. Here the student would need to understand that all operations within the parentheses must be completed before the parentheses can finally be cleared by multiplication of the understood minus one outside the parentheses. The various testing experiments show that the incidence of error is very high in the use and understanding of this algebraic situation. For this reason the typical text, including First Course in Algebra, gives intensive drills in antici-
pating this difficulty.

5. \(9\mu^2 - 30\mu + 25 - 9\mu^2 - 30\mu - 25 = 40\)

The steps taken in this part of the problem were the addition of two sets of signed numbers, i.e., \(-15\mu + 15\mu\); and \(15\mu + 15\mu\) in the second parentheses; the multiplication of the entire simplified parentheses by the minus one. In this case the law for multiplying unlike signed terms was used.

6. \(-60\mu = 40\)

In this step all like terms were combined; the skills used were the addition of signed numbers; i.e., the combining of \(9\mu^2\) and \(-9\mu^2\) to get 0, the addition of \(+25\) and \(-25\) to get 0, and the addition of \(-30\mu + 30\mu\) to get \(-60\mu\). Another skill was used in combining the three different kinds of terms separately, for instance, the \(9\mu^2\) with the \(-9\mu^2\) and not with the \(15\mu\). In this case the student would have to know the algebraic concepts that only similar terms can be combined, and he would have to understand just what constitute similar terms; i.e., the difference between a term in \(\mu^2\) and one in \(\mu\), and the unique character of the integrals \(25\).

7. \(\mu = -\frac{2}{3}\)

Several steps were taken at this point, involving different formal techniques. First the student divided both sides by \(-60\); i.e., \(-\frac{60\mu + 40}{-60} = -\frac{40}{60}\). Here the fundamental law of the equation was used. The first result in the student's mind was \(\mu = -\frac{40}{60}\), in which he correctly used the skill of
dividing both sides by the signed coefficient of the unknown, and used the algebraic laws for the division of signed numbers. Finally he used the basic arithmetic skill in reducing \(-\frac{40}{60}\) to \(-\frac{2}{3}\).

8. \((3x-5)^2 - (3x+5)^2 = 40\)

For the check, the student first reexpresses the problem in the original form, since a valid proof is one that proceeds from the original expression only.

9. \([3\left(-\frac{2}{3}\right) - 5]^2 - [3\left(-\frac{2}{3}\right) + 5]^2 = 40\)

In this step the proper idea of substitution was used. Further, it was necessary to arrange the system of parentheses and brackets and to keep the significance of the minus sign before the second set of brackets.

10. \((-2-5)^2 - (-2+5)^2 = 40\)

This step involved the multiplication of signed integrals and fractions and the use of the law which requires that the numbers inside parentheses must be completely simplified before the parentheses can be removed by multiplication.

11. \((-7)^2 - (3)^2 = 40\)

The skill used here was the combination by addition of the signed numbers; the addition of two minus numbers and the more difficult addition of a plus and a minus.

12. \(+49 - 9 = 40\)

The quantities were squared because of the exponent outside of the parentheses; the quantities were then multiplied
by the plus one and the minus one understood before each parentheses. The most important skill used was in squaring the \( +3 \) inside the second parentheses and getting a \( +9 \) and then resolving the \( +9 \) into \( -9 \) by the multiplication of the understood minus one; i.e., \(-3^2 = -(+9) = -9\). The student would of course have to know the law that a plus times a minus gives a minus.

13. \( y_o = 40 \)

Here the simple combination of the \( +9 \) with the \(-9\) gives the final step, the equation balances and hence checks.

The above analysis shows that the successful solving and checking of the problem required the use of 13 basic algebraic skills or techniques. Theissen-McCoy require the student to work through 369 problems in 13 different drills so that students will have "a sufficient mastery of technique to inspire confidence in their ability to solve the equation." Father Doyle, on the other hand, gives no drill work whatsoever in anticipation of the student's difficulties in solving the above typical equation of the first quarter's school work.

The more important difficulty in the solution and checking of the above equation arises not out of an inability to work the formal technical aspects, such as the addition and multiplication of signed numbers, etc., but out of the considerable difficulty of when to make the correct operation and especially the difficulty which arises from the complex
development of the problem. The number of different operations is considerable, but the correct performance of each single operation is relatively simple. It is difficult to see just how much confidence "in their ability to solve equations" is developed by a series of drills in the different operations, especially when each problem in the drill is brief and tests only one or two operations. It is apparent that the essential difficulty in solving the above equation is the need of working through the mass of details without becoming confused or distracted. Drill work in each of the operations involved is hardly adapted to develop the very essential ability of being able to work through the complex detail work.

14. **Neatness and form precede understanding.**

This characteristic feature of the book stands out because it is to all appearances contrary to much of the traditional thinking. The statement most generally agreed upon by textbook writers is that "Understanding comes first." Theissen-McCoy certainly agree to this principle, for several pages of explanation and the formulation of rules and directions precede actual drill work. Father Doyle's position is that actual mathematical work - i.e., the solution of equations - precedes any understanding. Thus, in the General instructions in *Exercises in Algebra* he remarks:
The student must make very much of neatness and order, because if what is written looks untidy and disorganized, so will be the thinking — and the answers. In the beginning those students make the most rapid progress who follow instructions blindly and do not try to develop their own methods.33

Again he remarks in a letter to the writer of this paper that "No amount of thinking, lecturing, or discussing will add one cubit to one's ability to mathematize."34 Further on in the same letter Father Doyle holds that "As far as the student is concerned he must learn it as mathematics as a language; that is, by use, not by reason. Thinking out (for instance) the quadratic formula is not mathematics."35 Father Doyle insists on neatness and form as a means of developing understanding. The traditional authors make neatness and form the result of understanding.

The fact that Exercises in Algebra is decidedly different from the typical ninth-grade algebra text is clear. An examination of the objectives of the author will show just why this text differs so radically.

We have already seen that Theissen-McCoy hold the ability to solve equations to be the core idea and objective of first-year algebra. It should also be clear that the de-

33 Doyle, 6.
34 Doyle, in a letter to the present writer dated August 14, 1945.
35 Loc. Cit.
talled inventory of the text leaves the impression that the book is poorly adapted to achieving this prime objective.

The analysis and criticism of First Course in Algebra was made on the basis of the authors' stated objectives. The same procedure will be followed in evaluating Exercises in Algebra. What are the educational and mathematical objectives of the author of Exercises in Algebra? They are to be found in three principle sources: (1) in the Report of the Committee on Mathematics of the Missouri Province, published in 1914 as the official educational creed for the high schools of the present Missouri and Chicago provinces of the Society of Jesus; (2) in the text itself together with the Teacher Manual written by Father Doyle and distributed in mimeographed form to the schools using Exercises in Algebra; (3) in private letters sent to the present writer in response to questions regarding objectives and certain phases of teaching the text.

The central point of the author's creed is stated in the Preface to Exercises in Algebra; namely, "The streamlined make-up of this booklet points strongly to the fact that mathematics is a discipline. Solid progress in it comes only as a result of hard work and concentrated study." However, since the word 'discipline' does have different educational connotations, it is necessary to examine the precise meaning

36 Doyle, 3.
Father Doyle gives in his prime educational objective. The author claims in the Teacher Manual that his final goal in writing the text was "to develop a text that is suitable to laboratory teaching and at the same time that contains a training that will result in better retention and transfer of knowledge to later courses that require mathematics as a background." In this statement Father Doyle is making direct reference to the teaching method outlined in the Missouri Province Report on mathematics. Since this teaching method is logical development of the objectives therein stated, it is necessary to know these objectives to understand Father Doyle's use of the word 'discipline'. The report states:

The purpose of High School Mathematics now hinges on the more fundamental question - "What is the chief purpose of High School?" If it be "culture" then "culture" subjects must predominate, and only so much of the "informational" studies will be admitted as is absolutely necessary; moreover even these must be so taught as to bring their own proper "cultural" effects to a maximum. This is the old ideal of the Society of Jesus; and it is now universally admitted as the true ideal of what a (Jesuit) High School should be ... the primary purpose of the High School is culture, and not information merely or the dissemination of knowledge.

37 Doyle, Teacher Manual, 1.
38 Report of the Committee on Mathematics, (Supplement to the Course of Studies of the Missouri Province. For Private Circulation.) Loyola University Press, 1914, 7.
The report holds, therefore, that the cultural objective is the ideal for the Jesuit high school. Father Doyle's conception of "discipline" is what the report means by the word "cultural". The ideas of the two are in complete harmony but the terminology differs.

The report, to supplement which Father Doyle wrote Exercises in Algebra, gives these main cultural objectives for High School Algebra:

1. A boy in solving exercises or problems, learns to master his resources, and use them independently; he acquires self-confidence in attacking or circumventing difficulties, learns to think correctly, learns how to check and test his conclusions when in doubt; he gains faith in his own powers, and his originality develops.

2. The symbols form a new language, very condensed and idiomatic; he learns to translate to and from these, and again acquires self-confidence and power.

3. The habit of generalizing gradually grows on him and gives him broader views, and the power of grasping and subordinating details.

4. Incidentally he is acquiring habits of mental accuracy, and of concentration of mind upon one point.

5. The sense of satisfaction in discovering new things or solving hard problems makes him eager to try his powers on other difficulties, independently; he will say, 'Don't tell me - I'll get this myself'.

39 Ibid., 10-11.
The formative, as opposed to informative, nature of this objective is clearly seen when the report adds, "That the student actually get the answer is not important culturally; but that he should go at the problem, investigate it, try to get it, try to think out new plans and methods, - all these efforts of spontaneous attack are the central educational feature of algebra." 40 How different the attitude of the authors of typical first-year algebra texts, with their "Understanding comes first," and Theissen-McCoy's confession of a "constant effort to explain underlying principles" sounds in comparison to the report's directive that "The great danger here lies in too much explanation and talk on the teacher's part; the student must not in general be passive, but active, and only the insuperable difficulties must be solved for him." 41

While Father Doyle is essentially interested in promoting teaching methods in harmony with the report's cultural objective, and not principally with basic objectives, he insists that:

Algebra does teach reasoning, but in a very different way from philosophy. Mathematics is the art of thinking with the aid of a pencil. It has to do with detail work, not with depth of thought. There is no such thing as a gentleman's knowledge in this field any more than there is a

40 Ibid., 11.
41 Loc. Cit.
talent for mathematics. It has to be acquired...42

With a similar thought in mind he further points out, "The processes become clear only when you do them; explaining them is like giving a learned lecture on working cross word puzzles."43

That the formative objective is primarily in Father Doyle's mind becomes even clearer in his directions to the teachers. The directions from Father Doyle to the teachers repeat over and over again that "Mathematics must be learned by the student, not talked about by the teacher." He illustrates this principle by the following description of a typical class period:

After handing out the new page that is to be worked on, the teacher gives a very brief talk on the matter at hand and rapidly works a problem in the Form that he wishes all to follow on their papers. No questions may be allowed during this explanation. Next the students are told to start the first problem of their assigned work. The teacher writes the first problem on the board and encourages the class to finish it before he does. Without saying anything more he proceeds at the board, working, slowly and carefully. When the problem is completed he spends the rest of the period walking from desk to desk giving help, suggestions and encouragement.44

42 Doyle, A Supplement for Elementary Algebra, an unpublished booklet distributed to teachers of Exercises in Algebra, 4.
43 Loc. Cit.
44 Ibid., 5.
Theissen-McCoy's lengthy explanations stand in sharp contrast to Father Doyle's directive to the teacher:

Be very rigid in the beginning about the matter of no questions during explanations. It is surprising how quickly the difficulties of individual students vanish into thin air once they have started to work ...45

He insists repeatedly on this directive:

Do not try to anticipate difficulties by trying to make things too clear. Let the student find out for himself and help him when the trouble comes. Your classroom explanations should not average more than about fifteen minutes a week!46

As the report directs, "... only the insuperable difficulties must be solved ..."47

How closely Father Doyle's directions to the teachers of Exercises in Algebra conforms in spirit to the Missouri Province report is seen from the following strongly worded appeal in the report:

The teacher should never use the lecture method; this belongs not to the High School, but to the college. It is a matter of experience that a teacher who explains and talks continually while the boys are passive and receptive, is unsuccessful; they may say and think that they understand the matter perfectly;

46 Loc. Cit.
the teacher may think that everything is now obvious and clear; but they are both mistaken. Things do not sink in when a boy is passive; he sheds the knowledge thus communicated. The old fashioned recitation method, in which the boy studies at home what the teacher explained today, and then recites it in class tomorrow, is equally profitless. These methods rest on the false idea that the acquirement of new knowledge is the main thing; on the contrary, as we have seen, it is the by-product: the main thing is that the boy learn how to think originally and how to study new problems with what he already has. The teacher who tells him the result, shows him how, and explains it all, is consequently defrauding of the essential feature of his education; when such a spoon-fed boy graduates, he will be helpless and uneducated still.

This is the formative objective; or, as the report puts it, the cultural objective of first-year algebra. It agrees completely with Father Doyle's idea of the purpose of high-school algebra. The marked bearing this idea has on the text is seen in the fact that there is never more than a few lines of explanation preceding any exercise, with the total printed explanation barely equaling four fully printed pages. In noticeable contrast, and also indicating a radically different point of view of the authors of First Course in Algebra, are the lengthy explanations in this text, equaling in fact 118 pages of the book.

The precise mathematical objective which Father Doyle

48 Ibid., 16.
feels should result from a thorough mastery of complex equations is what he calls a "feel for number." In correspondence with the present writer he elaborated at length on what he meant by this "feel for number." Regarding its importance, Father Doyle says:

This feel for number is probably the most useful acquirement of a course in mathematics as far as the education of an average student is concerned. In fact, the layman will rarely use anything from his mathematics course except this facility with numbers. Its importance in such ideas as correlation, statistics, averages, insurance, etc., are obvious. 49

His concept is made clearer by the following quotation from a letter:

Feel for number is a fundamental skill. Mathematics can be defined as the science of quantity, (i.e., in a mathematical context, and not in a philosophical context.) Quantity is usually expressed in terms of numbers. But a computational wizard is not necessarily a mathematician; in fact it seems that such an ability is an obstacle because it usually implies the lack of ability to generalize ... On the other hand great facility in the elementary processes of arithmetic is a skill that is practically necessary to good mathematizing, else one's progress is made too laboriously. Doing mathematics without a feel for number is like learning a language without memorizing a vocabulary ... Feel for number implies much more than just the ability to add,

49 Doyle, letter dated August 14, 1945.
multiply and divide. The next important needs are:

1. **A facile ability with numbers;** not only how to compute with them, but to **feel** their relative values, e.g. \( \frac{1}{2} \) is smaller than \( \frac{1}{3} \). \( \frac{5}{7} \) is smaller than \( \frac{5}{9} \). \( \frac{3}{5} \cdot \frac{9}{10} \) is greater than \( \frac{3}{5} \), etc., etc.

2. **Decimals are fractions but in practice they are very different.** This implies the whole idea of approximation, and irrational numbers.

3. **Every integer is either a prime or factorable into a prime uniquely.** When a mathematician sees 56, it is not just the number after 55. It is made up of 2 and 7, it is not a perfect square, but does contain 2 three times, etc.

In summary, therefore, the two texts differ considerably in their general mathematical and educational objectives, in the content and presentation of matter, and in teaching procedure. **First Course in Algebra** is an informational course in mathematics, while **Exercises in Algebra** has a formative educational aim. The content of the first text contains the traditional algebraic material for ninth-grade algebra with heavy emphasis on the technique or formal processes; the second text avoids, to a large extent, the teaching of the formal processes and gives emphasis to the equation. The authors of **First Course in Algebra** use as their principle teaching method very complete explanation and illustration; the author of **Exercises in Algebra** uses the method of practice in the exercises. **Loc. Cit.**
Exercises in Algebra proceeds on the basis that understanding comes as a result of hard work and consequently avoids almost entirely printed explanations of the laws and workings of algebra.
CHAPTER IV

THE TESTING PROGRAM

While the principal purpose of this study is to describe the differences between *Exercises in Algebra* and *First Course in Algebra*, the results of a modest testing program will be reported. The review of the related literature in Chapter II showed the different mathematical views of the authors. The detailed comparison of the two texts in Chapter III revealed the practical differences resulting from the disparate mathematical views of Doyle and Theissen-McCoy. Both in theory and in practice the authors differ in regard to objectives, content and teaching methodology of algebra.

However, as a sort of corollary to the main purpose of this study, the writer carried out a preliminary testing program to learn what, if any, differences develop in the actual classroom use of the two texts. Nevertheless, in undertaking even a modest testing program of the kind to be described, the writer was under considerable limitation. First of all, it was evident that two different tests would have to be given towards the end of the second semester to both the Doyle trained students and to those using Theissen-McCoy. Further, the groups would have to be large enough to insure an adequate basis of comparison. There was little difficulty in arranging for one test, but individual teachers were under-
standably hesitant to permit the time for a second test, especially in those classes where the particular test did not seem to connect closely with the review work at the time under study. Since only partial success was achieved in overcoming this difficulty, the results of the tests should be interpreted with a good deal of caution.

A great difficulty presented itself in administering a test for *Exercises in Algebra*. Because the content matter of this text is largely complex equations, an objective-type test in formal algebraic technique is inadequate. Yet a test made up of complicated equations would not only require more than the allotted forty minutes to take, but would place the Theissen-Mc McCoy students under a serious handicap. A compromise test was therefore prepared which reduced the complexity of the equation work and which was brief enough to administer in the arranged time. This test, however, does not pretend to be a fully adequate sampling of the work required of those students who have completed two semesters of *Exercises in Algebra*. However, under the circumstances it seemed to be the only fair test for both groups. It is simple enough so as not to seriously handicap the students who have spent most of the two semesters in the study of formal technique; it is complicated enough to test the skill of the Doyle trained students. However, since it is at best a compromise test, the results should be interpreted accordingly.
One further consideration is necessary for a fair interpretation of the results of the testing program. The desirability of testing at this time the Doyle trained students was considered inadvisable by some of the teachers. They pointed out that the text was still in an experimental form with plans already underway for revision, and that the difficult and largely unfamiliar laboratory method required for teaching the book was still in the developmental stage. It was reasonably thought by some of the teachers that any comparative study based on experimental evidence at this time would not only be somewhat inconclusive as far as the Doyle students were concerned, but might even bring discredit on the newer text. This objection is indeed valid and consequently it is in the light of these difficulties that the results should be studied. The conclusions reported in this study are at the best tentative. However, even though the results could not be considered final, the testing program was undertaken for whatever evidence it would offer regarding the comparative achievement of the two groups of students.

In May, 1946, the two tests were administered. One of these, an objective type test, was given in the eleven Jesuit high schools of the Midwest. 3172 freshmen students in 68 class sections took this test. The second test, administered a few days after the objective test, contained a series of equation problems. This test was given in six of the Jesuit
high schools in 32 class sections to a total of 999 boys. The results of the objective test will be reported first. Copies of the two tests have been placed in the appendix.

The Cooperative Algebra Test, Elementary Algebra Through Quadratics, Revised Series, Form Q,\(^1\) published under the aegis of the American Council on Education and distributed by the Cooperative Test Service was the objective test given to the 2172 Jesuit high school students. This test was chosen because it is considered as one of the most representative and reliable tests in general use at the present time. Since the norms of the test are set up on the basis of nation-wide use, the test provides a good instrument for comparing Jesuit high school students with others. The report of the Joint Commission referred to in the review of the literature says of the Cooperative tests that, "They are representative of the best modern achievement tests generally available."\(^2\)

This test is aimed at measuring the achievement of students who have completed two semesters work with a typical elementary algebra text. Hence, it is an adequate testing instrument for students using the First Course in Algebra. The

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1 John A. Long and L. D. Siceloff, Cooperative Algebra Test, Elementary Algebra Through Quadratics, Revised Series, Form Q, Cooperative Test Service, New York, 1940.
Cooperative Test Service in its Handbook advertises that these tests:

... represent a wide sampling of the materials covered in the typical mathematical courses as indicated by textbooks and courses of study.3

Regarding the elementary algebra test, the Handbook declares that:

It is composed of a section of short-answer items covering the application of the basic skills and principles included in the typical elementary algebra course ...4

Since the 63 items in the test are for the most part work in the formal operations, it could be presumed that the students using the formal operations text, First Course in Algebra, would have some notable advantage over those students who completed two semesters work in Exercises in Algebra. Students using the former text would have been explicitly prepared for just such a test, while those using the latter text would not. However, since the items cover the basic operations of algebra, the test was given with the purpose of determining which group had mastered better these fundamental operations, and of learning whether these fundamental operations were mastered to any adequate degree by those students who had

4 Ibid, 22.
studied them informally and implicitly in the equation work of *Exercises in Algebra*.

The 2172 students taking the Cooperative test were grouped into their regular 68 classes in the eleven schools. Both the school and the classes will be identified by letter and number and not according to their real names. Table IV gives a composite picture of the achievement of the eleven schools. The table lists the schools, identified by letter, according to rank in the Cooperative test. It also gives the median score of each school, the percentile of the Median, and for purposes of comparison the median I Q score of each school (for freshmen), and the median arithmetic score. Further, the letters D and T-M in parentheses after the rank number show the text used; D representing *Exercises in Algebra*, and T-M *First Course in Algebra*. Where both D and T-M are listed the meaning is that in the same school some classes used the former text, some the latter.

5 *In the Directions For Using the Cooperative Tests*, p. 14, the following description of the percentile score is given: "The percentile value corresponding to a given score shows what percentage of the students in that group achieve scores below that score."

6 The I Q test and the arithmetic test were administered to all freshmen at the beginning of the school year. *The Henmon-Nelson High School Examination*, Form C, was the intelligence test given, and the *Schrammel and Otterstrom Arithmetic Test*, Form A, was the arithmetic test.
COMPARATIVE ACHIEVEMENT OF THE ELEVEN SCHOOLS IN THE THREE TESTS

**TABLE IV**

<table>
<thead>
<tr>
<th>Rank &amp; Author</th>
<th>School</th>
<th>Median Coop. Score</th>
<th>Percentile Median</th>
<th>Median I Q</th>
<th>Median Arithmetic Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (D)</td>
<td>H</td>
<td>63.6</td>
<td>87.0</td>
<td>120</td>
<td>51</td>
</tr>
<tr>
<td>2 (T-M)</td>
<td>J</td>
<td>60.1</td>
<td>77.3</td>
<td>114</td>
<td>50</td>
</tr>
<tr>
<td>3 (T-M)</td>
<td>G</td>
<td>59.8</td>
<td>76.3</td>
<td>123</td>
<td>54</td>
</tr>
<tr>
<td>4 (D)</td>
<td>E</td>
<td>57.7</td>
<td>68.9</td>
<td>118</td>
<td>47</td>
</tr>
<tr>
<td>5 (D)</td>
<td>F</td>
<td>57.4</td>
<td>67.9</td>
<td>119</td>
<td>50</td>
</tr>
<tr>
<td>6 (D)</td>
<td>C</td>
<td>55.1</td>
<td>59.4</td>
<td>123</td>
<td>57</td>
</tr>
<tr>
<td>7 (D)</td>
<td>I</td>
<td>53.0</td>
<td>50.5</td>
<td>119</td>
<td>52</td>
</tr>
<tr>
<td>8 (T-M)</td>
<td>A</td>
<td>52.6</td>
<td>48.7</td>
<td>111</td>
<td>40</td>
</tr>
<tr>
<td>9 (D)</td>
<td>D</td>
<td>51.5</td>
<td>44.0</td>
<td>120</td>
<td>57</td>
</tr>
<tr>
<td>10 (D)</td>
<td>B</td>
<td>51.2</td>
<td>43.2</td>
<td>115</td>
<td>44</td>
</tr>
<tr>
<td>11 (T-M)</td>
<td>K</td>
<td>48.2</td>
<td>32.0</td>
<td>117</td>
<td>47</td>
</tr>
</tbody>
</table>

The Cooperative Test Service gives the percentile mean for the elementary algebra test as 52.9. This figure is based upon the actual achievement of 15,000 pupils in 130 public secondary schools of the East, Middle West and West. Table V gives the distribution according to scaled scores of all the students of each school, as well as each school's median and the percentile of the median.
### DISTRIBUTION IN THE ELEVEN SCHOOLS ACCORDING TO THE SCALED SCORES OF COOPERATIVE TEST

**TABLE V**

<table>
<thead>
<tr>
<th>Scaled Scores</th>
<th>H</th>
<th>J</th>
<th>G</th>
<th>E</th>
<th>F</th>
<th>C</th>
<th>I</th>
<th>A</th>
<th>D</th>
<th>B</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td>90-100</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>80-89</td>
<td>8</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>7</td>
<td>1</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>70-79</td>
<td>57</td>
<td>65</td>
<td>27</td>
<td>20</td>
<td>21</td>
<td>12</td>
<td>18</td>
<td>16</td>
<td>7</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>60-69</td>
<td>111</td>
<td>95</td>
<td>66</td>
<td>40</td>
<td>48</td>
<td>62</td>
<td>42</td>
<td>10</td>
<td>29</td>
<td>18</td>
<td>32</td>
</tr>
<tr>
<td>50-59</td>
<td>72</td>
<td>105</td>
<td>73</td>
<td>46</td>
<td>70</td>
<td>101</td>
<td>86</td>
<td>18</td>
<td>51</td>
<td>45</td>
<td>72</td>
</tr>
<tr>
<td>40-49</td>
<td>26</td>
<td>43</td>
<td>14</td>
<td>29</td>
<td>45</td>
<td>43</td>
<td>65</td>
<td>22</td>
<td>53</td>
<td>34</td>
<td>110</td>
</tr>
<tr>
<td>30-39</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>11</td>
<td>9</td>
<td>5</td>
<td>5</td>
<td>37</td>
</tr>
<tr>
<td>Total</td>
<td>275</td>
<td>316</td>
<td>186</td>
<td>140</td>
<td>192</td>
<td>233</td>
<td>222</td>
<td>82</td>
<td>146</td>
<td>106</td>
<td>263</td>
</tr>
</tbody>
</table>

Median: 63.6 60.1 59.8 57.7 57.4 55.1 53.0 52.6 51.5 51.2 48.5

Med. %ile: 87.0 77.3 76.3 68.9 67.9 59.4 50.5 48.7 44.0 43.2 32.0

Tables IV and V show that the performance of the students in Jesuit high schools compares favorably with that of the 15,000 public school freshmen whose percentile mean is 52.9. Six of the eleven Jesuit schools made median scores higher than the national median. Five Jesuit schools were below the 52.9 percentile. This was to be expected in view of the median I Q scores for each school which are probably somewhat higher than the national high-school median.

In the eleven schools, 68 freshmen sections or classes took the Cooperative test. Table VI gives the rank of each section, the number of pupils in the section and the school, the scaled score median, the corresponding percentile for each median, and the textbook used. The letter D represents
**Doyle's Exercises in Algebra** and T-M means Theissen-McCoy's First Course in Algebra.

**COMPARATIVE ACHIEVEMENT OF THE SIXTY-EIGHT CLASS SECTIONS IN THE COOPERATIVE ALGEBRA TEXT**

**TABLE VI**

<table>
<thead>
<tr>
<th>Rank</th>
<th>School</th>
<th>Text</th>
<th>Section</th>
<th>Pupils</th>
<th>Score</th>
<th>Median</th>
<th>Percentile of Median</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A</td>
<td>T-M</td>
<td>la</td>
<td>27</td>
<td>75.0</td>
<td>99.0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>H</td>
<td>D</td>
<td>la</td>
<td>36</td>
<td>72.5</td>
<td>98.2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>J</td>
<td>T-M</td>
<td>ld</td>
<td>33</td>
<td>72.2</td>
<td>98.1</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>I</td>
<td>D</td>
<td>lf</td>
<td>27</td>
<td>70.5</td>
<td>96.5</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>J</td>
<td>T-M</td>
<td>la</td>
<td>30</td>
<td>70.1</td>
<td>96.1</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>E</td>
<td>D</td>
<td>lc</td>
<td>30</td>
<td>68.5</td>
<td>94.5</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>F</td>
<td>D</td>
<td>le</td>
<td>31</td>
<td>68.2</td>
<td>94.2</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>H</td>
<td>D</td>
<td>lb</td>
<td>30</td>
<td>67.5</td>
<td>93.2</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>H</td>
<td>D</td>
<td>le</td>
<td>39</td>
<td>67.1</td>
<td>92.7</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>K</td>
<td>T-M</td>
<td>la</td>
<td>30</td>
<td>66.8</td>
<td>92.2</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>J</td>
<td>T-M</td>
<td>lb</td>
<td>39</td>
<td>66.7</td>
<td>92.0</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>J</td>
<td>T-M</td>
<td>le</td>
<td>37</td>
<td>65.7</td>
<td>90.5</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>H</td>
<td>D</td>
<td>lf</td>
<td>37</td>
<td>64.2</td>
<td>88.4</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>G</td>
<td>D</td>
<td>ld</td>
<td>40</td>
<td>64.0</td>
<td>88.0</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>H</td>
<td>D</td>
<td>lh</td>
<td>36</td>
<td>63.5</td>
<td>86.7</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>C</td>
<td>D</td>
<td>lb</td>
<td>35</td>
<td>63.0</td>
<td>85.5</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>E</td>
<td>D</td>
<td>le</td>
<td>32</td>
<td>62.5</td>
<td>84.2</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>G</td>
<td>D</td>
<td>lb</td>
<td>36</td>
<td>62.5</td>
<td>84.2</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>C</td>
<td>D</td>
<td>lc</td>
<td>35</td>
<td>61.0</td>
<td>80.0</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>G</td>
<td>T-M</td>
<td>le</td>
<td>34</td>
<td>60.7</td>
<td>79.1</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>J</td>
<td>T-M</td>
<td>lc</td>
<td>28</td>
<td>60.5</td>
<td>78.5</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>E</td>
<td>D</td>
<td>lb</td>
<td>30</td>
<td>60.0</td>
<td>77.0</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>D</td>
<td>D</td>
<td>la</td>
<td>31</td>
<td>60.0</td>
<td>77.0</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>D</td>
<td>D</td>
<td>lb</td>
<td>29</td>
<td>59.8</td>
<td>76.3</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>C</td>
<td>D</td>
<td>la</td>
<td>35</td>
<td>59.7</td>
<td>75.9</td>
<td></td>
</tr>
<tr>
<td>26</td>
<td>B</td>
<td>D</td>
<td>la</td>
<td>30</td>
<td>59.5</td>
<td>75.5</td>
<td></td>
</tr>
<tr>
<td>27</td>
<td>I</td>
<td>T-M</td>
<td>lg</td>
<td>38</td>
<td>59.3</td>
<td>74.5</td>
<td></td>
</tr>
<tr>
<td>28</td>
<td>G</td>
<td>T-M</td>
<td>le</td>
<td>37</td>
<td>59.2</td>
<td>74.2</td>
<td></td>
</tr>
<tr>
<td>29</td>
<td>H</td>
<td>D</td>
<td>ld</td>
<td>37</td>
<td>59.0</td>
<td>73.5</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>F</td>
<td>D</td>
<td>lb</td>
<td>32</td>
<td>56.8</td>
<td>65.8</td>
<td></td>
</tr>
</tbody>
</table>
The above figures from Table VI show that 27 of the 68
sections or approximately 40 percent of the sections have medians below the 50th percentile. In the 68 sections, 26 used Theissen-McCoy's text, and the remaining 42 used Doyle's *Exercises in Algebra*; of the 42 Doyle classes, 15 sections or 35 percent were below the 50th percentile, while, of the 26 Theissen-McCoy classes, 12 sections or 46 percent were below the 50th percentile. In the upper ranks of the sections, 18 Doyle classes or 42 percent were above the 75th percentile. On the other hand, seven Theissen-McCoy sections, or 27 percent were above the 75th percentile. On the basis, therefore, of the percentile norms set up by the Cooperative Test Service for public secondary schools, the achievement of the 68 sections in the eleven Jesuit high schools tested is superior. Further, it would appear from Table VI that in general the Doyle classes ranked higher as class groups than the Theissen-McCoy sections. A smaller percent of the Doyle classes were below the 50th percentile and a larger percent of the same classes were above the 75th percentile. However, the difference does not appear to be large enough to be greatly significant.

Table VII gives the distribution of the two groups of students. The Doyle pupils numbered 1328 and the Theissen-McCoy group had 844 students. The scaled score and the percentile ranking of both groups are listed.
COMPARATIVE DISTRIBUTION ACCORDING TO SCALED SCORES
OF DOYLE GROUP AND THEISSEN-MCCOY GROUP

TABLE VII

<table>
<thead>
<tr>
<th>Scale Score</th>
<th>Distribution of Doyle Students</th>
<th>Distribution of Theissen-McCoy Students</th>
<th>Percentile Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>90-91</td>
<td>1</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>88-89</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>86-87</td>
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<td>3</td>
<td></td>
</tr>
<tr>
<td>84-85</td>
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<td>2</td>
<td></td>
</tr>
<tr>
<td>82-83</td>
<td>7</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>80-81</td>
<td>13</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>78-79</td>
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<td>76-77</td>
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<td>99</td>
</tr>
<tr>
<td>74-75</td>
<td>29</td>
<td>21</td>
<td>98</td>
</tr>
<tr>
<td>72-73</td>
<td>59</td>
<td>36</td>
<td>96</td>
</tr>
<tr>
<td>70-71</td>
<td>76</td>
<td>50</td>
<td>94</td>
</tr>
<tr>
<td>68-69</td>
<td>60</td>
<td>36</td>
<td>91</td>
</tr>
<tr>
<td>66-67</td>
<td>75</td>
<td>34</td>
<td>88</td>
</tr>
<tr>
<td>64-65</td>
<td>36</td>
<td>28</td>
<td>83</td>
</tr>
<tr>
<td>62-63</td>
<td>102</td>
<td>51</td>
<td>77</td>
</tr>
<tr>
<td>60-61</td>
<td>54</td>
<td>27</td>
<td>70</td>
</tr>
<tr>
<td>58-59</td>
<td>100</td>
<td>63</td>
<td>63</td>
</tr>
<tr>
<td>56-57</td>
<td>119</td>
<td>63</td>
<td>55</td>
</tr>
<tr>
<td>54-55</td>
<td>114</td>
<td>69</td>
<td>46</td>
</tr>
<tr>
<td>52-53</td>
<td>102</td>
<td>47</td>
<td>38</td>
</tr>
<tr>
<td>50-51</td>
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<td>63</td>
<td>30</td>
</tr>
<tr>
<td>48-49</td>
<td>48</td>
<td>50</td>
<td>24</td>
</tr>
<tr>
<td>46-47</td>
<td>49</td>
<td>43</td>
<td>18</td>
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<td>44-45</td>
<td>29</td>
<td>28</td>
<td>13</td>
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<tr>
<td>42-43</td>
<td>22</td>
<td>25</td>
<td>9</td>
</tr>
<tr>
<td>40-41</td>
<td>12</td>
<td>19</td>
<td>6</td>
</tr>
<tr>
<td>38-39</td>
<td>11</td>
<td>15</td>
<td>4</td>
</tr>
<tr>
<td>36-37</td>
<td>4</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>34-35</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>32-33</td>
<td>3</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

Total 1328 844

The following significant facts are learned from an analysis of Table VII. The median scale score for the 844
Theissen-McCoy students is 55.7 and the percentile is 60.7. This compares with a median scale score of 56.1, percentile 63.4, for the 1328 Doyle students. Above the 75th percentile are 40 percent of the Doyle students while 35 percent of the Theissen-McCoy group are in the same range. Somewhat more meaningful is the fact that 24 percent of the latter group are below the 25th percentile while only 13 percent of the Doyle students are below.

In general, therefore, the results of the objective test show that the performance of students in the Jesuit high schools tested was superior to the 15,000 public school students tested. It further revealed that those classes using Exercises in Algebra ranked higher as class groups than those using First Course in Algebra. Also significant is the showing of the Doyle students compared with the Theissen-McCoy group. The median score of the former group is higher and in addition there is a higher percentage of the former group above the 75th percentile and a smaller proportion below the 25th percentile. This higher rating of the Doyle students becomes all the more significant in view of the type of course. Where the Theissen-McCoy students were specifically conditioned for just such an objective type test in the formal operations, and consequently should have rated a good deal higher than the Doyle students, the latter group were definitely not thus prepared in the two semesters of work in solving equations.
Whether a more thorough testing program would reveal any significant change in this picture is impossible to say. Consequently, as interesting as the data from the testing program is, it seems better to withhold any final judgment at the present time.

Mention has already been made of the difficulties in administering any suitable test for Exercises in Algebra. The test which was sent out to the schools was at best a compromise. It was made up of a series of eight equation problems ranging from very simple linear equations in one unknown to quadratics. The problems were arranged in progressively more difficult equations containing monomial and polynomial terms, both integral and fractional. One problem with a fractional answer had to be checked, the others did not have to be checked. The student was supposed to be able to finish in forty minutes.

For a student who had spent two semesters in solving the problems of Exercises in Algebra the test should have been quite easy. The reason is that such a student had concentrated almost exclusively on equations, many of them quite complex, and with fractional answers which had to be checked. On the other hand, the student who had spent two semesters in the formal operational type of text such as First Course in Algebra should be at some disadvantage in the test described. All the important operations which such a student has studied formally are indeed contained in the equations of the test, but not in
the context he has been accustomed to.

Six of the eleven Jesuit schools were able to give the test. The total number of students was 999 in 32 class sections. Twelve of these classes with a total of 379 students used First Course in Algebra while 620 students in 20 class sections used Exercises in Algebra. Table VIII gives the distribution of scores in the six schools. The schools are identified with the same letter as before in the reporting of the results of the objective test. The scoring was based on a grade of one-hundred for finishing all eight problems without an error. Each problem (and the check, when required) was worth 12.5 points.

COMPARATIVE STANDING OF THE SIX SCHOOLS IN THE EQUATION TEST

TABLE VIII

<table>
<thead>
<tr>
<th>Score</th>
<th>H</th>
<th>G</th>
<th>J</th>
<th>F</th>
<th>B</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>88</td>
<td>17</td>
<td>57</td>
<td>8</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>87.5</td>
<td>65</td>
<td>25</td>
<td>58</td>
<td>28</td>
<td>4</td>
<td>9</td>
</tr>
<tr>
<td>75.0</td>
<td>50</td>
<td>22</td>
<td>37</td>
<td>36</td>
<td>8</td>
<td>4</td>
</tr>
<tr>
<td>62.5</td>
<td>23</td>
<td>18</td>
<td>41</td>
<td>29</td>
<td>19</td>
<td>13</td>
</tr>
<tr>
<td>50.0</td>
<td>25</td>
<td>12</td>
<td>41</td>
<td>17</td>
<td>10</td>
<td>18</td>
</tr>
<tr>
<td>37.5</td>
<td>8</td>
<td>9</td>
<td>35</td>
<td>15</td>
<td>13</td>
<td>12</td>
</tr>
<tr>
<td>25.0</td>
<td>7</td>
<td>3</td>
<td>26</td>
<td>10</td>
<td>13</td>
<td>11</td>
</tr>
<tr>
<td>12.5</td>
<td>0</td>
<td>1</td>
<td>14</td>
<td>11</td>
<td>9</td>
<td>16</td>
</tr>
<tr>
<td>Total tested</td>
<td>266</td>
<td>117</td>
<td>309</td>
<td>154</td>
<td>78</td>
<td>84</td>
</tr>
</tbody>
</table>

Mean 80.7 74.2 66.0 62.5 48.5 45.2

As in the objective test, School H is in first place. School J, which ranked second in the former test, is in third
place. However, in School G only 117 students of a class of 186 took the test. These 117 students were in three class sections, two of which were rated as superior by the principal. This probably accounts for School G's superior performance. In School H and J all algebra sections took the test. In the other schools one or more sections did not take the test.

Since in most of the schools the algebra classes are arranged homogeniously, the figures in table VIII and in subsequent tables are to be interpreted with considerable caution. Indeed the writer is of the opinion that had all the 68 algebra sections in the eleven schools taken this test the figures would have been notably different. As it is some of the schools exempted either their very poor or their very good classes, with the result that the full picture is incomplete.

Table IX shows the performance of the 32 class sections which took the test. The sections are arranged according to rank, the schools and sections are listed by letter as before; the text used is indicated by D or T-M as in the other tables. The mean score for each section is listed as well as the number of pupils in the section. The 32 class sections are ranked in descending order according to their performance in the objective test. This comparative rank is given in parentheses immediately after the rank of each section in the equation test.
# Comparative Performance of the Thirty-Two Class Sections in the Equation Test

## Table IX

<table>
<thead>
<tr>
<th>Rank</th>
<th>School</th>
<th>Section</th>
<th>Text</th>
<th>Number of Pupils</th>
<th>Mean</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>J</td>
<td>a</td>
<td>T-M</td>
<td>30</td>
<td>92.50</td>
</tr>
<tr>
<td>2</td>
<td>H</td>
<td>a</td>
<td>D</td>
<td>35</td>
<td>91.78</td>
</tr>
<tr>
<td>3</td>
<td>J</td>
<td>d</td>
<td>T-M</td>
<td>33</td>
<td>84.46</td>
</tr>
<tr>
<td>4</td>
<td>H</td>
<td>d</td>
<td>D</td>
<td>37</td>
<td>84.43</td>
</tr>
<tr>
<td>5</td>
<td>H</td>
<td>e</td>
<td>D</td>
<td>36</td>
<td>83.68</td>
</tr>
<tr>
<td>6</td>
<td>H</td>
<td>h</td>
<td>D</td>
<td>35</td>
<td>83.57</td>
</tr>
<tr>
<td>7</td>
<td>H</td>
<td>b</td>
<td>D</td>
<td>32</td>
<td>80.46</td>
</tr>
<tr>
<td>8</td>
<td>J</td>
<td>e</td>
<td>T-M</td>
<td>34</td>
<td>79.04</td>
</tr>
<tr>
<td>9</td>
<td>H</td>
<td>f</td>
<td>D</td>
<td>33</td>
<td>78.41</td>
</tr>
<tr>
<td>10</td>
<td>G</td>
<td>c</td>
<td>T-M</td>
<td>34</td>
<td>77.57</td>
</tr>
<tr>
<td>11</td>
<td>F</td>
<td>e</td>
<td>D</td>
<td>29</td>
<td>76.72</td>
</tr>
<tr>
<td>12</td>
<td>G</td>
<td>e</td>
<td>T-M</td>
<td>36</td>
<td>75.69</td>
</tr>
<tr>
<td>13</td>
<td>H</td>
<td>g</td>
<td>D</td>
<td>29</td>
<td>74.56</td>
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<tr>
<td>14</td>
<td>J</td>
<td>c</td>
<td>T-M</td>
<td>27</td>
<td>72.58</td>
</tr>
<tr>
<td>15</td>
<td>A</td>
<td>a</td>
<td>D</td>
<td>25</td>
<td>70.00</td>
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<tr>
<td>16</td>
<td>J</td>
<td>k</td>
<td>T-M</td>
<td>29</td>
<td>65.17</td>
</tr>
<tr>
<td>17</td>
<td>F</td>
<td>b</td>
<td>D</td>
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<td>62.50</td>
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<tr>
<td>18</td>
<td>J</td>
<td>h</td>
<td>T-M</td>
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<td>D</td>
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<td>a</td>
<td>D</td>
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<td>62.18</td>
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<td>D</td>
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<td>D</td>
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<td>a</td>
<td>D</td>
<td>38</td>
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<td>24</td>
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<td>38</td>
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<td>F</td>
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<td>D</td>
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<td>26</td>
<td>J</td>
<td>f</td>
<td>T-M</td>
<td>40</td>
<td>46.97</td>
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<tr>
<td>27</td>
<td>J</td>
<td>j</td>
<td>T-M</td>
<td>16</td>
<td>46.55</td>
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<tr>
<td>28</td>
<td>B</td>
<td>c</td>
<td>D</td>
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<td>29</td>
<td>J</td>
<td>g</td>
<td>T-M</td>
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<td>38.60</td>
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<td>30</td>
<td>A</td>
<td>c</td>
<td>D</td>
<td>31</td>
<td>35.84</td>
</tr>
<tr>
<td>31</td>
<td>B</td>
<td>d</td>
<td>D</td>
<td>18</td>
<td>34.72</td>
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<tr>
<td>32</td>
<td>A</td>
<td>b</td>
<td>D</td>
<td>28</td>
<td>34.35</td>
</tr>
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</table>
SUMMARY OF DATA OF EQUATION TEST

TABLE X

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total students tested</td>
<td>999</td>
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<tr>
<td>Doyle students tested</td>
<td>620</td>
</tr>
<tr>
<td>T-McCoy students tested</td>
<td>379</td>
</tr>
<tr>
<td>Average of all students</td>
<td>66.66</td>
</tr>
<tr>
<td>Average of Doyle students</td>
<td>66.02</td>
</tr>
<tr>
<td>Average of T-McCoy students</td>
<td>65.07</td>
</tr>
<tr>
<td>Classes tested</td>
<td>32</td>
</tr>
<tr>
<td>Class Average</td>
<td>65.41</td>
</tr>
<tr>
<td>Average of Doyle classes</td>
<td>64.54</td>
</tr>
<tr>
<td>Average of T-McCoy classes</td>
<td>66.84</td>
</tr>
<tr>
<td>Doyle classes tested</td>
<td>20</td>
</tr>
<tr>
<td>T-McCoy classes tested</td>
<td>12</td>
</tr>
</tbody>
</table>

The most significant fact reported in Tables IX and X is the almost equal performance of the Doyle and Theissen-McCoy students. Sixteen sections have averages above the means for all the 32 sections. In this upper group, seven of the classes used First Course in Algebra, while nine used Exercises in Algebra. This means that approximately fifty percent of the T-M classes are better than average in the equation test; approximately the same percentage of Doyle classes are in this group. Seven of the Doyle sections performed notably better in the equation test than could be
expected from their rank in the objective test. Two T-M classes made better showings in the former test than their proportionate rank in the objective test. The mean for all the 999 students is almost the same as the mean of the 620 Doyle pupils and the 379 T-M students. The difference in these means is so small as to be insignificant. A slight margin is shown in favor of the T-M class sections over the Doyle classes when the class means are compared. Again, however, the difference is too slight to be meaningful. No matter from what angle the figures are studied, the scores show that the performance of the two groups is practically identical.

In the objective test the scores of the two groups were also fairly equal. The interpretation was made in the discussion of the objective test scores that the Doyle group actually showed up better in view of the different type course. A similar observation should be made here in favor of the T-M group. The equation test favored to some extent the Doyle group. Any prediction of performance would have declared that the Doyle students would achieve higher scores in this test than the T-M students. The writer, who prepared the test, felt that the Doyle students would indeed do so notably better that any comparison would be difficult or impossible to make. In fact the test was made considerably easier than would have been the case had all the testees been
Doyle students, in order that the T-M students would not appear to be placed at too great a disadvantage. In view of this, the standing of the T-M group becomes all the more remarkable. The scores show that this group did just as well as the Doyle group; a fair interpretation of these scores shows that the T-M students performed considerably better than the Doyle group.

This interpretation, however, is made only tentatively. The total number of students tested, especially in the T-M group appears to be too small to warrant any final conclusions. Again the small number of class sections in the two groups makes a valid comparison of performance questionable. The most that can be said is that on the basis of the test, incomplete and inadequate as it surely is as a valid testing instrument, the T-M students turned in papers as good as the Doyle students.

In summary, therefore, the testing program showed that both groups did pretty much the same type of work in formal operations regardless of the text used. The same thing is true in regard to equations. The two groups, using radically different texts, showed essentially the same mastery of both the formal operations and the equations. Whether a more thorough testing program under favorable conditions would show a different picture is impossible to say now.
CHAPTER V

CONCLUSION

This paper has attempted to describe the differences between the two elementary algebra textbooks presently in use in the eleven Midwest Jesuit high schools. These texts are *Exercises in Algebra* and *First Course in Algebra*.

The discussion was begun, as was explained in the Introduction, at the meeting of the Jesuit mathematics instructors, by the report of the Rockhurst teachers. There a challenge was made to the traditional view underlying the teaching of mathematics in the secondary school. A new and indeed revolutionary position was explained by the Rockhurst group and a new course of elementary algebra was suggested in the light of this position. The discussion of the new course, as it was exemplified in Doyle's text, was developed by comparing it with the older and typical text by Theissen-McCoy.

This comparison of the texts was preceded by a review of the major sections of the literature which describes the different viewpoints and which effect in an essential way the structure of the resulting texts. In this review the formal approach to algebra by the traditionalists was explained in the light of their own teaching, and the theoretical position underlying the equation approach to algebra was reported. In addition, the position of the Progressive Educational Association was outlined in order to give greater
clarity, by way of contrast, to both the teachings of the National Committee as well as the Rockhurst group.

Chapter III took up the comparison of the two texts. An inventory of the content of both texts was given, the different objectives and the teaching techniques of the authors were described. It was seen that in every aspect *First Course in Algebra* is a typical elementary algebra textbook which teaches the laws and operations of algebra formally through lengthy explanations, rules, and drills in the fundamental operations. On the other hand, *Exercises in Algebra* was shown to be a text largely made up of equations, almost entirely lacking in explanatory material, and teaching the essentials of algebra not through drills in the fundamental operations but through equations in which these operations are constantly used.

A small amount of evidence on the problem of whether high-school students can learn adequately these necessary essentials of algebra by the method outlined by the Rockhurst group was offered in the report of the testing program. There the tentative judgment was made that the results of the objective test given to all the pupils in the eleven Jesuit high schools of the Midwest show that students lose nothing by the equation approach. The achievement of the students using the typical Theissen-McCoy text was not notably different from the work of the Doyle students, as judged by the results of
the objective test, in formal operational work. The equation test was given to fewer students and was consequently a less complete study. The very tentative conclusion was made that the Theissen-McCoy students did just as well as the Doyle students in equation solving and in handling the necessary computational work.

Final judgment on whether the Rockhurst method is superior must be delayed several years until the course has been fully developed and until teachers have had sufficient experience with it. The main points to be determined are whether the Doyle students show a greater computational skill, whether they retain the power to work through a mass of complex mathematical data, and whether they have any greater enthusiasm and interest in mathematics as a result of the new course than the students trained according to the traditional methods.
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BOOKLETS

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PERIODICAL ARTICLES


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UNPUBLISHED MATERIAL

Doyle, William C., Selected letters addressed to the writer in 1945-46.

APPENDIX I

Equation Test

Name ____________________________
School __________________________
Class ____________________________
Teacher __________________________
Date ____________________________

I. Solve the following equation. Do not check.
   \[ 3x + 5 = 1 + 5x \]

II. Solve the following equation. Do not check.
    \[ 9 + 3(4x - 1) = 20 - 2(7x + 3) \]

III. Solve the following equation. Do not check.
     \[ (r - 3)(r + 1) = (r - 2)(r - 1) \]

IV. Solve and check the following equation.
    \[ (x + 2)^2 - (x - 2)^2 = 4 \]

V. Solve the following equation. Do not check.
    \[ \frac{4x}{3} + \frac{7}{12} = \frac{2x - 3}{4} - \frac{x - 1}{6} \]

VI. Solve for \( x \) in the following equation. Do not check.
    \[ 5x - 3a = 7a \]

VII. Solve the following quadratic equation by the factoring method. Do not check.
     \[ 2x(x + 3) + 8 = 3 - (2 - x)(2 + x) \]

VIII. Solve and check the following quadratic equation by the quadratic formula. The formula is:
     \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
     \[ 2x^2 + 5x - 3 = 0 \]

Note: Solve all the problems of this test. The time you will have to do so is 40 minutes.
Please print:

Name.____________________________________________________________________________________________________________

Last First Middle Date.________________________________________________________________________________________________

Grade or Class._____________________________________________________________________________________________________

Age._______________________________________________________________________________________________________Date of Birth.______________________________________________________________________________

Yrs. Mos.

School.______________________________________________________________________________________________________________

City.______________________________________________________________________________________________________________

Sex. _____________________________________________________________________________________________________________

M. or F.

Title of the algebra course you are now taking.________________________________________________________________________

Instructor._____________________________________________________________________________________________________________________________________________________

In what grade did you begin the study of algebra?________________________________________________________________________

Number of years you have studied algebra (one semester = ½ year; one quarter = ⅓ year): _______________

General Directions: Do not turn this page until the examiner tells you to do so. This examination consists of three parts, and requires 40 minutes of working time. The directions for each part are printed at the beginning of the part. Read them carefully, and proceed at once to answer the questions. DO NOT SPEND TOO MUCH TIME ON ANY ONE ITEM. ANSWER THE EASIER QUESTIONS FIRST; then return to the harder ones if you have time. There is a time limit for each part. You are not expected to answer all the questions in any part in the time limit; but if you should, go on to the next part. If you have not finished Part I when the time is up, stop work on that part and proceed at once to Part II. If you finish the last part before the time is up, you may go back and work on any part. No questions may be asked after the examination has begun.

You may answer questions even when you are not perfectly sure that your answers are correct, but you should avoid wild guessing, since wrong answers will result in a subtraction from the number of your correct answers.

<table>
<thead>
<tr>
<th>Part</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minutes</td>
<td>15</td>
<td>10</td>
<td>15</td>
<td>40</td>
</tr>
</tbody>
</table>

| Scaled Score | Percentile |
PART I

Directions: Each problem below is followed by five possible answers, only one of which is correct. By working each problem find the correct answer and put its number in the parentheses at the right.

1. \(-5x - x\) equals
   1-1 \(-6x\)
   1-2 \(5x^2\)
   1-3 \(6x\)
   1-4 \(-4x\)
   1-5 \(-5\) .......................... 1( )

2. \(-\dfrac{2a}{a}\) equals
   2-1 \(2a\)
   2-2 \(2\)
   2-3 \(-2a^2\)
   2-4 \(-2\)
   2-5 \(-\dfrac{2}{a}\) .......................... 2( )

3. If \(p = 9\) and \(r = -5\), the numerical value of \(2p + 3r\) is
   3-1 \(1\)
   3-2 \(17\)
   3-3 \(3\)
   3-4 \(-27\)
   3-5 \(33\) .......................... 3( )

4. If \(6r + 1 - (4r - 3) = 20\), \(r\) equals
   4-1 \(\dfrac{3}{5}\)
   4-2 \(8\)
   4-3 \(9\)
   4-4 \(11\)
   4-5 \(12\) .......................... 4( )

5. The sum of \(-15c^4\) and \(-3c^4\) is
   5-1 \(-18c^4\)
   5-2 \(-12c^4\)
   5-3 \(12\)
   5-4 \(12c^8\)
   5-5 \(-18c^8\) .......................... 5( )

6. If \(16 = \dfrac{n}{2^n}\), \(n\) equals
   6-1 \(8\)
   6-2 \(14\)
   6-3 \(18\)
   6-4 \(32\)
   6-5 \(\dfrac{1}{8}\) .......................... 6( )

7. \((-c)(-c)\) equals
   7-1 \(-2c^2\)
   7-2 \(-c^2\)
   7-3 \(0\)
   7-4 \(c^2\)

8. If \(\dfrac{3k + 8}{5} = 7\), \(k\) equals
   8-1 \(\dfrac{1}{3}\)
   8-2 \(14\)
   8-3 \(9\)
   8-4 \(-\dfrac{1}{3}\)
   8-5 \(5\) .......................... 8( )

9. Factor \(\pi R^2 - \pi r^2\).
   9-1 \(\pi(R - r)(R - r)\)
   9-2 \((R + r)(R - r)\)
   9-3 \((\pi R - r)(R - r)\)
   9-4 \(\pi(R + r)(R - r)\)
   9-5 \((\pi R - r)(R + \pi r)\) .......................... 9( )

10. If \(c = -5\), \(h = 3\), \(n = -4\), what is the numerical value of \(3c + 4h - 2n\)?
    10-1 \(35\)
    10-2 \(-11\)
    10-3 \(-19\)
    10-4 \(11\)
    10-5 \(5\) .......................... 10( )

11. What is the product of \(x^3\) and \(x^9\)?
    11-1 \(x^{27}\)
    11-2 \(x^3\)
    11-3 \(x^{12}\)
    11-4 \(x^{13}\)
    11-5 \(x^6\) .......................... 11( )

12. What is the remainder when \(4.2m\) is subtracted from \(9.75m\)?
    12-1 \(9.33\)
    12-2 \(5.55\)
    12-3 \(13.95\)
    12-4 \(5.55\)
    12-5 \(9.33\) .......................... 12( )

13. If \(3r - k = 30\), and \(r - k = 8\), \(r\) equals
    13-1 \(9\)
    13-2 \(5\)
    13-3 \(3\)
    13-4 \(11\)
    13-5 \(5\) .......................... 13( )
14. What is the remainder when $-3n^2 + 7$ is subtracted from $2n^2 - 5n + 3$?
   14-1 $-n^2 - 5n + 10$
   14-2 $5n^2 - 5n - 4$
   14-3 $-n^2 - 3n + 10$
   14-4 $-5n^2 - 5n + 10$
   14-5 $-n^2 + 2n + 3$ . . . . . . . 14( )

15. Reduce $\frac{x^2 - 2x - 15}{x^2 + 7x + 12}$ to lowest terms.
   15-1 $\frac{x - 3}{x + 3}$
   15-2 $\frac{(x + 5)(x - 3)}{(x + 3)(x + 4)}$
   15-3 $\frac{x + 4}{x - 5}$
   15-4 $\frac{(x - 5)(x + 3)}{(x + 6)(x + 2)}$
   15-5 $\frac{x - 5}{x + 4}$ . . . . . . . . . . . . . 15( )

16. If $\frac{r - 1}{r + 2} = \frac{r + 2}{r + 6}$, $r$ equals
   16-1 $\frac{2}{9}$
   16-2 $2$
   16-3 $3\frac{1}{3}$
   16-4 $2\frac{2}{3}$
   16-5 $10$ . . . . . . . . . . . . . 16( )

17. Simplify $9\sqrt{2} - 4\sqrt{2} + \sqrt{2}$.
   17-1 $6\sqrt{2}$
   17-2 $13\sqrt{2}$
   17-3 $6\sqrt{6}$
   17-4 $-36\sqrt{2}$
   17-5 $5\sqrt{2}$ . . . . . . . . . . . . . 17( )

18. If $R = A - \frac{2}{3}n^2$, one value of $n$ when $R$ is 4 and $A$ is 100 is
   18-1 8
   18-2 12
   18-3 32
   18-4 72
   18-5 156 . . . . . . . . . . . . . . . 18( )

19. If the graph of the equation $3x + 5y = 1$ passes through the point $(m, -4)$, the value of $m$ is
   19-1 $\frac{3}{5}$
   19-2 $-7$
   19-3 $\frac{6}{3}$
   19-4 $7$
   19-5 $-2\frac{1}{5}$ . . . . . . . . . . . . . 19( )

20. The positive square root of $100a^{36}$ is
   20-1 $10a^9$
   20-2 $50a^{18}$
   20-3 $100a^6$
   20-4 $10a^{72}$
   20-5 $10a^{18}$ . . . . . . . . . . . . . 20( )
i. Directions: Continue as in the preceding exercises.

2. A man gave his lawyer $x$ dollars to pay a speeding fine. After paying the fine, the lawyer had $15$ left. How many dollars was the fine, in terms of $x$?
   
   \[ 2-1 \quad 15 - x \]
   
   \[ 2-2 \quad 15x \]
   
   \[ 2-3 \quad x + 15 \]
   
   \[ 2-4 \quad \frac{15}{x} \]
   
   \[ 2-5 \quad x - 15 \]

3. A baseball diamond has the form of a square. If the distance between bases is $x$ feet, a player running the bases between first and third will have to run a distance of

   \[ 3-1 \quad x \text{ ft} \]
   
   \[ 3-2 \quad x^2 \text{ ft} \]
   
   \[ 3-3 \quad 2x \text{ ft} \]
   
   \[ 3-4 \quad 2x^2 \text{ ft} \]
   
   \[ 3-5 \quad 3x \text{ ft} \]

4. In a certain high school there are 200 more girls than boys. The total number of pupils in the school is 1876. How many boys are there?

   \[ 4-1 \quad 638 \]
   
   \[ 4-2 \quad 838 \]
   
   \[ 4-3 \quad 1038 \]
   
   \[ 4-4 \quad 1138 \]
   
   \[ 4-5 \quad 1676 \]

5. The baseball team in a certain town played 84 games during the last season. The team won 16 more games than it lost. If $x$ equals the number of games lost, the equation you would use to find $x$ is

   \[ 5-1 \quad x + x + 16 = 84 \]
   
   \[ 5-2 \quad x + x - 16 = 84 \]
   
   \[ 5-3 \quad x = 84 - 16 \]
   
   \[ 5-4 \quad x - x + 16 = 84 \]
   
   \[ 5-5 \quad x - x - 16 = 84 \]

6. On her first three algebra tests Grace had marks of 79, 92, and 81. What must her mark on the fourth test be if she wishes to get an average of 85 for all four tests?

   \[ 6-1 \quad 80 \]
   
   \[ 6-2 \quad 82 \]
   
   \[ 6-3 \quad 84 \]
   
   \[ 6-4 \quad 85 \]
   
   \[ 6-5 \quad 88 \]

7. According to the graph above, what was the average number of gallons of ice cream sold per day when the maximum temperature was 95°?

   \[ 7-1 \quad 35 \]
   
   \[ 7-2 \quad 50 \]
   
   \[ 7-3 \quad 55 \]
   
   \[ 7-4 \quad 60 \]
   
   \[ 7-5 \quad 65 \]
8. The sum of two numbers is 50, and the second is 16 more than the first. If \( x \) is the first number, the second may be expressed as

\[
\begin{align*}
\delta-1 & \quad 16 - x \\
\delta-2 & \quad x - 16 \\
\delta-3 & \quad x + 16 \\
\delta-4 & \quad 34 - x \\
\delta-5 & \quad 66 - x 
\end{align*}
\]

9. The sum of the angles of a triangle is 180 degrees. One angle of the triangle is 36 degrees. The second angle is half as large as the third. How many degrees are there in the third angle?

\[
\begin{align*}
\gamma-1 & \quad 36 \\
\gamma-2 & \quad 48 \\
\gamma-3 & \quad 72 \\
\gamma-4 & \quad 96 \\
\gamma-5 & \quad 144
\end{align*}
\]

10. The table above gives values of \( y \) for corresponding values of \( x \). The equation showing the relationship between \( x \) and \( y \) is

\[
\begin{align*}
\zeta-1 & \quad y = x - 1 \\
\zeta-2 & \quad y = x + 1 \\
\zeta-3 & \quad y = 2x \\
\zeta-4 & \quad y = \frac{1}{2}x \\
\zeta-5 & \quad y = \frac{3}{2}x
\end{align*}
\]

11. A new highway was constructed at a cost of $96,000 to be paid for jointly by the state, county, and township. The county agreed to pay as much as the state, and the township was to pay twice as much as the county. How much did the township have to pay?

\[
\begin{align*}
\eta-1 & \quad $16,000 \\
\eta-2 & \quad $24,000 \\
\eta-3 & \quad $32,000 \\
\eta-4 & \quad $48,000 \\
\eta-5 & \quad $64,000
\end{align*}
\]

12. A molding 50 inches long is to be cut into four parts so that each part after the first shall be one inch longer than the preceding part. The length \( x \) of the shortest piece can be found from the following equation:

\[
\begin{align*}
\iota-1 & \quad 4x + 6 = 50 \\
\iota-2 & \quad 4x + 10 = 50 \\
\iota-3 & \quad 4x + 1 = 50 \\
\iota-4 & \quad x + 3 = 50 \\
\iota-5 & \quad 4x - 4 = 50
\end{align*}
\]

13. A package weighing \( w \) pounds (where \( w \) is a whole number) is sent by parcel post. If the postage is 6 cents for the first pound and 2 cents for each additional pound, how many cents will it cost to mail the package providing it weighs more than 1 pound?

\[
\begin{align*}
\kappa-1 & \quad 8w \\
\kappa-2 & \quad 2w + 6 \\
\kappa-3 & \quad 6 + 2(w - 1) \\
\kappa-4 & \quad 6w + 2(w - 1) \\
\kappa-5 & \quad 6w + 2w
\end{align*}
\]

14. The numerator of a given fraction is less than the denominator by 8. If both numerator and denominator are increased by 5, the resulting fraction equals \( \frac{2}{3} \). What is the given fraction?

\[
\begin{align*}
\lambda-1 & \quad 28 \\
\lambda-2 & \quad 35 \\
\lambda-3 & \quad 16 \\
\lambda-4 & \quad 24 \\
\lambda-5 & \quad 52 \\
\lambda-6 & \quad 64 \\
\lambda-7 & \quad \frac{11}{19} \\
\lambda-8 & \quad 21 \\
\lambda-9 & \quad 29
\end{align*}
\]

15. A dealer wishes to mix hazelnuts worth 50¢ a pound and cashews worth 75¢ a pound to obtain 10 lb of mixed nuts worth 55¢ a pound. How many pounds of cashews should he use?

\[
\begin{align*}
\mu-1 & \quad 2 \\
\mu-2 & \quad 5 \\
\mu-3 & \quad 2 \\
\mu-4 & \quad 6 \\
\mu-5 & \quad 4.2 \\
\mu-6 & \quad 8
\end{align*}
\]
Directions: Continue as in the preceding exercises.

1. If \( n - 3 = 15 \), \( n \) equals
   \[
   \begin{align*}
   1-1 & \quad 12 \\
   1-2 & \quad -5 \\
   1-3 & \quad 18 \\
   1-4 & \quad -45 \\
   1-5 & \quad 5 . . . . . . . . . . . . 1( )
   \end{align*}
   \]

2. The second power of \( w \) is
   \[
   \begin{align*}
   2-1 & \quad w^{1/2} \\
   2-2 & \quad 2w \\
   2-3 & \quad \sqrt{w} \\
   2-4 & \quad w^2 \\
   2-5 & \quad \frac{w}{2} . . . . . . . . . . . . 2( )
   \end{align*}
   \]

3. \( 2r \) divided by \( 4r \) equals
   \[
   \begin{align*}
   3-1 & \quad \frac{1}{2} \\
   3-2 & \quad 2r \\
   3-3 & \quad 8r^2 \\
   3-4 & \quad \frac{1}{2}r \\
   3-5 & \quad 8r . . . . . . . . . . . . . . 3( )
   \end{align*}
   \]

4. \( 4h + 3 - (-2h) - 5h \) equals
   \[
   \begin{align*}
   4-1 & \quad -3h + 3 \\
   4-2 & \quad h + 3 \\
   4-3 & \quad 14h \\
   4-4 & \quad 5h \\
   4-5 & \quad 11h + 3 . . . . . . . . . . . . 4( )
   \end{align*}
   \]

5. The formula for the area of a triangle
   is \( A = \frac{bh}{2} \). If \( A = 20 \) and \( b = 8 \), \( h \) equals
   \[
   \begin{align*}
   5-1 & \quad \frac{1}{4} \\
   5-2 & \quad 2\frac{1}{2} \\
   5-3 & \quad \frac{1}{5} \\
   5-4 & \quad 80 \\
   5-5 & \quad 5 . . . . . . . . . . . . . . 5( )
   \end{align*}
   \]

7. If \( \frac{c}{n} = \frac{b}{2} \), then \( b \) equals
   \[
   \begin{align*}
   7-1 & \quad \frac{2c}{n} \\
   7-2 & \quad \frac{c}{n} \frac{b}{2} \\
   7-3 & \quad \frac{2}{n} \frac{b}{2} \\
   7-4 & \quad \frac{b}{c} \frac{c}{2} \\
   7-5 & \quad \frac{n}{2c} . . . . . . . . . . . . . . 7( )
   \end{align*}
   \]

8. The number of yards in \( n \) feet is
   \[
   \begin{align*}
   8-1 & \quad \frac{3}{n} \\
   8-2 & \quad n - 3 \\
   8-3 & \quad 3n \\
   8-4 & \quad 3 - n \\
   8-5 & \quad \frac{n}{3} . . . . . . . . . . . . . 8( )
   \end{align*}
   \]

9. If \( \frac{s}{a} = \frac{1}{h} \), then \( s \) equals
   \[
   \begin{align*}
   9-1 & \quad \frac{1}{h} - \frac{1}{a} \\
   9-2 & \quad \frac{a}{h} \\
   9-3 & \quad ah \\
   9-4 & \quad \frac{1}{ah} \\
   9-5 & \quad \frac{h}{a} . . . . . . . . . . . . . . 9( )
   \end{align*}
   \]

10. "When \( a \) is divided by \( b + c \), the quotient is \( n + 1 \) and the remainder is \( x \)," may be expressed algebraically as
    \[
    \begin{align*}
    10-1 & \quad \frac{a}{b + c} = n + 1 + \frac{x}{b + c} \\
    10-2 & \quad \frac{a}{b + c} = \frac{n + 1}{b + c} + x \\
    10-3 & \quad \frac{a}{b + c} = (n + 1)(b + c) + x \\
    10-4 & \quad \frac{a}{b + c} = n + 1 + x(b + c) \\
    10-5 & \quad \frac{a}{b + c} = n + 1 + x \\
   \end{align*}
   \]
11. \( \frac{4}{a} + \frac{7}{n} \) equals

11-1 \( \frac{4a + 7n}{a + n} \)
11-2 \( \frac{4n + 7a}{an} \)
11-3 \( \frac{11}{a + n} \)
11-4 \( \frac{4a + 7n}{an} \)
11-5 \( \frac{4n + 7a}{a + n} \)

12. How many minutes will it take a man to walk \( g \) yards if he walks at the rate of \( n \) yards a minute?

12-1 \( g + n \)
12-2 \( gn \)
12-3 \( 60gn \)
12-4 \( \frac{n}{g} \)

13. If \( K = Rx + n \), then \( x \) equals

13-1 \( \frac{K}{R} - n \)
13-2 \( R(K - n) \)
13-3 \( K - n - R \)
13-4 \( \frac{K - n}{R} \)
13-5 \( K - n - R \)

14. If \( Sx - 7 = cx \), then \( x \) equals

14-1 \( \frac{c}{S - 7} \)
14-2 \( \frac{c}{S} \)
14-3 \( \frac{7}{S - c} \)
14-4 \( \frac{-7}{Sc} \)
14-5 \( \frac{S - c}{-7} \)

15. Which one of the following is always true?

15-1 \( m(-m) = m^2 \)
15-2 \( \sqrt{a^2 - b^2} = (a + b)(a - b) \)
15-3 \( \frac{r + S}{r} = 1 + S \)
15-4 \( n(rn) = nr^2 \)
15-5 \( p + q = 1 \)

16. \( \frac{\sqrt{26}}{\sqrt{2}} \) equals

16-1 \( 24 \)
16-2 \( \sqrt{13} \)
16-3 \( \sqrt{24} \)
16-4 \( 2\sqrt{13} \)
16-5 \( 13 \)

17. "The difference between two numbers, increased by the difference between the squares of the numbers," may be expressed algebraically as

17-1 \( a - b + a^2 - b^2 \)
17-2 \( (a - b)(a - b)^2 \)
17-3 \( a - b - a^2b^2 \)
17-4 \( (a - b)(a^2 - b^2) \)
17-5 \( a - b + (a - b)^2 \)

18. If \( B = \frac{1}{4}d^2R \), then \( R \) equals

18-1 \( \frac{B}{4d^2} \)
18-2 \( \frac{d^2}{4B} \)
18-3 \( \frac{B - \frac{1}{4}d^2}{B} \)
18-4 \( \frac{4d^2}{B} \)
18-5 \( \frac{4B}{d^2} \)

19. A train has averaged \( d \) miles per hour for \( H \) hours on a run which is to be \( B \) miles in all. Which formula tells what distance \( M \) the train has yet to travel?

19-1 \( M = B - dH \)
19-2 \( M = H(B - d) \)
19-3 \( M = \frac{B}{d} \)
19-4 \( M = B - H - d \)
19-5 \( M = \frac{B}{H}(d) \)

20. The product of \( h^2 \) and \( h^{1/2} \) is

20-1 \( h^{3/2} \)
20-2 \( h^4 \)
20-3 \( h^{5/2} \)
20-4 \( h^{1/4} \)
Questions 21 through 25 refer to the graph below.

![Graph Image]

1. For line AA, when \( x = 2 \), \( y \) equals
   - 21-1 \( y = 2 \)
   - 21-2 \( y = 0 \)
   - 21-3 \( y = 4 \)
   - 21-5 \( y = 6 \)  \( \ldots \ldots \ldots . .21(\quad) \)

2. The solution of the pair of equations represented by lines AA and BB is
   - 22-1 \( x = 4, y = 2 \)
   - 22-2 \( x = 4, y = -2 \)
   - 22-3 \( x = 2, y = -4 \)
   - 22-4 \( x = -4, y = -2 \)
   - 22-5 \( x = -4, y = 2 \)  \( \ldots \ldots \ldots . .22(\quad) \)

3. \( 3x + y = 0 \) is the equation of line
   - 23-1 AA
   - 23-2 BB
   - 23-3 CC
   - 23-4 DD
   - 23-5 EE  \( \ldots \ldots \ldots . .23(\quad) \)

4. \( x - y = -4 \) is the equation of line
   - 24-1 AA
   - 24-2 BB
   - 24-3 CC
   - 24-4 DD
   - 24-5 EE  \( \ldots \ldots \ldots . .24(\quad) \)

5. \( x - 3y = -10 \) is the equation of line
   - 25-1 AA
   - 25-2 BB
   - 25-3 CC
   - 25-4 DD
   - 25-5 EE  \( \ldots \ldots \ldots . .25(\quad) \)

26. If \( x = \frac{n}{aB^2} \), then \( B \) equals
   - 26-1 \( \left( \frac{n}{ax} \right)^2 \)
   - 26-2 \( \pm \sqrt{\frac{1-a}{x}} \)
   - 26-3 \( \pm \sqrt{\frac{n}{ax}} \)
   - 26-4 \( \pm \sqrt{\frac{x-n}{a}} \)
   - 26-5 \( \pm \sqrt{\frac{ax}{n}} \)  \( \ldots \ldots \ldots . .26(\quad) \)

27. \( (-2\sqrt{5})^2 \) equals
   - 27-1 20
   - 27-2 -100
   - 27-3 50
   - 27-4 -20
   - 27-5 100  \( \ldots \ldots \ldots . .27(\quad) \)

28. If \( \frac{1}{A} = m + \frac{2}{t} \), then \( A \) equals
   - 28-1 \( \frac{m + 2}{t} \)
   - 28-2 \( \frac{1}{m} + \frac{t}{2} \)
   - 28-3 \( \frac{t}{mt + 2} \)
   - 28-4 \( \frac{t}{m} + \frac{2}{t} \)
   - 28-5 \( \frac{mt + 2}{t} \)  \( \ldots \ldots \ldots . .28(\quad) \)

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Amount to be subtracted

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Number right

Subtract
(See table above)

Raw Score = Difference

Scaled Score
(See table on key)
APPROVAL SHEET

The thesis submitted by James J. McWilliam, S.J. has been read and approved by three members of the Department of Education.

The final copies have been examined by the director of the thesis and the signature which appears below verifies the fact that any necessary changes have been incorporated, and that the thesis is now given final approval with reference to content, form, and mechanical accuracy.

The thesis is therefore accepted in partial fulfillment of the requirements for the Degree of Master of Arts.

4-21-48
Date

Signature of Adviser