A Linear Cephalometric Analysis: Its Description and Application in Assessing Changes in the Maxilla After Orthodontic Treatment

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A LINEAR CEPHALOMETRIC ANALYSIS:
ITS DESCRIPTION AND APPLICATION IN ASSESSING
CHANGES IN THE MAXILLA AFTER ORTHODONTIC TREATMENT

BY

F. PETER WALL

A Thesis Submitted to the Faculty of the Graduate School
of Loyola University in Partial Fulfillment of
the Requirements for the Degree of
Master of Science

JUNE
1965
Francis Peter Wall was born in Chicago, Illinois on January 5, 1936.

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ACKNOWLEDGEMENTS

My sincere appreciation is extended to all those who have aided in making this investigation possible, particularly to the following:

To Joseph R. Jarabak, D.D.S., Ph.D., Professor of Orthodontics, Loyola University, whose devotion to orthodontics and to the teaching profession was an inspiration, and whose knowledge, wisdom and personal words of encouragement made this work possible.

To James A. Fizzell, B.S., in E.E., Consultant for the Department of Orthodontics, Loyola University, whose understanding and guidance were invaluable in the formation and design of this research and for his many hours of patience in teaching this student.

To John J. Flanagan, M.S., Ph.D., Assistant Professor of Psychology, Loyola University, for his technical advice and the statistical discipline used in this study.

To J. Keith Grimson, D.D.S., my colleague and friend, without whose help this work could not have been completed.

To my classmates Drs. Gene Dongieux, Bud Ellis, Edgar Geigel, Gerald Jeffry, Dale Kostiwa and Robert Lokar, for their assistance throughout the past two years.
To Jerry Lerch, D.D.S. and Harold Arai, D.D.S. for their assistance in preparing the graphic illustrations.

To Misses Barbara Furmaniak, Evelyn Furmaniak, Irene Clancy and Frances Grohwin, for their help in typing this thesis.

To my loving Mother and Father in whose image I will aspire to live all my days.
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earliest classifications. He devised a method of orienting the dentition to anthropometric landmarks through the use of profile facial photographs and plaster study models. His recognition of the interdependence of teeth and occlusion, jaw relationships, craniofacial morphology and their effect on the ultimate concepts of occlusion formed the basis of a new study known as gnathostatics.

The first known attempt to study the relation of dental and facial structures from standard lateral head roentgenograms was made by Carrea in 1924. He used lead wire, stuck with adhesive tape to the face, to bring out the soft tissue profile. Carrea, emphasized the usefulness of such radiographs or "teleradiofacies" in classifying dental occlusion, especially in the antero-posterior direction and in distinguishing false prognathism from true prognathism.

Later Pacini (1926) perfected this roentgenographic technique by offering a method of correcting the enlargement of the image on the radiograph to its natural dimensions. It was not until 1931 that use of the lateral head roentgenograph was fully developed. During 1931, Broadbent and Hofrath independently published their techniques of taking standardized head radiographs. Both men used a head
positioning apparatus, cephalostat, and a fixed target to film distance. With this method, it was possible, for the first time, to duplicate head x-rays of the same person in both exactness of size and detail. This enabled the investigator to follow longitudinally the developmental pattern and the intricacies of tooth formation, eruption and adjustment. This new method of analysis was called cephalometric analysis.

The science of cephalometrics has developed in two phases. The first phase, the study of growth and development of the craniofacial complex, provided data for the scientific quantitation and objective orthodontic study of craniofacial growth. This phase was most strongly influenced by the research work of such men as Todd, Broadbent, Hofrath, and Brodie.

The second period in the development of the science of cephalometrics was the application of cephalometric techniques to clinical orthodontics. Downs (1948) was the first to select a series of cephalometric skeletodental landmarks for use in clinical diagnostic assessment. These landmarks enabled him to predict more closely the results that a given treatment would bring. After the work of Downs many systems of analysis were advanced using the Downs analysis as a working base.
Most of the cephalometric analyses currently used in orthodontics assess the relationship of dental and facial landmarks to each other and to the cranium through the use of angular measurements. A number of cephalometric analyses have been developed which employ both angular and linear measurements, but it was not until Wylie's study that a method of roentgenographic headplate analysis was proposed in which linear measurements were the sole criteria of assessment.

B. Review of Early Cephalometric Assessment Methods:

Broadbent (1931) opened new horizons in the field of orthodontic research with the development of his precision instrument, the cephalostat. By means of this head-positioning device he was able to study the growth of the dental and facial developmental pattern through the use of standardized roentgenograms taken along the same parallel lines at different time intervals.

Broadbent (1937) was of the opinion that landmarks in the cranial base were more stable and fixed than those in the growing face. Recognizing the fallacy of employing unstable landmarks in anthropometric technique intended to show dentofacial changes, Broadbent
took advantage of the fact that relatively little increase in growth occurs at the base of the cranium. He established the Bolton-nasion plane and a point midway on a perpendicular from this plane to the center of the sella turcica that was designated as "R", the registration point. He felt that these points, located within the cranium, were the "most fixed in the whole head". This correlation of common landmarks outside of the face permitted exact measurement of changes in the face itself in relation to these landmarks. For this reason he used the Bolton-nasion plane for orientation and superposition of x-ray tracings to study growth changes.

In order to facilitate measurement of facial and dental changes, Broadbent added the Frankfort horizontal plane (a line connecting the inferior border of the orbit and the superior border of the tragus) and then added a perpendicular to this plane at orbitale. The Frankfort plane and the perpendicular dropped at orbitale form a definite angle with the Bolton-nasion plane. The exact measurement of this angle was maintained in later studies. Subsequent measurements of dental changes in relation to the Frankfort horizontal and to the perpendicular at orbitale were, therefore, in fixed relation to the relatively more fixed Bolton-nasion plane.
Brodie (1941) measured growth of the face by means of cephalometric roentgenograms as used by Broadbent. He attempted to complement Broadbent's work by taking the various components of the human head and studying each as a separate entity. The areas of the head studied by Brodie were: the brain case, the nasal area, the maxillary and mandibular areas and the occlusal plane. By using angles and angular relationships in place of linear measurements, he found it possible to eliminate differences due to size and to obtain more accurate comparisons of growth.

He divided the brain case from the face by means of two straight lines which meet at sella turcica. A line from sella to nasion designates the anterior cranial base, and a line from sella to Bolton point the posterior cranial base. In a later study using composite tracings of the entire facial skeleton, Brodie used the sella-nasion plane or anterior cranial base plane as the plane of orientation for successive cephalometric radiographs. Brodie contended that Bolton's point became increasingly more difficult to decipher on the lateral headplate as the age of the patient increased and it was for this reason that he discarded the Bolton-nasion plane in favor of the more accurately determined sella-nasion plane.
Downs (1948) introduced a method of recording the skeletal and denture pattern by which the facial form could be measured. His study was prompted by the controversy that prevailed at that time in orthodontics and in the literature, concerning the extraction of dental units as a necessary adjunct in the treatment of some malocclusions. In his investigation he undertook "to determine the range of facial and dental pattern within which one might expect to find the normal, and further to discover whether any usable correlations existed in such normals."

In this study he used roentgenographic cephalometry. His material consisted of twenty boys and girls each possessing a clinically excellent occlusion. Tracings were made of all lateral head x-rays taken with the teeth in occlusion and the Bolton triangle was outlined on each tracing. Since this triangle represented the area at the base of the cranium to which the face is joined, Downs accordingly divided the head into the cranium and face. Further, he fractionated the face into upper face, teeth and alveolar area, and lower face or mandible. The first objective of Downs' study was to appraise the pattern of the facial skeleton exclusive of the teeth and alveolar process. His second objective was to determine the
relationship of the teeth and alveolar process to the facial skeleton.

Downs' fractionation of the cranio-facial complex made available for study certain areas which were of clinical interest. These were facial plane, nasion to pogonion; the angle of convexity, nasion-subspinale-pogonion; the A-B plane, subspinale-supramentale; the mandibular plane angle and the "Y" axis. He used the facial angle to assess the degree of recession or protrusion of the chin. The angle of convexity was used to determine the protrusion of the maxillary part of the face to the total profile. The A-B (A maxillary denture base, B mandibular denture base) relation to the facial plane measured the anterior limit of the denture bases to each other and to the profile. Using the mandibular plane angle to the lower border of the mandible and the Frankfort horizontal, he constructed another plane, the "Y" axis. The "Y" axis was drawn through sella and gnathion. He found that as the facial angle decreases the mandibular plane tends to increase. The angular relation of the "Y" axis to the Frankfort plane expressed the changes in the facial skeleton as the face grew in a downward and forward direction.

Downs also used five other measurements in analyzing the relationship of the denture to the skeletal pattern. They were: the
cant of the occlusal plane, axial inclination of mandibular incisors to the mandibular plane, axial inclination of mandibular incisors to the occlusal plane and the protrusion of maxillary incisors.

Downs placed emphasis on the fact that there are variations in the relationship of the denture to the skeletal pattern in individuals with excellent occlusions. Still in spite of this he feels the norms he suggests can be of great value in locating areas of disharmony in malocclusions. The work of Downs makes it possible for the present day orthodontist to have a better comprehension of where the teeth should be in regards to the surrounding cranio-facial structures.

Downs emphasized the importance of using the analysis in its entirety rather than depending upon just a few of the ten measurements.

Margolis (1943) described a maxillo-facial triangle made up of a cranial base line (from nasion through the cranial edge of the spheno-occipital synchondrosis), the mandibular plane drawn through the lower border of the mandible and the facial line drawn from nasion to pogonion. This facial triangle was constructed in order to appraise the proportional development of the lower third of the face and to describe a basic pattern in the balanced, well-developed face.
The use of this method of analysis also enabled the operator to orient the dentition within the face and to note growth subsequent to orthodontic treatment. The landmarks employed in the facial triangle were at easily discernible points on the profile roentgenogram. The interdependence of the size of the angles in the triangle made it a valuable aid in dentofacial studies, since it revealed the relative difference in size and relationship of specific maxillofacial areas to each other.

Tweed (1944, 1954) proposed the Frankfort-Mandibular Incisal Angle Analysis. This analysis was based upon the relationship of the inclination of the mandibular incisors to the mandibular plane. He stated that in order for the mandibular incisor teeth to be stable, they must exhibit an axial inclination to the mandibular plane of 90⁺ or -5 degrees. Tweed chose the Frankfort plane as his registration line and in his later study he extended a line representing the axial positioning of the mandibular central incisor, up to this plane. The angle thus formed was found to have a mean average of 65 degrees which Tweed contended was ideal for stable treatment results.

Wylie (1947) devised a method of assessing antero-posterior dysplasia by an analysis of the degree to which certain facial disharmonies were compensated for or reinforced by variations
in the area of the affected parts. His study differed from those previously presented in that his assessment of craniofacial relations was made by means of linear rather than angular measurements. He took several measurements on the Frankfort plane. These were: the most posterior point on the mandibular condyle to sella, sella to pterygomaxillary fissure, pterygomaxillary fissure to the buccal groove of the maxillary first molar, pterygomaxillary fissure to anterior nasal spine and mandibular length. These landmarks were projected as perpendiculars to the Frankfort horizontal plane. According to Wylie any of the first four dimensions, if larger than their respective mean value will tend to make the face more orthognathic, if less, they tend to make the face more prognathic.

Steiner in 1953, published his cephalometric analysis about which he stated that "much of this assessment method is made up of ideas of others." It was his prime effort to produce an assessment of cephalometric headplates that would be of greatest value to the clinical orthodontist. Steiner felt that because of the difficulty of locating porion on the lateral x-ray the Frankfort plane was not an acceptable one for the orientation of successive headplates. He chose the sella-nasion plane because the points sella and nasion are
both located in the midsagittal plane and they are clearly visible on the x-ray.

To study the inclination and relation of maxillary and mandibular central incisors, he related them to lines nasion-subspinale and nasion-supramentale respectively. According to Steiner the maxillary central incisor should lie on the line nasion-subspinale in such a way that the most mesially placed point of its crown is four millimeters in front of the line and its axial inclination to the line is 72 degrees. The mandibular central incisor was determined to be properly positioned if its most labial aspect was four millimeters in front of the nasion-supramentale line and if its long axis to this line was 25 degrees.

Bjork (1955) investigated the effects of variations in posterior cranial anatomy on facial prognathism. He also studied the relationship between facial build and occlusion. He devised a facial diagram to determine facial prognathism by means of linear and angular measurements. The millimeter length of selected parts of the cranium and of the jaws and the size of the angles formed by these parts were assessed in relation to one another and as integral parts of the whole diagram.
Bjork’s facial diagram includes the following: A line drawn from the apex of the anterior nasal spine to nasion, to the center of sella-turcica, to articulare (the point of intersection of the dorsal contour of the articular process of the mandibular condyle and the temporal bone), to the point of intersection between lines tangent to the ramus and base of the mandible, to a point of intersection between lines tangent to the base of the mandible and pogonion, and from there to infradentale. The angles formed by the intersection of these lines are the saddle angle, the joint angle, the gonion angle and the chin angle. Bjork used the sella-nasion plane as a reference plane and he measured points on the facial profile to this plane. This enabled him to orient the facial profile in relation to the cranial base by using nasion as a constant point of measurement.

C. Review of Recent Cephalometric Assessment Methods:

In 1952, Craig proposed a method of assessing the magnitude of differences between the skeletal pattern of Class I and Class II, Division I malocclusions. He related his tracings to a grid system of horizontal and vertical lines by laying each tracing over a sheet
of millimeter graph paper on which the centimeter lines were accentuated. The graph paper was divided into four quadrants by two coordinate axes, one horizontal, and one vertical, which intersected each other in the middle of the page. All tracings were oriented with the center of sella-turcica superposed at the intersection of the two coordinates and with the Frankfort plane parallel to the horizontal axis. The location of any anatomical point could be recorded as its distance from each of the vertical and horizontal coordinates. Horizontal readings to the right of the vertical coordinate were indicated by a plus value, those to the left by a minus value. Similarly, vertical readings above the horizontal axis were plus, those below, minus.

Williams (1953) devised a method of analysis to ascertain statistically, what changes occur with age in certain cranio-facial proportions in the horizontal and vertical planes. He used the sella-nasion plane with a perpendicular drawn to this plane at sella as the means of superimposing his serial roentgenograms. He made tracings of the lateral headplates and oriented these tracings on a millimeter graph paper. The distances in millimeters of the selected landmarks were measured from the horizontal axis of
reference, which he called "X", and from the vertical axis, which he called "Y". The mean relative distance of each point along the "X" and "Y" axis was determined for each subject both before and after the onset of puberty.

Altemus (1955) presented a study of dentofacial relationships in normal and Class II Division I malocclusions. He selected landmarks on the mandible and in the upper face and made measurements of these points in two planes of space, the horizontal and the vertical. Analysis for anteroposterior dysplasia of the upper face was made according to the technique described by Wylie. However, Altemus substituted articulare and Downs' "A" point for the points of glenoid fossa and anterior nasal spine. Fifteen linear and four angular measurements were used to assess mandibular morphology. He used the Frankfort horizontal plane as a reference line and he dropped perpendicu- lars from this plane to the selected landmarks in the mandible.

Coben (1955) attempted to appraise the face against a right angle coordinate system using effective rather than absolute dimensions. To correlate facial form and the postural position of the head, the Frankfort horizontal plane was employed as a plane of orientation.
The Frankfort plane represented the horizontal plane of reference which Coben called the abcissa. Lines perpendicular to this plane were called ordinates. Depth of the cranial base was analyzed and recorded as the horizontal length between points basion and nasion. To evaluate middle face prognathism, the depth of the middle face (basion to point "A") and its contributing segments (basion to sella, sella to PTM and PTM to point "A") were expressed as a percentage of the base depth, basion to nasion. The vertical height of the face was analyzed by measurements taken along ordinates drawn to the abscissa. All vertical measurements were expressed as a percentage of the anterior face height, nasion-menton. An index of the relationship of face height to face depth was then established by expressing anterior face height as a percentage of the cranial base depth.

Elsasser (1957) proposed a coordinate system of analysis employing linear measurements. He used the Frankfort plane as the horizontal plane of reference and a perpendicular drawn to this plane at a point twenty millimeters forward of nasion as the vertical plane of reference. The various landmarks were located on the headplate by placing a small hole through the x-ray at the appropriate point. The points were transferred to transparent coordinate graph
paper by placing the graph paper over an illuminated headplate and placing a mark at the spot where a pinpoint of light showed through. The horizontal and vertical distances from these points to the reference axes were measured and recorded. Three angular measurements were obtained from linear measurements and these angles were used to determine the relationship of the upper incisor to the Frankfurt plane, the relationship of the lower incisor to the Frankfurt plane, and the relationship of the Frankfurt plane to the mandibular plane.

Kean (1958) did a cephalometric study to determine the variations in facial depth relative to the type of occlusion. He used a coordinate system in the analysis of two groups of children, one with normal occlusion and the other with Class II, Division I malocclusion. Measurements of facial height were measured to the Frankfurt horizontal plane and those of facial depth were measured to a perpendicular from this plane to basion. Kean used a specially constructed glass scale to read the linear measurements from each tracing directly in millimeters.

In order to construct this scale, millimeter graph paper with accentuated centimeter lines was drymounted on a piece of thick
carboard which in turn was fixed to a bench. Fine stainless steel wires were stretched along the horizontal and vertical centimeter lines and kept in place with scotch tape. The resulting wire mesh was placed in the midline of the Margolis cephalostat, corresponding to the position of the mid-sagittal plane of the head, and radiographed. On the radiograph of enlarged centimeter squares, a standard series of squares having ten divisions per side was "fitted" by image projection. By removing the radiograph and in its place exposing a photographic glass plate to the fitted image, a permanent scale duplicating the enlarged millimeter squares was obtained. Thus, a millimeter on the scale was enlarged to the same extent as a given distance in the midsagittal plane is enlarged on a radiographic film since the midsagittal plane-film distance is constant. Any distance between two landmarks in the midsagittal plane of the subject could now be read directly from a tracing placed on the glass scale.

Merow (1962) proposed a method of analysis to determine, statistically, what changes occur with age in certain dento-facial proportions in both horizontal and vertical directions. His principal interest was to assess the amount of growth which occurred during the pubertal or "growth spurt" period. Merow oriented his before-
treatment and after-treatment tracings by using the Frankfort horizontal plane. He also employed the Bolton triangle in his analysis and point "R" was used at the base-point to which all measurements were related. The measurements were made on a sheet of millimeter graph paper which he divided into four quadrants by two axes - one horizontal which was called "X", and one vertical which was called "Y". Each anatomical point was then established in relation to the "X" and "Y" axes and recorded as horizontal and vertical distances from the respective axes.

The mean measurements were reduced to percentage as proportions of a base line. The distance along the "X" axis from "R" to "N" was used at the 100% base for the calculation of the horizontal proportions, and the vertical distance of "Gn" from the "X" axis used at the 100% base for the vertical proportions. By dividing the distance to each point into the length of the relating base line, the proportion that each point contributed in height and depth could be calculated. This was expressed as a percentage of the base line. By comparing the percentages of prepubertal and postpubertal proportions, any change in the proportion of a given area could be ascertained.
In 1963, Harris, Johnston, and Moyers undertook a study to determine the statistical parameters of craniofacial growth of an average population of children. This was a cephalometric study using a mixed longitudinal and cross-sectional series of roentgenograms. The landmarks used were identical to the set of cranial facial constructions used by the University of Toronto in the study of a population at Burlington, Ontario. The measurements taken from the headplates, called the Burlington measurements, provided the raw material from which a template was constructed. These investigators used the Bolton-nasion plane for orienting the tracings and extended the line representing the mean occlusal plane to this plane. Perpendicular lines were drawn to the Bolton-nasion plane and to the occlusal plane and measurements were taken by means of a vernier caliper. Means were then obtained for all measurements in each of the age-sex groupings and standard deviations from the means were calculated. The direction of growth was obtained for the points measured by fitting a line to the mean points of each age-sex grouping by means of a regression analysis. Rectangles were constructed around each mean point on the regression line. These rectangles represented the area in which a certain landmark would be found at a given age. A
vinyl plastic template was made to represent the geometric construction of selected statistical measurements applied to a selected sample. The template was a means of obtaining a rapid cephalometric analysis of an individual against a background of like subjects. This template was not used to diagnose the existence of a malocclusion, but rather to indicate the degree of harmony or disharmony of the craniofacial complex.

Wallis (1963) sought to determine if, within a framework of individual variation; Class II, Division II malocclusions represented a significantly distinct population. The technique of roentgenographic cephalometry used was that described by Broadbent and the method of analysis was the horizontal and vertical coordinate system devised by Coben. Twenty-seven measurements from each tracing were taken and their means and standard deviations were determined.

Wieslander (1963) investigated the influence of headgear treatment on the concurrent development of the craniofacial complex. By the use of cephalometric roentgenograms he attempted to evaluate the effect of occipital anchorage upon the dentofacial area in the mixed dentition period. The serial lateral headplates were oriented by superimposing on the spheno-ethmoidal plane. The Frankfort
horizontal plane was the horizontal plane of reference. The midline point of the two great wings of the sphenoid bone as it intersects the spheno-ethmoidal plane was selected as the registration point. A perpendicular drawn from this point through the Frankfort plane was the vertical plane of reference. Linear changes of horizontal and vertical measurements of certain anatomic landmarks were recorded in relation to this grid system and the data obtained was subjected to statistical analysis.

Huggins and Birch (1964) developed a linear cephalometric analysis to determine the stability of treatment results after retraction of procumbent maxillary incisor teeth. A transparent plastic measuring grid, scribed with one millimeter squares, was used to orient and measure the position of the upper incisors relative to the fixed adjacent anatomic plane (anterior nasal spine - posterior nasal spine). Changes in upper incisor position were recorded in both vertical and horizontal planes by direct measurements taken on the headplate. The horizontal plane of reference in this study was the palatal plane extending from anterior nasal spine to posterior nasal spine. The anterior nasal spine was selected as the registration point relative to which the upper incisor profile was determined.
Cephalometrics has evolved from the science of anthropometrics. This is evidenced by the many points, lines, and planes used in cephalometrics today which are the same as those used by anthropologists in their early studies on measurements of the dry skull.

Although primarily introduced to the specialty of orthodontics as a most important aid in research problems, cephalometrics has now become of great value in case analysis, treatment planning, and prognostic predictions. The use of lateral head radiographs enables the orthodontist to clearly visualize the bony framework of the facial structures. By assimilating the information derived from the roentgenogram with that received from other diagnostic aids the orthodontist is now able to analyze the facial pattern of each patient before inaugurating treatment procedures. Cephalometrics reveals handicaps that predict limitation of desirable results in treatment and it offers a scientific means of analyzing the results after treatment.

The method of taking serial roentgenograms used today is essentially the same as that proposed by Broadbent more than thirty years ago. The analysis of craniofacial relations developed by Broadbent has been employed in its entirety by some of the succeeding investigators while others have merely used his work
as a starting point. The major variation in all of the research in cephalometrics is the choice of a line or plane for the superimposition of serial tracings. The Frankfort horizontal, the Bolton-nasion plane, the sella-nasion plane, and the spheno-occipital synchondrosis-nasion plane are some of the planes recommended by the various investigators as planes of reference. Each investigator had valid reasons for selecting his particular plane instead of those chosen by others.

In this study the sella-nasion plane will be used as the plane of reference and sella will be the point of registration. The sella nasion plane was chosen because it was felt that this plane, representing the anterior cranial base, was the most stable plane in the head throughout growth. Another reason for selecting this plane is that the points sella and nasion are readily discernible on the lateral headplate.

A brief summary of some of the early works in cephalometrics was presented in the first part of the review. The initial studies of Broadbent and Brodie provided methods of analysis of cranio-facial morphology which served as models on which others were to improve. Perhaps the most significant improvements in cephalometric analysis were devised by Downs. This is substantiated by the fact that a
number of the later analyses incorporated points and angles proposed by Downs. The angular methods of analysis employed by these researchers in cephalometrics are the most popular methods used today by the clinician.

Within the past decade there has been considerable interest and study in the linear approach to cephalometric analysis. The advocates of linear measurements, as opposed to angular ones, are of the opinion that the former is a more accurate means of assessment. They contend that the coordinate system of analysis is less complicated for the clinician to use and that it is also more readily understood by the layman.

It is the opinion of this author that the pertinent landmarks in this research will be more accurately and meaningfully assessed by the linear method of measurement.

The method of evaluation in this study will be strictly linear. Changes in the maxilla occurring during treatment and in subsequent growth will be assessed on twenty-six Class I (Angle) four first premolar extraction malocclusions by measuring the distance from selected points to previously established reasonably stable reference axes within a system of rectangular coordinates. This method of
assessment is unique from those previously reviewed in that it will show changes in horizontal and vertical direction, as well as, changes in angles and intersection points.

The method of analysis employed in this study is the result of the combined efforts of this author and co-worker J. Keith Grimson.

D. Statement of the Problem:

The purpose of this thesis is twofold:

To devise and demonstrate the versatility of a linear system of measurements using analytic functions in rectangular coordinates.

To utilize this system of measurements in assessing the changes occurring in the morphology of the maxilla and in the positions of associated dental structures during the treatment of twenty-six Class I (Angle) first premolar extraction malocclusions.
CHAPTER II
MATERIALS AND METHODS

A. The Rectangular Coordinate System:

In this thesis, and in an associated thesis written by J. Keith Grimson, the use of a rectangular coordinate system for the evaluation of a cephalometric roentgenogram will be introduced. The rectangular coordinate system lends itself ideally to the use of a data reduction process performed by electronic computers. It is hoped that the coordinate method of cephalometric assessment and the application of computer analysis will someday be developed to the point of refinement whereby this combination can be employed in diagnosis and treatment planning and finally in assessing growth and treatment.

There is a theorem in mathematics which states that, "two points related to the same point are related to each other". This principle has been used in this study to locate related landmarks on lateral cephalometric headplates. Sella-turcica was selected as the point of origin to which all other points were to be related. Orthogonal reference axes were constructed to sella and all points were measured to these axes. A line from sella to nasion represented
the horizontal reference axis and a perpendicular to this line at sella represented the vertical reference axis. This system of measurement is known as a rectangular coordinate system or Cartesian coordinates.

It has been said that modern mathematics began with the development of analytic geometry. Analytic geometry is considered a separate branch of geometry because it uses algebraic methods and equations to study geometric problems. Although there are numerous instances of earlier uses of the techniques of analytic geometry, the first mathematicians to make systematic use of algebraic methods in geometry were Rene Descartes (1596-1650) and Pierre de Fermat (1601-1665).

The most basic concept of analytic geometry is the representation of geometric points by the use of numbers. A point in a plane can be fully determined if the distances from two given straight lines in the same plane are known. These mathematical concepts are the basis for the coordinate system of measurements to be used in this research.

In order to create a rectangular coordinate system, two mutually perpendicular lines (X'X) and (Y'Y) must be drawn intersecting at 0 (Figure 1). These reference lines, called axes, divide the plane into
FIGURE 1

THE RECTANGULAR COORDINATE SYSTEM
four parts, or quadrants. The quadrants are number I, II, III, and IV, starting with the upper righthand quadrant and reading counter-clockwise. The point of intersection of the axes is called the origin. The line $X'X$ is called the axis of abscissas, or x-axis, and is generally taken to be the horizontal. The line $Y'Y$ is called the axis of ordinates, or y-axis, and is generally taken to be vertical. All distances measured horizontally to the right or vertically upward are considered positive, and all distances measured horizontally to the left or vertically downward are considered negative. Positive distances are represented by positive numbers, and negative distances are represented by negative numbers. A landmark is a defined anatomical point which is represented in this study by a point on a drawing paper. Every landmark is located by two numbers which represent the distances of the landmarks from the axis of ordinates and the axis of abscissas. These two numbers are called the coordinates of the point. In analytic geometry, these two numbers are written in pairs separated by a comma and are enclosed in parentheses, as $(7, -5)$. The first number is called the abscissa and the second number is called the ordinate. Points of the plane located by these ordered pairs, have been related to a rectangular frame of reference and the numbers $X$ and $Y$ for
the point P are termed its coordinates relative to this reference frame. In Figure 1 three points in relation to their plane of reference have been located.

B. Selections of the Sample:

The material used in this study comprised pre- and post-treatment lateral cephalometric radiographs of twenty-six orthodontically treated children. The patients, seventeen female and nine male, were all treated in the private practice of Dr. Joseph R. Jarabak, Hammond, Indiana in accordance with the principles of the Jarabak "Light Differential Force Technique." The data used in this study were obtained from lateral cephalometric radiographs taken at the initiation of orthodontic treatment and upon completion of therapy. All malocclusions were clinically assessed as being Class I according to Angle’s classification; all required the extraction of four first premolar teeth. The selection of the sample followed a careful assessment of both models and lateral headplates of fifty treated patients. After evaluating the uniformity of the headfilms, the sample size was refined to a homogeneous group of twenty-six cases. Such criteria as density of the images, presence of large amalgam restorations in the posterior
teeth, and absence of third molars were used in selecting the cases to be studied. The distribution of the subjects by age at the initiation of treatment and at the completion of treatment is given in Tables I and II. The mean treatment time is found in Table III.

C. Roentgenographic Technique:

The lateral headplate radiographs used in this study were taken on a standard cephalometric machine (Figure 2). The tube housing of this machine incorporated a fixed anode and a high voltage generator providing a machine setting of 90 KVP and 45 m. amps. The exposure time for all films was 3/4 seconds.

The cephalostat (Figure 3) provided a fixed distance of forty-eight inches from the focal point of the roentgen tube to the midsagittal plane. The use of a head-holder was necessary in order to prevent distortion errors due to movements of the patient's head, and to allow the operator to replace the patient into the head-holder in the same spatial relation to the roentgen-ray tube each successive time a film was exposed. The mid-sagittal plane to film distance was kept at a constant dimension of fifteen centimeters.

Each subject was oriented in the cephalostat in
TABLE I

DISTRIBUTION OF THE SUBJECTS BY AGE
(PRETREATMENT)

<table>
<thead>
<tr>
<th>AGE (YEARS)</th>
<th>TOTAL</th>
</tr>
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<tbody>
<tr>
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<td>2</td>
</tr>
<tr>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>12</td>
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</tr>
<tr>
<td>15</td>
<td>2</td>
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</table>

MEAN 11.08     TOTAL 26
TABLE II
DISTRIBUTION OF THE SUBJECTS BY AGE
(POSTTREATMENT)

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<th>AGE (YEARS)</th>
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</thead>
<tbody>
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<tr>
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<tr>
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<td>5</td>
</tr>
<tr>
<td>16</td>
<td>0</td>
</tr>
<tr>
<td>17</td>
<td>2</td>
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</table>

MEAN 13.57     TOTAL 26
TABLE III

TREATMENT TIME

<table>
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<th>TOTAL</th>
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</thead>
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<td>1</td>
</tr>
<tr>
<td>1.0</td>
<td>4</td>
</tr>
<tr>
<td>1.5</td>
<td>11</td>
</tr>
<tr>
<td>2.0</td>
<td>5</td>
</tr>
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</tr>
<tr>
<td>3.0</td>
<td>1</td>
</tr>
<tr>
<td>3.5</td>
<td>1</td>
</tr>
</tbody>
</table>

MEAN 1.21  TOTAL 26
FIGURE 2

THE CEPHALOMETRIC APPARATUS
FIGURE 3.
THE CEPHALOSTAT
described by Broadbent. The ear rods and an orbitale marker were used to orient the head on Frankfort Horizontal plane. All patients were instructed to maintain their teeth in full occlusion and to avoid any movement during the exposure of the film.

The radiographic film used in this study was 8 x 10 inch high speed, blue brand, Kodak Medical X-Ray Film. Each cassette was equipped with double high speed intensifying screens to eliminate some secondary radiation and to provide greater contrast. The standard development procedure following the usual time-temperature method recommended by Eastman Kodak Company was adhered to for all films.

D. Spotting Technique:

Selected landmarks were spotted on each of the pretreatment and posttreatment lateral cephalometric headplates used in this study. The spotting was accomplished by piercing a small hole through the headplate where the landmark was determined to be. A definite sequence was followed for all spotting procedures.

All work was performed in a darkened room and the spotting session lasted no longer than thirty minutes. At the end of this
time the operator left the darkened room and entered an adequately lighted room for a period no shorter than fifteen minutes, after which time work was resumed. These precautions were taken in order to prevent eye strain and to reduce the amount of error due to eye fatigue.

The actual procedure was to place a sheet of cellulose acetate .002 of an inch thick on a transilluminated tracing table and then place the headplate to be spotted on top of the acetate. The acetate was used to provide a uniform thickness between the headplate and the plastic surface of the tracing table, so as to protect the surface and to make certain that the pinhole perforations through the film were the same diameter. The perforations were made by placing a phonograph needle test probe on the selected landmark and forcing the point of the probe through the film into the underlying cellulose acetate. Phonograph needles were replaced after spotting ten head-films in order to have a consistently sharp point with which to make uniformly small perforations.

To eliminate errors between investigators in locating Sella and Nasion, the points orienting the horizontal reference axis, one individual located these points on all lateral headfilms after which
their locations were corroborated by two-out-of-three vote basis with the original spotter and the two corroborators participating.

To locate the spotted points in Cartesian coordinates it was necessary to establish a rectangular grid system. This was provided by a coordinate paper laid off in major divisions of centimeters and minor divisions of millimeters. Drawing paper fulfilling these requirements was obtained from Keuffel and Esser Company (346-1510). These sheets were 8 1/2 x 11 inches in size and provided one major centimeter line for every ten minor millimeter lines.

Each spotted lateral headfilm was again returned to the tracing table, this time with a sheet of the drawing paper placed between the headplate and the cellulose acetate. Under the same conditions used in the spotting procedure, the graph paper was carefully oriented so that the point identifying Sella was located at the intersection of two major centimeter lines. For this study, Sella was located at the intersection of the tenth major vertical line from the left hand (8 1/2 inch) side of the graph sheet with the third major horizontal line from the top (11 inch) edge of the sheet (Figure 4). This provided a uniformity in the location of Sella on all sheets. When Sella was placed at its correct intersection, the graph sheet was rotated so
FIGURE 4

SLIDE RULE AND SPOTTED DRAWING PAPER
that Nasion was located on the same horizontal major centimeter line (third from the top). With these two points correctly located on the grid, the film and graph sheet were fixed together with "Scotch" tape. The phonograph needle test probe was again used to reenter the perforations in the film and to puncture the drawing paper at the precise location of each landmark. The same conditions and procedures were followed in marking the graph sheets as were used in the original spotting operation. After each graph sheet was perforated, the film was removed and each pinhole was carefully circled with a sharp pencil in order to easily locate all points. The axes of reference were marked for easy identification by placing pencil lines in segments in which no measurements from the axes to the landmarks would be made.

E. Description of Selected Landmarks:

The landmarks used in this study were selected because of their clinical interest and because of the ease and accuracy with which they were located and spotted (Figure 5). When possible, midsagittal plane structures were used. All points listed below were located by inspection except "A" point (Jarabak) and Posterior Bony Palate whose
FIGURE 5

SELECTED LANDMARKS
locating necessitated construction lines.

1. **Sella Turcica** - The geometric center of the pituitary fossa of the sphenoid bone.

2. **Nasion** - The middle point of the frontonasal suture.

3. **Anterior Nasal Spine (ANS)** - The most anterior point on the hard palate.

4. **Posterior Nasal Spine (PNS)** - The most posterior point on the hard palate.

5. **"A" Point (Jarabak)** - A point measured 2 mm. anterior to the intersection of a line drawn from the apex of the maxillary central incisor perpendicular to the line tangent to the root and parallel to the long axis of the tooth.

6. **Tip of Crown** (TC) - The incisal point of the most prominent maxillary central incisor.

7. **Apex of** (TR) - The apex of the root of the most prominent maxillary central incisor.

8. **Crown of** (C) - The most mesial-occlusal point of the maxillary left first molar.

9. **Root of** (R) - The apex of the mesial root of maxillary left first molar.
10. Crown of \(_8 (C_8)\) - The most mesial-occlusal point of the maxillary left third molar.

11. Pterygomaxillary Fissure (PTM) - The most inferior point of the pterygomaxillary fissure.

12. Posterior Bony Palate (PBP) - The intersection of the lingual cortical plate of the hard palate with the line, passing through "A" point (Jarabak), at an angle of fifteen degrees to the Sella-Nasion line.

F. Measuring Technique:

Preparatory to the measuring of points on the drawing paper, data sheets were designed so information could be recorded in tabular form (Table IV). A list of the spotted landmarks was entered in the left hand vertical column of each data sheet. Two sets of paired columns were oriented in vertical fashion to the right of the column of landmarks. These were labeled I and II for pretreatment and post-treatment readings respectively. Each of these was divided into an X and Y column to represent the abscissa and ordinate of each point. Since all vertical readings were below the axis of abscissas, all would be negative values. To avoid entering each vertical measurement
DATA SHEET

NAME: PATIENT #1

ALL MEASUREMENTS ARE IN MILLIMETERS

<table>
<thead>
<tr>
<th>NAME OF POINT</th>
<th>X</th>
<th>Y</th>
<th>X</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;S&quot; SELLA</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>&quot;N&quot; NATION</td>
<td>73.0</td>
<td>0</td>
<td>73.2</td>
<td>0</td>
</tr>
<tr>
<td>ANS</td>
<td>62.4</td>
<td>-47.0</td>
<td>60.8</td>
<td>-46.8</td>
</tr>
<tr>
<td>PNS</td>
<td>16.9</td>
<td>-43.9</td>
<td>17.5</td>
<td>-44.0</td>
</tr>
<tr>
<td>&quot;A&quot;(JARABAK)</td>
<td>59.2</td>
<td>-50.5</td>
<td>58.1</td>
<td>-50.2</td>
</tr>
<tr>
<td>TC1 CROWN</td>
<td>62.7</td>
<td>-76.0</td>
<td>59.1</td>
<td>-77.3</td>
</tr>
<tr>
<td>TR1 ROOT</td>
<td>54.6</td>
<td>-51.3</td>
<td>55.0</td>
<td>-51.0</td>
</tr>
<tr>
<td>C6 CROWN</td>
<td>33.3</td>
<td>-64.1</td>
<td>37.1</td>
<td>-66.0</td>
</tr>
<tr>
<td>R6 ROOT</td>
<td>38.9</td>
<td>-47.3</td>
<td>41.9</td>
<td>-47.7</td>
</tr>
<tr>
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<td>PTM</td>
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<td>PBP</td>
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<td>-53.4</td>
<td>47.2</td>
<td>-53.2</td>
</tr>
</tbody>
</table>

NOTE: FOR ANALYTICAL WORK, ALL VERTICAL DIMENSIONS MUST BE CONSIDERED NEGATIVE.

TABLE IV
as negative, all were entered positive for this procedure and a note regarding this fact was made at the bottom of each data sheet.

A vernier caliper measuring to 0.1 millimeter was employed in obtaining the readings for each point on the graph sheets (Figure 6). A systematic procedure of measuring was achieved by first reading all horizontal measurements and then rotating the graph sheet ninety degrees to read all vertical measurements. Great care was taken so that the beaks of the caliper were accurately located over each pinhole and were correctly oriented with respect to the reference axis. The lined graph paper was essential for this procedure. As each reading was made on the vernier scale, it was recorded in the correct location on the data sheet by an assistant. This eliminated any fatigue resulting from alternatingly measuring and then recording the data for each point. Measuring procedures were conducted under the same conditions as the two spotting sessions.

G. Testing the Precision of Measurement:

In order to evaluate the ability of individuals to measure accurately with the vernier caliper, a study of the precision of measurement was undertaken. Six graduate students from the Orthodontic Department,
FIGURE 6

TEST PROBE, CALIPERS, AND DRAWING PAPER
Loyola University, School of Dentistry, Chicago, Illinois using the same vernier caliper measured five landmarks on each of two different patients. All landmarks were located and spotted on graph sheets in the manner previously described, by another graduate student prior to this experiment. Each individual made measurements in both horizontal and vertical directions for both the pretreatment and post-treatment lateral headplates of each patient.

The 120 horizontal and 96 vertical measurements were analyzed statistically by Fisher's analysis of variance (Tables V, VI). These analyses were of interest because they demonstrated the degree of precision which can be obtained using this type of measurement technique.

In looking at the analysis of variance tables it can be seen that there was large enough variation in the measurements so that all four of the main sources were significant contributors. The students did not make their measurements identically alike but their sum of squares was much smaller than any of the other sources. Although four of the interactions were statistically significant they are not essential to this study of precision.

There are seven interactions that contain nothing but estimates
### ANALYSIS OF VARIANCE

(HORIZONTAL MEASUREMENTS)

<table>
<thead>
<tr>
<th>SOURCES</th>
<th>D.F.</th>
<th>S.S.</th>
<th>M.S.</th>
<th>V. RATIO</th>
<th>SIGNIFICANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5% 2.71</td>
</tr>
<tr>
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<td>.60</td>
<td>.12</td>
<td>4.06</td>
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<tr>
<td>TREATMENTS</td>
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<td>59.50</td>
<td>2016.94</td>
<td>5% 2.48</td>
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<td>PATIENTS</td>
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<td>834.77</td>
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<td>.026</td>
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</tr>
<tr>
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<td>.30</td>
<td>.040</td>
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<td>T X L</td>
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<td>142635.68</td>
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</table>

STANDARD DEVIATION of ERROR = ± 0.172 mm. and the 99% CONFIDENCE LIMITS ARE = (2.63 x 0.172) = ± 0.4523 mm.

XXX = HIGHLY SIGNIFICANT VARIANCE RATIO

N.S. = NON-SIGNIFICANT VARIANCE RATIO

TABLE V
ANALYSIS OF VARIANCE

(VERITCAL MEASUREMENTS)

<table>
<thead>
<tr>
<th>SOURCES</th>
<th>D.F.</th>
<th>S.S.</th>
<th>M.S.</th>
<th>V. RATIO</th>
<th>SIGNIFICANCE</th>
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<td>241.12</td>
<td>241.12</td>
<td>6968.78</td>
<td>1% 4.10</td>
</tr>
<tr>
<td>LANDMARKS</td>
<td>4</td>
<td>116171.05</td>
<td>29042.76</td>
<td>839386.12</td>
<td>1% 6.96</td>
</tr>
<tr>
<td>PATIENTS</td>
<td>1</td>
<td>218.97</td>
<td>218.97</td>
<td>6326.58</td>
<td>1% 6.96</td>
</tr>
<tr>
<td>S X T</td>
<td>5</td>
<td>.05</td>
<td>.010</td>
<td></td>
<td>N.S.</td>
</tr>
<tr>
<td>S X L</td>
<td>20</td>
<td>1.12</td>
<td>.056</td>
<td></td>
<td>N.S.</td>
</tr>
<tr>
<td>S X P</td>
<td>5</td>
<td>.13</td>
<td>.026</td>
<td></td>
<td>N.S.</td>
</tr>
<tr>
<td>T X L</td>
<td>4</td>
<td>98.89</td>
<td>24.72</td>
<td>717.45</td>
<td>1% 3.56</td>
</tr>
<tr>
<td>T X P</td>
<td>1</td>
<td>45.51</td>
<td>45.51</td>
<td>1315.31</td>
<td>1% 6.96</td>
</tr>
<tr>
<td>L X P</td>
<td>4</td>
<td>310.15</td>
<td>77.54</td>
<td>2241.04</td>
<td>1% 3.56</td>
</tr>
<tr>
<td>S X T X L</td>
<td>20</td>
<td>.58</td>
<td>.029</td>
<td></td>
<td>N.S.</td>
</tr>
<tr>
<td>S X T X P</td>
<td>5</td>
<td>.06</td>
<td>.012</td>
<td></td>
<td>N.S.</td>
</tr>
<tr>
<td>T X L X P</td>
<td>4</td>
<td>26.36</td>
<td>6.59</td>
<td>190.46</td>
<td>1% 3.56</td>
</tr>
<tr>
<td>S X L X P</td>
<td>20</td>
<td>.35</td>
<td>.018</td>
<td></td>
<td>N.S.</td>
</tr>
<tr>
<td>S X T X L X P</td>
<td>20</td>
<td>1.00</td>
<td>.05</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

TOTAL  119  117116.60

STANDARD DEVIATION OF ERROR = \( \pm 0.186 \) mm. and the 99% CONFIDENCE LIMITS ARE = \( (2.63 \times 0.186) = \pm 0.490 \) mm.

XXX = HIGHLY SIGNIFICANT VARIANCE RATIO

N.S. = NON-SIGNIFICANT VARIANCE RATIO

TABLE VI
of experimental error. When the weighted mean of their sums of squares is taken, an error variance based on 95 degrees of freedom is obtained. The square root of this is the standard deviation of a distribution of experimental errors. The 99% confidence limits of this distribution as estimated from each of the tables are nearly plus or minus 1/2 mm. These are the limits of experimental error.

H. Maxillary Changes Selected for Study:

This study investigated four morphologic changes and four dental changes in the maxilla. These changes were determined by the use of analytic geometry and trigonometry. The mathematical procedures were performed with the use of a slide rule, mathematical tables and an electric calculator (see page 41). The data for each of the changes studied were recorded in tabular form. The equations written to determine the amount or degree of change were different for each case. Therefore, it is necessary to include a sample problem for each of the changes studied. A data sheet containing the figures which were used in the sample calculations is shown in Table IV (see page 46).
MORPHOLOGICAL STUDY

SAMPLE 1

CHANGES IN THE PALATAL PLANE ANGLE

For this and succeeding computations, "I" indicates the beginning headplate and "II" the finish headplate. All measurements are in millimeters.

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th></th>
<th>II</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>62.4</td>
<td>Y</td>
<td>-47.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>X</td>
<td>60.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Y</td>
<td>-46.8</td>
</tr>
<tr>
<td>ANS</td>
<td></td>
<td>PNS</td>
<td></td>
</tr>
<tr>
<td>16.9</td>
<td>-43.9</td>
<td>17.5</td>
<td>-44.0</td>
</tr>
</tbody>
</table>

The first step in this problem is to determine, by means of trigonometry, the tangent of the pretreatment palatal plane angle to the horizontal.

\[
\frac{Y - Y}{A \ P} = \frac{-47.0 - (-43.9)}{45.5} = -3.1
\]

\[
\frac{X - X}{A \ P} = \frac{62.4 - 16.9}{45.5} = 45.5
\]

\[
\text{tangent} = \frac{\text{opposite side}}{\text{adjacent side}}
\]

\[
\frac{Y - Y}{A \ P} = \frac{-3.1}{45.5} = -0.0681
\]
arctan \(-0.0681 = -3.9^\circ\)

The second step is to determine the tangent of the posttreatment palatal plane angle to the horizontal.

\[
\frac{Y}{A} - \frac{Y}{P} = \frac{-46.8 - (-44.0)}{60.8 - 17.5} = \frac{-2.8}{43.3} = -0.0647
\]

\[
\arctan(-0.0647) = -3.7^\circ
\]

\[
\Delta II - \Delta I = \triangle \angle
\]

\[-3.7^\circ - (-3.9^\circ) = 0.2^\circ \]

The palatal plane angle was \(-3.9^\circ\) and it became \(-3.7^\circ\). Therefore, the angle decreased (in the direction of S-N) \(0.2^\circ\).

In a coordinate system all angles are measured from a common origin. This locates the angle in a particular quadrant. Angles read counter-clockwise are considered positive while those read clockwise are negative.
This problem is designed to determine the change in the vertical distance from the apex of the mesial root of the maxillary first molar (6) to the palatal plane.

The first step in this problem is to write an algebraic equation for the slope of the pre- and posttreatment palatal plane lines. This is accomplished by using the point-slope formula. The tangents for the pre- and posttreatment palatal plane lines, which were determined in the first morphological study, will be needed in this study.

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th></th>
<th>II</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANS</td>
<td>62.4</td>
<td>-47.0</td>
<td>60.8</td>
</tr>
<tr>
<td>PNS</td>
<td>16.9</td>
<td>-43.9</td>
<td>17.5</td>
</tr>
<tr>
<td>R 6</td>
<td>38.8</td>
<td>-47.3</td>
<td>41.8</td>
</tr>
</tbody>
</table>

\[ Y = M \times X + b \] (point-slope formula)

\[ M = \text{tangent} \]

\[ M = \text{tangent} \]
I. \[ Y = MX + b \]
   \[ b = M X - Y \]
   \[ Y = -.0681 X - 42.75 \]

This is the equation for the pretreatment palatal plane line.

II. \[ Y = M X + b \]
    \[ b = M X - Y \]
    \[ Y = -.0647 X - 42.87 \]

This is the equation for the posttreatment palatal plane line.

The second step in this problem is to insert the "X" values of the root of 6 into the point slope formula and solve for the value of "Y".

\[ Y = -.0681 \cdot 38.8 - 42.75 = -45.39 \]
\[ Y = -.0647 \cdot 41.8 - 42.87 = -45.57 \]
The third step in this problem is to solve for the actual value of "Y".

\[ Y \text{ (measured)} = 47.30 \]

\[ Y \text{ (calculated)} = -45.39 \]

\[ Y \text{ (measured)} - Y \text{ (calculated)} = \Delta Y \]

\[ 47.30 - 45.39 = 1.91 \text{ mm.} \]

\[ Y \text{ (measured)} - Y \text{ (calculated)} = \Delta Y \]

\[ 47.70 - 45.57 = 2.13 \text{ mm.} \]

\[ \Delta Y \text{ II} - \Delta Y \text{ I} = Y \text{ (actual)} \]

\[ 2.13 - 1.91 = .22 \text{ mm.} \]

The alveolar height in the maxillary first molar region has increased .22 mm. during treatment.

MORPHOLOGICAL STUDY

SAMPLE 3

CHANGES IN ALVEOLAR HEIGHT AT \text{1}\

This problem is designed to determine the change in the vertical distance from the apex of the root of the maxillary central incisor.
to the palatal plane. This problem was solved in the same manner as was the problem in Sample 2. The equations for the slope of the pre- and posttreatment palatal plane lines, which were determined in the second morphological study, will be needed in this study.

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>62.4</td>
<td>60.8</td>
</tr>
<tr>
<td>Y</td>
<td>-47.0</td>
<td>-46.8</td>
</tr>
</tbody>
</table>

ANS

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>16.9</td>
<td>17.5</td>
</tr>
<tr>
<td>Y</td>
<td>-43.9</td>
<td>-44.0</td>
</tr>
</tbody>
</table>

PNS

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>54.6</td>
<td>55.0</td>
</tr>
<tr>
<td>Y</td>
<td>-51.3</td>
<td>-51.0</td>
</tr>
</tbody>
</table>

TR1 (Root)

I. \[ Y = MX + b \]
   \[ Y = -0.0681 X - 42.75 \]

II. \[ Y = MX + b \]
    \[ Y = -0.0647 X - 42.87 \]
    \[ Y = -0.0681 \cdot 54.6 - 42.75 = -46.48 \]
    \[ Y = -0.0647 \cdot 55.0 - 42.87 = -46.42 \]

\[ Y \text{ (measured)} - Y \text{ (calculated)} = \Delta Y \]

\[ 51.30 - 46.48 = 4.82 \]

\[ Y \text{ (measured)} - Y \text{ (calculated)} = \Delta Y \]
51.00 - 46.42 = 4.58

\[ \triangle Y_{II} - \triangle Y_{I} = \triangle Y \text{ (actual)} \]

4.58 - 4.82 = -.24 mm.

The alveolar height in the maxillary central incisor region has decreased .24 mm. during treatment.

MORPHOLOGICAL STUDY

SAMPLE 4

CHANGES IN POSITION AND LOCATION OF PTM

This problem is designed to determine the direction in which the inferior point of the pterygomaxillary fissure moved and the distance it traveled.

\[
\begin{array}{c|c|c|c}
& I & & II \\
\hline
PTM & X & Y & X & Y \\
\hline
& 15.0 & -29.9 & 15.0 & -30.7 \\
\end{array}
\]

The first step in this problem is to determine, by means of trigonometry, the path along which the point moved. The path is represented by the tangent function.

\[
\text{tangent} \quad = \quad \frac{\text{opposite}}{\text{adjacent}}
\]

\[
\text{tangent} \quad = \quad \frac{Y_{II} - Y_{I}}{X_{II} - X_{I}} = \frac{-30.7 - (-29.9)}{15.0 - 15.0}
\]
tangent $\frac{-0.8}{0} = -\infty$

arctan $-\infty = -90^\circ$ (360° - 90° = 270°)

$\Delta = 270^\circ$ (all angles are read counter clockwise from the same point of origin)

The point moved along a line perpendicular to the horizontal.

The second step in this problem is to determine the distance the point moved along this line. This is accomplished by using the trigonometric sine function. The value of the sine function was taken from a trigonometric table.

\[
\text{sine} = \frac{\text{opposite}}{\text{hypotenuse}}
\]

\[
\text{hypotenuse} = \frac{\text{opposite}}{\text{sine}}
\]

\[
\text{sine} \Delta = -90^\circ = -1.000
\]

\[
\text{hypotenuse} = \frac{-0.8}{-1.0} = 0.8 \text{ mm.}
\]

The point moved straight down 0.8 mm. during treatment.

**DENTAL STUDY**

**SAMPLE 5**

**CHANGES IN POSITION AND LOCATION OF 8**

This problem is designed to determine the direction in which
the maxillary third molar (8) moved and the distance it traveled.

This problem was solved in the same manner as was the problem in Sample 4.

\[
\begin{array}{cccc}
\text{C8 (Crown)} & I & & II \\
23.9 & -41.8 & 24.2 & -45.0 \\
\end{array}
\]

\[
\text{tangent} = \frac{\text{opposite}}{\text{adjacent}}
\]

\[
\frac{Y}{X} = \frac{-45.0 - (-41.8)}{24.2 - 23.9}
\]

\[
\text{tangent} = \frac{-3.2}{0.3} = -10.666
\]

\[
\text{arctan} -10.666 = -84.6^\circ \quad (360 - 84.6 = 275.4^\circ)
\]

\[
\theta = 275.4^\circ
\]

The maxillary third molar moved along a path located at 275.4°.

\[
\text{sine} = \frac{\text{opposite}}{\text{hypotenuse}}
\]

\[
\text{hypotenuse} = \frac{\text{opposite}}{\text{sine}}
\]

\[
\text{sine} \theta = 84.6 = -0.9957
\]
hypotenuse \[= \frac{-3.2}{-0.9957} = 3.2 \text{ mm.}\]

The maxillary third molar moved downward a distance of 3.2 millimeters along a path located at 275.4°.

DENTAL STUDY

SAMPLE 6

CHANGES IN POSITION AND LOCATION OF 6

This problem is designed to determine the distance the maxillary first molar moved and the point about which it moved. This calculation required the writing of two equations and the solving of the resulting simultaneous equations.

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>Y</th>
<th>II</th>
<th>Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>C 6   (Crown)</td>
<td>33.3</td>
<td>-64.1</td>
<td>37.1</td>
<td>-66.0</td>
</tr>
<tr>
<td>C 6   (Root)</td>
<td>38.8</td>
<td>-47.3</td>
<td>41.8</td>
<td>-47.7</td>
</tr>
</tbody>
</table>

\[\frac{(Y - Y)}{c} = \frac{c}{X - X} = \frac{(X - X)}{c} \]

\[Y + 64.1 = \frac{-64.1 + 47.3}{33.3 - 38.8} (X - 33.3)\]

\[Y = \frac{-16.8}{-5.5} (X - 33.3) - 64.1\]

\[Y = 3.0546 (X - 33.3) - 64.1\]
The next step is to solve the simultaneous equations representing the pre- and posttreatment long axis of 6. In solving these two equations the intersection of these two lines is determined. This intersection represents the axis of rotation of the maxillary first molar.

\[
3.8936 \, x \, - \, 210.45 \, = \, 3.0546 \, x \, - \, 165.82 \\
0.839 \, x \, = \, 44.63 \\
x \, = \, 53.19 \, mm.
\]

Substitute the calculated value of "X" into the equation for the pretreatment long axis of 6.

\[
Y \, = \, 3.0546 \, x \, - \, 165.82
\]
\[ Y = 3.0546 \cdot 53.19 - 165.82 \]
\[ Y = 162.47 - 165.82 \]
\[ Y = -3.35 \text{ mm.} \]

\[ X - X \]
\[ \text{II I} \]
\[ = \text{distance moved} \]
\[ 37.1 - 33.3 = 3.8 \text{ mm.} \]

The maxillary first molar moved forward 3.8 mm. tipping about a point located at (53.19, -3.35)

DENTAL STUDY

SAMPLE 7 (A)

CHANGES IN INCLINATION OF \( 1 \)

This problem is designed to determine the degree of change in the inclination of the maxillary central incisor. This problem was solved in the same manner as was the problem in Sample 1. The tangent values determined in this study will be used in Sample 7 (B).

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
</tr>
</thead>
<tbody>
<tr>
<td>TC 1</td>
<td>X \hspace{1cm} Y</td>
<td>X \hspace{1cm} Y</td>
</tr>
<tr>
<td>Crown</td>
<td>62.7 \hspace{1cm} -76.0</td>
<td>59.1 \hspace{1cm} -77.8</td>
</tr>
<tr>
<td>TR 1</td>
<td>X \hspace{1cm} Y</td>
<td>X \hspace{1cm} Y</td>
</tr>
<tr>
<td>Root</td>
<td>54.6 \hspace{1cm} -51.3</td>
<td>55.0 \hspace{1cm} -51.0</td>
</tr>
</tbody>
</table>

I. \[ Y - Y \]
\[ \text{II I} \]
\[ = -24.7 \]
\begin{align*}
X_{II} - X_{I} &= 8.1 \\
tangent &= \frac{opposite}{adjacent} \\
tangent &= \frac{-24.7}{8.1} = -3.0494 \\
\arctan(-3.0494) &= -71.8^\circ \quad (All \ angles \ in \ this \ problem \ will \ be \ negative, \ since \ they \ are \ read \ clockwise \ from \ the \ point \ of \ origin.) \\

II. \quad Y_{II} - Y_{I} &= -26.8 \\
X_{II} - X_{I} &= 4.1 \\
tangent &= \frac{-26.8}{4.1} = -6.5366 \\
\arctan(-6.5366) &= -81.3 \\
\Delta \overrightarrow{Y}^{II} - \Delta \overrightarrow{Y}^{I} &= \Delta \overrightarrow{A} \\
-81.3 - (-71.8) &= -9.5^\circ \\

The \ tip \ of \ the \ maxillary \ central \ incisor \ moved \ lingually \ 9.5^\circ. \\

DENTAL \ STUDY \\
SAMPLE \ 7 \ (B) \\
CHANGES \ IN \ POSITION \ AND \ LOCATION \ OF \ I
This problem is designed to determine the distance the maxillary central incisor moved and the point about which it moved. This calculation required the writing of two equations and the solving of the resulting simultaneous equations.

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X</td>
<td>Y</td>
</tr>
<tr>
<td>I C 1 (Crown)</td>
<td>62.7</td>
<td>-76.0</td>
</tr>
<tr>
<td>I R 1 (Root)</td>
<td>54.6</td>
<td>-51.3</td>
</tr>
</tbody>
</table>

I.

\[
(Y - Y) = \frac{c}{X} - \frac{r}{X} = (X - X) \\
\left(\frac{c}{r}\right)
\]

\[
Y + 51.3 = \frac{-76.0 + 51.3}{62.7 - 54.6} (X - 54.6)
\]

\[
Y = \frac{-24.7}{8.1} (X - 54.6) - 51.3
\]

\[
Y = -3.0494 (X - 54.6) - 51.3
\]

\[
Y = -3.0494 X + 166.497 - 51.3
\]

\[
Y = -3.0494 X + 115.2
\]

II.

\[
Y + 51.0 = \frac{-77.8 + 51.0}{59.1 - 55.0} (X - 55.0)
\]

\[
Y = -6.5366 (X - 55.0) - 51.0
\]

\[
Y = -6.5366 X + 359.51 - 51.0
\]

\[
Y = -6.5366 X + 308.51
\]
Locate the center around which tipping occurred. Solve simultaneous equations.

\[-6.5366X + 308.51 = 3.0494X + 115.2\]
\[-3.4872X = -193.31\]
\[X = 55.43 \text{ mm.}\]

Substitute the calculated value of "X" into the equation for the pretreatment central axis of 1.

\[y = -3.0494X + 115.2\]
\[y = -3.0494 \cdot 55.43 + 115.2\]
\[y = -169.03 + 115.2\]
\[y = -53.83 \text{ mm.}\]

Point of intersection of center of rotation in relation to apex of 1.

\[Y \text{ (measured)} = 51.3\]
\[Y \text{ (calculated)} = 53.83\]
\[51.3 - 53.83 = -2.53 \text{ mm.}\]
\[Y - X = \text{distance 1 moved}\]
\[59.1 - 62.7 = -3.6 \text{ mm.}\]
The maxillary central incisor moved 3.6 mm. in a lingual direction tipping about a point located at (55.43, -53.83) which is 2.53 mm. below the tip of the apex.

DENTAL STUDY

SAMPLE 8
RELATION OF APEX OF 1 TO THE "A" POINT (JARABAK) AND THE POSTERIOR BONY PALATE

This problem is designed to determine the position of the root of the maxillary central incisor in relation to the lingual cortical plate of the palate.

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th></th>
<th>II</th>
</tr>
</thead>
<tbody>
<tr>
<td>&quot;A&quot; Point (Jarabak)</td>
<td>X</td>
<td>Y</td>
<td>X</td>
</tr>
<tr>
<td></td>
<td>58.2</td>
<td>-50.5</td>
<td>58.1</td>
</tr>
<tr>
<td>P B P</td>
<td>47.3</td>
<td>-53.4</td>
<td>47.2</td>
</tr>
</tbody>
</table>

The equations representing the slope of the long axis of 1 determined in Sample 7 (B) will be used in this problem.

The first step is to write an equation to represent the line fifteen degrees to the horizontal passing through "A".

\[
tangent 15^\circ = 0.26795
\]

\[
I. \quad \frac{(Y - Y)}{a} = 0.26795 \frac{(X - X)}{a}
\]
Insert the equation for the pretreatment long axis of 1.

\[ Y(\text{Tooth}) = -3.0494X + 115.2 \]

These two simultaneous equations are solved to find the intersection point.

\[ 0.26795X - 66.095 = -3.0494X + 115.2 \]
\[ 3.31735X = 181.295 \]
\[ X = 54.65 \text{ mm.} \]

II. \( \left( Y - Y \right) = 0.26795 \left( X - X \right) \)

\[ Y = 0.26795X - 15.57 - 50.2 \]
\[ Y = 0.26795X - 65.77 \]

Insert the equation for posttreatment long axis of 1.

\[ Y(\text{Tooth}) = -6.5366X + 308.51 \]

These two simultaneous equations are solved to find the intersection point.

\[ 0.26795X - 65.77 = -6.5366X + 308.51 \]
\[ 6.80455X = 374.28 \]
\[ X = 55.00 \text{ mm.} \]

\[ 100 \times \frac{58.2 - 55.0}{58.2 - 47.3} = 100 \times \frac{3.2}{10.9} = 29.3\% \]

The root of the maxillary central incisor is located 29.3\% of the way back along the alveolar base.
CHAPTER III
FINDINGS

In determining the changes which took place in the morphology of the maxilla and in the position and location of related dental structures, a considerable amount of data was amassed. These data for each of the changes studied were recorded in tabular form, but they will not be presented in this manner. It was decided to illustrate these data in graphic form rather than subject the reader to the cumbersome task of trying to interpret long columns of numbers. These data are expressed by means of a rectangular polygon or histogram. Studying these histograms, the reader will be able to visualize the findings of each of the changes investigated. The frequency, as well as the magnitude of size of each measurement will also be readily discernible without having to peruse the tabulated computations.

The histograms used to illustrate the data were constructed in the following manner. Two lines divided according to a scale of values were drawn perpendicular to each other. The size or magnitude of the measurements were plotted along the horizontal axis. Graduations along this axis are of a positive or negative value, or a combination of both of these. The number of occurrences or the frequency of each
measurement was plotted along the vertical axis. After plotting the particular data for each patient, a line was drawn connecting the heights of each interval and a rectangular frequency polygon or histogram was formed.

After plotting the findings for each change, the resulting distributions were evaluated. If the distribution was found to be symmetrical and seemed to imitate a "normal" curve, the measurements were analyzed statistically to find their arithmetic average. In statistics, this value is called the mean and it is designated by the symbol \( \bar{X} \). A symmetrical dispersion also warranted the calculation of the standard deviation. This is a term used in statistics to signify the degree of dispersion about the mean and it is designated by the symbol \( \sigma \).

Eighteen histograms will be shown, each representing a specific change related to the point studied. In the majority of the cases the number of measurements (sample size) included in the distribution will be twenty-six. The sample size in some of the cases was reduced slightly by omitting extreme or unusual values. The statistical symbol for the sample size is \( N \). All values were tested for significance by applying principles of student's "t" distribution. The
results are shown in Table VII.

Changes in Maxillary Morphology and in the Maxillary Teeth:

1. Change in the Palatal Plane Angle:

The change in the angle of the palatal plane was computed by subtracting the pretreatment occlusal plane angle from the posttreatment angle. In calculating these angles, a departure was made from the customary method of reading angles in the coordinate system. These angles were read from the same origin (the horizontal axis of reference to the right of sella) in a clockwise direction. It was stated earlier in the "Methods and Materials" that angles read clockwise from the point of origin are given negative values. Therefore, all of the angles in this study are negative. The change in angularity obtained by subtracting the pretreatment from the posttreatment angle, required the maintaining of the correct polarity. The positive angular differences indicated that the palatal plane moved toward the horizontal axis of reference (sella-nasion) while a negative angle indicated that the palatal plane moved away from this axis. The histogram in Figure 7 has been constructed with a positive angular change indicating that the palatal plane moved upward, and a negative angle indicating that the palatal plane moved downward during treatment. It was found
### MEANS, STANDARD DEVIATIONS AND "t" RATIOS

<table>
<thead>
<tr>
<th>Change</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Standard Error</th>
<th>&quot;t&quot; Ratios</th>
<th>Significance</th>
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<td>.27</td>
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<td>2.31</td>
<td>.45</td>
<td>8.80</td>
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</tr>
<tr>
<td>Location of 8</td>
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<td>.49</td>
<td>12.32</td>
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</tr>
<tr>
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<tr>
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<td>.82</td>
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<td>Apex of 1 to PBP</td>
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<td>.56</td>
<td>1.86</td>
<td>.39</td>
<td>1.43</td>
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</tbody>
</table>

* .05 level

TABLE VII
NUMBER OF OCCURRENCES

\[ N = 25 \]

\[ \bar{x} = -0.54 \quad \sigma = 1.912 \]

CHANGE IN THE PALATAL PLANE ANGLE

FIGURE 7
that the change in the palatal plane angle for all twenty-six cases was within a range of $-5.0^\circ$ to $+4.6^\circ$. The average change was calculated to be $-0.54^\circ$ and a projection of the sample to a similar population would suggest that 95% of the cases would present a change in the palatal plane angle from $-1.34^\circ$ to $+0.263^\circ$. At the .05 level of probability, the "t" value is not significantly different from zero.

2. Change in Alveolar Height (mesial root apex to ANS-PNS):

Alveolar height was measured by calculating the distance along a perpendicular to the ANS-PNS plane from its intersection with that plane to the mesial root apex of the maxillary left first molar. The histogram in Figure 8 has been constructed in such a way as to show increase in the distance from the apex of the first molar to the palatal plane in a positive direction and decrease in this dimension in a negative direction. The distribution of the measurements was considered symmetrical and a mean was calculated to be $+1.10$ mm. The range was from $-1.90$ mm. to $+4.40$ mm. The range of measurements of the change in alveolar height of one standard deviation of a similar population would be from $+0.486$ mm. to $+1.714$ mm.

3. Change in Alveolar Height (root apex to ANS-PNS):

Alveolar height was measured by calculating the distance along
NUMBER OF OCCURRENCES

CHANGE IN ALVEOLAR HEIGHT
(MESIAL ROOT APEX 6 TO ANS-PNS)

FIGURE 8
a perpendicular to the ANS-PNS plane from its intersection with that plane to the apex of the maxillary central incisor. The histogram in Figure 9 has been constructed in such a way as to show increase in the distance from the apex of the central incisor to the palatal plane in a positive direction and decrease in this dimension in a negative direction. The distribution of the measurements was considered symmetrical and an average was calculated to be $+.083$. The calculated "t" value of $0.27$ is not significantly different from zero for a sample size of twenty-six cases.

4. Change in the Location of PTM:

The change in the location of the pterygomaxillary fissure was assessed both linearly and angularly. Figure 10 shows that PTM moved within a range of $0.0$ mm. to $4.20$ mm. Since the change in the location of PTM is a vector quantity, that is, it has both magnitude and direction, the linear quantities must be associated with angular measurements. This is indicated in Figures 11 and 11A. In the histogram illustrating the millimeter change in the location of PTM the lengths of the vectors representing the distance of motion formed practically a "square" distribution. This suggests almost equal probability for any of the lengths within the range covered.
NUMBER OF OCCURRENCES

\[ N = 26 \]

\[ \bar{X} = 0.083 \]
\[ \sigma \approx 1.51 \]

CHANGE IN ALVEOLAR HEIGHT
(ROOTH APEX 1 TO ANS - PNS)

FIGURE 9
NUMBER OF OCCURRENCES

N = 26

CHANGE IN THE LOCATION OF PTM

FIGURE 10
NUMBER OF OCCURRENCES

CHANGE IN LOCATION OF PTM

FIGURE 11
NUMBER OF OCCURRENCES

N = 26

\[ \bar{X} = 93.96^\circ \]

\[ \bar{\omega} = 115.7^\circ \]

CHANGE IN LOCATION OF PTM

FIGURE 11A
In plotting the dimensions for the change in degrees, as shown in Figure 11, there appeared to be two different distributions, one centered around 90° and the other around 270°. It was decided to superimpose one distribution upon the other and the resulting histogram is shown in Figure 11A. Since this graph shows a symmetrical distribution the arithmetic average was calculated. The determined value for the direction of the change of PTM was 93.9°.

5. Change in the Location of 8:

The change in the location of the maxillary third molar was assessed both lineally and angularly. Figure 12 illustrates a symmetrical distribution and the mean value for the measurements was +6.16 mm. One standard deviation yielded a range of +5.15 mm. to +7.17 mm. The change in the location of the maxillary third molar is a vector quantity, having both magnitude and direction, and therefore, the linear quantities must be associated with angular measurements. The angular measurements for the change in the location of the maxillary third molar are shown in Figure 13. This graph was considered to be symmetrical, although slightly skewed to the left, so the mean was determined. The average change was +257.4°.
NUMBER OF OCCURRENCES

N = 26

\( \bar{X} = 6.16 \)

\( \sigma = 2.45 \)

CHANGE IN THE LOCATION OF 8

FIGURE 12
NUMBER OF OCCURRENCES

\[ N = 24 \]

\[ \bar{X} = 257.42 \quad \omega = 187.9^\circ \]

CHANGE IN THE LOCATION OF 8

FIGURE 13
6. Change in the Position of 6:

The change in the position of the maxillary first molar was calculated by subtracting the pretreatment horizontal distance of the most mesial occlusal point on the crown of this tooth to the vertical reference axis from the posttreatment distance. Figure 14 shows the distribution of these measurements in millimeters. This graph appears to represent two distributions, however, the value for the mean was still calculated. The average determined was +2.50 mm.

The axis of tipping of the maxillary first molar was calculated by determining where the central axis of the tooth in its pretreatment position intersected with the central axis of this same tooth in its posttreatment position. The histogram in Figure 15 reveals that the horizontal measurements locating these intersection points fell within a range of -16.0 mm. to +102.0 mm. The mean for this symmetrical distribution was +34.76 mm. This value represents the average measurement from the vertical reference axis to the central axis intersection point. Figure 16 shows the distribution of the measurements from the horizontal reference axis to the central axes intersection point. The average for these vertical measurements was -62.19 mm. These two measurements, one horizontal and one vertical,
NUMBER OF OCCURRENCES

\[ N = 26 \]

\[ \bar{X} = 2.50 \]

\[ \sigma \approx 0.75 \]

CHANGE IN THE POSITION OF 6

FIGURE 14
NUMBER OF OCCURRENCES

\[ N = 25 \]

\[ \bar{x} = 34.76 \]

\[ \sigma = 61.5 \]

LOCATION OF THE POINT OF TIPPING 6
(HORizontals)
NUMBER
OF OCCURRENCES

<table>
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<th>12</th>
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<tr>
<td>5</td>
<td>4</td>
<td>3</td>
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</tr>
</tbody>
</table>

N = 25

X = -62.19

\[ \sigma = 143.5 \]

LOCATION OF THE POINT OF TIPPING 6
(VERTICAL)

FIGURE 16
actually indicate the axis of the point of rotation of the maxillary first molar.

7. Changes Related to the Maxillary Central Incisor:

A. Changes in the Inclination of \( I \):

The change in the inclination of the maxillary central incisor was computed by subtracting the pretreatment angle of the long axis of the tooth to the horizontal axis from the posttreatment angle. All of the angles in this study were read from the same point of origin in a clockwise direction and therefore, they were negative. The change in the angularity obtained by subtracting the pretreatment from the posttreatment angle, required the maintaining of the correct polarity. The negative angular differences indicated that the central incisor tooth moved away from the horizontal reference axis (sella- nasion) while a positive angle indicated that the central incisor tooth moved toward this axis. The histogram in Figure 17 illustrates the symmetrical distribution for the measurements in this study. The arithmetic average of the measurements was calculated to be \(-13.27^\circ\) and the range for one standard deviation of a projected similar sample was \(-17.99^\circ\) to \(-8.45^\circ\).
NUMBER OF OCCURRENCES

\[ N = 26 \]
\[ \bar{X} = -13.27 \]
\[ \sigma = 11.7 \]

CHANGES IN THE INCLINATION OF \( \underline{1} \)

FIGURE 17
B. Shift in the Incisal Edge of I:

This calculation was made by subtracting the pretreatment horizontal value of the tip of the incisal edge of the maxillary central incisor from the posttreatment value. Figure 18 demonstrates the symmetrical distribution of the measurements. The average measurement was -5.18 mm. and the range established for 95% of the population of a similar sample was -6.83 mm. to -3.53 mm.

C. Location of the Axis of Tipping I:

The axis of tipping of the maxillary first molar was calculated by determining where the central axis of the tooth in its pretreatment position intersected with the central axis of this same tooth in its posttreatment position. The histogram in Figure 19 shows the distribution of the horizontal measurements locating these intersection points was symmetrical. The calculated mean for this sample was +57.20 mm. This value represents the average measurement from the vertical reference axis to the central axes intersection point. Figure 20 reveals the dispersion of the measurements from the horizontal reference axis to the central axes intersection point. The average for these measurements was -59.95 mm. These two measurements, one horizontal and one vertical, actually indicate the axis of tipping of the maxillary
SHIFT OF THE INCISAL EDGE OF 1

FIGURE 18

\[ N = 26 \]
\[ \bar{X} = -5.18 \]
\[ \sigma = 3.99 \]
LOCATION OF THE POINT OF TIPPING 1
(HORIZONTAL)

FIGURE 19
NUMBER OF OCCURRENCES

\[ N = 21 \]

\[ \bar{X} = 59.95 \]

\[ \sigma = 55.1 \]

LOCATION OF THE POINT OF TIPPING 1 (VERTICAL)

FIGURE 20
D. Shift in the Axis of Tipping $1$

This calculation was made by subtracting the determined value for the vertical axis of tipping from the vertical measurement of the pretreatment apex of the maxillary central incisor. This calculation reveals the position of the axis of rotation to the apex of the tooth. If the calculated value of the vertical axis of tipping is greater than the pretreatment value of the vertical location of the apex, the value obtained by subtraction will be negative. This indicates location of the point of tipping below the apex of the tooth. Therefore, negative values of the histogram in Figure 21 indicate location below the apex and positive values indicate location above the apex. The distribution in this graph was considered symmetrical and the mean was calculated. The mean of $-2.64$ mm. was used to determine the standard deviation for a similar sample.

8. Changes Related to Apex of $1$:

A. Relation of Apex of $1$ to ("A" Point and PBP):

This problem was designed to determine the position of the root of the maxillary central incisor in relation to "A" point (Jarabak) and PBP (posterior wall of the bony palate). The location of the apex of
NUMBER OF OCCURRENCES

\[
\begin{array}{ccccccccccc}
\text{NUMBER OF OCCURRENCES} & 12 & 11 & 10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \\
\text{Shift of the Point of Tipping} & -44.0 & -3.30 & -22.0 & -11.0 & 0 & 11.0 & 22.0 & 33.0 \\
\text{mm.} & N = 23 & \bar{X} = -2.64 & \alpha \sim 3.85 \\
\end{array}
\]

SHIFT OF THE POINT OF TIPPING

FIGURE 21
the root along the line from "A" to PBP is at the intersection of the long axis of the central incisor with a line which makes an angle of $15^\circ$ with the horizontal reference axis and passes through points "A" and PBP. The line connecting the labial cortical plate of the palate, represented by "A", to the lingual cortical plate of the palate, represented by PBP, is established as 100% and the location of the apex is expressed as a percentage of the distance along this line starting from "A". The measurements for this sample are represented in Figure 22. This graphic representation of the measurement reveals a symmetrical distribution. The mean was +26.99% and the range of one standard deviation was +18.3% to +35.5%.

B. Change in the Apex of $I$ (Horizontal):

This calculation was determined by subtracting the posttreatment horizontal value for the apex of the maxillary central incisor from the pretreatment horizontal value. Both of these horizontal values were determined previously. If the posttreatment horizontal value is greater than the pretreatment value the number obtained by subtraction will be positive. Therefore, positive measurements indicate movement anteriorly, that is, away from the vertical reference axis. The histogram in Figure 23 shows the distribution of the measurements.
NUMBER OF OCCURRENCES

\( N = 25 \)

\[ \bar{x} = 26.99 \]

\( \sigma = 16.63 \)

RELATION OF APEX OF 1 TO
d"A" POINT AND PBP

FIGURE 22
NUMBER OF OCCURRENCES

\[ N = 24 \]

\[ \bar{X} = 0.56 \quad \sigma = 1.86 \]

CHANGE IN APEX OF 1

FIGURE 23
This distribution was considered symmetrical and the mean calculated was +.56 mm. The calculated "t" value of 1.43 is not significantly different from zero for sample size of twenty-four cases.
CHAPTER IV

DISCUSSION

The purpose of this study was twofold. The first was to devise and demonstrate the use of a system of rectangular coordinates using linear measurements in cephalometric analysis. This was done with the hope that at some future time, data obtained by using this type of coordinate measuring system could be fed into a computer and that the computer could be programmed to produce information which would aid in diagnosis and assessment of treatment. The second was to test the versatility of this method by applying it to the appraisal of changes in the morphology of the maxilla and of the maxillary teeth which occurred during the orthodontic treatment of twenty-six Class I first premolar extraction malocclusions. Each of these two purposes will be discussed separately.

The initial studies in cephalometrics performed by Broadbent (1931) and Brodie (1941) provided methods of analysis of craniofacial morphology which served as models on which succeeding investigators were to improve. Perhaps the most significant improvements in cephalometric analysis were devised by Downs (1948). This is substantiated by the fact that a number of the later analyses incor-
 incorporated points and angles proposed by Downs. The angular methods of analysis employed by these researchers in cephalometrics are the most popular methods used today by the clinician.

The use of straight line measurements in the linear assessment study of cephalometrics, was first proposed by Wylie (1947). The fact that the Wylie analysis using linear measurements was the only analysis of this type to be proposed for a period of over five years indicates that the use of linear measurements in the assessment of cephalometric roentgenograms was not universally received by the clinical orthodontist. Within the past decade, however, there has been considerable interest and study in the linear approach to cephalometric analysis. The advocates of linear measurements, as opposed to angular ones, are of the opinion that straight line measurements are a more accurate means of assessing the craniofacial complex than the more popularly used angular methods.

The mathematic principle of relating two points to the same point and thereby relating them to each other has been employed in this study to locate landmarks on lateral cephalometric headplates. A system of measurement known as the rectangular coordinate system or Cartesian coordinates is the embodiment of this mathematic
principle. The rectangular coordinate system is predicated on the fact that all points are referred to the intersection of a set of perpendicular axes. A point in a plane can be fully determined if the distances from two given intersecting straight lines are known. These principles incorporated in a Cartesian coordinate system have been used by many investigators and for various reasons.

One of these reasons is the application of this type of a measuring system employed by the United States Army to teach map and aerial photography reading. A military grid system is a network of squares formed by north-south and east-west grid lines placed on a map. Grid lines are identified by a specific number which is printed on the margin of the map directly opposite the line it indicates. A point on the map is identified by reading the grid coordinates of this point from the intersection point in the lower left hand corner of the map. The Army employs the mnemonic device "Read Right and Up" in instructing recruits to remember to read the horizontal distance of the point from the intersection before reading the vertical distance. If desired, this word association could be applied to the reading of measurements in this study since all of the landmarks to be located were in front of and below the intersection point (sella).
The application of Cartesian coordinates to cephalometrics is of a relatively recent date (Craig, 1951 and Williams, 1953). Later investigators such as Elsasser (1957) and Kean (1958) also employed a rectangular coordinate system to locate landmarks on the lateral roentgenogram. The major variation in the research work of these investigators was the selection of a registration point and the construction of reference lines to that point. Some of the investigators used sella as the registration point while others used nasion. The Bolton-nasion-plane, the sella-nasion plane and the Frankfort horizontal plane were some of the planes of reference used.

In this study the use of the coordinate system of measurements was somewhat different than in previous investigations. The sella-nasion plane was used as the vertical plane of reference and sella was selected as the point of registration. A line drawn perpendicular to sella was employed as the horizontal axis of reference. The sella-nasion plane was chosen because it was felt that this plane, representing the anterior cranial base, was the most stable plane in the head throughout growth. The findings of studies on the stability of facial planes performed by Brodie (1941) and Downs (1948) support this statement. Another reason for selecting this plane was the
relative ease with which the points sella and nasion can be located on a lateral headplate.

The selection of sella as the registration point, the sella-nasion plane as the vertical axis of reference and a perpendicular erected to sella as the horizontal axis of reference makes this study different from those previously mentioned. The use of a vernier caliper to measure the distance of the selected landmarks from the axes of reference was another unique feature of this investigation. Furthermore, an evaluation of the precision of measurement was incorporated into this study (see "Materials and Methods"). These features make this study of the use of a rectangular coordinate system in the location of cephalometric landmarks novel.

The Cartesian coordinate system is a working model of the principles of analytic geometry.

By using the Cartesian coordinate system to locate landmarks on a lateral headplate, we have, in effect, introduced the science of analytic geometry to the field of cephalometrics. It is hoped that this addition of analytic geometry to the field of cephalometrics will be as helpful and far-reaching in its impact on this field as the addition of biomechanics and bioengineering principles, as described
by Jarabak and Fizzell (1963), have been in the field of clinical orthodontics.

The final advantage of the coordinate system of measurement to be noted is the versatility of this method. Several mathematical problems were shown in Chapter III. These problems were designed to demonstrate this versatility. It was shown how angles and intersection points, as well as lengths of straight lines, can be computed by knowing the coordinates of a landmark. These problems may seem to be difficult, yet with a basic understanding of algebra, trigonometry and analytic geometry, they can be readily solved.

A rectangular coordinate system lends itself ideally to the use of a data reduction process performed by electronic computers. The coordinates for the landmarks on the cephalometric headplate selected for study can be permanently recorded on small cards, called punch cards, designed for use in a computer. Any of the calculations made by using the problems cited in the previous chapter can be performed by simply correctly programming the computer. The punch cards made for use in the computer would act as a permanent records of all of the data and would enable the investigator to obtain forgotten, lost, or newly sought information at any time by simply returning
the cards to the computer.

Having outlined the usefulness of Cartesian coordinates to the field of orthodontics, what needed to be shown was its immediate application and the validity of this application. This will be dealt with in the second part of this discussion.

An important factor that must be considered in evaluating the changes in the morphology of the maxilla and the maxillary teeth is that of growth. Growth is usually defined as increase in size. This increase in size has different increments during certain stages and at different times. Since there was no provision made in this investigation for determining how much of each change was due to growth, and how much was the result of orthodontic treatment, there was no possible way of separating or distinguishing one from the other. Therefore, the actual interest in this study was to evaluate the changes which occurred as a result of both growth and orthodontic treatment. The findings discussed will still be of clinical interest to the reader, since growth will also be an undetermined factor in the resulting changes of his treated patients.

Before proceeding with a discussion of each change, a brief description of the treatment employed to correct these malocclusions
will be presented. The treatment of a Class I first premolar extraction malocclusion as described by Jarabak and Fizzell (1963) consists of three objectives:

1) Simultaneous canine distal-driving
2) Space consolidation
3) Correct seating of inclined planes and establishment of a functional overbite-overjet relationship.

Force systems, both intrinsic and extrinsic, are used to accomplish all three objectives. The resulting effect is one in which the canines have been moved distally into the premolar extraction sites, the overbite reduced by uprighting the teeth in the buccal segments, the four incisors in each arch retracted and any remaining spaces closed, and finally the teeth aligned in their ideal functional relationship. If this treatment plan is kept in mind it will aid in the understanding of the findings of this investigation.

The remainder of this discussion will deal with the changes resulting from orthodontic treatment. The changes in the palatal plane angle will be the first of eight changes to be discussed. The palatal plane is represented by a line drawn from the anterior nasal spine (ANS) to the posterior nasal spine (PNS). The angle which the palatal plane makes with the horizontal reference axis (S-N) was
determined before and after treatment. The change in the angle was obtained by subtracting the pretreatment from the posttreatment angle. The average change was determined to be \(-0.54^\circ\) (Figure 7). This means that the palatal plane moved in a downward direction \(-0.54^\circ\) away from the S-N plane. In applying student's "t" distribution, it was determined that at the .05 level of probability, the value obtained could be explained by chance. This indicates that no real change occurred.

Change in alveolar height in the region of the maxillary first molar was the second consideration. The histogram in Figure 8 has been constructed in such a way as to show increase in the distance from the apex of the maxillary first molar to the palatal plane in a positive direction and a decrease in this dimension in a negative direction. The average increase in this dimension was +1.10 mm. which seems to indicate an increase in alveolar height occurred. Still, it must be remembered that the tipping of the molar teeth would have an influence on the position of the mesial root apex of this tooth. This, in effect, would change the distance between the apex and the palatal plane. If the molar tooth was tipped in an anterior direction, the root apex would move posteriorly. In tipping this tooth posteriorly, the root apex also moves upward, thus causing a seeming decrease
in the alveolar height. If the crown of the molar tooth was tipped in a posterior direction the apex of the root of this tooth would move forward and downward. This movement could account for the increase in the root apex to palatal plane distance. An increase in the vertical dimension might also be the result of further eruption of the molar tooth. An interesting speculation as to a decrease in the apex to palatal plane distance can be noted from a study by Sicher (1952) of the development of the nasal cavity. He states that the size and shape of the nasal cavity is altered by the apposition of bone on the orbital floor and the resorption of bone on the nasal floor. The latter is compensated for by apposition of bone on the oral surface of the palate.

The next change to be investigated was that of the alveolar height in the region of the maxillary central incisor. Alveolar height in the region of the maxillary central incisor as illustrated in Figure 9 was found to increase only slightly less than 1 mm. during treatment. The "t" value of .27 at the .05 level indicates that this change could have been due to chance.

Change in the location of the pterygomaxillary fissure was the fourth morphologic change considered. The pterygomaxillary fissure
is an oval-shaped radiolucency representing the fissure between the anterior margin of the pterygoid process of the sphenoid bone and the profile outline of the posterior surface of the maxilla. Figure 10 illustrates that the inferior tip of this point moved anteriorly in all twenty-six of the patients within a range of 0 to 4.2 mm. In the histogram (Figure 10) illustrating the metric change in the location of PTM, the lengths of the vectors representing the distance of motion formed practically a "square" distribution. This indicated that any one of the changes was as likely as any other. The pterygomaxillary fissure is greatly affected by growth (Wylie, 1948) and it is also a very difficult landmark to located accurately on the lateral cephalometric headplate. These two factors might account for the wide range of movement which this landmark seems to have experienced. The mean age of treatment for the patients in this sample was eleven years for the beginning of treatment and 13.5 years at the completion of treatment. Therefore, these patients were treated in that age level generally accepted to be the pubertal period. It is during this period that the adolescent child experiences considerable growth. Since the degree of accelerated growth experienced by any one child is peculiar to that child, the amount of growth during
this period can not be projected to another child. Therefore, when changes in the position of the pterygomaxilllary fissure are investigated, pubertal growth spurts must be considered.

The direction of movement of the pterygomaxilllary fissure is illustrated in Figures 11 and 11A. In plotting the latitudes of change in degrees (Figure 11) there appeared to be two different distributions, one centered around $90^\circ$ and the other, around $270^\circ$. It was decided to superimpose one distribution upon the other and the resulting histogram (Figure 11A) illustrated a symmetrical distribution. The determined value for the direction of the change of PTM was $93.9^\circ$. This indicates that the pterygomaxilllary fissure moved almost directly downward and slightly backward or upward and slightly forward.

The first of the changes of the maxillary teeth in this study was the change in the location of the maxillary third molar. The change in location of this point is a vector quantity and therefore, the linear measurements obtained must be associated with the determined angular measurements. The third molar moved anteriorly in all twenty-six cases (Figure 12) and the average change in position was $6.16 \text{ mm}$. The average position for the location of this tooth (Figure 13) was $257.4^\circ$. This indicates that the third molar moved
downward and backward. A great deal of the movement of the maxillary third molar can be attributed to the growth of the sutures which surround this tooth. Growth in these areas projects the maxilla forward. The third molar tooth at the ages of eleven, twelve and thirteen is generally located very close to the pterygomaxillary fissure which indicates that this tooth has only begun to make its downward descent into the oral cavity.

The change in the position of the maxillary first molar tooth was the second tooth change considered. The average change in the position of the maxillary first molar was determined to be +2.50 mm. (Figure 14). This indicates that this tooth moved anteriorly an average of 2.50 mm. during orthodontic treatment. The maxillary first molar moved posteriorly in only three of the twenty-six patients studied. The fact that this tooth moved anteriorly in twenty-three of the patients suggests that part of the space resulting from the extractions of the first premolar teeth was taken up by the mesial migration of the first molar. It must be remembered that here, as in the previous studies, the amount of change which is attributable to growth cannot be adequately determined.

The axis of tipping of the maxillary first molar was calculated
by determining where the central axis of the tooth in the pretreatment position intersected with the central axis of this same tooth in the posttreatment position. The histogram (Figure 15) which illustrates the horizontal location of the axis of tipping reveals that the average distance for the point of rotation from the vertical reference axis was +34.76 mm. The average length of the measurements for the distance from the horizontal reference axis to the point of intersection was -62.19 mm. (Figure 16). The coordinates for the average point of intersection of the pretreatment and posttreatment central axes of the tooth were +34.76 mm. and -62.19 mm. This means that the axis of rotation of the maxillary first molar was located within the root of this tooth near the cemento-enamel junction.

The location of the axis of tipping of the molar tooth was not consistent, since it was found to be in the root of some patients and in the crown of others. In seven of the patients the point of intersection was found to be considerably above the apex of the tooth. Points of intersection located above or below the tooth, indicate the tooth is translating rather than tipping. Gantt (1960) and Kemp (1961) investigated changes in the position of the mandibular first molar teeth resulting from treatment. The teeth in their sample
tipped about an axis in the roots. There has not been a comparable study investigating the tipping of maxillary first molar teeth. Findings from the measurements taken in this study tend to indicate that these teeth either tipped or translated and that tipping was more prevalent than translation.

Selected changes of the maxillary central incisor were next studied. The change in the inclination of the maxillary central incisor was computed by subtracting the pretreatment angle of the long axis of the tooth to the horizontal reference axis from the posttreatment angle. The average change in the inclination of this tooth was $-13.27^\circ$ (Figure 17). This means that the maxillary central incisor tooth was tipped in a lingual direction an average of $13.27^\circ$. By subtracting the pretreatment measurement of the tip of the incisal edge of the crown from the posttreatment measurement, the shift of the incisal edge was calculated. The histogram indicating this change (Figure 18) shows that in twenty-four of the patients the incisal edge of the crown moved lingually and in four patients it moved labially. The average movement of the crown to the lingual was $-5.18$ mm. In the patients where the incisal edge of the central incisor tooth moved labially it was determined that this tooth was lingually positioned before
treatment was begun. Therefore, in these patients the labial movement of the maxillary central incisor was desired. The lingual movement of the central incisor tooth during treatment can be attributed to the use of Class II elastic bands which were utilized in the fulfillment of the first treatment objective. These elastic bands exerted a pull in a lingual direction on the maxillary canines and were used to distal drive these teeth into the extraction sites. Since the maxillary incisor teeth are attached to the arch wire, a component of this force effects a lingual movement of these teeth as well as the canine teeth. The average lingual movement of the tip of the crown of the incisal edge of the maxillary central incisor indicates that the labial pro-cumbency of the central incisor teeth in this sample was reduced.

The axis of tipping of the maxillary central incisor was calculated in the same manner as was that of the maxillary first molar. The histogram illustrating the horizontal location of the point of tipping (Figure 19) shows that the point of tipping centered between 55.0 mm. and 57.8 mm. The actual arithmetic average of the measurements was 57.2 mm. The vertical location of the point of tipping of the maxillary central incisor was determined to be 59.95 mm. (Figure 20). The coordinates for the axis of tipping for
the average patient of 57.2 mm. and 59.95 mm. locate the axis of tipping within the root of the central incisor.

Since almost all of the patients showed that the axis of tipping was somewhere within the root of the central incisor, it was decided to determine the actual shift of the axis of tipping in relation to the apex of the root. The average shift of the axis of tipping was determined to be -2.64 mm. (Figure 21). This means, that in the majority of the patients studied, the point about which the central incisor tooth tipped was located in the apical one-third of the tooth. The extreme values on the histogram indicated that four of the patients exhibited a degree of translation of this tooth.

The final consideration in this study was to determine the position of the root of the maxillary central incisor relative to the labial and lingual cortical plates of the palate. The labial cortical plate of bone was designated as "A" point (Jarabak) and the lingual cortical plate was designated as PBP (posterior bony palate). The location of the apex of the root along the line from A to PBP is at the intersection of the long axis of the central incisor tooth with a line which makes an angle of 15° with the horizontal reference axis (S-N) and passes through points A and PBP (see "Methods and Materials").
The line connecting these two points is established as 100% and the location of the apex is expressed as a percentage of the distance along this line starting from A. The histogram illustrating the measurements for this study exhibits a symmetrical distribution (Figure 22). The mean was 26.99% which indicates that on the average the root of the maxillary central incisor was nearer the labial cortical plate of bone at the end of treatment than it was before treatment was begun. As was stated earlier, in the majority of the patients, the crown of the maxillary central incisor tooth was tipped in a lingual direction. It was further stated that this tooth experienced mostly tipping and little translation. Since the axis about which the central incisor tooth tipped was within the apical one-third of the root, it can be stated that the apex of this tooth moved labially very little in relation to the amount of posterior or lingual movement of the crown. In order to further investigate this observation, it was decided to calculate the actual amount of movement of the apex of the maxillary central incisor. This movement was determined by subtracting the calculated posttreatment value for the horizontal distance from the apex to the vertical reference axis from the pre-treatment value. The measurements obtained yielded an average
movement of the apex of +.56 mm. (Figure 23). The "t" value of 1.43 at the .05 level indicates that this change could have been due to chance.

Since there was virtually no change in the position of the apex from the beginning of treatment to the completion of treatment, one might assume that the apex was ideally located in the alveolar trough.

Steiner (1953) proposed a linear measurement for the ideal positioning of the maxillary central incisor to alveolar process. He stated that the most anterior point on the crown of this tooth should be 4 mm. in front of the line NA (nasion - "A" point - Downs). In reviewing some of the results of this study dealing with changes of the maxillary central incisor, it was thought that a measurement indicating the position of the apex of the root of the maxillary central incisor along the line "A" (Jarabak) PBP could be proposed, as Steiner has proposed, to indicate the ideal or stable position of the maxillary central incisor.

The rectangular coordinate system and the use of linear measurements have been employed in this study to locate selected landmarks on a cephalometric headplate. These data obtained by the use of this method of measurement were utilized in a series of problems designed to demonstrate the changes in the morphology of
the maxilla and of the maxillary teeth which occurred during, or as a result of, orthodontic treatment. By knowing the coordinates of the landmarks in the dental-facial structure one can readily obtain a considerable amount of information as to the position, location and interrelation of these landmarks. It is believed that an accurate analysis of the cranio-facial complex can be designed by the use of the Cartesian coordinate system. It is also believed, that the data thus received can be fed into a computer and that by so doing, vast amounts of information about the cranio-facial complex can be obtained through the extraordinary feats of memory and calculation of the computer.
CHAPTER V
SUMMARY AND CONCLUSIONS

The purpose of this thesis was twofold. The first was to devise and demonstrate the versatility of a linear system of measurements using analytic functions in rectangular coordinates. The second was to utilize this system of measurements in assessing the changes occurring in the morphology of the maxilla and in the positions of the maxillary teeth after treatment of twenty-six Class I (Angle) extraction malocclusions.

The mathematic principle of relating two points to the same point, and thereby relating them to each other, has been employed in this thesis to locate landmarks on lateral cephalometric headplates. This was accomplished by employing a system of measurement known as the rectangular coordinate system or Cartesian coordinates. Sella-Turcica was used as the point of origin to which all of the landmarks were to be related. Orthogonal reference axes were constructed to sella and all of the landmarks were measured to these axes. A line from sella to nasion represented the horizontal reference axis and a perpendicular to this line at sella represented the vertical reference axis.
5. Change in the position and location of the maxillary third molar.

6. Change in the position of the maxillary first molar.
   A. Location of the point of tipping of the maxillary first molar.

7. Changes related to the maxillary central incisor.
   A. Change in the inclination.
   B. Shift of the incisal edge.
   C. Location of the point of tipping.
   D. Shift of the point of tipping.

8. Location of the apex of the maxillary central incisor along the alveolar base line.
   A. Change in the apex of the maxillary central incisor.

The material used in this study comprised pre- and posttreatment lateral cephalometric radiographs of twenty-six orthodontically treated malocclusions. The patients, seventeen female and nine male, were all treated in the private practice of Dr. Joseph R. Jarabak, Hammond, Indiana in accordance with the principles of the Jarabak "Light Differential Force Technique."

It may be concluded that the following changes occurred as
a result of orthodontic treatment:

1. There was no significant change in the palatal plane angle during orthodontic treatment.

2. The alveolar height in the region of the maxillary first molar increased 1.10 mm.

3. There was no significant change in the alveolar height in the region of the maxillary central incisor during orthodontic treatment.

4. The pterygomaxillary fissure moved anteriorly in all twenty-six patients within a range of 0.0 to 4.2 mm. The direction of movement of the pterygomaxillary fissure was downward and backward or upward and forward.

5. The maxillary third molar moved anteriorly 6.16 mm. The direction of movement of the maxillary third molar was downward and backward.

6. The maxillary first molar moved anteriorly 2.50 mm. The axis of tipping of the maxillary first molar was located within the root of the tooth, near the cemento-enamel junction.

7. The crown of the maxillary central incisor moved
posteriorly 5.18 mm.

There was no significant change in the location of the apex of the root of the maxillary central incisor during orthodontic treatment.

The maxillary central incisor moved in a lingual direction 13.27 degrees.

The axis of tipping of the maxillary central incisor was 2.64 mm. below the apex of the tooth.

8. The apex of the root of the maxillary central incisor after treatment was located 26.99% of the way back along the alveolar base.
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APPROVAL SHEET

The thesis submitted by Dr. Francis Peter Wall has been read and approved by members of the Departments of Anatomy and Oral Biology.

The final copies have been examined by the director of the thesis and the signature which appears below verifies the fact that any necessary changes have been incorporated, and that the thesis is now given final approval with reference to content, form, and mechanical accuracy.

The thesis is therefore accepted in partial fulfillment of the requirements for the Degree of Master of Science.

May 19, 1965

DATE

Signature of Advisor